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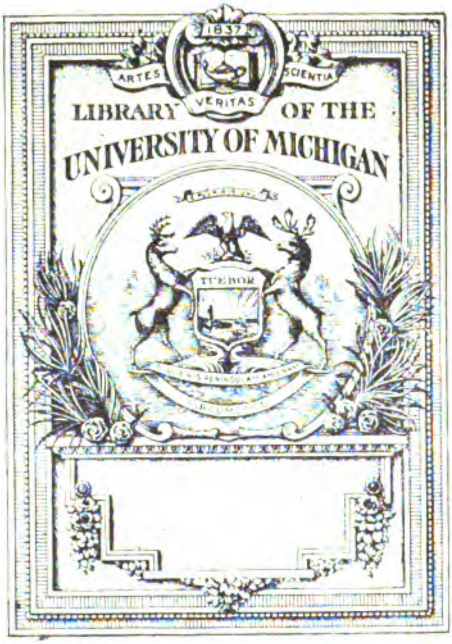
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A SYSTEM OF THE MATHEMATICS,

CONTAINING

The EUCLIDEAN GEOMETRY, Plane *and* Spherical
TRIGONOMETRY; the Projection of the SPHERE, both
Orthographic and *Stereographic*, ASTRONOMY, the
USE of the GLOBES and NAVIGATION:
The Manner of Computing the APPULSES of the MOON to
the *Fixed Stars*, and their Occultations by the Interpoſition of
Her Body, very uſeful for determining the Difference of *Longitude*
between Places. With an Account of the ſeveral METHODS
Propoſed and made Uſe of, by the moſt celebrated *Aſtronomers*
for aſcertaining the ſame.
New SOLAR TABLES, with their Conſtruction and Uſe. Tables
of the *Sun's Place*, *Right Aſcenſion*, *Declination*, Equation of Natural
Days for every Four Years; with *Tables* of *Variation* to make them
ſerve for a Hundred Years to come; and a *Catalogue* of the *Right Aſcenſions*,
Declinations, &c. of the moſt Eminent *Fixed* Stars: Deduced from the
FLAMSTEDIAN OBSERVATIONS.
The Conſtruction of the *Meridional Parts*, *Logarithms*, *Sines*, *Tangents*
and *Secants*, both *Natural* and *Artificial*, by the NEWTONIAN SERIES.
With an Account of the *Cycles*, *Periods*, *Epoch's*, *Epacts*, *Kalendars*, &c.

ALSO

A TABLE of *Meridional Parts* for every *Degree* and *Minute*
of *Latitude* to the Ten Thouſandth Place in *Decimals*, calculated *de Novo*.
Together with a Large and very Uſeful TABLE of the *Latitudes*
and *Longitudes* of Places; the whole being deſigned for the Uſe of the
MATHEMATICAL SCHOOL, founded by King *CHARLES* II.

VOL. II.

By *JAMES HODGSON*,
Maſter of the Royal *Mathematical School* in *Chriſt*'s *Hoſpital*,
and Fellow of the ROYAL SOCIETY.

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TO THE

Right Worſhipful Sir *Francis Forbes*, K^t.
ALDERMAN, PRESIDENT;

THE

Right Worſhipful Sir *George Mertins*, K^t.
ALDERMAN, TREASURER,

And the reſt of the Worſhipful

GOVERNOURS

OF

Chriſt's Hoſpital, *London*;

This SECOND VOLUME of the
SYSTEM of the MATHEMATICS, deſigned for the Uſe of the *Royal Mathematical School*,

As a grateful Acknowledgment of the many Favours received, is moſt humbly Dedicated,

By Your Worſhips moſt Humble and
Moſt Obedient SERVANT,

James Hodgſon.

THE CONTENTS.

PART IV.

SECTION I.

Contains a Demonſtration of ſeveral Methods for the laying down or projecting ot any Great or Small Circle upon any Plane *Stereographically*; and for meaſuring any Part or Portion of a Great or Small Circle when projected. From *Page* 1 to *p.* 19

Definitions *Page* 1

Some Properties of the *Cone* demonſtrated *Page* 3

Methods thence deduced and demonſtrated for projecting any Great or Small Circle, and finding their Poles *Page* 6

Stereographical Theorems *Page* 10

Other Methods thence deduced for drawing of any Great or Small Circle upon a given Plane *Page* 12

Methods for laying off any number of Degrees in a Great or Small Circle, and for meaſuring any Part or Portion of it when projected; deduced from preliminary *Theorems*, and demonſtrated *Page* 15

SECTION II.

Contains Twelve *Stereographical Problems*, of great Uſe in the *Projection of the Sphere*: Conſtructing of *Spherical Triangles* and *Aſtronomical Problems* From *Page* 9 to *p.* 46

How to draw a Great Circle that ſhall paſs thro any two points howſoever ſcituated, upon the Plane of the Projection *Page* 20

Various ways of deſcribing a Great Circle about any given Point conſidered as a Pole, and in all the various Caſes *Page* 20
Various ways of finding the Pole of any projected great Circle, in all the various Caſes *Page* 22
Various ways of drawing a leſſer or ſmall Circle about any given Point, conſidered as a Pole, in all the various Caſes; or which is the ſame thing, of drawing a ſmall Circle parallel to any given great Circle, at any given Diſtance *Page* 23
Various ways of finding the Pole of any projected ſmall Circle (*Page* 27
To lay off any Number of Degrees in a given great Circle from a given point *Page* 27
Another way of drawing a ſmall Circle, about any given Point as a Pole *Page* 29
To meaſure any Part or Portion of a projected great Circle *P.* 30
To lay off any Number of Degrees on any projected ſmall Circle from any given point, and to meaſure any Part or Portion of the ſmall Circle when projected *Page* 31
Thro a given point to draw a great Circle, that ſhall form a given Angle with a given great Circle, various ways, and in all the Varieties *Page* 34
To meaſure any Spherical Triangle when projected, *Page* 40
Thro a given point to draw a great Circle, that ſhall be perpendicular to a given great Circle *Page* 43
Two great Circles given by Poſition, forming with each other a given Angle, to draw another great Circle that ſhall form two Angles with the two given great Circles, equal to two given Angles, each to each reſpectively; provided that the three given Angles be greater than a Semi-Circle *Page* 45

SECTION III.

Contains a Demonſtration of ſome Affections or Properties of Spherical Triangles, from *Page* 46 to *p.* 54

SECTION IV.

Contains a Demonſtration of ſeveral *Theorems*, commonly called *Axioms*, for the Solution of the Caſes of *Right Angled Spherical Triangles* From *Page* 54 to *p.* 64

A Demonſtration of the firſt *Axiom* *Page* 54
A Demonſtration of the ſecond *Axiom* *Page* 55
Various Properties of *Spherical Triangles*, deduced from the two former *Axioms* *Page* 56, 57, 58.
A Demonſtration of the Lord *Nepers Theorem*, for the Solution of *Right-angled Spherical Triangles* *Page* 59
Directions for applying the ſame *Theorem* to Practice *Page* 63

SECTION V.

Contains a great Variety of *Stereographical* and *Trigonometrical* Solutions of the ſixteen Caſes of *Right-angled Spherical Triangles* From *Page* 64 to *p.* 120
Various ways of Conſtructing the *6th* Caſe of *Right-angled Spherical Triangles* *Page* 120

SECTION VI.

Contains a Demonſtration of ſeveral *Theorems* commonly called *Axioms*, for the Solution of the ſeveral Caſes of *Oblique-angled Spherical Triangles* From *Page* 120 to *p.* 155
Various Properties of *Spherical Triangles* demonſtrated, very applicable to the Solution of the firſt 10 Caſes of *Oblique-angled Spherical Triangles* *Page* 120, 121, *&c.*
The 3*d Axiom* demonſtrated *Page* 125
A Demonſtration of ſeveral Properties and Affections of the *Sines*, *Tangents*, &c. of Arches *Page* 126 *&c.*
An eaſy Demonſtration of the 3*d Axiom* of *Plane Trigonometry P.* 129
The 4*th Theorem* or *Axiom* demonſtrated *Page* 133
The 5*th Theorem* or *Axiom* demonſtrated *Page* 135
A Demonſtration of the *6th Theorem* or *Axiom* *Page* 136
A Demonſtration of the 7*th Theorem* or *Axiom* *Page* 137
Several Methods for the Solution of the 11*th* Caſe, wherein the three Sides are given, to find the Angles; deduced from the former *Axiom* *Page* 139, 140, *&c.*
A Demonſtration of the 8*th Theorem* or *Axiom* *Page* 143
Several other Methods deduced from this *Axiom*, for finding the Angles from the three Sides of a *Spherical Triangle* given *P.* 144, 145, *&c.*

A Demonſtration of the ninth Theorem or Axiom p. 151
Various Methods deduced from the former Axioms for finding a Side of a Spherical Triangle, the three Angles being given. p. 152, &c.
The tenth Theorem or Axiom demonſtrated. p. 153, &c.
Great Variety of other Methods deduced from this Axiom for the Solution of the former Caſe p. 154

SECTION VII.

Contains a very great Variety of Stereographical and Trigonometrical Solutions of the twelve Caſes of Oblique Angled Spherical Triangles in all their Varieties and Ambiguities, from p. 155 to p. 216.
The Way and Manner of reſolving any of the firſt ten Caſes of Oblique-Angled Spherical Triangles, with or without letting fall the Perpendicular. p. 161
General Laws for letting fall the Perpendicular in any one of the Caſes p 163
The 11th Caſe reſolved 12 different ways by Trigonometrical Calculations. p. 198, &c.
Twelve different Solutions of the 12th Caſe p. 207
A great Variety of different Ways of conſtructing the 12th Caſe Stereographically p. 208

PART V.

SECTION I.

Contains Aſtronomical Definitions p. 217

SECTION II.

Contains the Stereographical Projection of the Sphere upon the Planes of the principal Great Circles, from p. 222 to p. 244
The Stereographic Projection of the Sphere upon the Plane of the Meridian. p. 222
The Stereographic Projection of the Sphere upon the Plane of the Horizon. p. 255

The

The Stereographical Projection of the Sphere upon the Plane of the Equator. p. 230
The Stereographic Projection of the Sphere upon the Plane of the Ecliptic p. 233
The Uſe of the Projection of the Sphere upon the Planes of the ſeveral Great Circles in accounting for the various Cæleſtial *Phænomena*, and the Solution of Aſtronononomical Problems p. 237

SECTION III.

Contains Stereographical and Trigonometrical Solutions of the chief and primary Problems ariſing from the Diurnal Motion of the Sun, or the Rotation of the Earth about her Axis, from p. 245 to p. 417.
The Stereographical Solution of the firſt *Problem* (*viz.* from the Sun's Longitude and greateſt Declination, &c.) upon the Planes of the four principal Great Circles. p. 246
General Rules for finding the Right Aſcenſion p. 249
Some Uſes of the 2d Caſe of the 2d Problem p. 252
Some Uſes of the 3d Caſe of the 2d Problem. p. 255
The reſt of the Caſes of the ſame Problem, with their Anſwers. p. 256
The Quantity of the Obliquity of the Ecliptic as it has been determined by the moſt eminent Aſtronomers from *Ariſtarchus*, who lived about 280 years before our Saviour, to the preſent Times. p. 257
The Stereographical Solution of the 2d *Problem* (*viz.* from the Latitude of the Place and Declination of the Sun, &c.) upon the Planes of the principal Great Circle p 269
Various Methods for determining the Amplitude, Oblique Aſcenſion, Deſcenſion, Aſcentional Difference, Time of the Riſing and Setting of the Sun in all the Various Situations of Places, from p. 263 to p. 272
The Beginning, End, and Duration of the Longeſt Day and Night at the *North Cape*, inveſtigated with the Way and Manner of finding it in all Places within the Artic and Antartic Circle, p. 273
That in the Latitude of 65 Deg. 53 Min. North, on the 14th of *June* at 12 H. *PM.* the Sun was ſeen two of his Diameters above

above the Horizon, (by reaſon of the Refraction) when he was really depreſſed 40 min. below the Horizon, and that for this Cauſe probably at the *North Cape*, the longeſt Day is increaſed 15 Natural Days, and that the longeſt Day is greater than the longeſt Night by 37 Natural Days. p. 274
The Manner of finding the Beginning and End of Morning and Evening Twilight ſhewn p 277
How to determine the Time when the Twilights continue, from Sun Set to Sun Riſe. p. 277
That the Twilights do not increaſe and decreaſe at the ſame time and in the ſame manner, as the Days increaſe and decreaſe. p. 279
That the Duration of Twilight is of different Lengths, at the ſame Diſtance from, but on contrary ſides of the Equator. p. 279
An Inveſtigation of the Rule for finding the Time when the ſhorteſt Twilights happen in all Places p. 280
That the Twilights in all Places under the Equator, are of ſhort Duration, greateſt when the Sun is in the Solſtice, and leaſt when he is in the Equator p. 281
That at certain times of the Year, in all Places that are ſcituated at a greater Diſtance than 48 deg. 31 min. from the Equator, there is no dark Night p. 281
That the Cauſe of Twilights proceeds from the Reſiſtance that the Solar Rays meet with in paſſing thro the Atmoſphere. p. 282
How to find the time of the Riſing and Setting of the Sun, by allowing for the Refraction p. 283
At *London* on the *7th* of *May*, the Sun appears to Riſe upon the Account of the Refraction ſooner than he would otherwiſe do, by 04 min. 10 ſec. ½, and Sets ſo much later; and that the Apparent Day is thereby increaſed 8 min. 21 ſec. p. 284
That the Refraction cauſes an Error in the Amplitude, as well as in the Length of the Day p. 284
The Manner how it may be found p. 284
Another and more eaſy Way, of finding the Apparent Time of the Riſing and Setting of the Sun, &c. by allowing for the Refraction p. 285
In all Computations the Sun's Declination muſt be found at the Time as near as poſſible, when any Appearance will happen p. 285
How to find when either Limb of the Sun begins to appear at the Riſing,

Riſing, or diſappear at the Setting, by allowing for the horizontal Semi-diameter of the Sun. *p.* 286.

Of what Uſe theſe Allowances for the Refraction and Semi-diamiter of the Sun in computing the time of the apparent Riſing and Setting are *p.* 287

The Solution of the reſt of the Caſes of this Problem *p.* 289

The Solution of the 3*d Problem* and its ſeveral Caſes for finding the Height and Azimuth of the Sun, at the Hour of Six. *p.* 294.

The Solution of the 4*th Problem* and its ſeveral Caſes for finding the Heighth of the Sun, and time when he will appear upon the Prime Vertical. *p.* 302

The Solution of the 5*th Problem* for finding the Sun's Height and Azimuth at any time, when he is in the Equator, and its ſeveral Caſes. *p.* 312.

The Solution of the 6*th Problem* for finding the Sun's Height and Azimuth when he is in any point of the Ecliptic, and its ſeveral Caſes. 323

Various Methods of inveſtigating the Hour of the Day and Azimuth of the Sun in all the Variety of Caſes *p.* 327, *&c.*

The Solutions of the doubtful Caſes in Aſtronomy *p.* 340.

In all places between the Tropics, when the Latitude of the place is leſs than the Sun's Declin. and both the ſame way, the Sun will appear upon the ſame Azimuth at two different times in the Morning and Afternoon, the Solution of one Example at *Barbadoes* at the time of the Summer Solſtice *p.* 348.

An Inveſtigation of the time when the Sun's Azimuth will be the greateſt, and how much it will be on the ſame Day and in the ſame Place. *p.* 351

A Solution of the 6th *Problem*, and its various Caſes for finding the Latitude of the Place from two unequal Altitudes or Azimuths of the Sun, *&c.* *p.* 377

The Reaſon why the Quantities of the ſeveral Component parts of the foregoing and following Problems have been given in Seconds and Parts of Seconds *p.* 390

By help of the Tables of the Sun's Place, Declination, *&c.* to find the true Place of the Sun, his Declination, *&c.* at any given Hour and Minute, and in any given place. *p.* 391

Tables of the Sun's true Place every Day at Noon, for the Years 1725, 1726, 1727, 1728. *p.* 393

A Table ſhewing how much the Sun's place alters any Day of the Year, for any time leſs than a Hundred Years p. 402
Tables of the Sun's Declination every Day at Noon, for the Years 1725, 1726, 1727, 1728. p. 403
A Table ſhewing how much the Sun's Declination alters every Day of the Year for any Time leſs than a hundred Years p. 412
Tables ſhewing the Sun's Right Aſcenſion every Day at Noon, for the Years 1725. 1726, 1727, 1728. p. 413
A Table ſhewing how much the Sun's Right Aſcenſion alters for any time leſs than an hundred Years p. 422
NB. The Tables being printed laſt, and there being not ſufficient Room left, is the Reaſon that the next Page begins with 417

SECTION IV.

Contains Methods for calculating the Longitudes, Latitudes, Right Aſcenſion, and Declination of the Moon, or of any Fixed Star or Planet, the Times of their Riſing, Setting, *&c.* from *p.* 417 to *p.* 513
Definitions p. 417
Given the Latitude and Longitude of a Fixed Star, or Planet to find the Right Aſcenſion and Declination p. 418
Various Examples of the ſame Problem p. 421
Given the Right Aſcenſion and Declination of a Fixed Star or Planet to find its Longitude and Latitude p. 422
Various Examples of the ſame Caſe p. 424
The Equable and Uniform Increaſe of the Star's Longitude muſt cauſe an unequal Increaſe of their Right Aſcenſions, *&c.* p. 429
The Method made uſe of by Mr. *Flamſteed* for the ready obtaining of the Right Aſcenſion and Declination of a Star at any time in his Catalogue p 430
Given the Meridional Altitude of an unknown Star or Planet, and its diſtance from a Known Star to find the Unknown Star's Right Aſcenſion p. 431
Various Examples of the ſame Problem p. 433
The Altitude of an Unknown Star or Planet taken at any time, together with its Diſtance from a Known Fixed Star being given to find

find the Right Aſcenſion, &c. of the Unknown Star p. 436
Various Examples of the ſame Problem p. 441
Given the Longitude and Latitude of two Fixed Stars, and the Diſtance of each of them from a third, to find the Longitude, &c. of the unknown Fixed Star p. 443
Various Examples of the ſame Problem p. 451, &c.
The Uſe of theſe Problems in finding of the Places of the Fixed Stars and Planets p. 455
An Account of the Inſtrument contrived by Mr. *Flamſteed*, and the Method made uſe of by him for making his large Catalogue of the Fixed Stars p. 455
How to find the Time when the Moon or any Star will paſs the Meridian. p. 456
To find the Amplitude of any Star, and the Time of its Riſe, Setting, &c. from the Latitude of the Place, &c. p. 459
General Rules for finding the Time of the Riſing and Setting of the Stars, &c. p. 462
To find what Stars never Riſe nor Set, which are viſible and which are inviſible. p. 463
Various Examples of the ſame Caſe. p. 464
To find the Time of the Riſing and Setting of the Moon p. 467
That the Motion of the Sun in the Ecliptic in the ſpace of 12 hours will cauſe no ſenſible Error in computing the time of the Riſing and Setting. p. 470
The Method made uſe of by Aſtronomers for finding the time of the Moon's Southing. p. 471
To compute the Time when the Center of the Moon will Riſe and Set, by allowing for her Horizontal Parallax, or which is the ſame thing, to find when the Center of the Moon will arrive at the Eaſtern and Weſtern ſide of the ſenſible Horizon, p. 471
To compute the time when the Center of the Moon will Riſe or Set, regard being had to her Horizontal Parallax, and the Horizontal Refraction. p. 472
Of what Uſe this Method of allowing for the Horizontal Parallax and Refraction is in computing the time of the Riſing and Setting of the Luminaries. p. 473
To find when any Star will appear upon the Six a Clock Hour Circle, and what will be its Height and Azimuth at that time, p. 473

Various Examples of the ſame Caſe. p. 475

To find at what time any Star will appear upon the Prime Vertical, and what will be its Height at that time. p. 477

Several Examples of the ſame Caſe. p. 479

To find the Hour of the Night, and Azimuth of a Star from the Latitude of the Place, &c. p. 480

Several Examples of the ſame Caſe. p. 483

The Uſe of the two former Caſes. p. 484

To find the Longitude of the Mid Heaven and Nonageſſima, what Points of Ecliptics are Riſing Setting, &c. p. 485

To find the Coſmical and Achronical Riſing and Setting of a Fixed Star, &c. uſually called the Poetical Riſing and Setting, &c. 490

How from the Horizontal Parallax of the Moon, or any Planet, to compute the Parallax in Altitude, and from thence the Parallax in Longitude and Latitude, &c. p. 496

How from the Parallax in Altitude to find the Diſtance of the Planet from the Earth and the contrary. p. 499

Of what Uſe the Parallaxes are. p. 499

Examples in the various Caſes for finding the Parallax in Longitude and Latitude from the Horizontal Parallax. p. 504

How to find the time when the Moon will Cover or Eclipſe any Fixed Star that lies within the Zodiac. p. 507

Example the firſt, of the Moons covering a Fixed Star in the Forehead of the *Whale*, on the 23d of *October* 1721. Another Example of the Occultation of the South Horn of the *Bull* by the Moon, on the 16th of *January* 1722. p. 510

The Uſe of theſe Occultations in determining the Difference of Longitude between Places. p. 513

How the Beginning, Middle, and End of a Solar Eclipſe may be determined p. 513

An Account of the Aſtronomical Works of the late Reverend Mr. FLAMSTEED the King's Aſtronomer. p. 514

A Table containing the Right Aſcenſions, Declinations, Longitudes, and Latitudes of above 60 eminent Fixed Stars, deduced from the Flamſteedian Obſervations, and fitted to the Year 1726. p. 515

The Latitudes, Longitudes, Right Aſcenſions and Declinations of the four remarkable Southern Stars, commonly called the *Croſiers*. p. 518

A Table ſhewing the Latitudes and Longitudes of the principal Harbours, Headlands, Iſlands, and Cities in the World *p.* 521
A Table of Logarithims, from 1 to 10000
A Table of Sines, Tangents and Secants

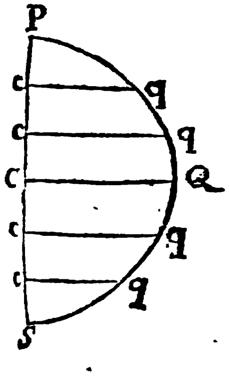
PART

PART IV.

Section I.

Of the Stereographic Projection of the Sphere.

IF the Semicircle *P Q S*, be turned about upon its Diameter *P S*, (the Diameter *P S* remaining fixed as an Axis) until it return again to the Place from whence it began to move, each Point as *Q*, *q*, *&c.* will deſcribe the Circumference of a Circle, whoſe Radius or Semi-diameter is *Cq*, *cq*, &c. and the Arch *P q Q q S*, will generate the Surface of a Sphere.



And ſince the Radij or Semi-diameters of the ſeveral Circles deſcribed by the Points *Q*, *q*, &c. are as the Sines of the reſpective Diſtances of the Points *Q*, *q*, &c. from the Points *P* and *S*, and that the Radius or Semi-diameter *CQ*, of the Circle generated by the Point *Q*, the Sine of 90 Degrees is the greateſt of all the Sines, it follows, that the Circle generated by the Point *Q*, is the greateſt of all the Circles deſcribed about the Axis *PCS*; and inaſmuch as this Circle divides the Surface of the generated Sphere into two equal parts, becauſe the Point Q divides the Arch PQS equally. Hence it is, that every Circle that divides the Surface of the Sphere into two equal parts is called a great Circle.

Again, becauſe the ſeveral Circles deſcribed by the Points q, q, &c. become leſſer and leſſer as the Points q, q, approach nearer and nearer to the Points P and S, but yet retain their Paralleliſm one to another, and to the great Circle deſcribed by the Point Q; and foraſmuch as each of theſe divide the Surface of the generated Sphere into two unequal Parts, becauſe the generating Points q, q, &c. divide the Arch PQS unequally. Hence it is, that every Circle that Surface of the Sphere divides into two unequal parts, is called a *Leſſer*, *Parallel*, or *Small Circle*.

The two Extream Points P and S of the Axis PS, about which the Semi-circle is ſuppoſed to revolve, are called the Poles; and ſince each of theſe Points are equally diſtant from every Circle deſcribed by the Points Q, q, q, &c. Hence it is, that that Point upon the Superficies of the Sphere which is every way equally diſtant from any Circle deſcribed upon it, is called the Pole of that Circle.

Hence it is, that every Circle whatſoever, deſcribed upon the Superficies of the Sphere has two Poles; and ſince every ſmall Circle is equidiſtant from ſome one great Circle, it follows,

That the Poles of every Small Circle are the ſame with the Poles of that Great Circle to which they are parallel. Wherefore,

All Small Circles parallel to each other, have the ſame common Poles.

General Definition

Conceive the Eye placed ſome where in the Superficies of the Globe, and at the ſame time a Plane to cut the Globe, and to ſtand at Right-angles to the Line connecting the Centers of the Globe and Eye, if infinite Right Lines be imagined to flow from the Eye, to every Point in the Circumference of every Circle deſcribed upon the Spherical Surface, they will trace out upon the cutting Plane, what we call a *Stereographic* Projection of the Sphere.

Whence and from the preceding it follows,

1. That the Eye is placed in one of the Poles of that Circle thro' which the Plane paſſes, and upon which the *Projection* is to be

formed, which for diſtinction ſake we call the *Plane* of the *Projection*, alſo the Pole oppoſite to the Eye we call the remoteſt Pole.

2. That the *Pole* when Projected falls in the *Center* of the *Plane of Projection*,

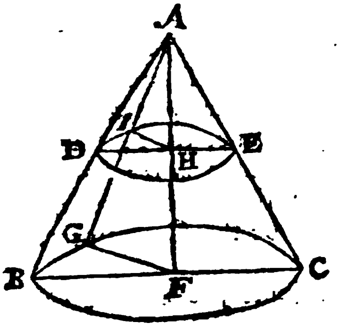
3. That all Great Circles which interſect each other in the Eye Point (inaſmuch as they lie in the ſame Plane with the Eye, and that the common interſection of two Planes is a ſtreight Line) will when Projected, become Right Lines, interſecting each other in the Center or Pole of Projection.

4. That the Poles of every ſuch Great Circle will be found in the Periphery of the Plane of Projection, at the diſtance of 90 Degrees or a Quadrant, from either Interſection of the Great Circle, with the Periphery of the Plane of Projection.

5. That every ſmall Circle which paſſes thro' the Eye, will be repreſented in the Projection by a Right Line, at Right Angles to that projected Great Circle which paſſes thro' the Eye, and cuts the Great Circle to which the Small Circle is parallel, at Right Angles.

6 That the Lines flowing from the Eye to the Circumference of every other Circle will form a Conical Superficies, and in order to inquire into the Nature and Properties of theſe Circles when projected, it is neceſſary to premiſe the following *Lemma's*.

Lemma I.



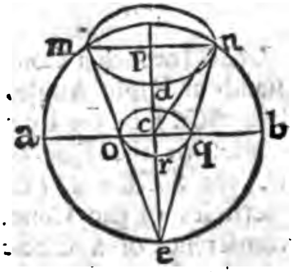
If a Right Cone *ABC* (or ſuch a Cone whoſe Axis *AF* ſtands at Right Angles to its circular Baſe *BGC*) be cut by a Plane *DIEH*, parallel to the Baſe of the Cone *BGC*, the Section *DIE*, made in the Superficies of the Cone, will be the Circumference of a Circle whoſe Center will be found in that Right Line *AF*, which connects the Vertex *A* of the Cone, and Center *F* of its Baſe.

Let the Cone be cut by a Plane thro' the Axis *AF*, then will the common Interſection of this Plane, with the Plane *DIE* parallel to the Baſe *BGC*, be the Right Line *DE*; which Right Line *DE* will be parallel, to the Diameter of the Baſe of the Cone *BC*, and both will lye in the ſame Plane *ABC*; therefore (by *Prop. the* 18th *of Part the* 1ſt) it will be, as *AF* : *FB* :: *AH* : *HD*; and as *AF* : *FC* :: *AH* : *HE*. Wherefore, (by *the* 13th *and* 11th *of Prop. the* 15th, *Part the* 1ſt) it will be, as *FB* : *FC* :: *HD* : *HE*; but *FC* is equal to *FB*, therefore *HD* is equal to *HE*. Again to any other Point as *G*, in the Circumference of the Baſe of the Cone, from the Vertex *A*, draw the Right Line *AG*; this being in the Superficies of the Cone, ſhall cut the Plane thro' *DIE* ſome where as in *I*, and draw *HI*, which will be parallel to *FG*, and in the ſame Plane with it, therefore, as *AF* : *FG* :: *AH* : *HI*. Therefore, whereas *FG* equal to *FB* is equal to *FC*, *HI* equal to *HD* is equal to *HE*; and the ſame will hold good wherever the Point *I* falls, wherefore the Section *DIE* will be the Circumference of a Circle, and *H* its Center (by *Def. the* 15th *and* 16th *of Part the* 1ſt.) W. W. D.

If the Cone *ABC* had been a Scalene Cone, that is, ſuch a Cone whoſe Axis does not ſtand at Right-angles to its Baſe, yet the Section made in the Superficies of this Cone, being cut by a Plane parallel to its Baſe, will be the Circumference of a Circle, as is evident from the preceding Demonſtration.

Hence and from the General Definition it follows,

1. That all Small Circles parallel to the Plane of Projection will become Circles in the Projection.



For let *e* repreſent the Eye, *m d n* any Small Circle parallel to the Plane of Projection *a b*; becauſe the Line *e p* drawn from *e* the Vertex of the Cone, to *p*, ſtands at Right-angles to the Plane *m d n*, the Section *o r q*, made in the Superficies of this Cone by the Plane *a b* parallel to *m d n*, will be the Circumference of a Circle by the former *Lemma*.

2. That they will all have one common Center, *viz.* the Center or Pole of Projection.

For ſince the Center of every parallel Circle will be found in the Axis of the Cone *ep*, their common Repreſentation in the Plane of the Projection will be in the Point *c*, the Center of the Projection.

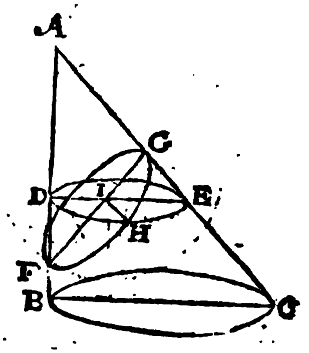
3. That their common Center, *viz*. the Center of the Projection will be their common Pole.

For the Axis of the Cone which connects the Eye at *e* and *p*, their common Pole, cuts the Plane of Projection in *c* the Center.

4. That their Radius's or Semi-diameters, will be equal to the Semi-tangents of their diſtances from the remoteſt Pole of the Projection.

For, becauſe the Triangle *emn* is the Section of the Cone thro' the Vertex and Center of its Baſe, therefore *oq* is the Diameter of the Circle to be projected; and *cq* equal to *oc* its Radius or Semi-diameter, is the Tangent of the Angle *ceq* equal to half the Angle *pen*, equal to the Arch *pn*, the diſtance of the Small Circle *mdn*, to be projected, from P the remoteſt Pole.

Lemma II.



If a Scalene Cone *ABC*, be cut by a Plane *FHGI*, not parallel to the Baſe, yet ſo that it cuts off a part *AFG*, ſimilar to the whole, the Section *FHG* made in the Superficies of the Cone, will be the Circumference of a Circle.

Thro' any Point *I*, of the Line *FG*, or common Interſection of the two Planes *ABC* and *FHE*. Let the Cone be cut by a Plane *DHE*, parallel to the Baſe of the Cone *BC*, then will the Section *DHE* be a Circle by the firſt *Lemma*; and *HI* the common Interſection of the two Planes *DHI* and *FHG* be perpendicular to the Lines *DE* and *FG*, and becauſe the Triangles *AFG*, *ABC*, and *ADE* are ſimilar, the Triangles *GIE*, and *DIF* are ſimilar, (for the Angles *IEG*, *IFD* are equal, as are the Angles *GIE* and *DIF*) therefore, as *GI* : *IE* :: *ID* : *IF*; therefore, $GI\times IF=IE\times ID$, but becauſe *H* is in the Circumference of the Circle *DHE*, and *IH* perpendicular to *DE*, therefore $HIq = DI \times IE$, and conſequently $FI \times IG = HIq$; therefore H is in the Circumference of a Circle whoſe Diameter is FG, and this will hold good in whatſoever part of the Line FG the Point I be taken; wherefore the Section FHG made in the Superficies of the Cone, will be the Circumference of a Circle. W. W. D.

Whence it follows,

1. That all Circles whatſoever, (except ſuch as lye in the ſame Plane with the Eye, which have been already conſidered) tho' they are not parallel to the Periphery of the Plane of Projection, yet become Circles in the Projection.

Fig. 1.

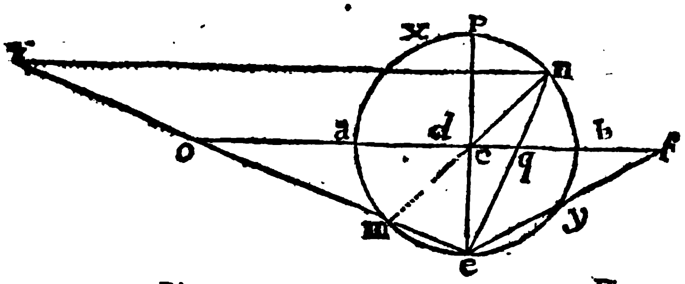


Fig. 2.

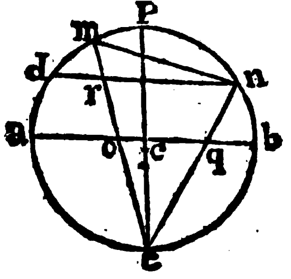
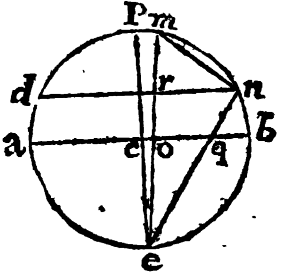


Fig. 3.



For, let e repreſent the Eye, ab the Plane of the Projection, p its remoteſt Pole, and mn the Diameter of the Circle to be Projected; then will the Triangle enm be the Section of the Cone, thro' the Vertex and Center of the Baſe.

Thro' n draw nr parallel to ab, then is the Angle eqo equal to enr (by *Prop. the* 5th *Part the* 1ſt) equal to emn (by *Cor. the* 1ſt *of Prob. the* 11th *Part the* 1ſt) and the Angle nem common to the Triangles nem and oeq; therefore the Angle enm is equal to eoq, (by *the*

the 2d *Cor. of Prop. the* 6th *Part the* 1ſt) and the Triangles *c n m* and *c o q* are ſimilar, (by *the* 28th *Def. of Part the* 1ſt) but ſubcontrarily placed, wherefore (by *Lemma the* 2d) if *n m* be the Diameter of a Circle deſcribed upon the Superficies of the Sphere, *o q* ſhall be the Diameter of a Circle, and its Repreſentative upon the Plane of the Projection *a b*.

2. That the Center and Poles of every Great Circle will be found in that Diameter when projected, which connects the two Interſections of the Circle to be projected, and a Great Circle paſſing thro' the Eye, and cutting the Periphery of the Plane of Projection at Right Angles, or which is the ſame thing, in the Repreſentative of that Great Circle which cuts the Circle to be projected, and the Plane of the Projection at Right Angles.

Thus in *Fig.* the *1ſt*, the Center and Poles of the Great Circle *m c n*, will be found in *o q*, the Repreſentative of the Diameter *m n* when projected, which connects the two Interſections *n* and *m*, of the Great Circle *n c m* to be Projected, with the Great Circle *p n b e m a*, which Great Circle *p n b e m a* paſſes thro' the Eye, and cuts the Circle *n e m* to be projected, and the Periphery of the Plane of Projection at Right Angles; which projected Diameter for Diſtinction ſake, I call the projected Axis or Line of Meaſures.

3. That every Small Circle will have its projected Axis in that Line, in which lies the Projected Axis of the Great Circle to which it is parallel.

4. That the Semi-tangents, of the greateſt and leaſt diſtances of any Circle from the remoteſt Pole of Projection, ſet off on the ſame or contrary ſides of the Center of the Projection, as the Caſe directs, will give the Interſections or Extremities of the projected Axis.

5. That the Projected Diameter or Axis of every Great or Small Circle, within which the Pole of Projection lies, is equal to the Sum of the Semi-tangents of the greateſt and leaſt diſtances of that Circle from the remoteſt Pole of the Projection; thus, in *Fig.* the *1ſt* and *2d*, *o q* the projected Diameter or Axis of the Circle *n m*, is equal to the Sum of *o c* and *c q*, the Semi-tangents of the Arches *m p* and *p n*, the greateſt and leaſt diſtances of that Circle from *p*, the remoteſt Pole of the Projection.

6. That the Diameter or Projected Axis of every Small Circle, without which the remoteſt Pole of Projection lies, is equal to the difference of the Semi-tangents of the greateſt and leaſt diſtances

of that Circle from the remoteſt Pole of the Projection; thus, in *Fig.* the 3*d*, *o q* the projected Diameter or Axis of the Small Circle *m n*, is equal to the difference between *c q* and *c o*, the Semi-tangents of *p n* and *p m*, the greateſt and leaſt diſtances of that Circle from *p*, the remoteſt Pole of the Projection.

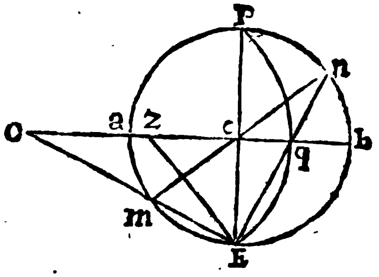
7. That of the two Poles of every Great Circle, and conſequently of all Small Circles parallel to it, that which falls within the Plane of the projection, will be found in the projected Axis, diſtant from the Center of the Projection, by the Semi-tangent of the Complement of the neareſt diſtance of that Circle from the remoteſt Pole of the Projection; thus, in *Fig.* the 1*ſt*, *d* the Pole of the Circle *n m*, is diſtant from the Center of the Projection by the Space of *dc*, the Semi-tangent of the Arch *x p*, the Complement to a Quadrant of the Arch *p n*, the neareſt diſtance of the Circle *n m*, from *p* the remoteſt Pole of the Projection.

8. That its other Pole will be found in the projected Axis, on the contrary ſide of the Center of the Projection, diſtant from it by the Semi-tangent of the projected Circle's neareſt diſtance, to the remoteſt Pole of the Projection increaſed by a Quadrant; thus, in *Fig.* the 1*ſt*, the exterior Pole *f* of the Circle *n m*, is diſtant from the Center *c* of the Projection, by the diſtance of *c f*, the Semi-tangent of *p y*, the neareſt diſtance of *p n*, of the projected Circle from the remoteſt Pole *p*, of the Projection increaſed by a Quadrant, or 90 deg. *n y*.

9. By *Cor.* the 7*th* we are taught, how to find the interior Pole of any projected Great Circle, and conſequently of all Small Circles parallel to it, by ſetting off the Semi-tangent of the Complement of its neareſt diſtance from the remoteſt Pole of the Projection, in its projected Axis, from the Center of the Projection on the contrary ſide to that in which the projected Circle falls.

10. By *Cor.* the 8*th* we are taught, how to find the exterior Pole of any Great Circle, by ſetting off the Semi-tangent of its neareſt diſtance from the remoteſt Pole of the Projection, increaſed by a Quadrant in its projected Axis from the Center, on the ſame ſide in which the projected Circle falls.

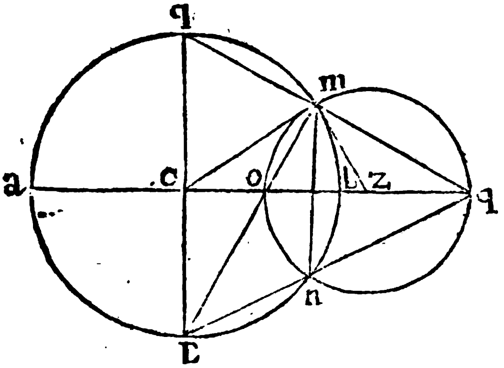
It has been ſhewn in *Cor.* 11. that if mn be the Diameter of a Great Circle deſcribed upon the Superficies of the Sphere, that oq ſhall be the Diameter of its Repreſentative upon the Plane of Projection.



Let oq be biſected in z, and draw ze, then will z be the Center of the Great Circle to be projected, and its Radius ze equal zq, equal to zo; becauſe ze is equal to zo, the Angle zeo is equal to zoe (*by the 4th Cor. of Prop. the 2d, Part the 1ſt*) equal to the Angle ceq (*by the 4th Cor. of Prop. the 18th Part the 1ſt*) for the Triangles oeq, cqe, are ſimilar) equal to half the Angle pcn (*by Prop. the 11th Part the 1ſt*) wherefore the Angles $oez + ceq$ are equal to pcn, and conſequently the Angle zec equal to ncb (*by the 3d Axiom*) now becauſe the Angle zce is a Right Angle, zc will be the Tangent, and ze the Secant of the Angle zec equal to ncb, the Complement of the neareſt diſtance of the Great Circle ncm, to p the remoteſt Pole of the Projection, or Inclination of the Plane of the Great Circle to be projected, to the Plane of Projection, to the Radius ce.

12. Wherefore, if the Semi-tangent of the Complement of the neareſt diſtance of any Great Circle to be projected, from the remoteſt Pole of the Projection, be ſet off from the Center in the Plane of Projection, in the projected Axis of that Great Circle, on the contrary ſide of the Center to which the projected Circle falls, it will give the Center of the Circle to be projected, and the Secant of the ſame diſtance will be its Radius or Semi-diameter.

13. Let mn be the Diameter of any Small Circle deſcribed upon the Surface of the Sphere, cutting the Plane of the Projection at Right-angles, or which is the ſame thing, whoſe Poles b and a lie in the Periphery of the Plane of Projection, then will oq be the Diameter of its Repreſentative upon the Plane of Projection ab; biſect oq in z, and draw zm, and mc, and mq, then will z be the Center of the Small Circle to be projected, and zm equal to zq, its Semi-diameter. Becauſe cm is equal to ce, the Angle cme is equal to cem, (*by Cor. the 4th of Prop. the 2d Part the 1ſt*) equal to cqm (by *Cor. the 2d*,

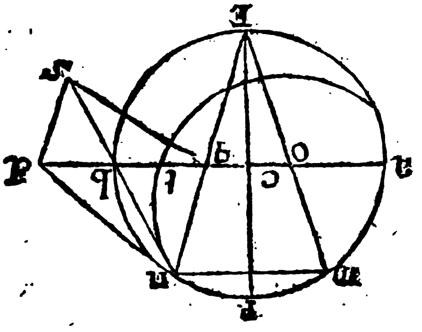


of Prop. the 6th *Part the* 1ſt) for the Angles *coe*, *moq*; alſo *eco*, and *omq*, of the Triangles *eco*, and *omq*, are equal) equal to *qmz*, for *zq* and *zm* are equal, therefore, whereas the Angle *omq* is a Right Angle, the Angle *cmz* ſhall be a Right Angle alſo, wherefore *mz* is the Tangent, and *cz* the Secant of the Angle *mcb*, or Arch *mb*, the diſtance of the Small Circle *mon*, to be projected from its Pole *b*, to the Radius *cm*.

14. Wherefore, if the Secant of the diſtance of any Small Circle which cuts the Periphery of the Plane of Projection at Right-angles from its Pole, be ſet off from the Center of the Projection on the ſame ſide on which the Small Circle falls, it will give the Center of the Circle to be projected, and the Tangent of the ſame diſtance will be its Radius or Semi-diameter.

Prop. I. *A Theorem*

In *Stereographic Projection*, the Angles made by the Circles on the Surface of the Sphere, are equal to the Angles made by their Repreſentatives on the Plane of the Projection.



Suppoſe the Eye at *e*, projecting the Spherical Angle *tnb*, upon the Plane *acb* at Right-angles to the Plane *apbe* at the Point *q*, I ſay the Angle *sqd* is equal to the Angle *tnb*, equal to the Angle *snd*, made by the two Planes *pndb* and *nstm*.

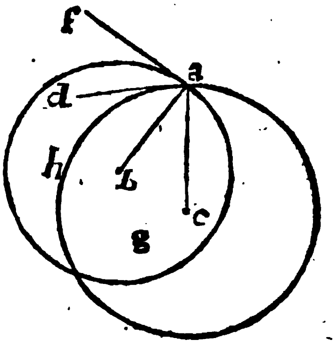
From the Angular Point *n*, draw *n d* and *n s*, tangents to the two Circles *n b e* and *n t m*.

Becauſe the Planes *a c t b* and *s n d* are both perpendicular to the Plane of the Circle *a p b e* by Conſtruction, their common Interſection *d s*, will be perpendicular to the Right Line *a c b* continued, and conſequently the Angles *q d s*, and *n d s* are Right Angles.

From *n* draw *n m* parallel to *a b*, and joyn the Points *e* and *m*, then will the Angle *d n e* be equal (*by the* 7th *Cor. of Prop. the* 11*th Part the* 1*ſt*) to the Angle *n m e*, equal (*by Cor. the* 4*th of Prop. the* 2*d of Part the* 1*ſt*) to *m n e* equal to (*by Prop. the* 5*th Part the* 1*ſt*) to *n q d*; therefore is *n d* equal to *d q*, wherefore in the Triangles *s n d*, *s q d* are *n d* equal to *q d*, the Angles *n d s* and *q d s* are Right, and the ſide *d s* common, wherefore (*by the* 2*d Prop. Part the* 1*ſt*) the Angle *s q d* upon the Plane of the Projection, is equal to the Angle *s n d* made by the two Planes, forming the Angle *t n b* upon the Spherical Surface, which was to be Demonſtrated.

Prop. II. *A Theorem.*

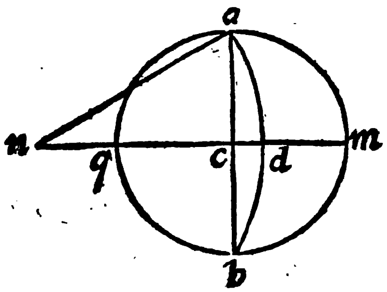
If two Circles *c a b* and *b a g* interſect each other in the Point *a*, the Angle *g a h* formed by them at their interſection, ſhall be equal to the Angle *b a c* made by the Radij *a b* and *a c*, drawn to the Point of Interſection *a*.



Conſtruction. To the Point of Interſection *a* draw *f a*, a Tangent to the Circle *g a b*, and *d a* a Tangent to the Circle *b a c*.

Dem. Becauſe the infinitely ſmall Portions of the Circles *g a b* and *b a c*, do coincide with the Tangents *f a* and *d a*, and conſequently have the ſame Direction, therefore the curved Lined Angle *g a h* formed by the two Circles, is equal to the Right-lined Angle *f a d*, formed by the Tangents *f a* and *d a*; and becauſe the Angle *f a b* is a Right-angle equal to the Angle *d a c*, take away from each the interjacent Angle *b a d*, and there will remain the Angle *b a c* equal to the Angle *f a d*, equal to the Angle *g a h*, which was to be Demonſtrated.

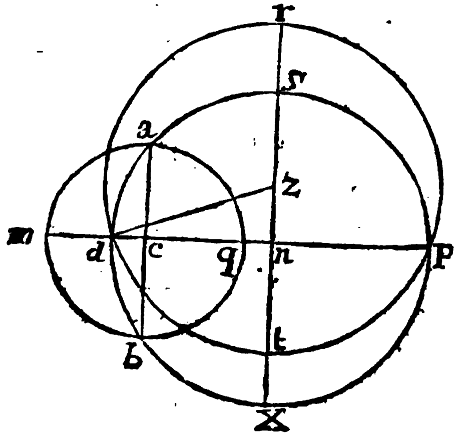
Hence and from the preceding Propoſition we are taught how to Project any Great or Small Circle, from the conſideration of the Angle that the Plane of the Circle to be projected, forms with the Plane of the Projection, or with the Plane of any other projected Great Circle.



1. Suppoſe it were required to project the Great Circle *a d b*, that ſhall form a given Angle with the Periphery of the Plane of the Projection.

It has been already ſhewn, that the Centers of all Great Circles paſſing thro' the Points *a* and *b*, ſhall be found in the Right Line *mdc* produced. Wherefore, Draw the Line *a n* forming an Angle with *a c*, equal to the given Angle, where this Line cuts *m d c q*, as in *n*, will give the Center.

Becauſe the Angle *a c n* is a Right-angle, *c n* will be the Tangent of the Angle *c a n*, and *a n* the Secant of the ſame Angle; wherefore, if the Tangent of the Angle that any Great Circle forms with the Periphery of the Plane of the Projection, be ſet off in its projected Axis, it will give the Center of the Circle to be projected, and the Secant of the ſame Angle will be the Semi-diameter. Again,



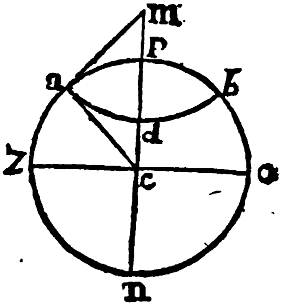
2. Suppoſe it were required to project a Great Circle, that ſhall form a given Angle with the projected Circle *a d b*, conſidered as the Periphery of the Plane of a Projection. Thro' *n* the Center of the given Great Circle *a d b p*, draw *x n s*, at Right-angles to *d p*, becauſe all Great Circles that paſs thro' the Point *d*, of the Circle *x d s p*, will interſect the Periphery of the ſame Circle. Again, in *p*, at the diſtance

of a Semi-circle from the Point *d*, therefore the Center of the Circle to be projected, ſhall be found ſomewhere in the Line *x s*.

Thro' the given Point *d* draw *d z*, forming an Angle equal to the given Angle, where this interſects the Line *x s* as in *z*, it will give the Center.

Becauſe the Angle *d n z* is a Right-angle, *z n* will be the Tangent, and *d z* the Secant of the Angle *z d n*, to the Radius *d n*.

Wherefore, if the Tangent of the Angle that any Great Circle forms with the Great Circle *a d b*, be ſet off in the Line *n s*, from *n* the Center of the given Circle, it will give the Center of the Circle to be projected, and the Secant of the ſame Angle will be its Semi-diameter to the Radius *d n*, of the Circle firſt given.



3. Suppoſe it were required to project a Small Circle, that ſhall paſs thro' the Point *a*, at the diſtance of *p a* from its Pole, and ſtand at Right-angles to the Periphery of the Plane of the Projection *z p o n*.

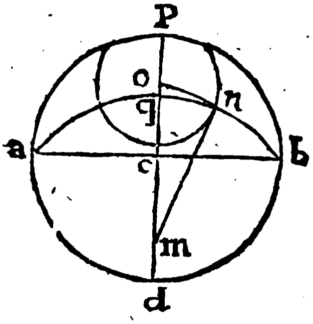
From *c* the Center of the Plane of the Projection, to *a* the given Point, draw the Line *c a*, and at Right-angles to it the Line *a m*, where this interſects the Line *n c p*, paſſing thro' *p* the Pole of the Circle to be drawn, it will give the Point *m*, the Center of the Circle to be projected.

Becauſe the Angle *c a m* is a Right-angle, *a m* will be the Tangent of the Arch *p a*, the diſtance of the Small Circle *a* from its Pole *p*, and *c m* the Secant of the ſame diſtance to the Radius *c a* or *c p*.

Wherefore, if the Secant of the diſtance of any Small Circle (that cuts the Plane of the Projection at Right-angles) from its Pole, be ſet off in its projected Axis from the Center of the Plane of Projection, it will give the Center of the Circle to be projected, and the Tangent of the ſame diſtance will be its Semi-diameter. Again,

4. Suppoſe it were required to project a Small Circle that ſhall paſs thro' the Point *n*, at the diſtance of *q n*, from its Pole *q*, and to ſtand at Right-angles to the Plane of the projected Great Circle *a q b*.

From

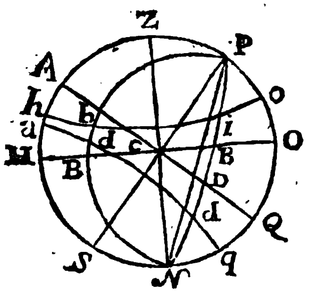


From m the Center of the given Great Circle to n, draw the Right-line mn, and at Right angles to it the Line no, where this interſects the projected Axis dp, of the given Great Circle as in o, gives the Center of the Circle to be projected.

Becauſe the Angle onm is a Right-angle, mo will be the Secant of the Angle qmn, and on the Tangent of the ſame Angle to the Radius mn, or mq, of the Circle aqb; wherefore if the Secant of the diſtance of the Point n from its Pole q, be ſet off from m the Center of the given Circle in its projected Axis to o, it will give the Center of the Circle to be projected, and the Tangent of the ſame diſtance will be its Semi-diameter.

And ſince the Angle formed by the Interſection of the Planes paſſing thro' any two Great Circles, is the ſame with the Inclination of the two Planes, and this being meaſured by an Arch of a Great Circle, intercepted between the two Planes, at a Quadrantal diſtance from the Interſection of the Peripherys of the two Circles, which Arch of Meaſure is ever equal to the Complement of the neareſt diſtance, of the Periphery of one of the Circles from the Pole of the other, as any one may readily apprehend, it will be eaſy from what has been ſaid, to project any Great or Small Circle, that ſhall paſs thro' any Point taken at pleaſure, in any given projected Great or Small Circle, with any poſſible Conditions, and from any poſſible *Data*, as the Reader will ſee more at large in the next Section.

Prop. III. *A Theorem.*



If the two remoteſt Poles P and N, of the two Great Circles HO and AQ, be connected together by the Right-line PN, and about this Right-line as a Diameter, a Small Circle $PBDN$, be deſcribed upon the Surface of the Sphere, the Arches BO and dQ, of the two Circles HO and AQ, intercepted between the Great Circle $ZPOQNH$, paſſing thro' the Poles P, N, S, Z, of the Circles *HO* and *AQ*, and the Small Circle *PBDN* will be equal one to another, alſo the Arches *bo* and *dq*, of the parallels *po* and *aq*, equally diſtant from the Poles *Z* and *N*, contained between the Small Circles *PBDN*, and the Great Circle *ZONH*, will be likewiſe equal one to another.

For becauſe *PQ* is equal to *NO*, and the Arch *OQ* common, therefore is the Arch *PO* equal to *NQ* (*by the 3d Axiom*) again, becauſe the Arches *PD* and *NB* of the Small Circle *PBDN* are equal, and the Arch *DB* common, therefore is the Arch *PB* equal to *ND* (*by the 3d Axiom*) and the Angle *BPO* equal to *DNQ*, it being the Inclination of the two Planes, therefore the Triangles *OPB* and *QND* are equal, and conſequently the Arch *BO* is equal to *DQ*.

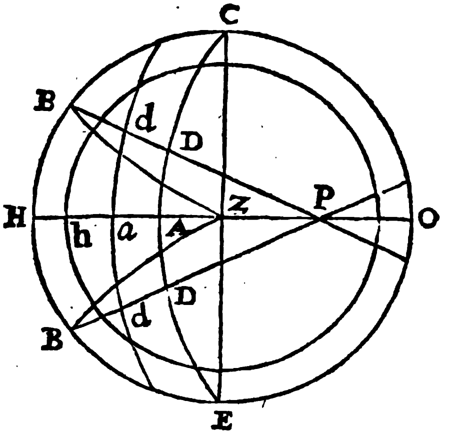
After the ſame manner may the Triangles *Pbo* and *Ndq* be proved equal, and conſequently the Arches *bo* and *dq* of the Small Circles *Hbo* and *adq* equal one to another.

If the Small Circle *PBDN*, infinitely extended every way from the Center, be carried about upon this Line *PN* as an Axis, the common Interſection of this Plane with the Surface of the Sphere will always be a Circle; and becauſe the Arches *PO* and *NQ* always remain the ſame, and the Arches *PB* and *ND* are conſtantly equal between themſelves, and the Angle *BPO* is continually equal to the Angle *DNQ*, each being the ſame with the Angles formed by the two Planes, it follows that the Arches *BO* and *DQ*, *AD*, and *HB*, of the Great Circles (*HO* and *AQ*) and the Portions *bo* and *dq*, *ad* and *hb* of the Small Circles (*ho* and *aq*) intercepted between this Plane (in all its various Scituations) and the Great Circle (*ZONH*) will be equal one to another; alſo the Arches *DD* and *BB* of the Great Circles, and the Portions *dd* and *bb* of the Small Circles, intercepted between this Plane in any two different Scituations are equal each to each.

Cor. Whence it follows, that if from *P* the Pole of any projected Great Circle *CDADE*, two Right Lines as *PDB*, *PDB*, be drawn, they will cut off Arches *HB*, *HB*, in the Periphery of the Plane of Projection, equal to the Arches *DA*, *AD*, in the projected Great Circle, intercepted between the Lines *PB* and the Great Circle *PZQO*; alſo an Arch *DD* of the projected Great Circle *CDDC*, equal to the Arch *BB*, of the Periphery of the Plane of the Projection intercepted between themſelves.

In the following Figure, let *HCOE* repreſent the Plane of the Great Circle *HCO* projected, the Eye being placed in *N*, then will the Line

Np



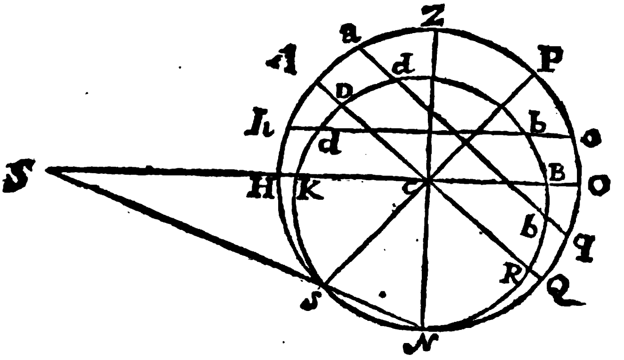
NP in the former Figure, be projected in the Point *P* in this Figure at the diſtance of *zp*, equal to the Semi-tangent of the Arch *zp*; and becauſe the Small Circle deſcribed about the Line *NP* in the former Figure, paſſes thro' the Eye at *N*, it will be repreſented in this Figure by the Right-Line *PDB* infinitely extended both ways, (by the 5*th Cor. of the General Definition*) alſo the upper Semi-circle of the Great Circle *HZON* in the former Figure, will be repreſented by the projected Diameter *OZH* in this Figure; wherefore by the preceding Propoſition, the Arches *DA*, *AD*, of the projected Great Circle *CDADC*, will be equal to the Arches *BH*, *HB*, of the Periphery of the Plane of Projection, and conſequently the whole Arch *DD* of the projected Great Circle, will be equal to the whole Arch *BB*, of the Periphery of the Plane of Projection, cut off by the two Right Lines *PDB*, *PDB*, drawn from *P* the Pole of the Circle *CDADE*, thro' the extremities *D*, *D*, of the Arch *DD*. W. W. D.

Alſo for the ſame Reaſon the Arches *bb*, *bb*, and *bb*, of the Small Circle *bbh* parallel to the Plane of Projection, cut off by the Right Lines *PB*, *PH*, *PB*, are equal to the Arches *da*, *ad*, and *dd*, of the Small Circle *dad*, (which is parallel to the projected Great Circle *CDADE*, and as far diſtant from its upper Pole *P*, as the given Parallel is diſtant from the under Pole *N*) cut off by the ſame Right Lines *PB*, *PH*, *PB*.

If to aſſiſt the Imagination, the *Reader* conceives the external Portions of the infinite Right Lines *PB*, *PH*, *PB*, to return again, and unite in the Pole or Center of the Projection Z, he will readily ſee the Reaſon of this whole *Corollary* without the Aſſiſtance of the former *Demonſtration*.

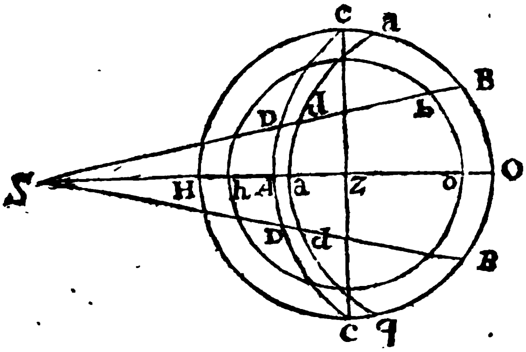
Prop. IV. *A Theorem.*

If the two neareſt Poles *N* and *S*, of the two Great Circles *HO*, *AQ*, be connected together by the Right Line *NS*, and about this Right Line as a Diameter a Small Circle be deſcribed, if this Small Circle be infinitely extended every way from its Center, and this infinite Plane be carried about upon the Right Line *N S* as an Axis, the common Interſection of this Revolving Plane with the Surface of the Sphere, will always be a Circle repreſented by *s K D B R N*, and will cut off equal Arches *A D* and *BO*, *H R* and *R Q*, in thoſe Great Circles *AQ* and *HO*, whoſe Poles are connected together; and it will alſo cut off equal Arches *a d* and *b o*, *h d* and *b q* from any Parallels or Small Circles *a q* and *b o*, equally remote from the Poles *S* and *N*, between it ſelf and the Great Circle *ZONH*, paſſing thro' the Poles Z, *P*, *N*, and *S*, of the Great Circles *HO* and *A Q*, and of the Small Circles *bo* and *aq*, but on the contrary Sides.



For the Triangles *SAD* and *NBO*, *SHK* and *NRQ*, *s a d* and *Nbo*, *s k d*, and *Nbq*, have each two Sides and the Angle contained of the one, equal to two Sides and the contained Angle of the other, and therefore equal in all reſpects; wherefore, &c. as was to be Demonſtrated.

Cor. Whence it follows, that if from *S* the exterior Pole of the Great Circle *CDAQN*, Lines as *SB*, *SO*, *SB*, be drawn, they will cut off equal Arches *BO*, *bo*, *Bb*, in the Periphery of the Plane of Projection, equal to the Arches *DA*, *AD*, *DD*, in the Great Circle *CDADC*; alſo Arches *bo*, *ob*, *bb*, in the Parallel *bob*, equal to the Arches *da*, *ad*, *dd*, in the Parallel *dad*, each Parallel being equally diſtant from the connected Poles *Z* and *S*.



For if the *Reader* conceives the Sphere repreſented by the Figure belonging to the 4*th Prop.* to be actually Projected, upon the Plane *HCOC*; in the adjacent Figure the Eye being ſuppoſed in *N*, he will readily perceive, that the Great Circle *ACQ* in the firſt Figure, will be repreſented by the projected Circle *CAC* in the latter. Alſo the Parallels *aq* and *no*, in the former by the Parallels *aq* and *bo* in the latter; the Line *NS* in the former will be projected in the Point *S* in the latter, and the Small Circle deſcribed about the Line *NS* will be repreſented by the Right Line *SDdB*, in its different Scituation, and the Great Circle *HZON* by the Right Line *SHAZO* infinitely produced; wherefore by the preceding Propoſition, the Arches *DA*, *AD*, of the projected Great Circle *CAC*, will be equal to the Arches *BO*, *OB*, of the Periphery of the Plane of Projection, and conſequently the whole Arch *DD* of the one, will be equal to the whole Arch *BB* of the other; and the ſame will hold good for the Arches *dd* and *bb*, of the Small Circles *aq* and *bo*. Wherefore, *&c.* as was to be Demonſtrated.

Whence it follows univerſally,

1. That if from either of the Poles of any projected Great Circle, Lines be drawn thro' the Extremitys of any Part or Segment of it, till they meet the Periphery of the Plane of Projection, they will cut off from it on that Part which is on the contrary Side of the Center of Projection with the Pole, an Arch equivalent to the Segment or Portion of the projected Arch.

2. That if from either of the Poles of any projected Small Circle, Right Lines be drawn to the Extremitys of any Segment or Portion

of it (and continued if need be) they will cut off an Arch equivalent to it, in that Small Circle which is parallel to the Plane of the Projection, and as far diſtant from the under Pole of the Projection, as the given Small Circle is diſtant from the Pole taken.

Hence we are taught univerſally, how to divide any projected Great or Small Circle according to any given Ratio, or to meaſure any Part or Portion of either of them when projected.

Section. II.

Containing a few eaſy Stereographical Problems, *whoſe Solutions naturally flow from the Principles laid down in the former Section, of great Uſe in the* Projection *of the* Sphere, Conſtructing *of* Spherical Triangles *and* Aſtronomical Problems.

GREAT *Circles* with reſpect to their mutual Inclinations or Poſitions are diſtinguiſhed into

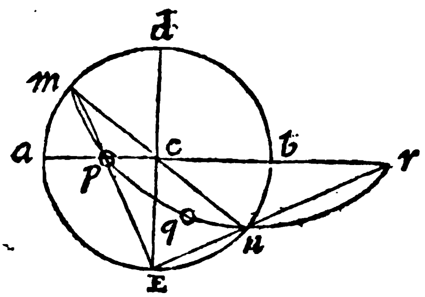
1. *Right Circles*, that is, ſuch Circles as cut the Periphery of the Plane of Projection at Right Angles, and when projected, are repreſented by Right Lines, interſecting each other in the Center or Pole of the Projection. Or,

2. *Oblique Circles*, that is, ſuch Circles as cut the *Periphery* of the *Plane* of *Projection* at unequal Angles, and in the Projection are repreſented by Circles that cut the Periphery of the Plane of the Projection into two equal Parts; and conſequently a Right Line connecting their Interſections, ſhall paſs thro the Center of the Projection.

3. The Periphery of the Plane of the projection, is uſually called the *Primitive Circle*.

Problem 1.

To draw a Great Circle that ſhall paſs thro' any two given Points in the Plane of the Projection.



Caſe the 1*ſt*. If one of the Points be in the Center, and the other in the Primitive Circle as *a*, or in ſome other Point as *p*, then thro' *C* the Center, and the given Point *a* or *p*, draw the Right Circle *a p c b*, and the thing is done.

Caſe the 2*d*. If neither of the Points *p* or *q* be in the Center.

1. Thro' one of the Points as *p*, draw a Diameter as *a p c b*, and croſs it at Right Angles with the Diameter *d c E*.

2. Thro' *E* and *p* draw the Line *E p m*, to cut the Primitive Circle in *m*, and draw the Diameter *m c n*.

3. Thro' *E* and *n* draw the Right Line *E n*, till it meet the Diameter *a p c b* produced in *r*.

4. Thro' the Points *p*, *q*, and *r*, draw a Circle (by *Cor. the* 9*th of Prop. the* 14*th Part the* 1*ſt*) and the thing is done.

Dem. For becauſe *m n* is the Diameter of a Great Circle deſcribed upon the Superficies of the Sphere, *p r* ſhall be the Diameter of its Repreſentative upon the Plane of the Projection *a b*; and conſequently every Circle that paſſes thro' the Points *r* and *q* ſhall be the Repreſentative of ſome Great Circle upon the Superficies of the Sphere, wherefore the Circle *p q r* ſhall be a Great Circle. *W.W.D.*

Problem 2.

About any Point as a Pole, to deſcribe a Great Circle.

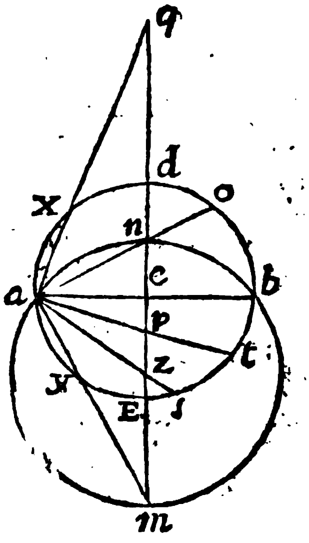
Caſe 1.

If the Point given be in the Center, then the Primitive Circle is the Great Circle required.

Caſe 2.

If the Point given be in the Primitive Circle as *d*.

Thro'



1. Thro' *d* and the Center *c* draw the Diameter *d E*, and draw the Right Circle *a c b*, and the thing is done. Or,

2. Set off 90 Degrees or a Quadrant from *d* to *a* and *b*, and draw the Right Circle *a c b*; which will be the Circle required.

Caſe 3.

If the Point be neither in the Center nor in the Primitive Circle, but in ſome other Place as *p*.

1. Thro' *p* the given Point and *c* the Center, draw the Diameter, or Right Circle *E p c d* and croſs it at Right-angles with the Diameter *a c b*.

2. Thro' *a* and the Point *p* draw the Line *a p t*; and ſet off 90 Degrees or a Quadrant on both ſides of it to *o* and *y*.

3. Thro' *a* and the Point *o* and *y*, draw the Lines *a n* and *a y*, till they interſect the Diameter *d c E* produced in *n* and *m*.

4. Biſſect *n m* in the Point *z*, then will *z* be the Center; and *z n* or *z m* the Radius of the Circle to be drawn.

Or,

2. Having drawn the Right Line *a p t*, ſet off the Arch *b t* from *t* to *s*, and draw the Line *a z s*, then will *z* be the Center, and *z a* the Radius of the Circle to be drawn.

Or,

3. If the diſtance from *p* the Point given, to *c* the Center of the Circle to be drawn be given, ſet off the Tangent of that diſtance from *c* to *z*, and the Semi-tangent of its Complement from *c* to *n*, then will *z* be the Center, and *z n* the Radius of the Circle to be drawn.

Or,

4. Set off the Secant of the diſtance from *a* to *z*, then will *z* be the Center, and *z a* the Radius of the Circle to be drawn, as before.

Or,

5. Set off the Chord of the double Diſtance from *b* to *t*, and draw *a z s* for the Center *z*, alſo ſet off the Chord of the Complement from *b* to *o*, and draw *a n o* for the Interſection or Extremity of the Diameter, then will *z* be the Center, and *z n* the Radius of the Circle to be drawn.

Problem 3.

To find the Pole of any projected Great Circle, ſee the Figure belonging to the preceding Problem.

Caſe 1.

If the Circle given be the Primitive Circle *a d b E*.

Find the Center *c* (by *Cor. the 6th of Prop. the 10th Part the 1ſt*) and you have its Pole.

Caſe 2.

If the Circle given be a Right Circle, or ſuch a Circle as is repreſented in the Figure by the Right Line *a b*.

Solution. Croſs it at Right-angles with the Diameter *d E*, and the Points *d* and *E* will be the Poles.

Or,

Set off 90 Degrees or a Quadrant, from either of the Extremitys *a* and *b* of the given Circle *a b*, in the primitive Circle to *d* and *E*, then will *d* and *E* be the Poles.

Caſe 3.

If the Circle given be an Oblique Circle as *a n b*.

Solution. 1. Thro' *a* one of the Interſections of the Oblique Circle with the Primitive Circle, and *n* the Interſection of the ſame Oblique Circle with its projected Axis, draw the Right Line *a n o*, cutting the Primitive Circle in *o*.

2. Set off 90 Degrees or a Quadrant in the Primitive Circle, both ways from *o* to *x* and from *o* to *t*, and draw the Lines *a p t* and *a x q*, cutting the projected Axis produced if need be, in *p* and *q*, then will *p* and *q* be the Poles of the Circle to be found.

Or,

If *c n* the neareſt diſtance of the Oblique Circle, from *c* the Center of the Primitive Circle or Pole of Projection be given, ſet off the Semi-tangent of that diſtance plus 90 Degrees, from *c* the Center of the Primitive Circle, on the ſame ſide of the Center that the Oblique Circle *a n b* falls in the projected Axis, to *q*, and you have one of the Poles, and the Semi-tangent of the Complement of the ſame diſtance, ſet off from the Center on the contrary ſide in the Axis to *p*, will give the other Pole *p*.

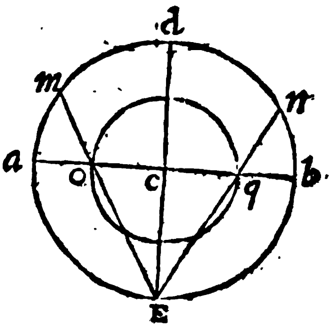
Problem 4.

About any Point as a Pole, to deſcribe a Leſſer or Small Circle, or which is the ſame thing, to draw a Small Circle parallel to any given Great Circle.

Caſe 1.

If the Point given be the Center of the Primitive Circle, or which is the ſame thing, if the Circle to be drawn be a parallel to the Primitive Circle.

Solution.



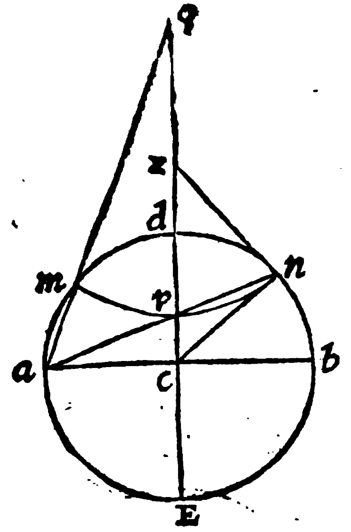
1. *By the Chords*, ſet off the Chord of the diſtance of the Small Circle from its Pole in the Primitive Circle, from *d* to *m*, or from *d* to *n*, or its diſtance from the Great Circle to which it is required to be parallel, from *a* to *m*, or from *b* to *n*, and draw the Line *E m* or *E n*, where either of theſe Lines interſect the Diameter *a b* as in *o* or *q*, gives, one Interſection of the Diameter of the Circle to be drawn, of which *c* is the Center, and *c o* or *c q* the Radius.

2. *By the Semi-tangents*, with the Semi-tangent of the diſtance of the Small Circle from the Pole of the Projection, or the Semi-tangent of the Complement of its diſtance from the Great Circle, to which it is required to be parallel, between the Points of the Compaſſes, ſetting one Foot in the Center or Pole *c*, with the other deſcribe the Leſſer *o q*, and the thing is done.

Caſe 2.

If the Point given be in the Primitive Circle at *d*, or which is the ſame thing, if the Small Circle to be drawn be required to be parallel to the Right Circle *a b*, and which muſt therefore of neceſſity cut the Plane of Projection at Right-angles.

Solution.



Solution.

1. *By the Chords*, ſet off the Chord of the Small Circle's diſtance from its Pole *d*, to *m* and *n*, or the Chord of its diſtance from the Great Circle *a c b*, to which it is required to be parallel, from *a* to *m*, and from *b* to *n*, and draw the Lines *a m q* and *a p n*, where theſe Lines interſect the Axis *c q* in *q* and *p*, will give the two Extremitys of the Diameter of the Circle *b* to be projected, the middle Point *z* of which will be the Center, and *z q* or *z p*, the Radius of the Circle to be drawn.

Or,

2. Draw *z n* a Tangent to the Point *n*; or at Right-angles to the Radius *c n*, of the Primitive Circle *a d b E*, where this interſects the projected Axis in *z*, gives the Center of the Small Circle to be drawn, of which *z n* will be the Radius.

Or,

3. Having found the Points of Interſection *m* and *n* as before, with the Tangent of the Small Circle's diſtance from its Pole *d*, or the Tangent of the Complement of its diſtance from the Great Circle *a c b*, to which it is required to be parallel, between the Points of the Compaſſes, ſetting one Foot in *m* or *n*, with the other croſs the projected Axis in *z*, then will *z* be the Center, and the diſtance between the Points of the Compaſſes, the Radius of the Circle to be drawn.

Or,

4. Having ſet off the Semi-tangent of the Small Circle's diſtance from the Great Circle to which it is to be parallel, or the Complement of its diſtance from the given Pole in the projected Axis from *c* to *p*, it will give one Interſection of the Circle to be drawn, and the Secant of the diſtance of the Small Circle from the Great Circle to which it is to be parallel; or of the Complement of its diſtance from the given Pole, ſet off from *c* the Center of the Primitve Circle,

in the projected Axis the ſame way to *z*, will give the Center of the Circle to be drawn, whoſe Radius will be *zp*.

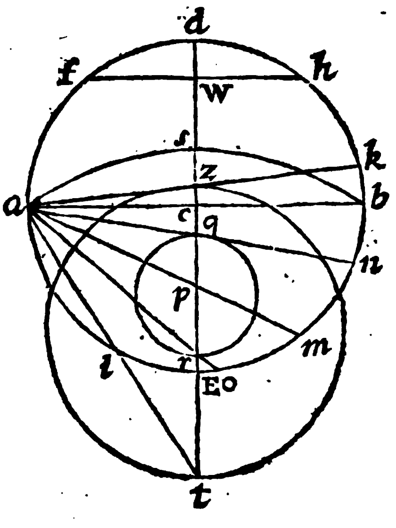
5. Having found the Interſection *p*, by the help of the Semi-tangent of the Small Circle's diſtance from the Great Circle to which it is to be parallel, ſet off from thence the Tangent of the Small Circle's diſtance from its Pole in the Axis from *p* to *z*, then will *z* be the Center, and *zp* the Semi-diameter of the Circle to be drawn.

Or,

6. Having ſet off the Semi-tangent of the Small-Circle's diſtance from the Great Circle to which it is to be parallel, or the Semi-tangent of the Complement of its diſtance from its Pole, in the Axis from *c* to *p*, it will give one Interſection of the Diameter, and the Semitangent of the ſame diſtance plus 90 Degrees or a Quadrant, ſet off the ſame way in the Axis from *c* to *q*, will give the other Extremity of the Diameter the middle Point wherefore *z*, will be the Center, and the diſtance *zp* or *zq*, will be the Semi-diameter.

Caſe 3.

If the Point given be neither in the Center nor in the Primitive Circle, but ſomewhere within the Circle as at *p*, or which is the ſame thing, ſuppoſe it were required to draw a Small Circle parallel to a given Oblique Circle *a s b*.



Solution.

1. *By the Chords.* 1. Thro' *p* the Point given, and *a* the Pole of the projected Axis *d c E*, draw the Line *a p m* to interſect the Primitive Circle in *m*.

2. Set off the Chord of the Small Circle's diſtance from the given Point or Pole or the Complement of its diſtance from the Great Circle to which it is required to be parallel, from *m* to *n*, and from *m* to *o*, or

from m to l, and from m to k, and draw the Lines alt, aro, aqn, and azk; where theſe interſect the projected Axis in r and q, or in t and z, will give the Extremities of the Diameters rq and tz, of the Circles to be drawn, the middle Point of each of which will be the Centers; or,

If the diſtance of the Point p from c the Center of the Circle or Pole of the Projection be known, and the diſtance of the Small Circle from its Pole or given Point p, be leſs than the diſtance between the given Point p and the Center c;

Then ſet off the Semi-tangent of the Sum and Difference of the diſtances of the Small Circle from its Pole, and of the Pole or given Point, from the Center of the Projection, on the ſame Side of the Center of the Primitive Circle that the Pole is of in the projected Axis, from c to r and q, and you will have the Extream Points r and q, of the Diameter cq, of the Small Circle to be drawn. But,

If the diſtance of the Small Circle from its Pole be greater than the diſtance of the Pole, or given Point from the Center of the Projection.

Then ſet off the Semi-tangent of the Sum of the Small Circle's diſtance from its Pole, and the diſtance of the two Poles in the projected Axis, from c the Center of the Primitive Circle, on the ſame ſide as the Pole is of, to t; and you have one Interſection, and the Semi-tangent of the difference between the diſtance of the Small Circle from its Pole, and the diſtance between the two Poles ſet off from the Center c, on the contrary Side to z, will give the other Interſection of the Diameter zt, of the Circle to be drawn, the middle Point of which will be the Center

Caſe 4.

If the diſtance of the Small Circle to be drawn from its Pole p, be equal to the Supplement of the diſtance of the Point p from the Center of the Projection c, then in this Caſe, the Small Circle paſſes thro' the Eye Point, and will be repreſented by a ſtreight Line, as fwh (by *Cor. the 5th, of the General Difinition of the former Section*) at Right-angles to the projected Axis, at the diſtance of cw, from the Center of the Primitive Circle, equal to the Tangent of the Complement of the diſtance of p the given Point, from c the Center, and conſequently it interſects the projected Axis in the Center of the Great Circle that paſſes thro' the given Point p.

Whence it is obvious, that all Small Circles whoſe diſtance from the Pole p, is greater than the Supplement of the diſtance of the Point p from the Center c, or Pole of the Primitive Circle, will have their Centers on the contrary Side of the Small Circle swk, that the Point is of, and will revolve about the Exterior Pole.

Problem 5.

To find the Pole of any projected Small Circle.

General Solution. Find the Poles of the Great Circles to which the Small Circle is parallel, and you have the Poles of the Small Circle required. But,

If the Great Circle to which the Small Circle is parallel, be not given,

Then (*See the preceding Figure*) Thro' a the Pole of the Small Circle's projected Axis, and the two Extremitys q and r, and z and t, of the Small Circle's projected Diameters qr or zt, draw the Lines aqn, and aro, or azk, and alt, and biſect the Arches no or kl, in the Point m, and draw the Line apm, where this interſects the projected Axis in p, will give the Pole of the Small Circle qr, or zt, required to be found.

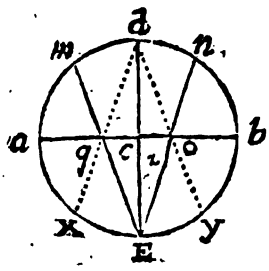
Problem 6.

From any given Point in a projected Great Circle, to cut off an Arch equal in quantity to any given Arch, or which is the ſame thing, to lay off any Number of Degrees in a given Great Circle, from a given Point.

General Solution. 1. Thro' the Pole of the given Great Circle, and the given Point, draw a Right Line to interſect the Primitive Circle.

2. From this Interſection ſet off the Number of Degrees in the Primitive Circle, on that Side as the Caſe requires.

3. From the Point laſt found, to the Pole of the given Great Circle, draw a Right Line, where this Right Line interſects the given projected Great Circle, will give the other Extremity of the Arch to be cut off, and the Portion of the Great Circle contained between the Point firſt given and the Point laſt found, will contain the given Number of Degrees.



Caſe 1.

If the given Great Circle be the Primitive Circle, ſet off the Number of Degrees in the Periphery, from *n* the given Point to *m*, and the thing is done; and *n m* is the Arch required.

Caſe 2.

If the given Great Circle be a Right Circle as *a c b*, and the given Point be *o*.

Solution.

1. From *E* or *d*, the Poles of the given Great Circle *a c b*, thro' the given Point *o*, draw the Right Lines *E o n*, or *d o y*, to interſect the Periphery in *n* or *y*.

2. From the Interſection *n* or *y*, ſet off the Number of Degrees the Arch muſt contain, from *n* to *m*, or from *y* to *x*, and draw the Lines *E m* or *d x*, where either of theſe interſect the given Great Circle as in *q*, will give the other Extremity of the Arch *q o*, equal in Quantity to the given Arch, or that ſhall contain the ſame Number of Degrees.

If the Point given were the Center of the Primitive Circle as *c*, ſet off the Semi-tangent of the Arch from *c* to *o*, or from *c* to *q*, according as the Caſe directs; and the Points *o* or *q*, will give the other Extremity of the Arch to be found, for *c o* is the Semi-tangent of the Arch *d n*.

If the given Point were *o*, and the diſtance of the Point *o* from the Center *c* were given, and the length of the given Arch greater than the Arch *c o*. In this Caſe ſet off the Semi-tangent of the Exceſs from *c* to *q* on the contrary Side of the Center, and the Point *q* will be the other Extremity of the Arch *q o* to be cut off.

But if the Length of the given Arch be greater than the diſtance of the Point *o* from the Center of the Projection *c*, in this Caſe ſet off the Semi-tangent of the Exceſs of the Arch *c o*, above the given Arch *z o*, from *c* to *z*, on the ſame Side of the Center *c*, and the Point *z* will be the other Extremity of the Arch *z o*, to be laid off.

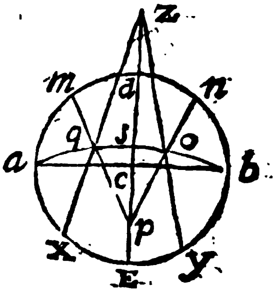
Caſe 3.

Caſe 3.

IF the given Great Circle be an Oblique Circle, as *a s b*, and the given Point be *o*.

Solution. 1. From *p* or *z* the Poles of the given Great Circle *a s b*, thro' the given Point *o*, draw the Right Lines *p o n*, or *z o y*, to interſect the Periphery in *n* or *y*.

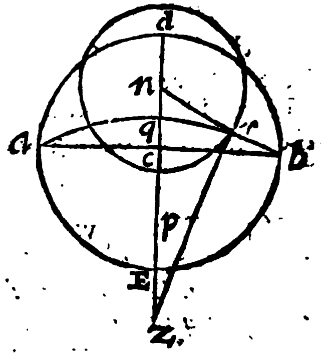
2. From the Interſections *n* or *y*, ſet off the Number of Degrees the Arch muſt contain, from *n* to *m*, or from *y* to *x*, and draw the Lines *p q m*, or *z q x*, where either of theſe interſect the given Great Circle as in *q*, will give the other Extremity *q*, of the Arch *q o*, equal in Quantity to the given Arch, or that ſhall contain the Number of Degrees required.



Hence and from *Cor. 4th*, of *Prop. the 2d* of the former *Section*, we are taught a very ready way of deſcribing a Parallel or Small Circle about any Point *q* as a Pole.

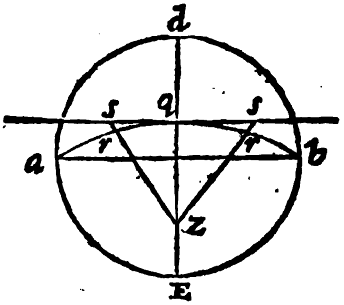
1. For Having drawn a Great Circle *a q b*, thro' the given Point *q*, ſet off the diſtance of the Small Circle from its Pole from *q* to *r*.

2. From *z* the Center of the Great Circle *a q b* to *r*, draw the Right Line *z r*, and at Right-angles to it *n r*, then will *n* be the Center, about which if a Small Circle be deſcribed at the diſtance of *n r*, the thing is done.



This Method is very uſeful in the drawing of Parallels in any *Stereographic* Projection of the Sphere, about any Pole, when the Pole falls within the Plane of the Projection.

For



For Having drawn the infinite Line sqs a Tangent to the Great Circle aqb, in the Point q, lay off the Degrees of diſtance of the Small Circle from its Pole q, in the Great Circle from q to r, by the former *Problem*, this will give one Interſection of the Small Circle to be drawn; and if from z the Center of the Great Circle aqb, thro' the Point r the Right Line zrs be drawn, it will interſect the Tangent sqs in the Point s, at the diſtance of qs from the Pole q, the Semi-diameter or Radius of the ſame Circle.

Problem 7.

To Meaſure any Part or Portion of a projected Great Circle.

General Solution. Find the Pole of the given Great Circle, and draw a couple of Right Lines from the Pole thro' the two Extremities of the Arch to be meaſured, and the Arch cut off in the Primitive Circle, will give its Meaſure.

Caſe 1.

If the Circle given be the Primitive Circle, then the Center is the Pole, and any Arch of the Circumference as nm, applied to the Line of Chords, will give the number of Degrees it contains: See the Figures belonging to the former *Problem*.

Caſe 2.

If the Circle given be a Right Circle as acb, and the Arch to be meaſured be qo.

Solution. Thro' E or d, the Poles of the Circle acb, and the Extremities q and o of the Arch qo, draw a couple of Right Lines as Eon and Eqm, or doy and dqx, and the Arch mn or xy of the Primitive Circle, cut off by the two Lines, is equal to its Meaſure, and applied to the Line of Chords will give the number of Degrees it contains. Or,

The Sum of the Arches co and cq, meaſured upon the Line of half Tangents, will give the Quantity of the Arch qo.

Caſe

Caſe 3.

If the Circle given be an Oblique Circle as $a s b$, and the Arch to be meaſured be $q o$.

Solution. Thro' p or z the Poles of the Circle $a s b$, and the Extremities q and o of the Arch $q o$ to be meaſured; draw a couple of Right Lines as $p o n$ and $p q m$, or $z o y$ and $z q x$, and the Arch $m n$, or $x y$, of the Primitive Circle, cut off by the two Right Lines, is equal in quantity to the given Arch $q o$; and being applied to the Line of Chords, will give the number of Degrees it contains.

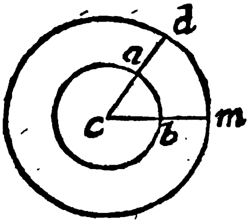
There are other ways for meaſuring or laying down an Arch of a projected Great Circle, but theſe that I have made Uſe of are the readieſt for Practice, and the propereſt for my preſent Buſineſs.

Problem 8.

To lay down any Number of Degrees on any projected Small Circle, from any given Point, and on the contrary, to meaſure any Part or Portion of a Small Circle when projected.

Caſe 1.

If the Small Circle given be parallel to the Primitive Circle, and the given Point be a, from whence the Degrees are to be laid off.



Solution. Thro' c the Center of the Primitive Circle, and a the given Point, draw the Line $c a d$, to cut the Primitive Circle in d, from whence lay off the Number of Degrees to m, and draw the Line $c m$, where this cuts the Small Circle as in b, will give the other Extremity of the Arch $a b$, which ſhall contain the Number of Degrees required.

On the contrary, by drawing two Lines from c thro' a and b, the Extremities of any Part or Portion of the Small Circle $a b$, we ſhall have an Arch $d m$, cut off in the Primitive Circle, equal in Value to the given Arch $a b$, and which applied to the Line of Chords will give the Number of Degrees it does contain.

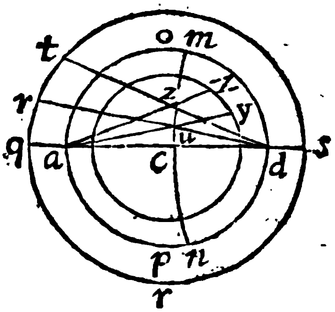
If the Radius $c b$ be applied to the Line of Chords upon the Sector, the Arch $a b$ applied to the ſame Line of Chords, will ſhew the Number of Degrees it contains.

On the contrary, the Number of Degrees the Arch is to contain, taken off from the ſame Sector, and ſet off in the Periphery of the Small Circle, from *a* the given Point to *b*, will give the other Extremity of the Arch *a b*, that ſhall contain the given Number of Degrees.

Caſe 2.

If the Small Circle given *m u c n*, be parallel to a Right Circle and the given Point be *u*,

1. Conſider the diſtance of the Small Circle *m z u n* from its Pole *d*, and deſcribe a Small Circle *q r t s*, at the diſtance of its Supplement from the Point *c*.



2. From *d* the Pole, thro' *u* the given Point, draw the Right Line *d u t*, and ſet off the Number of Degrees the Arch is to contain from *r* to *t*, and draw the Line *t z d*, where this interſects the Small Circle, *m n* in *z*, will give the other Extremity of the Arch *u z*, that ſhall contain the given Number of Degrees.

Or,

Having drawn the Small Circle *x y*, at the ſame diſtance from *c* as the given Small Circle *m n* is, diſtant from its Pole *d*.

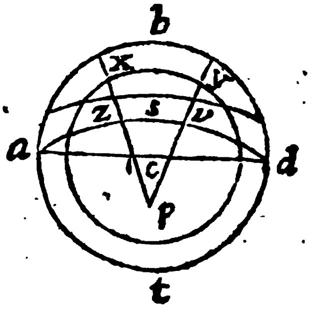
Thro' *a* the other Pole of the given Small Circle and the given Point *u*, draw the Right Line *a u y*, and ſet off the Number of the Degrees the Arch is to contain, in the Periphery of the Small Circle laſt drawn from *y* to *x*, and draw *a x*, which will interſect the Small Arch *m n* in *z*, the other Extremity of the Arch *u z* required, that ſhall contain the Number of Degrees given.

On the contrary, by drawing a couple of Right Lines from *d*, the Pole of the Small Circle *m n*, thro' the Extremities *z* and *u* of any Arch *z u*, they will cut off an Arch *r t*, equal in quantity to the Arch *z u* in that Small Circle *q r t*, whoſe diſtance from the Center *c*, is equal to the Supplement of the diſtance of the Point *d* from the Small Circle given. Or,

By drawing a couple of Lines from *a*, the remoteſt Pole of the Small Circle *m n*, thro' the Extremities *z* and *u* of any Arch *z u* of any Arch *z u*, they will cut off an Arch *x y*, equal in quantity to the Arch *z u* in that Small Circle *x y*, which is as far diſtant from the Center *c* of the Primitive Circle, as the neareſt Pole *d* is diſtant from the Small Circle *m u*.

Caſe 3.

If the Small Circle given, be parallel to an Oblique Circle as *a s d* and the given Point be *u*.



1. As in the former *Caſe*, ſo in this conſider the diſtance of the Small Circle given from either of its Poles, and deſcribe a Small Circle as *x y*, as far diſtant from the under Pole of the projected Circle.

2. Thro' *p* and the given Point *u*, draw the Right Line *p y u* to cut the Periphery of the Small Circle *x y* in *y*, and ſet off the Degrees the Arch is to contain from *y* to *x*, and draw the Line *p z x*, where this cuts the Small Circle *z u* in *z*, it will give the other Extremity *z* of the Arch *z u*, that ſhall contain the Number of Degrees given.

On the contrary, by drawing Lines from *p* the Pole of the Small Circle *z u*, thro' the two Extremities *z* and *u* of any Arch *z u*, they will cut off an Arch *x y*, equal in Value to the given Arch *z u*, in that Small Circle which is drawn about the Center *c* as a Pole, and as far diſtant from it as is the Supplement of the diſtance of the given Small Circle *x y*, from its Pole *p*, Or

Univerſally, thro' the two given Poles of the given Small Circle and the given Point, draw a Great Circle; and ſet off the Number of Degrees from the Interſection of this Great Circle, with the Great Circle to which the given Small Circle is parallel in the given Great Circle; a Great Circle ſtruck thro' the given Poles and the Point laſt found, will cut off an Arch in the Small Circle, equal in Quantity to the given Arch, or which ſhall contain the given Number of Degrees.

On the contrary, by drawing two Great Circles thro' the Poles of the given Small Circle and the Extremities of the given Arch, they will cut off an Arch in that Great Circle to which the given

Small Circle is parallel, equal in Value to the given Arch of the given Small Circle.

Hence we are taught, How to divide any given Parallel or Small Circle, in any given Ratio; or how to meaſure any Part or Portion of it when projected.

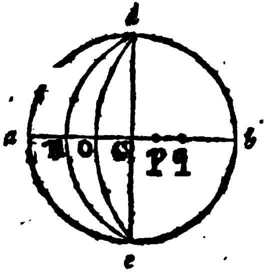
Problem 9.

Thro' a given Point to draw a Great Circle, that ſhall form a given Angle with a given Great Circle.

This *Problem* being of great Uſe in the Conſtructing of *Spherical Triangles* and *Aſtronomical Problems*, I ſhall exemplify it thro' all the Varieties, and that it may be the better underſtood, it is neceſſary to explain what is meant by a *Spherical Angle*.

A Spherical Angle *is formed by the meeting together or Interſection of two Great Circles upon the Surface of the Sphere, and is meaſured by an Arch of a Great Circle, at a Quadrantal diſtance from the Angular Point; wherefore, if about any Angular Point as a Pole, a Great Circle be deſcribed, the Arch of that Great Circle intercepted or contained between the two Circles which conſtitute the Angle, is the meaſure thereof.*

In the adjacent Figure, let *d a e b* repreſent a Great Circle, *d n e*, *d o e*, two other Great Circles, interſecting in the Points *d* and *e*, and forming with it two Spherical Angles *a d n* and *a d o*, and between themſelves the Spherical Angle *n d o*, it is manifeſt that the Arches *a n*, *a o*, and *n o*, of the Great Circle *a c b*, deſcribed about the Angular Point *d* as a Pole, and intercepted between the Arches *a d*, *n d*, and *o d*, is the Meaſure of the Angles *a d n*, *a d o*, and *n d o*.



Let *p* and *q* repreſent the Poles of the Circles *d n e* and *d o e*, wherefore, becauſe *c a* is equal to *p n*, and the Arch *c n* common, *c p* will be equal to *a n*, and for the ſame reaſon *c q* will be equal to *a o*, and *p q* will be equal to *n o*. Whence it follows,

That the Arch of the Great Circle deſcribed about the Angular Point as a Pole, and intercepted between the Poles of the two Circles that conſtitute the Angle, is equal to the Meaſure or Quantity of the contained Angle.

Hence the Pole of one Circle being given, we are taught how to find the Pole of another Circle, that ſhall form any Angle with the given Great Circle, *viz.* by laying off the Number of Degrees the Angle muſt contain, (in that Great Circle which is at a Quadrantal diſtance from the Angular Point) from the given Pole, and the Point laſt found will be the Pole of the Circle to be drawn.

Hence we are taught a general Solution of the former *Problem*, as follows,

About the Point given as a Pole, deſcribe a Great Circle, and if the Point given be in the Great Circle with which the Angle is to be formed,

Set off the Number of Degrees the Angle muſt contain in that Great Circle, from the Pole of the given Circle, and the Point laſt found will be the Pole of the Circle to be drawn. But

If the given Point be not in the Great Circle with which the Angle is to be formed,

About the Pole of the given Circle, deſcribe a Small Circle at the diſtance of the Quantity of the given Angle, where this parallel interſects the Great Circle (drawn about the Angular Point as a Pole) it will give the Pole of the Circle to be drawn.

Caſe 1.

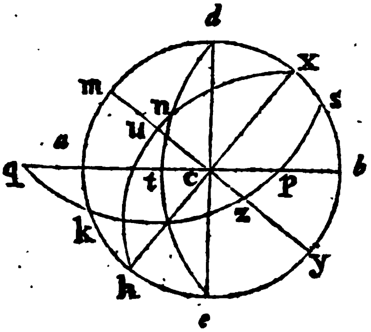
If the given Great Circle be a Right Circle as *a c b*, and the given Point be the Center of the Primitive Circle as *c*, and the Angle to be Conſtructed be *a c m*.

Set off the Quantity of the Angle or Number of Degrees it muſt contain, in the Primitive Circle, from *d* the Pole of the Right Circle to *x*, and about *x* as a Pole deſcribe the Right Circle *m c y*, then will *m c a* be the Angle to be formed. Or,

The Angle *m c a* may be Conſtructed as a Plane Angle, by ſetting off the Degrees it muſt contain from *a* to *m*, and drawing the Diameter or Right Circle *m c y*.

Caſe 2.

If the given Great Circle be a Right Circle as *a c b*, and the given Point be *p*, taken in the Right Circle, and the Angle to be formed be *x p b*.



About the given Point *p*, deſcribe a Great Circle *d n e*, and ſet off the Number of Degrees the Angle muſt contain, from *d* the Pole of the Right Circle to *n*, then will *n* be the Pole of the Circle to be drawn, about which Point *n*, if the Great Circle *s p k* be deſcribed the thing is done; and *s p b* is the Angle to be Conſtructed.

Caſe 3.

If the given Circle be the Primitive Circle *a d b e*, the given Point be *d*, taken in the Primitive Circle, and the Angle to be Conſtructed be *m d n*.

Set off the Number of Degrees the Angle muſt contain, in the Right Circle *a c b*, (which is the Great Circle deſcribed about the Point *d*) from *c* to *p*, about which Point *p*, if a Great Circle as *d n e* be drawn the thing is done, and *m d n* is the Angle to be Conſtructed.

Or the Quantity of the Angle might have been ſet off in the Great Circle *a c b* from *a* to *t*, and then a Great Circle drawn thro' the Points *d*, *t*, and *e*, would have formed the Angle *m d n*.

If the Angle to be Conſtructed be an Obtuſe Angle as *x d n*, the Quantity of the Angle ſet off from *c* to *q*, will give the Exterior Pole *q* of the Circle *d t e* to be drawn, and this being the ſame Great Circle as is deſcribed about the Point *p*, whoſe diſtance from the Center *c*, is equal to the Angle *m d n*, the Supplement of the Angle *n d x*, it comes to the ſame thing whether you make Uſe of the Supplement *m d n* as in the former *Example*, or of the Angle *n d x* as in this, but the former being moſt apt for Practice is generally made Uſe of.

Caſe 4.

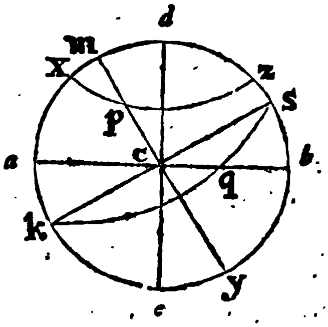
If the given Great Circle be an Oblique Circle as *d n e*, and the given Point *n* be taken in the Oblique Circle, and the Angle to be Conſtructed be *d n x*.

Having deſcribed the Great Circle *s p z k* about the given Point *n*, as a Pole, ſet off the Quantity of the Angle *d n x* from *p* to *z*, about which Point *z* as a Pole, if a Great Circle as *x n u b* be drawn, the thing is done; and *d n x* is the Angle to be Conſtructed

Caſe 5.

If the given Great Circle be a Right Circle as *a c b*, and the given Point *s* be taken without it, and the Angle to be formed be *s q b*.

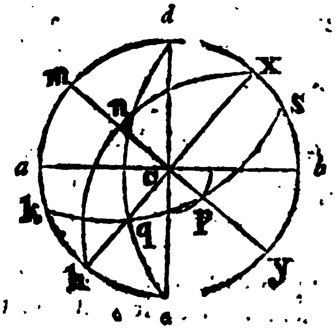
About the given Point *s* as a Pole, deſcribe the Great Circle *m c y*, and about the Point *d* the Pole of the given Circle *a c b*, deſcribe a Parallel or Small Circle *x p z*, at the diſtance of *d x*, the Quantity of the given Angle, where theſe two Circles interſect each other, as in the Point *p*, it will give the Pole of the Circle to be drawn, about which if a Great Circle as *s q k* be drawn, the thing is done; and *s q b* is the Angle to be Conſtructed.



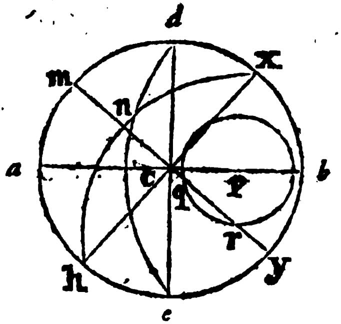
Caſe 6.

If the given Great Circle be the Primitive Circle, and the given Point be *n*, taken at pleaſure within the Projection, and the Angle to be formed be *d x n*.

About the given Point *n* as a Pole, deſcribe the Great Circle *k p s*, and about *c* the Pole of the Primitive Circle, deſcribe a Small Circle *q p*, at the diſtance of *c q*, equal to the Quantity of the Angle given, where theſe two Circles interſect as in *p*, is the Pole of the Circle to be drawn, about which Point, if the Great Circle *b n x* be deſcribed, the thing is done; and *d x n* is the Angle to be Conſtructed.



Caſe 7.



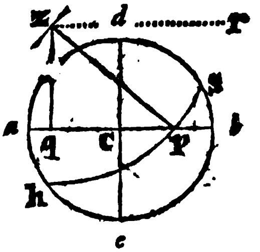
If the given Great Circle be an Oblique Circle *d n e*, and the given Point be taken without the Circle as at *x*, and the Angle to be Conſtructed be *x n d*.

About *x* the given Point, draw the Great Circle *m c y*, and about *p* the Pole of the given Circle *d n e*, deſcribe the Small Circle *q r*, where this interſects the Great Circle before deſcribed as in *q*, it will give the Pole of the Circle to be drawn; about which Pole, if the Great Circle *x n h* be drawn, the thing is done; and *d n x* is the Angle to be Conſtructed.

If in either of theſe laſt Caſes, the Small Circle interſects the Great Circle in two Points as *q* and *r*, either of theſe Points may be taken for the Pole of the Circle to be drawn. If it only touch the Great Circle the *Problem* is limited; but if it neither cut nor touch the Great Circle the *Problem* is impoſſible.

Each of theſe Examples have been adapted as near as poſſible, to the uſual way of projecting *Spherical Triangles*, and the Conſtructing of *Aſtronomical Problems* upon the Plane of the *Meridian*.

By the help of the 2*d Prop.* of the former *Section* and its *Corollarys*, we are taught a direct Method for finding the Centers of the Circles to be drawn in any of the former Examples.



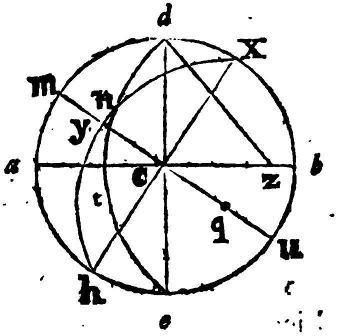
Thus in *Caſe* the 2*d*, where the given Great Circle is a Right Circle as *a c b*, the given Point is *p*, and the Angle to be formed *s p b*.

1. Thro' *q* the Center of the Great Circle paſſing thro' the Points *d*, *p*, and *e*, draw the Line *q z*, perpendicular to the Right Circle *a c b*, and make the Angle *q p z* equal to the Complement of the given Angle *s p b*, where the Lines *q z* and *p z* interſect as in *z*, it will give the Center *z*, of the Circle *s p h* to be drawn. Or,

2. Having made *cq* equal (to the Tangent of the Arch *pb*) to the Radius *cb*, ſet off the Tangent of the Complement of the Angle *spb* to the Radius *pq*, from *q* to *z*, then will *z* be the Center of the Circle to be drawn. Or,

3. With the Secant of the Complement of the Angle *spb*, between the Points of the Compaſſes, ſetting one foot in *p* the given Point, with the other croſs the Line *qz* in *z*, then will *z* be the Center, and *zp* the Radius of the Circle to be drawn. Or,

4. With the Tangent of the Complement of the given Angle *spb*, between the Points of the Compaſſes, ſetting one foot in *q*, deſcribe a ſmall Arch at *z*, or (if the Point *q* be not given) at the ſame diſtance draw *zr* parallel to *acb*, and with the Secant of the Complement of the ſame diſtance between the Points of the Compaſſes, ſetting one foot in *p*, with the other foot croſs the former Arch or parallel Line *zr* in the Point *z*, then will *z* be the Center, and *zp* the Radius of the Circle to be drawn.



Again in the 3*d Caſe*, where the given Great Circle is the Primitive Circle, the given Point *d*, and the Angle to be formed is *adt*.

1. From the given Point *d* draw *dz*, forming an Angle with the Diameter *dce*, equal to the given Angle *adt*, where this Line interſects the projected Axis *acb* in *z*, it gives *z* the Center of the Circle *dte* to be drawn, whoſe Radius or Semidiameter is *dz*. Or,

2. The Tangent of the given Angle *adt*, to the Radius *cd*, ſet off from *c* to *z*, will give the Center of the Circle to be drawn.

Or,

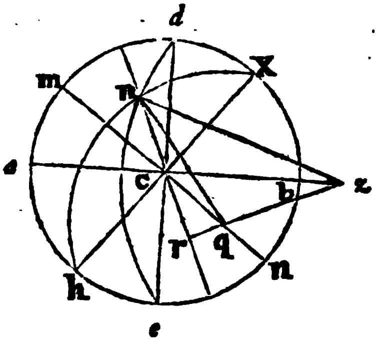
3. The Secant of the ſame Angle ſet off from *d* to *z*, will give *z* the Center of the Circle to be drawn, of which the Radius or Semidiameter is *dz*. Or,

4. With the Tangent of the Angle given, ſetting one foot of the Compaſſes in *c*, ſweep a ſmall Arch at *z*, with the Secant of the ſame diſtance, one foot of the Compaſſes being placed in *d*, croſs the former Arch in *z*, the Center of the Circle to be drawn, of which the Semidiameter will be the diſtance *dz*.

Likewiſe in *Caſe* the *6th*, where the given Great Circle is the Primitive Circle, the given Point is *n*, and the Angle to be projected is *d x n* (ſee the former Figure.)

1. With the Tangent of the given Angle *d x n*, to the Radius *c b*, ſetting one foot of the Compaſſes in *c* the Center of the Projection, with the other foot deſcribe a ſmall Arch at *q*, with the Secant of the ſame diſtance, ſetting one foot in *n* the given Point, croſs the former Arch in the Point *q*, then will *q* be the Center of the Circle *b n x* to be drawn, and the Secant of the Angle or diſtance *n q*, the Radius or Semidiameter. Or,

2. Make the Angle *e d z* equal to the given Angle *a d t*, then will *c z* be equal to *c q*, the diſtance of the Circle to be found, from the Center of the Primitive Circle, and *d z* the Radius or Semidiameter of the Circle to be drawn.



Again, in *Caſe* the *4th*, where the given Great Circle is an Oblique Circle, as *d n e*, the given Point is *n*, and the Angle to be formed is *d n x*.

From *n* the given Point, to *z* the Center of the given Circle, draw the Line *n z*, and let *z r* be perpendicular to the Line *n c r*, drawn from *n* the given Point, thro' *c* the Center of the Primitive Circle.

Make the Angle *z n q* equal to the given Angle *d n x*, where the Line *n q* cuts *z r* before drawn, it gives *q* the Center of the Circle *x n b* to be drawn, of which *n q* is the Radius or Semi-diameter.

Problem 10.

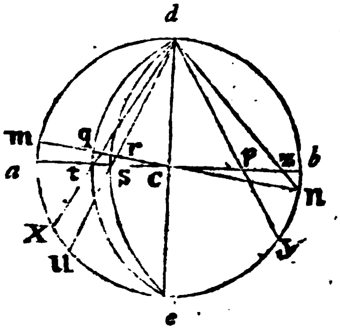
To meaſure or find the Quantity of any Spherical Angle when projected.

It has been ſhewn in the former *Problem*, that if the two Sides or Arches that contain any Spherical Angle, be produced till they are Quadrants, that then the Arch of the Great Circle (whoſe Pole is the Angular Point) contained or intercepted between them is the Meaſure of the contained Angle; and this being always equal to the Arch of the ſame Great Circle, contained between the Poles of the two

Great Circles which conſtitute the Angle. Hence, and from the 7th *Problem* we are taught, how to find the Quantity of any Spherical Angle, *viz.* by laying a Ruler over the angular Point, and the two Extremities of the Quadrantal Arch, contained or intercepted between them: Or by laying a Ruler over the angular Point, and the two Poles of the two Circles that conſtitute the Angle, the Arch (in either Caſe) cut off in the Primitive Circle, is equal in quantity to the contained given Angle, and being applied to the *Line of Chords* will give the Number of Degrees it contains; as ſhall be illuſtrated in its ſeveral Varieties.

Caſe 1.

If the angular Point be in the Center of the Primitive Circle at *c*, then the Arches *m c*, *c a*, that conſtitute the Spherical Angle *mc a*, are ſtreight Lines, the Great Circle deſcribed about the angular Point *c*, is the Primitive Circle *a d b e*, and the Arch *m a* of the Primitive Circle intercepted between the Quadrantal Arches *m c*, *c a*, is the Meaſure of the Angle *m c a*, and being applied to a *Line of Chords* whoſe Radius is *c a*, will give the Number of Degrees it contains.



Caſe 2.

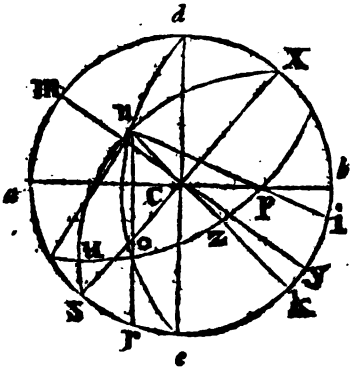
If the angular Point be in the Periphery of the Primitive Circle at *d*, then the Arches *d m*, *d q*, that conſtitute the Spherical Angle *m d q*, being produced to *a* and *t*, till *d a* and *d t* are Quadrants, will cut off the Arch *a t* of the Great Circle *a c b*, (of which the angular Point *d* is the Pole) the Meaſure of the Angle *a d t*, and therefore (by *Prob.* the 7*th*) a Ruler laid over the angular Point *d*, and the Extremities *a* and *t* of the Arch *a t*, will cut off an Arch *a x* of the Primitive Circle, equal in quantity to the given Angle *a d t*, and which applied to the *Line of Chords*, will give the Number of Degrees it does contain.

And becauſe the Arch *cp* of the ſame Great Circle *acb*, intercepted between the Poles *c* and *p*, of the Arches *md*, *dq*, that conſtitute the Angle *mdq*, is equal to the Arch *at*, the Meaſure of the ſame Angle: If a Ruler be laid over the angular Point *d*, and the Poles *c* and *d*, or a couple of Lines be drawn from the Point *d* thro' the Pole *c* and *d*, they will cut off an Arch *ey* in the Primitive Circle, equal to the Arch *ax*, equal in quantity to the given Angle *mdq*.

Again, becauſe the Great Circle *acb*, deſcribed about the angular Point *d* as a Pole, is a Right Circle; therefore the Arch *cp* contained between the Poles *c* and *p*, of the conſtituent Arches *md*, and *dq*, applied to the *Line of Semi-tangents* will give the quantity of the given Angle *mdq*: Or, the Arch *ct* applied to the ſame Line of *Semi-tangents*, will give the Complement of the ſame Angle *mdq* to be meaſured.

Again, it is manifeſt, that the Arch *rs* of the Great Circle *acb*, contained between the Quadrantal Arches *dt*, *ds*, that form the Angle *qdr*, equal to the Arch *pz* of the ſame Great Circle *acb*, intercepted between the Poles *p* and *z*, of the Conſtituent Arches *ds*, *dt*, is equal in quantity to the given Angle *qdr*; therefore, Lines *dx*, *du*, *dy*, *dn*, drawn from the Point *d*, thro' *t* and *s*, *p* and *z*, the Extremities of the Arches *ts* and *pz*, will cut off Arches *xn* and *yn*, in the Primitive Circle, equal in quantity to the Angle *qdr* to be meaſured.

Caſe 3.



If the angular Point be within the Primitive Circle at *n*, about the Point *n* as a Pole, Let the Great Circle *uozp* be drawn;

Then becauſe the Arch *uo* equal to *zp*, is the Meaſure of the Angle *uno* equal to the Angle *dnx*, if Lines *nus*, *nor*, and *nzk*, and *npi* be drawn, from the angular Point *n*, thro' the Extremities *u* and *o*, *z* and *p*, of the Arches *uo* and *zp*, they will cut off the Arches *sr* and *ki* in the Primitive

Circle, equal in quantity to the Angle *d n x*; which arches applied to the *Line of Chords*, will give the Number of Degrees it does contain.

And becauſe the Great Circle *u o z p*, deſcribed about the angular Point *u* as a Pole, will always paſs thro' the Poles *z* and *p*, of the Conſtituent Circles *x n*, *d n*, if the Great Circle *u o z p* be not drawn, yet if Lines *n i*, *n k*, be drawn thro' the angular Point *n*, and the Poles *z* and *p* of the Circles *x n*, *d n*, that conſtitute the Angle *d n x*, the Arch *k i* cut off in the Primitive Circle, is equal in Value to the Angle *d n x* to be meaſured; and being applied to the *Line of Chords*, it will give the Number of Degrees it does contain; and this is the uſual Way of meaſuring Angles that fall within the Primitive Circle.

Problem 11.

Thro' a given Point, to draw a Great Circle that ſhall be Perpendicular to a given Great Circle; or which is the ſame thing, that ſhall cut the given Great Circle at Right-angles.

This *Problem* is only one particular *Caſe* of the *9th Problem*, where the Angle to be projected is a Right Angle, but becauſe it frequently occurs in the projecting of *Right-angled Spherical Triangles*, and is of Uſe in the Solution of *Oblique-angled Spheric Triangles*, I ſhall give it here a Place by its ſelf, and ſhall Exemplify it thro' the various Poſitions of the given Point, with regard to the given Great Circle. And,

Becauſe the Pole of any Great Circle is diſtant from that Circle by the Arch of 90 Degrees or a Quadrant, meaſured in a Great Circle paſſing thro' the given Pole and the given Great Circle. Hence it is,

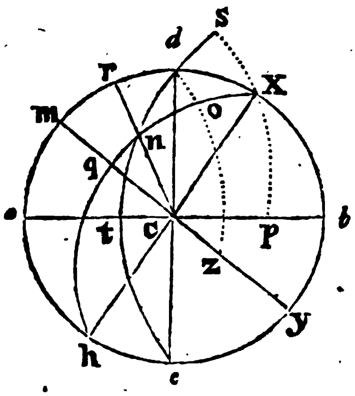
1. That if one Great Circle paſs thro' the Poles of another Great Circle, it will cut that Great Circle at Right-angles; and on the contrary.
2. That if one Great Circle cut another Great Circle at Right-angles, it will paſs thro' that Circle's Poles, and hence we learn a general Solution of this *Problem*, viz. Thro' the given Point and the Poles of the given Great Circle, draw a Great Circle, and the thing is done.

Caſe 1.

If the given Great Circle be the Primitive Circle *a d b e*, and the given Point be *r* or *n*.

Thro' the given Point *r* or *n* and the Pole *c* of the Primitive Circle, draw the Right Circle *c n r* and the thing is done; and *n r* is the Perpendicular required to be drawn.

Caſe 2.



1. If the given great Great Circle be a Right Circle as *a c b*, and the given Point be *c* the Center of the Primitive Circle.

Thro' the Poles *d* and *e* of the given Right Circle, and the given Point *c*, draw the Right Circle *d c e* and the thing is done; and *d c e* is the Perpendicular required to be drawn.

2. If the given Great Circle be a Right Circle as *a c b*, and the given Point be *t* or *n*.

Thro' *d* and *e* the Poles of the given Great Circle, and the given Point *t* or *n*, draw the Oblique Circle *d n t e* and the thing is done; and *n t* is the Perpendicular required to be drawn.

Caſe 3.

If the given Great Circle be an Oblique Circle as *x q b*, and the given Point be *d* or *o*.

Thro' the given Point *d* or *o* and the Pole *z* of the given Great Circle *x q b*, draw the Arch of the Great Circle *d o z*, then will *d o* be Perpendicular to the Oblique Circle given *x q b*.

Again, If the given Point be *x*, and the given Great Circle be *e t d*.

Thro' *p* the Pole of the given Great Circle, and *x* the given Point, draw the Great Circle *p x s*, till it meet the Great Circle *e t d* produced in *s*, then will *x s* be a Perpendicular to the Great Circle *e t d* drawn from the Point *x*.

Problem 12.

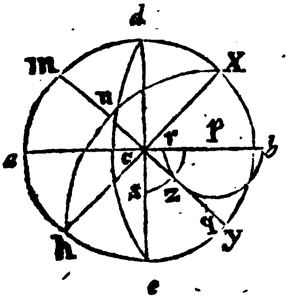
Two Great Circles given, by Poſition forming with each other a given Angle, let it be required to draw another Great Circle that ſhall form two Angles with the two given Great Circles, equal to two given Angles, each to each reſpectively, provided that the three given Angles be greater than a Semicircle.

It is a known Property of a *Sphepical Triangle* (and will be demonſtrated in the next *Section Page* 49, and which the *Reader* ought to be acquainted with before hand) that if the three nearer Poles of the Sides of any Spherical Triangle be connected together by Arches of Great Circles, they will with thoſe Arches conſtitute a Spherical Triangle, whoſe Sides will be equal to the Angles of the Triangle firſt given, except that the greateſt Side will be the Supplement of the greateſt Angle.

And hence we are taught a ready Solution of the *Problem* before us, *viz. by drawing a Parallel about each of the Poles of the two given Great Circles, at the diſtance of the Quantity of the Angle, that the Great Circle to be drawn muſt form with the reſpective Great Circle, whoſe Pole is taken, and where theſe two Parallels interſect each other, it will give the Pole of the Circle to be drawn.*

But if either or both of the given Angles be Obtuſe, then inſtead of the Angles you muſt make Uſe of their Supplements.

Example.



Let *d b e* and *d n e* be the two given Great Circles, forming with each other the given Angle *n d x*, Let *x n h* be the Circle required to be drawn, forming with the Great Circle *d b e* the Angle *d x n*, and with the Circle *d n e* the Angle *d n x*, each equal to any aſſigned quantity.

Conſtruction. About *p* the Pole of the Circle *d n e*, deſcribe the Small

Circle *r z*, at the diſtance or quantity of the Angle *d n x*, alſo about *c* the Pole of the other Circle *d b e*, deſcribe the Small Circle *s z*, at the diſtance of the quantity of the Angle *d x n*, where theſe two Small Circles interſect each other in the Point *z*, is the Pole of the Circle *x n h* to be drawn, and if the Tangent of the Angle *d x n* be ſet off in the Line *c z* from *c* to *q*, then will *q* be the Center of the ſame Circle *x n h* to be drawn.

Whoſoever is well acquainted with what has been ſaid of Projections in the former *Section*, will readily ſee the Reaſon of every Solution in this *Section*; and 'tis for this Reaſon that I have not referred to every particular *Theorem* or *Corollary* in the *1ſt Section*, upon which the ſeveral Solutions given in this *Section* depend; but having ſhewn a great Variety of ways of deſcribing any Great or Small Circle in any Projection, and of meaſuring any Part or Portion of it when projected, I ſhall proceed in the next *Section*, to conſider the Affections and Properties of the ſeveral Triangles made upon the Surface of the *Globe*, by the mutual Interſections of the Great Circles deſcribed upon it.

Section III.

Of Spherical Trigonometry.

1. As a *Plane Triangle*, the Subject of *Plane Trigonometry* is formed by the mutual Interſection of three ſtriat Lines upon a *Plane Surface*, ſo a *Spherical Triangle* the Subject of *Spherical Trigonometry*, is formed by the mutual Interſection of three Great Circles on the *Surface* of the *Sphere*.

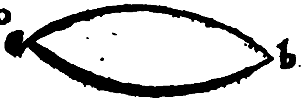
2. So that as each Angle of a Plane Triangle is formed by the meeting together of two Right Lines on a Plane Surface, each Angle of a Spherical Triangle is formed by the meeting together or Interſection of two Great Circles on a Spherical Surface.

3. Again, as a Plane Angle is meaſured by the Arch of a Circle, deſcribed about the angular Point as a Center, and intercepted between the Lines which form the Angle; ſo a Spherical Angle is meaſured by the Arch of a Great Circle, deſcribed about the angular Point, on the Surface of the Sphere, and intercepted between the Circles which conſtitute the Angle.

4. Hence it is, that if the Sides that contain the Spherical Angle are continued till they are Quadrants, then the Arch of the Great Circle (of which the angular Point is the Pole) contained or intercepted between them, is the Meaſure of the Angle.

5. Hence it is, that a Spherical Angle is the ſame with the Inclination of the Planes of the two Great Circles which conſtitute the Angle.

6. Hence it is, that the oppoſite Angles *a* and *b* at the Interſections of any two Great Circles are equal.



And ſince the Arch of a Great Circle is the neareſt diſtance between two Points upon the Surface of the Sphere, as a ſtrait Line is the ſhorteſt Tract between two Points upon a Plane Surface, it will be proved after the ſame manner as in *Plane Triangles.*

7. That one Arch of a Great Circle falling upon another, makes two Angles equal to two Right-angles.

8. That the oppoſite Angles made by the Interſection of two Great Circles are equal.

9. That if one Angle and the two containing Sides of one Spherical Triangle, be equal to one Angle and the two contrary Sides of another Triangle, or if one Angle and the two adjacent Sides of one Triangle, be equal to one Angle and the two adjacent Sides of another Triangle, or if the three Sides of one Triangle be equal to the three Sides of another Triangle, each to each reſpectively, then the remaining parts of one Triangle are equal to the remaining parts of the other Triangle, each to each reſpectively, and the two Triangles are equal.

10. That an *Equicrural Spherical Triangle* hath its two Angles at the Baſe mutually equal; and on the contrary, if the Angles at the Baſe are equal, the oppoſite Sides are equal alſo.

11. Triangles mutually *Equilateral,* are alſo mutually *Equiangular.*

12. That the greateſt Angle of every Triangle is oppoſite to the greateſt Side, and on the contrary, the greateſt Side ſubtends the greateſt Angle.

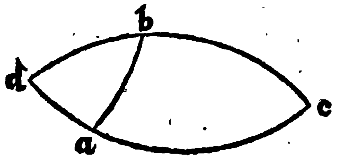
That

13. That any two Sides of a Spherical Triangle taken together are greater than the third that remains.

Again,

Since all Great Circles cut each other into two equal parts, as has been prov'd in the *firſt Section* it follows,

14. That every Side of a Spherical Triangle is leſs than a Semicircle or 180 Degrees.



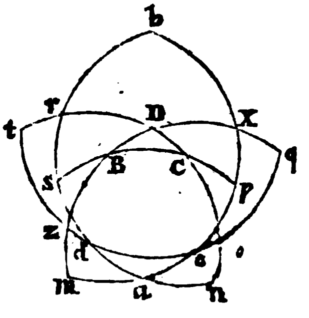
For *d b* one Side of the Spherical Triangle *d b a*, is leſs than the Semicircle *d b c*, and *d a* is leſs than *d a c*.

15. That the Sum of the three Sides of any Spherical Triangle, is leſs than a whole Circle or 360 Degrees.

For *b a* is leſs than *b c* and *a c* taken together, (by the 13*th* preceding *Art.*) and conſequently *b a*, *b d*, and *d a*, &c. taken together are leſs than *d b c* and *d a c* (by the 16*th Geom. Axiom*) leſs than two Semicircles or 360 Degrees.

16. From the 4*th* preceding *Art.* it follows, that the Poles *b*, *d*, *c*, of the Spherical Triangle *BDC* when connected together by Arches of Great Circles, will with thoſe Arches conſtitute a Spherical Triangle *b d c*, whoſe

$\left\{\begin{array}{l}\text{Angles}\\\text{Sides}\end{array}\right\}$ are equal to the Supplement of the $\left\{\begin{array}{l}\text{Sides}\\\text{Angles}\end{array}\right\}$



About the Points *B*, *D*, and *C*, as Poles, deſcribe the Great Circles *b p c a m*, *q o c d t*, and *b s d a n*.

Now *c* being the Pole of *BD*, *c m* is a Quadrant, and equal to *p b*; *b* being the Pole of *B C*, add to each part the interjacent Arch *p c*, and the whole Arch *c b* will be equal to the whole Arch *p m*, the Supplement of *p x*, the Meaſure of the Angle at *B*.

Again, *d* being the Pole of *d c*, *d n* equal to a Quadrant is equal to *s b*, add to each the interjacent Arch *s d*, and it will make *b d* equal to *s n*; the Supplement of the Arch *r s*, the Meaſure of the Angle at *C*.

Laſtly, From the Arch do equal to a Quadrant, equal to the Arch cq, take away the Arch co, and there will remain dc equal to oq, the Meaſure of the Supplement of the Angle at D.

17. Hence it is manifeſt, that the Triangle dac conſtituted between the three nearer Poles d, a, and c, of the Triangle BDC, hath its Sides da, ac, and dc, equal to the Angles C, B, and D, of the Triangle BDC, and its Angles d, a, and c, equal to the Sides DC, DB, and BC, of the ſame Triangle (except that the greateſt Side dc of the Triangle dac, is equal to the Supplement of the greateſt Angle D of the Triangle BDC, and the greateſt Angle at a the Supplement of the greateſt Side BC.)

18. Wherefore, Since the three Angles of any Spherical Triangle being given, the three Sides of another Spherical Triangle are known, whoſe Angles are equal to the Sides of the former Triangle (except that the greateſt Angle of the Triangle given, is equal to the Supplement of the greateſt Side of the Triangle found; and the greateſt Angle of the Triangle found, is equal to the greateſt Angle of the Triangle given) it follows, that the three Angles of any Spherical Triangle being given, the Sides may be eaſily found, and on the contrary, the Sides of a Spherical Triangle being given, the Angles may be eaſily had.

19. Hence and from the 15*th* it follows, that the three Angles of any Spherical Triangle taken together, are greater than two Right-angles, or 180 Degrees.

For by the 15*th*, bd, dc, bc, the Supplements of the Angles C, D, and B, of the Triangle BDC, taken in one Sum is leſs than four Right-angles, that is ſix Right-angles leſſened by the Sum of the Angles C, D, and B, of the Triangle BDC, are leſs than four Right-angles; that is, two Right-angles leſſened by the Sum of the Angles C, D, and B, is leſs than nothing. Wherefore, the Sum of the Angles C, D, and B, is greater than two Right-angles or 180 Degrees.

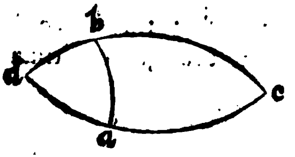
Or thus, da and ac the Meaſures of the Angles C and B, taken together in one Sum, are greater than dc the Supplement of the Angle at D; and ſince dc and its Supplement make but two Right-angles, da and ac and the Supplement of dc ſhall make more than two Right-angles, that is, the Angles B, D, and C, taken together, make more than two Right-angles, or 180 Degrees.

Or thus, cq (the Supplement of dc) da and ac the Meaſures of the Angles *D*, *C*, and *B*, of the Triangle *BDC* together with the Sides of the Triangle *d b c*, make three Semicircles or 540 Degrees, but the Sum of the Sides of the Triangle *d b c* is leſs than two Semicircles or 360 Degrees, (by the 15*th Ar.*) therefore the Meaſures of the Angles *D*, *C*, and *B*, are greater than two Right-angles; that is, the Sum of the Angles of the Triangle *BDC* is greater than two Right-angles or 180 Degrees.

20. And becauſe (by the 7*th Ar.*) the Sum of the Internal and External Angles of any Spherical Triangle is equal to ſix Right-angles, it follows, that the Sum of the Angles of any Spherical Triangle is leſs than ſix Right-angles or 540 Degrees.

21. Hence it is manifeſt, that in any Spherical Triangle, the Sum of any two Angles is greater than the Supplement of the third that remains, and conſequently the External Angle is leſs than the Sum of the two inward and oppoſite Angle.

22. If the Sum of the Legs of any Spherical Triangle be equal to a Semi-circle or 180 Degrees, the Internal Angle at the Baſe ſhall be accordingly equal to the outward and oppoſite Angle.



For If *d b* and *b a* taken together be equal to the Semi-circle *d.b c*, then *b a* is equal to *b c* (by the 3*d Geom. Axiom*) and conſequently the Angle *b a c* is equal to the Angle *c*, (by the 10*th* preceding *Ar.*) which is equal to the Angle *d* (by the 6*th* preceding *Ar.*) Wherefore the Angle *d* is equal to *b a c*.

23. If the Sum of the Legs be greater than a Semicircle, the Internal Angle at the Baſe is accordingly greater than the outward and oppoſite Angle.

For if *d b* and *a b* taken together be greater than the Semicircle *d b c*, then is *b a* greater than *b c* (by the 18*th Geom. Ax.*) and conſequently the Angle *b a c* is leſs than the Angle *c* (by the 12*th* preceding *Ar.*) which is equal to the Angle *d* (by the 6*th* preceding *Ar.*) Wherefore, the Angle at *d* is greater than *b a c*.

24. If the Sum of the Legs be leſs than a Semicircle, the Angle at the Baſe is accordingly leſs than the outward and oppoſite Angle.

If *d b* and *b a* taken together be leſs than *d b c* then *b a* is leſs than *b c* (by the *18th Geom. Ax.*) and conſequently the Angle *b a c* is greater than the Angle at *c* (by the *12th* preceding *Ar.*) which is equal to the Angle *d* (by the *6th* preceding *Ar.*) Wherefore, the Angle *d* is leſs than the Angle *b a c*.

25. If the Sum of any two Sides of a Spherical Triangle be equal to a Semicircle, the Sum of the oppoſite Angles ſhall be accordingly equal to a Semicircle or two Right-angles.

For, if *d b* and *b a* taken together, be equal to the Semicircle *d b c*, then the Angle at *d* equal to the Angle at *c* is equal to the Angle *b a c* (by the *10th* preceding *Ar.*) but the Angles *d a b* and *b a c* make together two Right-angles (by the *7th* preceding *Ar.*) Wherefore, the Angles *b d a* and *b a d* do together make two Right-angles or 180 Degrees.

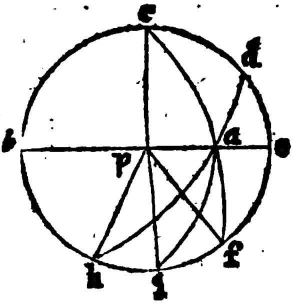
26. If the Sum of the two Sides be greater than a Semicircle, the Sum of their oppoſite Angles ſhall accordingly be greater than a Semicircle or two Right-angles.

For, if *d b* and *b a* taken together be greater than the Semicircle *d b c*, the Angle at *d* is greater than *b a c* (by the *23d* preceding *Ar.*) but *b a c* and *b a d* taken together make two Right-angles (by the *7th* preceding *Art.*) Wherefore, *b d a* and *b a d* taken together, ſhall make more than two Right-angles or 180 Degrees (by the *16th Geom. Ax.*)

27. If the Sum of the two Sides taken together be leſs than a Semicircle, the Sum of their oppoſite Angles ſhall accordingly be leſs than a Semicircle.

If *d b* and *b a* taken together be leſs than the Semicircle *d b c*, then the Angle at *d* is leſs than the Angle *b a c* (by the *23d* preceding *Ar.*) but the Sum of the Angles *d a b* and *b a c* make juſt a Smicircle (by the *7th* preceding *Art.*) Wherefore, the Angles *d* and *d a b* taken together ſhall make leſs than two Right-angles or 180 Degrees (by the *16th Geom. Ax.*)

28. Hence it is manifeſt, that in an Equicrural Spherical Triangle, if one of the equal Legs be greater than, equal to, or leſs than a Quadrant, the oppoſite Angles at the Baſe ſhall be each accordingly, greater than, equal to, or leſs than a Right-angle, or 90 Degrees.



29. If from any Point a, (except the Pole p of the Circle $bhgec$, ſeveral Arches of Great Circles as ab, ab, ag, af, ae, &c. be drawn, till they cut the Circumference of the great Circle $bhgec$, the greateſt is ab, which paſſes thro' p the Pole of the given Great Circle, and the leaſt is ae the part contained between the Point a and the Circumference; and of all the other Arches the Arch ab neareſt to that which was drawn thro' the Pole p, is greater than ag which is removed farther off, and ag greater than af, &c. but the Angle hab is leſs than gab, leſs than fab, &c. and if g be the Pole of the Circle bae, the Angle bah is Acute, and ab greater than a Quadrant; alſo the Angle gab is a Right-angle, and ag equal to a Quadrant, and the Angle fab Obtuſe, and af leſs than a Quadrant.

For becauſe p is the Pole of the Circle $bgec$, the Arches pb, ph, pg, are equal, therefore $ap+pb$ is equal to $ap+ph$ (by the 2d *Geom. Ax.*) but $ap+ph$ is greater than ah (by the 13th.) Wherefore, $ab=ap+pb$ is greater than ah.

Again, $ap+ph$ is equal to $ap+pg$, therefore, whereas the Angle aph is greater than apg, the Baſe ah is greater than ag

After the ſame manner it may be prov'd that ag is greater than af, &c.

Again, $ba+ae$ is equal to pe, which is equal to pf, but pf is leſs than $pa+af$ (by the 13th) Therefore, ae is leſs than af (by the 18th *Geom. Ax.*) and the ſame will happen whereſoever the Point f is taken; wherefore ae is the leaſt of all the Arches that can be drawn from the Point a upon the Circumference $bhec$.

Again, becauſe g is the Pole of the Circle $bpae$, the Angle gab is a Right-angle, and bg equal to a Quadrant, wherefore, the Angle hab is an Acute Angle, and bh leſs than a Quadrant, but ah greater, alſo the Angle fab is an Obtuſe Angle, and the Arch fb greater than a Quadrant, but af leſs, and this will happen whereſoever the Points f and h are taken, provided they be placed on contrary Sides of the Points g.

30. Hence it is that the Legs of a Right-angled Spherical Triangle are of the ſame kind with their oppoſite Angles. For in the Tri-

angle *b h a*, becauſe *b a* is greater than the Quadrant *b p*, the Angle *b h e* is greater than the Right-angle *b h p*, and in the Triangle *h a e* becauſe *a e* is leſs than the Quadrant *p e*, the Angle *e h a* is leſs than the Right-angle *e h p*.

31. Again, If the Legs, and conſequently the Angles are of the ſame kind, that is, both greater or both leſs than Quadrants, the Hypothenuſe ſhall be leſs than a Quadrant, thus in the Triangles *f a b* and *f a e*, where the Legs *a b* and *b f* in one, are greater than Quadrants, and the Legs *a e* and *e f* in the other leſs than Quadrants; the Hypothenuſe *a f* is leſs than a Quadrant: But if the Legs are of a different kind, that is, one greater and the other leſs than a Quadrant, the Hypothenuſe ſhall be accordingly greater than a Quadrant; thus in the Triangle *a b h*, where *a b* is greater and *b h* leſs than a Quadrant, the Hypothenuſe *a h* is greater than a Quadrant.

32. Wherefore, If the Hypothenuſe of a Right-angled Triangle is leſs than a Quadrant or 90 Degrees, the Legs and conſequently the Angles are ſimilar; that is, both greater or leſs than Quadrants: and on the contrary, if the Hypothenuſe be greater than a Quadrant the Legs and conſequently their oppoſite Angles ſhall be one leſs the other greater than a Quadrant or 90 Degrees.

33. If the Hypothenuſe be leſs than a Quadrant or 90 Degrees, each of the Legs ſhall be ſimilar to its adjacent Angle; that is, if the Leg be greater than 90 Degrees or a Quadrant, the adjacent Angle ſhall be greater than 90 Degrees or a Quadrant, and on the contrary, thus in the Triangle *f a b*, where the Legs *b a* and *b f* are each greater than Quadrants, the adjacent Angles *b a f* and *b f a* are each greater than Quadrants; alſo in the Triangle *a f e*, where the Legs *a e*, *e f*, are each leſs than Quadrants, the Angles at *a* and *f* are Acute. Wherefore, *Vice Verſa*, if the Leg of a Right-angled Triangle be ſimilar, or of the ſame kind with its adjacent Angle, the Hypothenuſe ſhall be leſs than a Quadrant or 90 Degrees.

34. If the Hypothenuſe be greater than a Quadrant or 90 Degrees, each Leg ſhall be diſſimilar to its adjacent Angle; thus in the Triangles *b a b* and *h a e*, in each of which the Hypothenuſe *a h* is greater than a Quadrant, the Angles *e a h* and *a h b* adjacent to the Sides *a e* and *b h*, each of which are leſs than Quadrants, are Obtuſe, but the Angles *a h e* and *h a b*, which are adjacent to the Legs *h e* and *b a*, each of which are greater than Qua-

drants are Acute; and on the contrary, if either of the Legs be diſſimilar to its adjacent Angle, the Hypothenuſe ſhall be greater than a Quadrant.

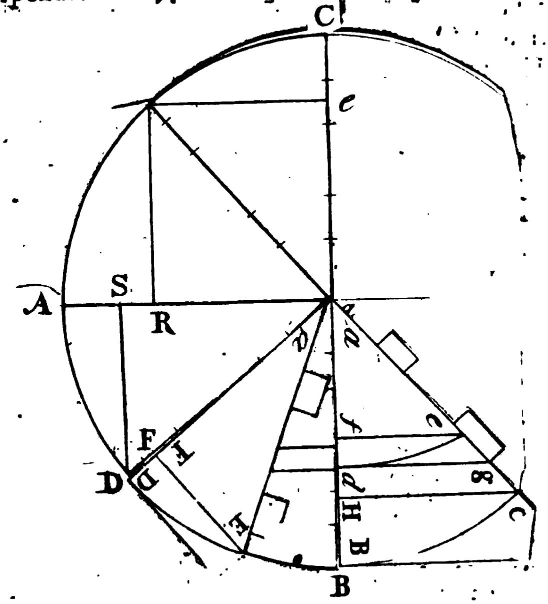
Section IV.

Containing a Demonſtration of ſome uſeful Theorems *commonly call'd* Axioms, *by which the ſeveral Caſes of* Right-angled Spherical Triangles *are reſolved.*

Propoſition the *1ſt or* Axiom *the 1ſt.*

IN Right-angled Spherical Triangles having the ſame Acute Angle at the Baſe

The Sines of the Hypothenuſes are in direct Proportion to the Sines of the Perpendiculars; and on the contrary.



Let the upper Quadrant of the Semicircle *A E C* be turned about upon its Axis *A a*, till the Sectors of the Circles *a D E*, *a B C* being erected perpendicularly over the Lines *a D*, *a B*, the Points *E* and *C* of the Quadrant *AEC*, coincide with the Points *E* and *C* of the Sectors *a D E* and *a B C*, then will be formed two Spherical Triangles *ABC* and *ADE* Right-angled at *B* and *D*, having the Acute Angle *BAC* or *DAE* at the Baſe common to both:

From the Point *E* draw *E R* a Perpendicular to the Axis *a A*, (and *E e* a Parallel to it) then will *e a* equal to *ER*, be the Sine of the Hypothenuſe *EA* of the Triangle *AED*, as is *a e* is the Sine of the Hypothenuſe *AC* of the Triangle *ABC*, alſo from *e* (in the Sector *a B C*) let fall the Perpendicular *ef*, which will be equal to *E F* (in the Sector *a D E*) the Sine of the Perpendicular *D E* of the Triangle *ADE*) for the Right-lined Triangles *REF* and *a e f* are equal, by the *1ſt Geom. Prop Part* the *1ſt.*) as is *c H* the Sine of the Perpendicular *c B* of the Triangle *ABC*.

Wherefore in the Sector *a B C*, becauſe *ef* and *c H* are each perpendicular to *a B* and conſequently parallel, it will be as *a C* to *a e*, ſo is *CH* to *ef*. That is,

As the Sine of the Hypothenuſe *AC* of the Triangle *ABC*,
To the Sine of the Hypothenuſe *AE* of the Triangle *ADE* ;
So is the Sine of the Penpendicular *BC* of the Triangle *ABC*,
To the Sine of the Perpendicular *DE* of the Triangle *ADE*.

Or, As *c H* to *ef*, ſo is *a C* to *a e*. That is;

As the Sine of the Perpendicular *BC* of the Triangle *ABC*;
To the Sine of the Perpendicular *DE* of the Triangle *ADE* ;
So is the Sine of the Hypothenuſe *AC* of the Triangle *ABC*,
To the Sine of the Hypothenuſe *AE* of the Triangle *ADE*.

Propoſition *the* 2d, or Axiom *the* 2d.

In Right-angled Spherical Triangles having the ſame Acute Angle at the Baſe, the Sines of the Baſes are in Direct Proportion to the Tangents of the Perpendiculars; and on the contrary.

From the Point *D* (in the former *Scheme*) draw *DS* a Perpendicular, and *D d* a Parallel to the Axis or Radius *A a*, then will *ad* equal to *DS*, be the Sine of the Baſe *AD* of the Triangle *A D E*, as is *A B* the Sine of the Baſe *AB* of the Triangle *ABC*.

From *d* in the Sector *aBC* draw *dg* a Perpendicular to the Baſe *aB*, which will be equal to *DG* in the Sector *aDE*, the Tangent of the Arch *DE* the Perpendicular of the Triangle *ADE*. (For the Right-lined Triangles *RDG* and *adg* are equal (by the 2*d Geom. Prop. of Part the* 1*ſt.*) as is *BK* a Perpendicular to *aB* from the Point *B*, a Tangent of the Arch *BC* the Perpendicular of the Triangle *ABC*.

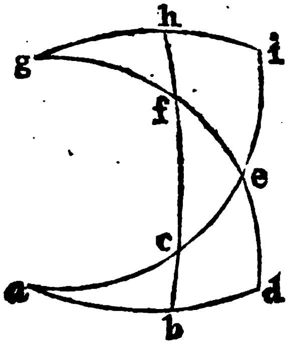
Wherefore, becauſe in the Sector *aBK*, *dg* and *BK* are each Perpendicular to *aB*, and conſequently parallel to each other, it will be, As *AB* to *Ad*, ſo is *BK* to *dg*. That is,

As the Sine of the Baſe *AB* of the Triangle *ABC*,
To the Sine of the Baſe *AD* of the Triangle *ADE*;
So is the Tangent of the Perpendicular *BC* of the Triangle *ABC*,
To the Tangent of the Perpendicular *DE* of the Triangle *ADE*.

And on the contrary, As *BK* to *dg*, ſo is *AB* to *Ad*. That is,

As the Tangent of the Perpendicular *BC* of the Triangle *ABC*,
To the Tangent of the Perpendicular *DE* of the Triangle *ADE*,
So is the Sine of the Baſe *AB* of the Triangle *ABC*,
To the Sine of the Baſe *AD* of the Triangle *ADE*.

Let *ab* repreſent the Baſe of a Spherical Triangle *abc* Right-angled at *b*, then will *bc* be the Perpendicular, *ac* the Hypothenuſe, *bac* the Angle at the Baſe, and *acb* the Vertical Angle, or Angle at the Perpendicular.



Produce the Sides *ab*, *ac*, and *bc*, till *ad*, *ae*, *ci*, and *ch*, are Quadrants, and about the Points *a* and *c* as Poles, deſcribe the Great Circle *defg*, and *ihg*, then will the Angles at *d*, *e*, *i*, and *h*, be Right-angles, *f* the Pole of the Circle *abd*, and *g* the Pole of the Circle *acei*; alſo *bd* the Complement of *ba*, and *gf* the Complement of *fe* equal to *ed*, the Meaſure of the Angle at *a*, in like manner *hf* the Complement of *fc* is equal to *cb*, and *ie* the Complement of *ce* equal to *ac*; alſo *gh* the Complement of *hi*, the Meaſure of the Angle *hci* equal to *acb*; theſe things being premiſed it follows,

1. That the Radius, the Sine of the Hypothenuſe, the Sine of the Angle at the Baſe, and the Sine of the Perpendicular, are directly proportional.

Or,

The Radius, the Sine of the Hypothenuſe, the Sine of one of the adjacent Angles, and the Sine of the oppoſite Side or Leg, are directly proportional.

For in the two Spherical Triangles abc and ade, Right-angled at b and d, becauſe bc is leſs than a Quadrant, its oppoſite Angle a at the Baſe common to both Triangles will be Acute. Wherefore, by the *1ſt Propoſition* it will be, As S.ae : S.ac :: S.ed : S.cb. That is, becauſe ae is a Quadrant, As the R : S.ac :: S.bac : S.bc.

2. The Radius, the Sine of the Baſe, the Tangent of the Angle at the Baſe, and the Tangent of the Perpendicular, are directly proportional.

Or,

The Radius, the Sine of one of the Legs, the Tangent of the adjacent Angle, and the Tangent of the other Leg, are directly proportional:

For becauſe bc is leſs than a Quadrant, the two Spherical Triangles abc and ade Right-angled at b and d, will have the ſame Acute Angle bac at the Baſe; wherefore, by *Prop.* the *2d* it will be, As the Sine of ad to the Sine of ab, ſo is the Tangent of de to the Tangent of bc; that is, becauſe ad is a Quadrant, As R : S.ab :: t,bac : t,bc.

3. The Radius, the Co-ſine of the Baſe, the Co-ſine of the Perpendicular, and the Co-ſine of the Hypothenuſe, are directly proportional.

Or,

The Radius, the Co-ſine of one of the Legs, the Co-ſine of the other Leg, and the Co-ſine of the Hypothenuſe, are directly proportional.

For in the two Spherical Triangles fdb, fec, Right-angled at d and e, becauſe ce the Complement of ac is leſs than a Quadrant, the Angle cfe at the Baſe common to both Triangles will be Acute; wherefore by the *1ſt Prop.* it will be, As S.fb : S.bd :: S.fc : S.ce. That is (becauſe fb is 90 Degrees) As R : c s,ab :: c s,cb : c s,: ac.

4. The Radius, the Co-ſine of the Angle at the Baſe, the Co-

tangent of the Baſe, the Co-tangent of the Hypothenuſe, are directly proportional.

Or,

The Radius, the Co-ſine of one of the Angles, the Co-tangent of its adjacent Leg, and the Co-tangent of the Hypothenuſe, are directly proportional.

For in the two Spherical Triangles *f d b*, *f e c*, Right-angled at *d* and *e*, becauſe *c e* the Complement of *a c* is leſs than a Quadrant, the Angle *c f e* at the Baſe common to both the Triangles will be Acute. Wherefore by the 2*d Prop.* As S. *f d* : S. *f e* : : t, *b d* : t, *c e*. That is becauſe *f d* is a Quadrant, As *R* : c s, *b a c* : : c t, *a b* : c t, *a c*.

5. The Radius, the Co-ſine of the Perpendicular, the Sine of the Angle at the Perpendicular, and the Co-ſine of the Angle at the Baſe, are directly proportional.

Or,

The Radius, the Co-ſine of one of the Legs, the Sine of its adjacent Angle, and the Co-ſine of its oppoſite Angle, are directly proportional.

For in the two Spherical Triangles *c i h*, *c e f*, Right-angled at *i* and *e*, becauſe *f e* the Complement of *e d*, the Meaſure of the Angle *a* is leſs than a Quadrant, the Angle at the Baſe *f c e* equal to *a c b*, the Angle at the Baſe *c e* common to both the Triangles will be Acute; wherefore by *Prop.* the 1*ſt*, As S. *c h* : S. *c f* : : S. *h i* : S. *f e*; that is, becauſe *c h* is a Quadrant, As *R* : c s, *b c* : : S. *a c b* : c s, *b a c*.

6. The Radius, the Co-ſine of the Hypothenuſe, the Tangent of the Angle of the Perpendicular, and the Co-tangent of the Angle at the Baſe, are directly proportional.

Or,

The Radius, the Co-ſine of the Hypothenuſe, the Tangent of one of its adjacent Angles, and the Co-tangent of the other adjacent Angle are directly proportional.

For in the two Spherical Triangles *c i h*, *c e f*, Right-angled at *i* and *e*, becauſe *f e* the Complement of *e d*, the Meaſure of the Angle *a* is leſs than a Quadrant, the Angle at the Baſe *f c e* common to both is Acute; therefore by *Prop.* the 2*d* it will be, As S. *c i* : S. *c e* : : t, *i h* : t, *e f*; that is, becauſe *c i* is a Quadrant, As *R* : c s, *a c* : : t, *b c a* : c t, : *b a c*.

7. Hence we are taught, how from any two *Parts* of a Right-angled Spherical Triangle (beſides the Right-angle) to find the reſt,

for *Example*, Suppoſe the Hypothenuſe and Angle at Baſe were given, to find the Perpendicular, it will be by the 1*ſt Cor.* As the Radius to the Sine of the Hypothenuſe, ſo is the Sine of the Angle at the Baſe, to the Sine of the Perpendicular. Again, if the Perpendicular and Angle at the Baſe were given, to find the Hypothenuſe, it will be by the ſame *Cor.* As the Sine of the Angle at the Baſe to the Sine of the Perpendicular, ſo is the Radius to the Sine of the Hypothenuſe; alſo if the Perpendicular, and Hypothenuſe were given, to find the Angle at the Baſe, it will be by the ſame *Cor.* As the Sine of the Hypothenuſe to the Radius, ſo is the Sine of the Perpendicular to the Sine of the Angle at the Baſe.

After the ſame manner, by the help of the other *Corollarys*, may all the Caſes of *Right-angled Spherical Triangles* be Reſolved; as ſhall be Exemplified more at large in the next *Section*.

From a diligent Conſideration of the preceding *Corollarys*, the Lord *Napier*, the firſt Inventor of the *Logarithms*, contrived two General Rules, eaſy to be retained in Memory, by the help of which the ſeveral Caſes of *Right-angled Spherical Triangles* might eaſily be reſolved. For ſince in every Triangle, beſides the Right-angle there are three Sides and two Angles; the two Sides comprehending the Right-angle, together with the Complements of the other three he called the five Circular Parts; and becauſe in every Triangle there are always two things given to find a third, one of theſe three which he called the *Middle Part* is either adjacent to the other two, in which Caſe they are called *Extreams Adjunct*, or ſeparated from each of them by ſome Side or Angle, which is neither given nor required, and are therefore called *Extreams Disjunct*; thus, if the Complement of the Hypothenuſe *a c* (See the former Figure) is taken for the Middle Part, the Complements of the two adjacent Angles *a* and *c* are the Extreams Adjunct, and the two Legs *a b* and *b c* containing the Right-angle, are the Extreams Disjunct.

Again, if the Complement of the Angle *a* or Angle at the Baſe, be taken for the Middle Part, the Complement of the Hypothenuſe *a c*, and the adjacent Leg or the Baſe *a b*, are the Extreams Adjunct, but the Complement of the other Angle *c* or Angle at the Perpendicular, and the Perpendicular or Leg *b c*, are the Extreams Disjunct; in like manner, if the Baſe or Leg *a b* be taken for the Middle Part, the Complement of the Angle *a* or Angle at the Baſe, and the Perpendicular, are the Extreams Adjunct; for the Right-angle *b* being always given makes no ſeparation, but the Complements of the Hypothenuſe *a c*, and the oppoſite Angle *c*, or Angle at the Perpendicular are the Extreams Disjunct; theſe things being premiſed, the Reaſon of the two Rules will evidently appear, *viz.*

Rule 1.

That in every Right-angled Spherical Triangle, the Rectangle or Product made of the Radius and the Sine of the Middle Part, is equal to the Rectangle or Product made of the Tangents of the Extreams Adjunct.

Caſe 1. Let the Complement of the Hypothenuſe *a c* be taken for the Middle Part, then will the Complements of the two Adjacent Angles *a* and *c* be the Extreams Adjunct; and by the *6th* preceding *Corollary* it will be, As $R : \mathrm{cs},ac :: \mathrm{t},bca : \mathrm{ct},bac$. That is, by permutation, As $R : \mathrm{t},bca :: \mathrm{cs},ac : \mathrm{ct},bac$; but by the known Property of the Tangents, prov'd in *Page* 79, the $R : \mathrm{t},bca :: \mathrm{ct},bca : R$; therefore, As $\mathrm{ct},bca : R :: \mathrm{cs},ac : \mathrm{ct},bac$; and conſequently (by the 15*th Geom. Theor.*) $R \times \mathrm{cs},ac = \mathrm{ct},bca \times \mathrm{ct},bac$. That is, the Radius multiplied into the Co-ſine of the Middle Part, is equal to the Product of the Tangents of the Extreams Adjunct, for as the Co-ſine of *a c* is the Sine of the Middle Part, ſo the Co-tangents of the Angles *a* and *c* are the Tangents of the Extreams Adjunct.

Caſe 2. If the Complement of the Angle *a* at the Baſe be taken for the Middle Part, the Complement of the Hypothenuſe *a c*, and the Baſe *ab* will be the Extreams Adjunct, and by the *4th* preceding *Cor.* it will be, As $R : \mathrm{cs},bac :: \mathrm{ct},ab : \mathrm{ct},ac$. That is by permutation, As $R : \mathrm{ct},ab :: \mathrm{cs},bac : \mathrm{ct},ac$; but $R : \mathrm{ct},ab :: \mathrm{t},ab : R$; wherefore, $\mathrm{t},ab : R :: \mathrm{cs},bac : \mathrm{ct},ac$; and conſequently, $\mathrm{t},ab \times \mathrm{ct},ac = R \times \mathrm{cs},bac$; that is the Radius multiplied into the Sine of the Complement of the Angle *a* or Sine of the Middle Part, is equal to the Product of the Tangent of *a b* one of the Extreams, into the Tangent of the Complement of *a c* the other Extream.

By the ſame parity of Reaſon, if the Complement of the Angle *c* be taken for the Middle Part, the Complement of the Hypothenuſe *a c* will be one of the Extreams adjacent, and the Perpendicular the other; and conſequently by changing *a* for *c* the $R \times \mathrm{cs},bca = \mathrm{t},cb \times \mathrm{ct},ac$.

Caſe 3. If one of the Legs *a b* be taken for the Middle Part, then the Complement of the adjacent Angle at *a* will be one of the Extreams Adjunct, and the Perpendicular *b c* the other Extream; but by the *2d* preceding *Cor.* it will be, As $R : \mathrm{S}.ab :: \mathrm{t},bac : \mathrm{t},bc$; that is, by permutation, As $R : \mathrm{t},bac :: \mathrm{S}.ab : \mathrm{t},bc$; but becauſe the $R : \mathrm{t},bac :: \mathrm{ct},bac : R$; therefore, $\mathrm{ct},bac : R :: \mathrm{S}.ab : \mathrm{t},bc$; and conſequently $R\ \mathrm{S}.ab = \mathrm{t},bc \times \mathrm{ct}\,bac$. That is, the Radius multiplied into the Sine of *ab* of the Middle Part, is equal to the Product of the Tangent of *bc* one of the Extreams, multiplied into the Tangent of the Complement of the Angle at *a* the other Extream.

After the ſame manner it may be proved, that if *bc* the other Leg be taken for the Middle Part, that the Radius multiplied into the Sine of the Middle Part *bc*, will be equal to the Product of the Tangent of *ab* one of the Extreams Adjunct, multipled into the Tangent of the Complement of the Angle *c* the other Extream; wherefore, *&c*, as was to be proved.

Rule 2.

The Rectangle under the Radius and the Sine of the Middle Part, is equal to the Product of the Co-ſines of the Extreams Disjunct, thus if the Complement of *ac* be taken for the Middle Part, the Legs *ab* and *bc* will be the Extreams Disjunct, but by the 3*d* preceding *Corollary*, As $R : \mathrm{cs},ab :: \mathrm{cs},bc : \mathrm{cs},ac$. Wherefore, $R \times \mathrm{cs},ac = \mathrm{cs},ab \times \mathrm{cs},bc$; that is the Radius multiplied into the Sine of the Complement of the Hypothenuſe *ac* the Middle Part, is equal to the Product of the Co-ſines of the Legs *ab* and *bc* the Extreams Disjunct.

Caſe 2. Let the Complement of the Angle *a* be taken for the Middle Part, then will the Complement of the Angle at *c* be one of the Extreams Disjunct, the Leg *bc* the other, but by the 5*th* preceding *Corollary* the $R : \mathrm{cs},bc :: \mathrm{S}.bca : \mathrm{cs},bac$; wherefore, $R \times \mathrm{cs},bac = \mathrm{cs},bc \times \mathrm{S}.bca$. That is, the Radius multiplied into the Sine of the Complement of the Angle *a* the Middle Part, is equal to the Product of the Co-ſine of *bc* one of the Extreams, multiplied into the Sine of the Angle *c*, the Complement of the other Extream.

After the ſame manner it may be proved, that if the Complement of the Angle at *c* be taken for the Middle Part, that the Radius multiplied into the Sine of the Complement of the Angle *c* the Middle Part, will be equal to the Product of the Sine of the Angle *a*, the Complement of one of the Extreams, multiplied into the Co-ſine of *ab* the other Extream.

Caſe 3. If *ab* one of the Legs be taken for the Middle Part, then the Complement of *ac* the Hypothenuſe, will be one of the Extreams

Disjunct, and the Complement of the oppoſite Angle *c* will be the other Extream, but by the 1*ſt* preceding *Cor.* As $R : S.ac :: S.acb : S.ab$; wherefore, $R \times S.ab = S.ac \times S.acb$. That is, the Radius multiplied into the Sine of *a b* the Middle Part, is equal to the Sine of *a c* the Complement of one of the Extreams, multiplied into the Sine of the Angle at *c*, the Complement of the other Extream.

After the ſame Method of Reaſoning it may be proved, that if *b c* the other Leg be taken for the Middle Part, that the Radius multiplied into the Sine of the Middle Part *b c*, will be equal to the Product of the Sine of *a c*, the Complement of one of the Extreams, multiplied into the Sine of the Angle *a*, the Complement of the other Extream. Wherefore, *&c.* as was to be Demonſtrated.

Whence it follows univerſally, that the Radius multiplied into the Sine of the Middle Part, is equal to the Product of the Tangents of the Extreams Adjunct, or to the Product of the Co-ſines of the Extreams Disjunct; and conſequently if each of theſe Equations are reſolved into Proportions, by the help of the 15*th Prop.* of the 1*ſt Part*, the things required will be readily found.

For in every Equation where the Middle Part is ſought, the Radius becomes the firſt Term in the Equation, and the Co-ſines or Tangents of the Extreams the ſecond and third Terms; but where one of the Extreams is required, the Co-ſine or Tangent of the other Extream will be the firſt Term, the Radius and the Sine of the Middle Part the ſecond and third Terms, and in each Proportion the thing required will be the fourth or laſt Term.

Let us ſuppoſe for *Example*, that the Hypothenuſe *a c* and the Angle *a* at the Baſe were given, to find the Perpendicular *b c*, becauſe in this Caſe the Perpendicular *b c* is the Middle Part, and the Complements of the Hypothenuſe *a c* and the Angle *a* at the Baſe are Extreams Diſjunct, the $R \times S.bc = S.bac \times S.ac$, by the third *Caſe* of the 2*d Rule*, and conſequently becauſe *b c* the Middle Part is ſought, it will be by the 18*th Geom. Theor.* of the 1*ſt Part*, As *R* to S. *a c* the Complement of one of the Extreams, ſo is the Sine of the Angle *a*, the Complement of the other Extream, to the Sine of *b c* the Middle Part; that is, As the Radius to the Sine of the Hypothenuſe, ſo is the Sine of the Angle at the Baſe, to the Sine of the Perpendicular.

Again, if the Perpendicular *b c* and the Angle *a* at the Baſe were given, to find the Hypothenuſe *a c*, becauſe in this *Caſe* the Perpendicular *bc* is the Middle Part, and the Complements of the Hypothenuſe *ac*, and the Angle at the Baſe *a* are Extreams Disjunct; then $R\times S.\,bc = S.\,bac \times S.\,ac$ by the third *Caſe* of the 2*d Rule*, and becauſe the Hypothenuſe *ac* one of the Exrreams is ſought, it will be by the 15*th Geom. Theor.* of the 1*ſt Part*, As the Sine of the Angle *a*, the Complement of the other Extream, to the Radius; ſo is the Sine of *bc* the Middle Part, to the Sine of *ac*, the Complement of the Extream required. That is, As the Sine of the Angle at the Baſe, to the Sine of the Perpendicular; ſo is the Radius, to the Sine of the Hypothenuſe.

Again, if the Hypothenuſe *ac* and Perpendicular *bc* were given, to find the Angle *a* at the Baſe, becauſe the Component Parts of the Queſtion are ſtill the ſame, as in the two former *Caſes*, $R\times S.\,bc = S.\,bac \times S.\,ac$, by the 3*d Caſe* of the 2*d Rule*, and becauſe one of the Extreams, *viz.* the Angle *a* is ſought, it will be by the 15*th Geom. Theor.* of *Part* the 1*ſt*, As the S. *ac* the Complement of the other Extream, to the Radius; ſo is the Sine of the Middle Part *bc*, to the Sine of the Angle *a*, the Complement of the Extream required. That is, as the Sine of the Hypothenuſe *ac*, to the Radius; ſo is the Sine of the Perpendicular *bc*, to the Sine of the Angle at the Baſe *a*; agreeing exactly with the Proportions in the 7*th* preceding *Corollary*, deduced from the 1*ſt*.

For the Application of this General Rule to Practice, take theſe following Directions,

In every Right-angled Spherical Triangle

1. The Middle Part is always a Sine,
2. The Extreams Adjunct are Tangents,
3. The Extream Disjunct Sine Complements,
4. The Hypothenuſe and the two adjacent Angles are Complemental Parts.
5. If the Middle Part is ſought begin with the Radius.
6. If one of the Extreams be ſought begin with the other.

And for the more ready diſcovering at firſt ſight which is the Middle Part, and whether the Extreams are Adjunct or Disjunct, obſerve, that if the three Parts of the Triangle which make up the Queſtion, have no Interruption between them, but are all joyned together, then the middle Part of the three is the Middle Part, and the other two are the Extreams Adjunct; thus, if the three Parts that make the Queſtion were *ac* the Hypothenuſe, the Angle *a* the Angle at the Baſe, and

ab

a b the Baſe, then in this Caſe becauſe the three parts are all joyned together, the Angle at the Baſe is the Middle Part, and the other two, *viz.* the Hypothenuſe and the Baſe are Extreams Adjunct, in like manner, if the Parts that make up the Queſtion are the Angle at the Baſe, the Baſe, and the Perpendicular, (for it matters not which are given or which is required) then in this Caſe, for as much as the three things are all joyned together (for the Right-angle being known makes no Interruption) the Baſe or middle of the three is the Middle Part, and the other two Parts are Extreams Adjunct, but if the Hypothenuſe, Angle at the Baſe, and Perpendicular, were the three Parts to be conſidered, becauſe the Perpendicular is ſeparated from the Hypothenuſe, on one Side by the Angle at the Perpendicular, which is neither given nor required, and on the other Side from the Angle at the Baſe, by the Baſe, which is likewiſe neither given nor required, in this Caſe the Perpendicular or Part ſeparated from the reſt is the Middle Part, and the Hypothenuſe and Angle at the Baſe, the other two parts of the Queſtion are Extreams Disjunct, and the ſame Law is to be obſerved throughout all the reſt of the Caſes.

◆

SECT.

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Section V.

Containing the Application of the Four former Sections, *to the actual Solution of the Sixteen Caſes of* Right-angled Spherical Triangles.

IN every Spherical Triangle there are ſix Parts, *viz.* three Sides and three Angles, any three of which being known or given, the reſt may readily be found, and the various Queſtions ariſing from changing the things given and required are called the ſeveral Caſes; and in Spherical Trigonometry are reduced to 28, *viz.* 16 in Right-angled Spherical Triangles, and 12 in Oblique-angled Spherical Triangles, and are as follow.

Caſe 1.

Given the two Legs to find the Hypothenuſe.

Or,

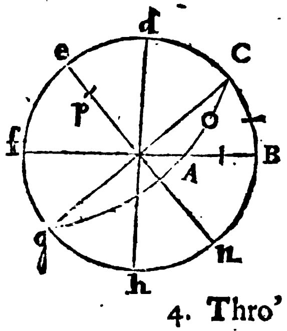
Given the Baſe and Perpendicular, to find the Hypothenuſe

Example.

In the Spherical Triangle *ABC*, Right-angled at *B*,

$$\text{are given}\begin{cases}AB=50^\circ\ 30'\ 30''\\ BC=40^\circ\ 30'\ 20''\end{cases}\text{req. } AC.$$

The Stereographic Solution.



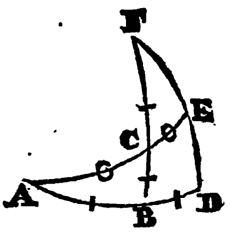
1. Draw the Primitive Circle *f d B h*, the Diameter *f B*, and at Right-angles to it *d h*.

2. Set off the Chord of *BC* 40 deg. 30 min. 20 ſec. in the Primitive Circle from *B* to *C*, and draw the Diameter *Cg*, and at Right-angles to it *e n*.

3. Set off *BA* equal to 50 deg. 30 min. 30 ſec. in the Right Circles from *B* to *A*, by *Caſe* the 2*d Prob.* the 6*th*, *Sect.* the 2*d*.

4. Thro' the Points *C* and *A*, draw a Great Circle as *C A g*, by *Problem* the *1ſt* of *Section* the *3d*, and the thing is done; and *A C* may be meaſured by the *3d Caſe* of *Problem* the *7th*, *Section* the *2d.*

Logarithmically, or by *Calculation.*



Produce the Sides *AB*, *BC*, and *AC*, to *D*, *F*, and *E*, till *AD*, *BF*, and *AE*, are Quadrants, and about the Point *A* as a Pole, deſcribe the Arch *DEF*; then in the two Spherical Triangles *FBD* and *FCE* Right-angled at *D* and *E*, and having the ſame Acute Angle *F* at the Baſe are given, *B D* the Complement of *AB*, *CF* the Complement of *BC*, whence to find *CE* the Complement of *AC*, it will by *Ax.* the *1ſt.*

As the S. *FB* : S. *FC* : S. *BD* : S. *CE*. That is,

Becauſe *FB* is a Quadrant, and that the Sine of 90 deg. is equal to the Radius (which Law muſt be obſerved in all the following Caſes.)

As the Radius ———————————— 10.0000000

To the Co Sine of the Perpendicular *CB*=40.30.20 9.8810095

So is the Co-ſine of the Baſe *AB*=50.30.30. ———— 9.8034338

To the Co-ſine of the Hypothenuſe *AC*=61.04.56½—9.6844433

Or,

As the Secant of the Perpendicular *BC*=40° 30′ 20″—10.1189905

To the Radius ———————————— 10.0000000

So is the Co-ſine of the Baſe *AB*=50.30.30 ———— 9.8034338

To the Co-ſine of the Hypothenuſe *AC*=61.04.56—9.6844433

Or,

As the Secant of the Baſe $AB=50.30.30$ ———— 10.1965662

To the Radius———————— 10.0000000

So is the Co-ſine of the Perpendicular $BC=40.30.20$—— 9.8810095

To the Co-ſine of the Hypothenuſe $AC=61.04.56$—— 9.6844433

Caſe 2.

Given the two Legs, to find either of the oppoſite Angles.

Or,

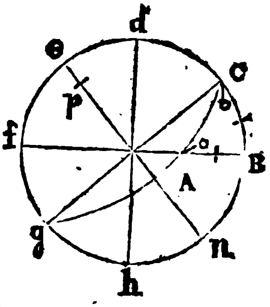
The Baſe and Perpendicular being given, to find the Angle at the Baſe, or the Angle at the Perpendicular.

Example.

In the Spherical Triangle ABC Right-angled at B,

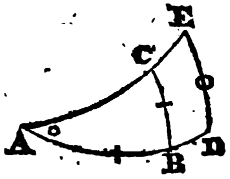
are given $\left\{\begin{array}{l} AB=50^\circ\ 30'\ 30'' \\ BC=40\ \ \ 30\ \ \ 20 \end{array}\right\}$ req. $<A$ and $<C$.

The Stereographical Conſtruction of this Caſe is the ſame with the former, and the Angle at C may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Sect.* the 2*d*, and the Angle at *A* by *Caſe* the 3*d* of the ſame *Problem*.



Logarithmically, or by *Calculation.*

Produce the Sides AB and AC, to D and E, till AD and AE are Quadrants, and joyn the Points D and E by the Arch of a Great Circle DE, of which A is the Pole; then in the two Spherical Triangles ABC and ADE, Right-angled at B and D, are given AB and BC, whence to find DE the Meaſure of the Angle A, it will be by *Ax.* 2*d*.



As

As the S. *AB* : S. *AD* : : t, *BC* : t, *DE*. That is,

As the Sine of the Baſe or Leg *AB*=50° 30′ 30″ ——9 8874582

To the Radius—————————————10.0000000

So is the Tang. of Perpendicular or Leg *BC*=40.30.20 9.9315842

To the Tang. of the Angle at the Baſe *A*=47.54.20 10.0441260

Or,

As the Radius—————————————10.0000000

To the Co-ſecant of the Baſe *AB*=50.30.30 ———10.1125418

So is the Tangent of the Perpendicular *BC*=40.30.20—9.9315842

To the Tangent of the Angle at Baſe *A*=47.54.20—10.0441260

Or,

Becauſe the Tangents and Co-tangents of Arches are receiprocally proportional it will be,

As t, *BC* : *R* : : S. *AB* : c t, < *A*. That is,

As the Tangent of the Perp. or Leg *BC*=40.30.20 ——9.9315842

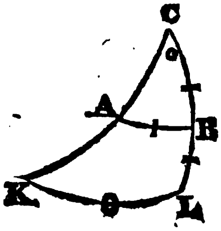
To the Radius —————————— 10.0000000

So is the Sine of the Baſe or Leg *AB*=50.30.30———9.8874582

To the Co-tangent of the Angle at Baſe *A*=47.54.20 9.9558740

Agreeing exactly with the former, and is the ſame Proportion as is deduced from the Lord *Napiers Theorem.*

Again;



Produce the Sides *CB* and *CA* to *L* and *K*, till *CL* and *CK* are Quadrants, and about the Point *C* as a Pole, deſcribe the Arch *KL*; then in the two Spherical Triangles *CBA* and *CLK* Right-angled at *B* and *L*, are given, *CB* and *BA*, whence to find *KL* the Meaſure of the Angle at *C*, it will be by *Ax.* the 2*d*.

As S. *CB* : S. *CL* : : t, *BA* t, *LK*. That is,

As the S. of the Perpendicular or Leg $CB=40^\circ\ 30'\ 20''$ 9.8125937

To the Radius ———————— 10.0000000

So is the Tangent of the Baſe or Leg *AB*=50.30.30 —— 10.0840242

To the Tangent of the Angle at Perp. *C*=61.50.29 —— 10.2714305

Or,

As the Radius ———————— 10.0000000

To the Co-ſecant of the Perpendicular *CB*=40.30.20 10.1874063

So is the Tangent of the Baſe *AB*=50.30.30 —— 10.0840242

To the Tangent of the Angle at Perp. *C*=61.50.29 10.2714305

Or according to the Lord *Napiers Theorem*,

As the t, *AB* : *R* : : S. *BC* : c t, <*C*. That is,

As the Tangent of the Baſe or Leg *AB*=50.30.30 — 10.0840242

To the Radius ———————— 10.0000000

So is the Sine of the Baſe Leg or *BC*=40.30.20 —— 9.8125937

To the Co-tangent of the Angle at Perp. *C*=61.50.29 9.7285695

Caſe 3.

Given the Hypothenuſe and one of the adjacent Angles, to find the other adjacent Angle.

Or,

Given the Hypothenuſe and Angle at the Perpendicular to find the Angle at the Baſe.

Example.

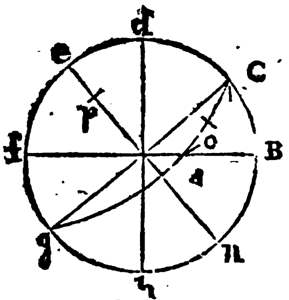
In the Spherical Triangle *ABC*, Right-angled at *B*,

are given $\left\{\begin{matrix} AC=61^\circ\ 04'\ 56'' \\ <C=61\ \ 50\ \ 29 \end{matrix}\right\}$ req. <*A*

The Stereographic Solution.

1. Having drawn the Primitive Circle *f d B b* thro' the Point *C*, taken at pleaſure, draw the Great Circle *C A B*, forming an Angle

of



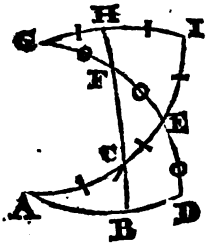
of 61 deg. 50 min. 29 ſec. with the Primitive Circle in the Point *C*, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Sect.* the 2*d*, and find its Pole *p* by *Caſe* the 3*d* of *Problem* the 3*d*.

2. Lay off 61 deg. 04 min. 56 ſec. the length of the Hypothenuſe in the Oblique Circle *CAg* from *C* to *A*, by *Caſe* the 3*d* of *Problem* the 6*th*.

3. Thro' the Center of the Primitive Circle *o*, and the Extremity *A* of the Arch *CA*, draw the Diameter or Right Circle *f o b*, and at Right-angles to it the Diameter *d o h*, to find its Poles *d* and *h*, and the thing is done; and the Quantity of the Angle at *a* may be found by *Caſe* the 3*d* of *Problem* the 10*th* of *Section* the 2*d*.

Logarithmically, or by *Calculation*.

1. Having produced the Sides *AB*, *AC*, *BC*, to *D*, *E*, and *F*, till *AD*, *AE*, *BF*, are Quadrants, and drawn the Great Circle *DEF*, about *A* as a Pole.



2. Produce the Sides *CE*, *CF*, *EF*, of the Triangle *CEF*, to *I*, *H*, *G*, till *CI*, *CH*, *EG*, are Quadrants, and draw the Great Circle *IHG*, about the Point *C* as a Pole, then in the two Spherical Triangles *CIH* and *CEF*, Right-angled at *I* and *E*, and having the ſame Acute Angle *C* at the Baſe, are given, *CE* the Complement of the Hypothenuſe *AC*, *HI* the Meaſure of the Angle at Perpendicular *C*, whence to find *FE* the Complement of *ED*, the Meaſure of the Angle at the Baſe *A*, it will be by *Axiom* the 2*d*.

As S. *CI* : S. *CE* : : t, *IH* : t, *FE*. That is,

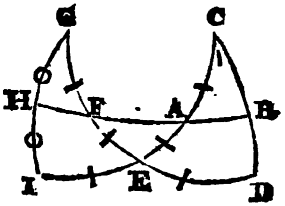
R : (S.*CE*=) Comp. *AC* : : t, (*IH*=∠*C* : t, (*FE*=) Comp. (*ED*=) ∠*A*.

As the Radius ——————————— 10.0000000

To the Co-ſine of the Hypothenuſe *AC*=61.04.56 — 9.6844449
So is the Tang. of the Angle at Perp. *C*=61° 50′ 29″ — 10.2714307

To the Co-tangent of the Angle at Baſe *A*=47.54.20 — 9.9558756

Or,



Having produced the Sides *CB*, *CA*, and *BA*, to *D*, *E*, and *F*, till *CD*, *CE*, and *BF* are Quadrants, and deſcribed the Great Circle *DEFG*, about the Point *C* as a Pole, produce the Sides *AE*, *AF*, and *EF*, to *I*, *H*, and *G*, till *AI*, *AH*, and *EG* are Quadrants, and about the Point *A* as a Pole, draw the Great Circle *IHG*, then in the two Spherical Triangles *AIH* and *AEF*, Right-angled at *I* and *E*, and having the ſame Acute Angle at the Baſe *A*, common to both, we have given *AE* equal to the Complement of *AC*, *FE* the Complement of *ED*, the Meaſure of the Angle at *C*; whence to find *HI* equal to the Meaſure of the Angle at *A*, it will be by *Axiom* the 2*d*.

As S. (AE=) Comp. AC. S, AI=R : : t,(FE=) Comp. (ED=<C : t, HI=)<A. That is,

As the Co-ſine of the Hypothenuſe AC=61.04.56 ——— 9.6844449

To the Radius ——————————— 10.0000000
So is the Co-tangent of the Ang. at Perp. C=61° 50′ 29″ 9.7285693

To the Tang. of the Ang. at the Baſe A=47.54.20 — 10.0441244

Or,

As the Radius ——————————— 10.0000000

To the Secant of the Hypothenuſe AC=61.04 56 — 10.3155551
So is the Co-tangent of the Angle at Perp. C=61.50.29 — 9.7285693

To the Tangent of the Ang. at the Baſe A=47.54.20 10.0441244

Or according to the Lord *Napiers Theorem*,

Becauſe the Hypothenuſe AC is the Middle Part, and the Angles at the Baſe and Perpendicular the Extreams Adjunct, it will be,

As ct,$<$C : R : : c s, AC : ct,$<$A. That is,

As the Co-tangent of the Angle at the Perp. C=61.50.29 9.7285693

To the Radius ———————————— 10.0000000
So is the Co-ſine of the Hypothenuſe AC=61.04.56——9.6844449

To the Co-tangent of the Angle at Baſe A=47.54.20 9.9558756

Caſe 3:

Example 2.

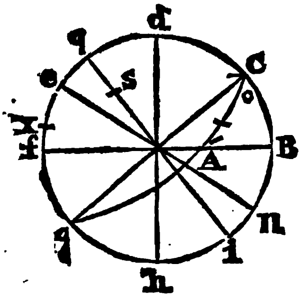
Given the Hypothenuſe and Angle at the Baſe, to find the Angle at the Perpendicular. That is,

In the Spherical Triangle ABC Right-angled at B,

$$\text{are given}\left\{\begin{array}{l} AC=61.04.56 \\ <A=47^{\circ}\ 54'\ 20'' \end{array}\right\}\text{req.} <C.$$

The Stereographic Solution.

1. Having drawn the Primitive Circle *f d B h*, draw the Diameter *f B*, and at Right-angles to it the Diameter or Right Circle *d h*.

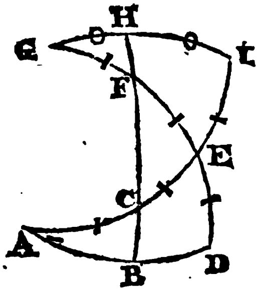


2. Set off the Hypothenuſe *AC* equal to 61 deg. 04 min. 56 ſec. from *f* to *q*, and draw the Diameter *q i*.

3. Set off 42 deg. 05 min. 40 ſec. the Complement of the Angle at the Baſe *A* in the Primitive Circle, from *q* to *x*, and draw *x s* perpendicular to *q i*, and *s C* parallel to *f B*.

4. Thro' *C* the Interſection of *s C*, with the Primitive Circle, draw the Diameter *C g*, and at Right-angles to it the Diameter *e n*, and draw the Oblique Circle *C A g*, forming an Angle of 47 deg. 14 min. 20 ſec with the Right Circle in the Point *A*, by *Caſe* the 2*d* of *Problem* the 9*th*, *Sect.* the 2*d*, and the thing is done; and the Angle at the Perpendicular *C* may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, *Sect.* the 2*d*.

Logarithmically, or by Calculation.



Having made the ſame Preparation as in the former Caſe, that is, having produced the Sides *A B*, *B C*, *A C*, &c. in the two Spherical Triangles *CEF* and *CIH*, Right-angled at *E* and *I*, and having the Angle at *C* common, we ſhall have given, *FE* equal to the Complement of *ED*, the Meaſure of the Angle at *A*, *CE* equal to the Complement of *AC* the Hypothenuſe, whence to find IH the Meaſure of the Angle *FCE* equal to the Angle *ACB*, it will be by *Ax.* the 2*d.*

As S. *CE* : S. *CI* : : t, *EF* : t, *IH*. That is,

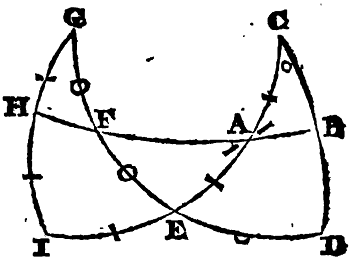
AsS.(*CE*=)Comp *AC* : *R* : : t,(*EF*)Comp. (*ED*=)< *A* : t,*IH*=< *C*

As the Co-ſine of the Hypothenuſe *AC*=61.04.56——9.6844449

To the Radius —— —— —— —— —— — 10.0000000

So is the Co-tang. of the Angle at the Baſe *A*=47.54.20 9.9558735

To the Tang. of the Angle at the Perp. *C*=61.50.29 10.2714286



Or,

Having produced the Sides *CB*, *BA*, *CA*, to *D*, *E*, *F*, till *CD*, *CE*, and *BF* are Quadrants, and deſcribed the Great Circle *DEF*, about the Point *C* as a Pole, &c. as in the former Caſe, in the two Spherical Triangles *AIH* and *AEF* Right-angled at *E* and *I*, and having the ſame Acute Angle at the Baſe *A* common to both, are given *AE* equal to the Complement of *AC*, *IH* equal to the Meaſure of the Angle *HAI*, equal to the Angle *CAB*, whence to find *FE* the Complement of *ED*, equal to the Meaſure of the Angle *C*, it will be by *Ax.* the 2*d.*

As S. *AI* : S. *AE* : : t, *IH* : t, *EF*. That is,

R : S.(*AE*=)Comp.*AC* : : t,(*IH*=)< *A* : t,(*FE*=)Com. (*ED*=)< *C*

As the Radius ———————————— 10 0000000

To the Co-ſine of the Hypothenuſe *AC*=*61.04.56*——*9.6844449*
So is the Tang. of the Angle at the Baſe *A*=47.54.20——10.0441265

To the T. Comp. of the Angle at Perp. *C*=*61.50.29* —*9.7285714*

Which is the ſame Proportion with that in the firſt *Example* of this *Caſe*, only changing *A* for *C*, or the Angle at the Baſe for the Angle at the Perpendicular.

Or, according to the Lord *Napiers Theorem*, becauſe AC the Hypothenuſe is the Middle Part, the Angles at the Baſe and Perpendicular the Extreams Adjunct, and the Angle at the Perpendicular required, it will be,

As ct.<A : R : : c s, AC : c t,<C. That is,

As the Co-tang. of the Angle at the Baſe A=47.54.20—*9.9558735*

To the Radius ———————————— —10.0000000
So is Co-ſine of the Hypothenuſe AC=*61.04.56* ——— *9.6844449*

To the Co-tangent of the Angle at Perp. *C*=*61.50.29* *9.7285714*

Agreeing exactly with the two former Solutions.

Caſe 4.

Given the Hypothenuſe and one of the Angles, to find the adjacent Leg.

Or,

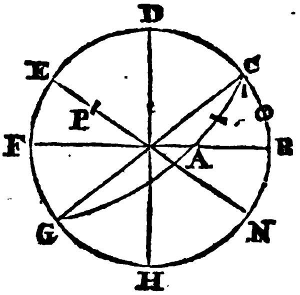
Given the Hypothenuſe and the Angle at the Perpendicular, to find the Perpendicular.

Example.

In the Spherical Triangle *ABC*, Right-angled at *B*,

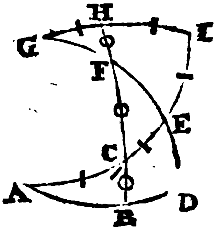
are given $\left\{\begin{array}{l} AC=61.04.56 \\ <C=61^\circ\ 50'\ 29'' \end{array}\right\}$ req. BC.

The Conſtruction of this *Caſe* is the ſame with the firſt *Example* of the third *Caſe*, and the Perpendicular BC may be meaſured by *Caſe* the *1ſt* of *Problem* the *7th Section* the *2d.*



But to find its length by Calculation,

Produce the Sides AB, AC, BC, *&c.* as in *Caſe* the *3d*, then in the 2 Spherical Triangles GIE and GHF, Right-angled at H and I, and having the ſame Acute Angle at the Baſe G common to both, we have given GH equal to the Complement of HI, the Meaſure of the Angle at C, IE equal to the Hypothenuſe AC whence to find HF equal to the Perpendicular CB, for CH, CI, FB, and FD are Quadrants, it will be by *Ax.* the *2d.*

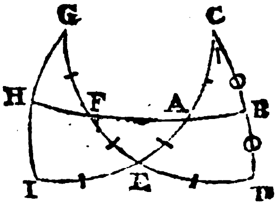


As S. IG : S. GH : : t, IE : t, HF. That is,
R : S (HG=) Comp. (HI=)< C : : t, (IE=Comp. CE=Comp.) AC : t, (HF=Comp. FC=Comp.) CB. That is,

As the Radius —————————— 10.0000000
To the Co-ſine of the Angle at Perp. C=61.50.29——9.6738628
So is the Tangent of the Hypothenuſe AC=61.04.56——10.2577192
To the Tangent of the Perpend. BC=40.30.20——9.9315820

Or,

Having produced the Sides CA, CB, BA, to E, D, F, till CE, CD, BF, are Quadrants, *&c.* as in the *1ſt Example* of the *3d Caſe*, in the 2 Spherical Triangles FDB and FEA, Right-angled at D and E, and having the ſame Acute Angle at the Baſe F common to both, we have



given FE equal to the Complement of ED, the Meaſure of the Angle at C, EA equal to the Complement of the Hypothenuſe AC, whence to find DB equal to the Complement of the Perpendicular BC, it will be by *Ax.* the 2*d*.

As S. (FE=) Comp. (ED=)<C : S. FD=R : : t, (EA=) Comp. AC : t, (DB=) Comp. BC. That is,

As the Co-ſine of the Angle at Perpen. C=61.50 29——9.6738628

To the Radius————————————10.0000000
So is the Co-tangent of the Hypoth. AC=61 04.56——9.7422808

To the Co-tangent of the Perp. BC=40.30.20——10 0684180

Or,

According to *Napiers Theorem*, becauſe the Angle C is the Middle Part, and the Hypothenuſe AC, and Perpendicular BC, the Extreams Adjunct, it will be,

As c t, AC : R : : c s, <C : t, BC. That is,

As the Co-tangent of the Hypothenuſe AC=61.04.56—9.7422808

To the Radius ———————— 10.0000000
So is the Co-ſine of the Angle at the Perp. C=61.50.29—9.6738628

To the Co-tangent of the Perpendicular BC=40.30.20 9.9315820

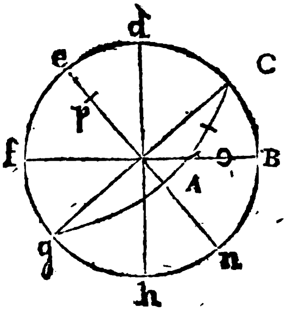
Caſe 4.

Example 2.

Given the Hypothenuſe and Angle at the Baſe, to find the Baſe.
That is,
In the Spherical Triangle *ABC*, Right-angled at *B*,

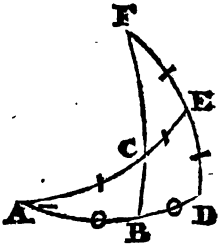
$$\text{are given}\left\{\begin{array}{l} AC=61^\circ\ 04'\ 56'' \\ \angle A=47\ \ 54\ \ 20 \end{array}\right\}\text{req. } AB$$

This



This *Caſe* is Conſtructed like the 2*d* *Example* of the 3*d* *Caſe*, and AB may be meaſured by the 2*d* *Caſe* of *Problem* the 7*th* of *Section* the 2*d*.

But to determine its length by Calculation.



Produce the Sides AB, BC and AC, to D, E, and F, *&c.* as in *Caſe* the 1*ſt*, then in the two Spherical Triangles FDB, FEC, Right-angled at D and E, having the ſame Acute Angle F at the Baſe common to both, we have FE equal to the Complement of DE, the Meaſure of the Angle A, EC equal to the Complement of AC the Hypothenuſe, and DB equal to the Complement of AB the Baſe, and it will be by *Ax.* the 2d.

As S.(FE=) Comp. of (DE=) <A : (S. FD=) R :: t, (EC=) Comp. AC : t, (BD=) Comp. of AB. That is,

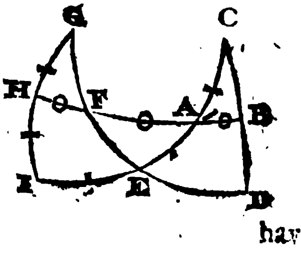
As the Co-ſine of the Angle at the Baſe A=47.54.20——9.8263046

To the Radius———————————— 10.0000000

So is the Co-tangent of the Hypoth. AC=61.04.56——9.7422808

To the Co-tangent of the Baſe AB=50.30.30.————9.9159762

Or,



Having produced the Sides CB, BA, and CA, to D, E, and F, *&c.* as in the 1*ſt* *Example* of the 3*d* *Caſe* in the two Spherical Triangles G I E and G H F, Right-angled at I and H, and having the ſame Acute Angle G at the Baſe common to both, we have given *GH* equal to the Complement of *HI*, the Meaſure of the Angle at *A*, *IE* equal to the Hypothenuſe *AC* (for *IA* and *EC* are Quadrants) and *HF* equal to the Baſe *AB*, (for *HA* and *FB* are Quadrants,) and it will be by *Ax.* the 2*d*.

As S. *GI*=*R* : S. (*GH*=)Comp. (*HI*=)<*A* : : t,(*IE*=Comp. of *EA*=Comp.)*AC* : t,(*HF*=Comp. of *FA*=Comp.) *AB*. That is,

As the Radius ———————————————— 10.0000000

To the Co-ſine of the Angle at the Baſe *A*=47.54.20 9.8263046
So is the Tangent of the Hypothenuſe *AC*=61.04.56—10.2577192

To the Tangent of the Baſe *AB* =50.30.30 ———10.0840238

Or,

According to *Napiers Theorem*, becauſe the Angle at the Baſe *A* is the Middle Part, and *AC* the Hypothenuſe, and *AB* the Baſe Extreams Adjunct, it will be,

As c t, *AC* : *R* : : c s,<*A* : t, *AB*. That is,

As the Co-tangent of the Hypothenuſe *AC*=61.04 56 —9.7422808

To the Radius ———————————— 10.0000000
So is the Co-ſine of the Angle at the Baſe *A*=47.54.20—9.8263046

To the Tangent of the Baſe *AB*=50.30.30 ———10 0840238

Caſe 5.

Given the Hypothenuſe and one of the adjacent Angles, to find the Side or Leg oppoſite.

Or,

Given the Hypothenuſe and Angle at the Perpendicular to find the Baſe.

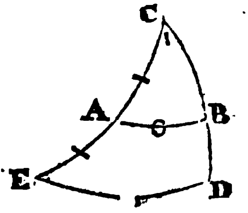
Example.

In the Spherical Triangle *ABC* Right-angled at *B*,

are given, $\left\{\begin{array}{l} AC=61.04.56 \\ < C=61.50.29 \end{array}\right\}$ req. *AB*.

The *Stereographical Conſtruction* of this *Caſe* is the ſame with the 1*ſt Example* of the 3*d Caſe*, and *AB* the Baſe may be meaſured by *Caſe* the 2*d*, of *Prob.* the 7*th*, *Sect.* the 2*d*.

But to determine the length by Calculation.



Produce the Sides *CB* and *CA*, to *D* and *E*, till *CD* and *CE* are Quadrants, and join the Points *E* and *D* by the Arch *DE*, of which the Point *C* is the Pole, then in the two Spherical Triangles *ECD* and *ACB*, Right-angled at *D* and *B*, and having the ſame Acute Angle *C* at the Baſe, we have given *CA*, and *ED* equal to the Meaſure of the Angle at *C*, whence to find *AB* it will be by *Axiom* the 1ſt.

As $S.\,CE=R : S.\,AC :: S.\,(ED=)\angle C : S.\,AB$. That is,

As the Radius —————— 10.0000000

To the Sine of the Hypothenuſe *AC*=61.04.56 ——— 9.9421641
So is the Sine of the Angle at the Perpend. *C*=61.50 29 9.9452936

To the Sine of the Baſe *AB*=50.30.30 —————— 9.8874577

Or,

As the Co-ſecant of the Hypoth. *AC*=61.04.56 ——— 10.0578359

To the Radius —————— 10.0000000
So is the Sine of the Angle at the Perp. *C*=61.50.29 — 9.9452936

To the Sine of the Baſe *AB*=50.30.30 —————— 9.8874577

Or,

As the Co-ſecant of the Angle at the Perp. *C*=61.50.29 10.0547064

To the Radius —————— 10.0000000
So is the Sine of the Hypothenuſe *AC*=61.04.56 ——— 9.9421641

To the Sine of the Baſe *AB*=50 30.30 —————— 9.8874577

The firſt Proportion is the ſame with that deduced from the Lord *Napiers Theorem*, for becauſe the Baſe *AB* is the Middle Part, the Hypothenuſe and Angle at the Perpendicular are the Extreams Disjunct, and becauſe the Middle Part is ſought, it will be as the Radius, &c.

Caſe

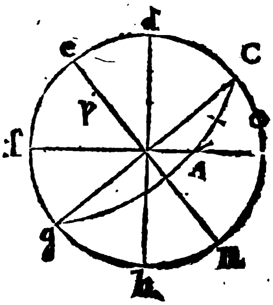
Caſe 5.

Example 2.

Given the Hypothenuſe and Angle at the Baſe, to find the Perpendicular. That is,

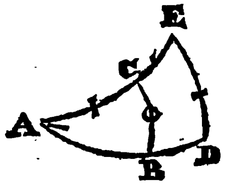
In the Spherical Triangle *ABC*, Right-angled at *B*.

are given $\left\{\begin{matrix}\angle A=47.54.20\\ AC=61.04.56\end{matrix}\right\}$ req. *BC*.



The *Stereographical* Projection of this *Caſe* is the ſame with the 2*d Example* of the 3*d Caſe*, and *BC* may be meaſured by *Caſe* the 1*ſt*, of *Problem* the 7*th*, *Section* the 2*d*.

And to find the length by Calculation,



Produce *AC* and *AB* to *E* and *D*, till *AD* and *AE* are Quadrants, and draw the Arch *DE*, which will be equal to the Meaſure of the Angle at *A*.

Then in the 2 Spherical Triangles *ADE* and *ABC*, Right-angled at *B* and *D*, and having the ſame Acute Angle *A* at the Baſe, are given *AC*, and *DE* equal to the Meaſure of the Angle at *A*, whence to find *BC* it will be by *Ax.* the 1*ſt*.

As the S. $AE=R$: S. AC :: S. $(DE=)\angle A$: S. BC. That is,

As the Radius	10.0000000
To the Sine of the Hypothenuſe *AC*=61.04.56	9.9421641
So is the Sine of the Angle at the Baſe *A*=47.54.20	9.8704278
To the Sine of the Perpendicular *BC*=40.30.20	9.8125919

Or,

As the Secant of the Hypoth. AC=61.04.61 ——— 10.0578359

To the Radius ——— 10.0000000
So is the Sine of the Angle at the Baſe A=47.50.20. —9.8704278

To the Sine of the Perpendicular BC=40·30.20——9.8125919

Or,

As the Co-ſecant of the Angle at the Baſe A=47.54.20 10.1295722

To the Radius——— 10.0000000
So is the Sine of the Hypothenuſe AC=61.04.56 ——9.9421641

To the Sine of the Perpendicular BC=40.30.20——9.8125919

The firſt of theſe Proportions is according to the Lord *Napiers Theorem.*

Caſe 6.

Given one Side and an Angle adjacent, to find the Hypothenuſe.

Or,

Given the Perpendicular and the Angle at the Perpendicular, to find the Hypothenuſe.

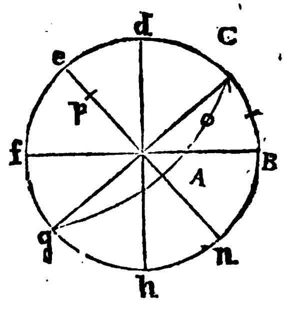
Example.

In the Spherical Triangle ABC, Right-angled at B,

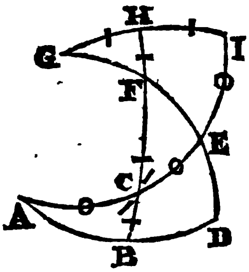
are given $\left\{\begin{array}{l}\angle C=61.50.29\\ BC=40.30.20\end{array}\right\}$ req. AC.

The Stereographic Solution.

1. Having drawn the Primitive Circle f d B h, the Diameter f B, and at Right-angles to it d h, ſet off the Chord of BC equal to 40 deg. 30. min. 20 ſec. from B to C, and draw the Diameter C g, and and at Right-angles to it e n.



2. Draw the Circle C A g forming an Angle with the Primitive Circle in the Point C, equal to 61 deg. 50 min. 29 ſec. by *Caſe* the 3*d* of *Prob.* the 9*th*, *Sect.* the 2*d*. and the thing is done; and AC may be meaſured by *Caſe* the 3*d* of *Prob.* the 7*th*, *Sect.* the 2*d*.



Logarithmically, or by Calculation.

Having produced the Sides AB, AC and BC, to D, E, and F, till AD, AE, BF, are Quadrants, *&c* as in the former Caſes, in the two Spherical Triangles GIE and GHF, Right-angled at H and I, and having the ſame Acute Angle G at the Baſe common to both, we have given GH equal to the Complement of HI the Meaſure of the Angle at C, HF equal to CB, whence to find IE equal to AC, it will be by *Ax*. the 2*d*.

As S. (GH=)Comp. of (HI)=<C : (S. GI=) R : : t, (HF=) BC : t, (IE=) AC. That is,

As the Co-ſine of the Angle at the Perp. C=61.50.29——9.6738628

To the Radius———————————— 10.0000000

So is the Tangent of the Perpendicular BC=40.30.20 —9.9315842

To the Tangent of the Hypothenuſe AC 61.04.56 10.2577214

Or,

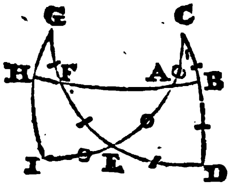
As the Radius————————————10.0000000

To the Secant of the Angle at the Perp. C=60.50.29 10.3261372

So is the Tangent of the Perpendicular BC=40.30 20 —9.9315842

To the Tangent of the Hypothenuſe AC=61.04.56 10.2577214

Or,



Having produced the Sides CB, CA and BA, *&c*. as in the 1*ſt Example* of *Caſe* the 3*d*, in the two Spherical Triangles FDB and FEA Right-angled at D and E, and having the ſame Acute Angle F at the Baſe common to both, we have given FE equal to the Complement of ED the Meaſure of the Angle at C, BD equal to the Complement of the Perpendicular BC, whence to find EA equal to the Complement of the Hypothenuſe AC, it will be by *Ax*, the 2*d*.

As (S.FD=)R : S.(FE=)Comp.(ED=)<C : : t,(DB=)Comp. BC : t,(EA=)Comp. AC. That is,

As the Radius —————— 10.0000000

To the Co-ſine of the Angle at the Perp.C=*61.50. 299.6738628*
So is the Co-tangent of the Perpend. BC=40.30.20—10.0684158

To the Co-tangent of the HypothenuſeAC=*61.04 56 9.7422786*

Or,

According to *Napiers Theorem*, becauſe the Angle at the PerpendicularC is theMiddle Part, and the Perpendicular BCand the Hypothenuſe AC Extreams Adjunct, and becauſe the Hypothenuſe is required it will be,

As the T. BC : R : : cs,<C : ct, AC. That is,
As the Tangent of the Perpendicular BC=40.30.20 ——*9.9315842*

To the Radius—————— 10.0000000
So is the Co-ſine of the Angle at the Perpend.C=*61.50.29 9.6738628*

To theCo-tangent of the HypothenuſeAC=*61.04.56 9.7422786*

Caſe *6*.

Example 2.

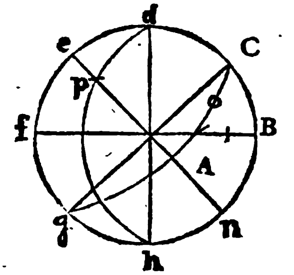
Given the Baſe and the Angle at the Baſe, to find the Hypothenuſe. That is,

In the Spherical Triangle ABC, Right-angled at B.

are given $\left\{\begin{array}{l} AB=50.30.30 \\ <A=47.54.20 \end{array}\right\}$ req. AC.

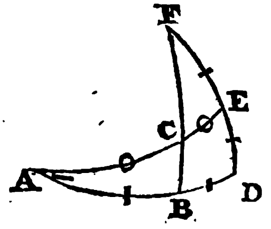
The Stereographic Solution.

1. Having drawn the Primitive Circle f d B h, the Diameter f B, and at Right-angles to it d h, ſet off 50 deg. 30 min. 30 ſec. the Length of AB from B to A, by *Caſe* the *2d* of *Prob.* the *6th Sect.* the *2d.*



2. Thro' the Point A draw the Oblique Circle CA g forming an Angle with the Right Circle f AB, equal to 47 deg. 54 min. 20 ſec. by *Caſe* the 2*d* of *Prob.* the 9*th* *Sect.* the 2*d*, and the thing is done; and AC may be meaſured by *Caſe* the 3*d* of *Prob.* the 7*th*, *Sect.* the 2*d*.

But to determine its Length by Calculation.



Having produced the Sides AB, AC, *&c.* as in *Caſe* the 1*ſt*, in the two Spherical Triangles FDB and FEC, Right-angled at D and E, and having the ſame Acute Angle at the Baſe F common to both, are given FE equal to the Complement of ED the Meaſure of the Angle at A, DB equal to the Complement of the Baſe AB, whence to find CE equal to the Complement of the Hypothenuſe AC, it will be by *Ax.* the 2*d*.

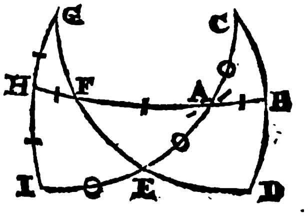
As S. (FD=)R : S.(FE=)Comp.(ED=)<A: : t,(DB=)Comp. AB : t, (CE=) Comp. of AC. That is,

As the Radius——————————10.0000000

To the Co-ſine of the Angle at the Baſe A=47.54.20 —9.8263046
So is the Co-tangent of the Baſe AB=50.30.20———9 9159758

To the Co-tangent of the HypothenuſeAC=61.04.56 9.7422804

Or,



Having produced the Sides CA, CB, *&c.* and made the ſame Preparation as in the 1*ſt* *Example* of *Caſe* the 3*d*.

In the two Spherical Triangles GHF and GIE, Right-angled at H and I, and having the ſame Acute Angle G at

G at the Baſe common to both, we have given GH equal to the Complement of HI equal to the Angle A, HF equal to AB, whence to find IE equal to AC it will be by *Ax.* the *2d.*

As S. (GH=) Comp. (HI=) < A : R : : t. (HF=) AB : t, (IE=) AC. That is,

As the Co-ſine of the Angle at the Baſe A=47.54.20 —9.8263046

To the Radius ———————————————— 10.0000000
So is the Tangent of the Baſe AB=50.30.30———10.0840242

To the Tangent of the Hypothenuſe AC=61.04.56 10.2577196

Or,

As the Radius———————————————— 10.0000000

To the Secant of the Angle at the Baſe AB=47.54.20 10.1736954
So is the Tangent of the Baſe AB=50.30.30 ———10.0840242

To the Tangent of the Hypothenuſe AC=61.04.56 10.2577196

Or,

According to *Napiers Theorem*, becauſe the Angle at the Baſe A is the Middle Part, and AB the Baſe, and AC the Hypothnuſe Extreams Adjunct, and the Hypothenuſe required, it will be,

As t, AB : R : : c s, < A : c t, AC. That is,

As the Tangent of the Baſe AB=50.30.30————10.0840242

To the Radius ———————————————10.0000000
So is the Co-ſine of the Angle at the Baſe A=47.54.20—9.8263046

To the Co-tangent of the Hypoth. AC=61.04.56—9.7422804

Caſe 7.

Given one Side and an Angle adjacent to find the other oppoſite Angle.

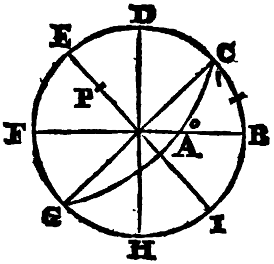
Or,

Given the Perpendicular and the Angle at the Perpendicular, to find the Angle at the Baſe.

Example

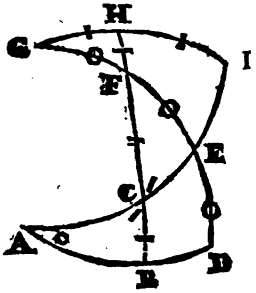
Example.

In the Spherical Triangle *ABC*, Right-angled at *B*, are given, $\left\{\begin{array}{l} BC=40.30.20 \\ <C=61.50.29 \end{array}\right\}$ req. $<A$.



The *Stereographical* Projection of this Caſe is the ſame with the 1ſt *Example* of the *6th Caſe*, and the Angle at *A*, may be meaſured by *Caſe* the 3*d* of *Prob.* the 10*th Sect.* the 2*d.*

Logarithmically, or by *Calculation.*



Having produced the Sides *AB*, *AC*, *BC*, &c. and made the ſame Preparation as in the 3*d Caſe*, in the two Spherical Triangles *CIH* and *CEF*, Right-angled at *E* and *I*, and having the ſame Acute Angle *C* at the Baſe common to both, we have given *CF* equal to the Complement of *BC*, *HI* equal to the Meaſure of the Angle at *C*, whence to find *FE* equal to the Complement of *ED* the Meaſure of the Angle at *A*, it will be by *Ax.* the 1ſt.

As S.(*CH*=)*R* : S.(*CF*=(Comp.)*BC* : : S.(*HI*=)<*C* : S.(*FE*=) Comp.(*ED*=)<*A*. That is,

As the Radius —————————— 10.0000000

To the Co-ſine of the Perp. *BC*=40.30.20 ———9.8810095
So is the Sine of the Angle at the Perpend.=61.50.29 —9.9452936

To the Co-ſine of the Angle at the Baſe *A*=47.54.20 9.8263031

Or,

As the Secant of the Perpendicular *BC*=40.30.20 —— 10.1189905

To the Radius —— —— —— —— —— —— 10.0000000
So is the Sine of the Angle at the Perpend.*C*=61.50.29 9.9452936

To the Co-ſine of the Angle at the Baſe *A*=47.54.20 9.8263031

Or,

As the Co-ſecant of the Angle at the Perp. *C*=61.40.29 10.0547064

To the Radius —————————— 10.0000000
So is the Co-ſine of the Perp. *BC*=40.30.20 ———— 9.8810095

To the Co-ſine of the Angle at the Baſe *A*=47.54.20 9.8263031

Caſe 7.

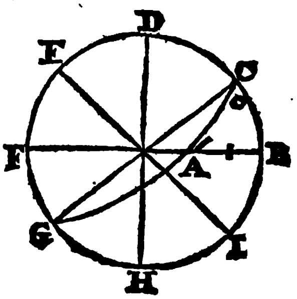
Example 2.

Given the Baſe and the Angle at the Baſe, to find the Angle at the Perpendicular. That is,

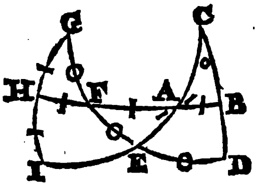
In the Spherical Triangle *ABC*, Right-angled at *B*.

are given $\left\{\begin{array}{l} AB=50.30.30 \\ <A=47.54.20 \end{array}\right\}$ req. $<C$.

The *Stereographical* Projection of this *Caſe* is the ſame with the 2*d Example* of the 6*th Caſe*, and the Angle at *C* may be meaſured by *Caſe* the 2*d* of *Prob.* the 10*th Sect.* the 2*d.*



Logarithmically, or by *Calculation.*



Having produced the Sides *CB*, *CA*, &c. and made the ſame Preparation as in *Caſe* the 3*d*, in the two Spherical Triangles *AHI* and *AFE* Right-angled at *I* and *E*, and having the ſame Acute Angle at the Baſe *A* common to both, are given *AF* equal to the Complement of *AB*, *HI* equal to the Angle at *A*, whence to find *FE* equal to the Complement of *ED* the Meaſure of the Angle at *C*, it will be by *Ax*. the 1*ſt*.

As (S. $AH=$)R : S.($AF=$)Comp.AB : : S·($HI=$)$<A$: S·($FE=$) Comp.($ED=$)$<C$. That is,

As the Radius ———————— 10.0000000

To the Co-ſine of the Baſe $AB=50{\cdot}30.30$ ——— 9.8034338

So is the Sine of the Angle at the Baſe $A=47{\cdot}54{\cdot}20$ —— 9.8704278

To the Co-ſine of the Angle at the Perp. $C=61.50.29$ 9.67338616

Or,

As the Secant of the Baſe $AB=50.30{\cdot}30$ ——— 10.1965662

To the Radius ———————— 10.0000000

So is the Sine of the Angle at the Baſe $A=47{\cdot}54{\cdot}20$ —— 9 8704278

To the Co-ſine of the Angle at the Perp. $C=61{\cdot}50.26$ 9.67338616

Or,

As the Co-ſecant of the Angle at the Baſe $A=47.54{\cdot}20$ 10.1295722

To the Radius ———————— 10.0000000

So is the Co-ſine of the Baſe $AB=50\ 30.30$ ——— 9.8034338

To the Co-ſine of the Angle at the Baſe $C=61{\cdot}50.29$ 9.67338616

Caſe

Caſe 8.

Given one Side and an Angle adjacent to find the other oppoſite Side.

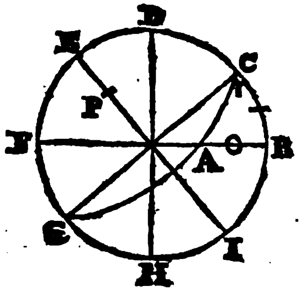
Or,

Given the Perpendicular and the Angle at the Perpendicular, to find the Baſe.

Example

In the Spherical Triangle *ABC*, Right-angled at *B*,

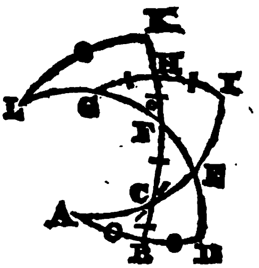
$$\text{are given} \left\{ \begin{array}{l} BC=40.30.20 \\ <C=61.50.29 \end{array} \right\} \text{req. } AB.$$



This *Caſe* is Conſtructed like the 1*ſt* *Example* of the *6th Caſe*, and the Baſe *AB* may be meaſured by the 2*d* *Caſe* of *Problem* the *7th*, *Section* the 2*d*.

Logarithmically, or by *Calculation*.

Having produced the Sides *AB*, *AC*, &c. as in *Caſe* the 3*d*, produce the Sides *FH* and *FG* of the Triangle *FHG*, to *K* and *L*, till *FK* and *FL* are Quadrants, and joyn the Points *L* and *K*, by the Arch of a Great Circle, then in the two Spherical Triangles *FHG* and *FKL*, Right-angled at *H* and *K*, and having the ſame Acute Angle at the Baſe *F* common to both, are given *FH* equal to the Complement of *FC*, equal to *CB*; *GH* equal to the Complement of *HI*, equal to the Angle at *C*, whence to find *LK* equal to the Angle at *F*, equal to *DB* equal to the Complement of *AB*, it will be by *Ax*. the 2*d*.



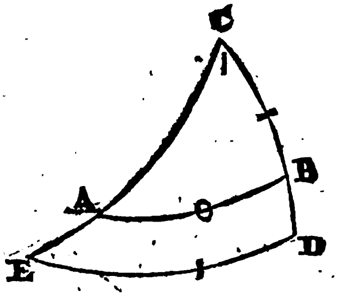
As S. (*FH*=) *CB* : *R* :: t, (*GH*=) Comp, (*HI*=) $<C$: t, (*LK*= $<F$=*BD*=) Comp. *AB*. That is,

As the Sine of the Perpendicular *CB*=40.30.20 ——— 9.8125937

To the Radius ——————————— 10.0000000
So is the Co-tang. of the < at the Perpend. *C*=61.50.29 9.2285693

To the Co-tangent of the Baſe *AB*=50.30.30 ——— 9.4159756

Or,



Having produced the Sides *CB*, *CA*; to *D* and *E*, till *CD* and *CE* are Quadrants, and joyned the Points *E* and *D* by the Arch *ED*, in the two Spherical Triangles *ABC*, *EDC*, Right-angled at *B* and *D*, and having the ſame Acute Angle at *C* common to both, are given *BC* and *ED* equal to the Meaſure of the Angle at *C*; whence to find *AB*, it will be by *Ax.* the 2*d*.

As (S. *CD*=)*R* : S. *CB* : : t, (*ED*=) < *C* : t, *AB*. That is,
As the Radius ——————————— 10.0000000

To the Sine of the Perpendicular *BC*=40.30.20 ——— 9.8125937
So is the Tangent of the Angle at the Perp. *C*=61.50.29 10.2714307

To the Tangent of the Baſe *AB*=50.30.30 ——— 10.0840244

Or,

According to *Napiers Theorem*, becauſe *BC* is the Middle Part and a Sine, and the Angle *C* and *AB* the Baſe Extreams Adjunct, and conſequently Tangents, it will be,

As c t, < *C* : *R* : : S, *BC* : t, *AB*. That is,

As the Co-tang. of the Angle at the Perp. *C*=61.50.29 9.7285693

To the Radius ——————————— 10.0000000
So is the Sine of the Perpendicular *BC*=40.30.20 ——— 9.8125937

To the Tangent of the Baſe *AB*=50.30.30 ——— 10.0840244

Caſe

Caſe 8.

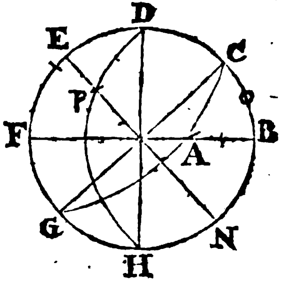
Example 2.

Given the Baſe and the Angle at the Baſe, to find the Perpendicular. That is,

In the Spherical Triangle *ABC*, Right-angled at *B*.

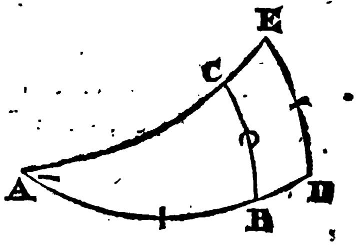
$$\text{are given}\begin{cases}AB=50.30.30\\ <A=47.54.20\end{cases}\text{req. } BC.$$

This *Caſe* is Conſtructed like the 2*d* *Example* of the *6th Caſe*, and *BC* may be meaſured by *Caſe* the 1*ſt*, of *Prob* the 7*th*, *Section* the 2*d*.



Logarithmically, or by *Calculation.*

Produce the Sides *AB* and *AC*, to *D* and *E*, till *AD* and *AE* are Quadrants, and joyn the Points *D* and *E* by the Arch *DE*, then in the two Spherical Triangles *ABC* and *ADE*, Right-angled at *B* and *D*, and having the ſame Acute Angle at *A* common to both, are given, *AB* and *DE*, equal to the Angle at *A*, whence to find *BC*, it will be by *Ax*. the 2*d*.



As (S. $AD=$)R : S. AB : : t, ($DE=$) $<A$: t, BC. That is;

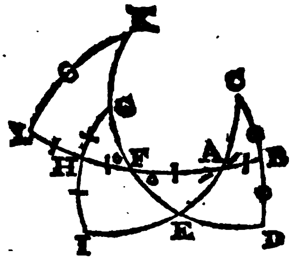
As the Radius —————————— 10.0000000

To the Sine of the Baſe $AB=50.30.30$ ———— 9.8874582

So is the Tangent of the Angle at the Baſe $A=47.54.20$ 10.0441265

To the Tangent of the Perpendicular $BC=40.30.20$ — 9.9315847

Or,



Having produced the Sides *CB*, *CA*, &c. as in *Caſe* the 3*d*, produce the Sides *FG* and *FH*, to *L* and *K*, and joyn the Points *L*, *K*, by the Arch *LK*, then in the two Spherical Triangles *FHG* and *FLK*, Right-angled at *H* and *L*, and having the ſame Acute Angle at *F* common, are given *HF* equal to the Complement of *FA* equal to *AB*, *GH* equal to the Complement of *HI* equal to the Angle at *A*, whence to find *KL*, equal to the Angle *F*, equal to *BD*, the Complement of *CB*, it will be by *Ax*. the 2*d*.

As S. (*FH*=Comp.*FA*=)*AB* : (S.*FL*=)*R* : : t,(*GH*=)Comp.(*HI* =)<*A* : t, (*KL*=<*F*=*BD*=)Comp. *BC*. That is,

As the Sine of the Baſe *AB*=50.30.30 ———————— 9.8874582

To the Radius ———————————————— 10.0000000

So is the Co-tang. of the Angle at the Baſe *A*=47.54.20 9.9558735

To the Co-tangent of the Perpend *BC*=40.30.30 — 10.0684153

Or,

According to *Napiers* Rule, becauſe *AB* is the Middle Part, and the Angle *A* and Perpendicular *BC* Extreams Adjunct, it will be,

As c t,<*A* : *R* : : S. *AB* : t, *BC*. That is,

As the Co-tangent of the Angle at the Baſe *A*=47.54.20 9.9558735

To the Radius ———————————————— 10.0000000

So is the Sine of the Baſe *AB*=50.30 30 ———————— 9.8874582

To the Tangent of the Perpendicular *BC*=40.30.20 10.9315847

Caſe 9.

Given one Side and an Angle oppoſite, to find the Side adjacent.

Or,

Given the Angle at the Baſe and the Perpendicular, to find the Baſe.

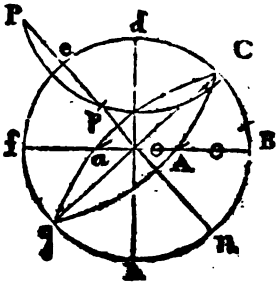
Ex-

Example.

In the Spherical Triangle *ABC*, Right-angled at *B*,

are given, $\left\{ \begin{array}{l} < A = 47.54.20 \\ BC = 40.30.20 \end{array} \right\}$ req. *AB*.

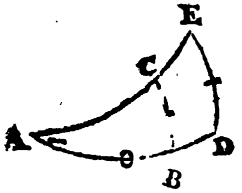
The Stereographic Solution.



1. Having drawn the Primitive Circle *fdBb*, the Diameter *fB*, and at Right-angles to it *db*, ſet off the Chord of *BC* 40 deg. 30 min. 20 ſec. from *B* to *C*, and draw the Diameter *Cg*, and at Right-angles to it *en*.

2. About the Point *d* as a Pole, deſcribe a Small Circle as *Pp*, equal to the Angle *A*, equal to 47 deg. 54 min. 20 ſec. (by *Caſe* the 2*d* of *Problem* the 4*th*, *Section* the 2*d*.) where this interſects the Great Circle *en* as in *P* and *p*, it will give the Poles of the Circles *CAg*, *Cag*, to be drawn, about which if the Circles *CAg* and *Cag* be drawn, (by *Caſe* the 3*d* of *Problem* the 2*d*, of *Sect.* the 2*d*.) the thing is done.

This *Caſe* is doubtful, as appears from the *Conſtruction*, for the Parallel *Pp* cuts the Great Circle *en* in the Points *P*, and *p*, the Poles of the Circles *CAg*, and *Cag*, whence it is manifeſt, that in each of the Triangles *CBA*, *CBa*, the Side *BC* is common to both, and the Angle *A* in the Triangle *BAC* is equal to the Angle *a* in the little Triangle *BaC*, and conſequently *aB* the Baſe of the Triangle *aBC*, is the Supplement of *AB* the Baſe of the Triangle *ABC*, (for the Triangles *FaG* and *CAB* are equal) to find which by Calculation,



Produce the Sides *AB*, and *AC*, to *D* and *E*, till *AD* and *AE* are Quadrants, and draw the Arch *DE*, then in the two Triangles *ADE* and *ABC*, Right-angled at *B* and *D*, &c. are given *BC*, and *DE*, the Meaſure of the Angle at *A*, whence to find *AB* it will be,

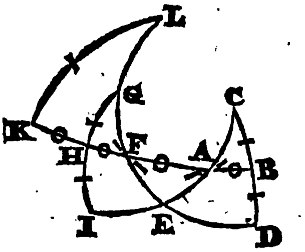
As t, (DE=) <A : t, BC : : (S. DA=) R : S. AB. That is,
As the Tangent of the Angle at the Baſe A=47.54.20 10.0441265

To the Tangent of the Perpendicular BC=40.30.20 9.9315842
So is the Radius——————————10.0000000

To the Sine of the Baſe AB=50.30.30——————9.8874577

Whoſe Supplement 129.29.30, is equal to the Baſe AB.

Or,



Having produced the Sides CB, CA, and BA, *&c.* as in the 2*d Example* of the 8*th Caſe*, in the two Spherical Triangles LKF and GHF, Right-angled at K and H, *&c* are given KL equal to the Angle F, equal to BD the Complement of CB, GH equal to the Complement of HI, the Meaſure of the Angle at A, whence to find HF, equal to the Complement of FA, equal to AB, it will be by *Ax.* the 2*d.*

As t, (LK= <F=BD=) Comp. CB : t, (GH=) Comp. (HI) = <A : : (S. KF=) R : S. (HF=Comp. FA=) AB. That is,
As the Co-tangent of the Perpend. BC=40.30.20 —10.0684158

To the Tangent of the Angle at the Baſe A=47.54.20 9.9558735
So is the Radius—————————— 10.0000000

To the Sine of the Baſe AB=50.30.30——————9.8874577

Or,

According to *Napiers Theorem*, becauſe AB is the Middle Part and required, and the Angle at A and BC are Extreams Adjunct it will be, R : c t, <A : : t, BC : S. AB. That is,
As the Radius —————————— 10.0000000

To the Co-tang. of the Angle at the Baſe A=47.54.20 9.9558735
So is the Tangent of the Perpendicular BC=40.30.20—9.9315842

To the Sine of the Baſe AB=50.30.30——————9.8874577

Caſe

Caſe 9.

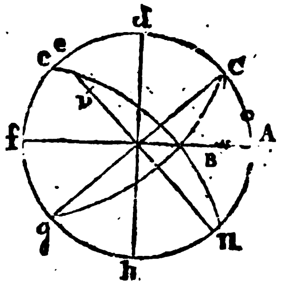
Example 2.

Given the Baſe and the Angle at the Perpendicular, to find the Perpendicular. That is,
In the Spherical Triangle ABC, Right-angled at B.

are given $\left\{ \begin{array}{l} AB = 50.30.30 \\ \angle C = 61.50.29 \end{array} \right\}$ req. BC.

The Stereographic Solution.

1. Having drawn the Primitive Circle FDBH, the Diameter FB, and at Right-angles to it DH, Set off AB 50 deg. 30 min. 30 ſec. from B to A (by *Caſe* the 2*d*, of *Problem* the 6*th*, *Sect.* the 2*d*.)

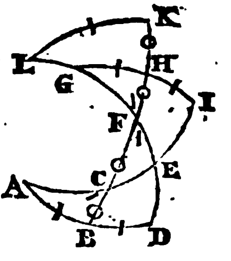


2. About the Point A as a Pole, draw the Great Circle d P p h (by *Caſe* the 3*d*, of *Problem* the 2*d*, *Sect.* the 2*d*.

3. About the Center of the Primitive Circle draw a Parallel of 61 deg. 50 min. 29 ſec. equal to the Angle C, (by *Caſe* the 1*ſt* of *Problem* the 4*th*, *Sect.* the 2*d*.) where this interſects the Great Circle d P p h, in the Points P and p, it gives the Poles of the Circles CA g and c A g, to be drawn.

This *Caſe* is likewiſe Ambiguous, as appears from the Conſtruction it ſelf, and B c is the Supplement of BC, for the Triangles CBA and c BA are equal, and to find it by Calculation,

Having produced the Sides AB, AC, *&c.* as in the 2*d Example*, of the 8*th Caſe*.



In the two Spherical Triangles LKF and GHF, Right-angled at K and H, *&c.* are given LK equal to the Angle at F, equal to BD, equal to the Complement of AB, GH equal to the Complement of HI, equal to the Angle C, whence to find HF equal to the

Complement of FC, equal to CB; it will be, by *Ax.* the 2*d*.
As (t, LK=<F=BD=) Comp. AB : t, (GH=) Comp. (HI=) <C : : (S. KF=)R : S.(HF=Comp. FC=) CB. That is,

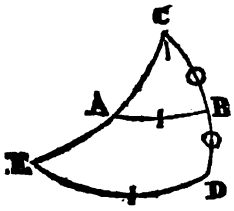
As the Co-tangent of the Baſe AB=50.30.30 ——— 9.9159758

To the Co-tang. of the Angle at the Perp.C=61.50.29 9.7285693
So is the Radius ——————————— —10.0000000

To the Sine of the Perpendicular BC=40 30.20 ——9.8125935

Whoſe Supplement 139.29.40 is the Length of CB.

Or,



Produce the Sides CA, CB, *&c.* as in *Caſe* the 2*d*, then in the two Spherical Triangles ACB and ECD, Right-angled at D and B, *&c.* are given AB and ED, equal to the Angle at C, whence to find BC it will be by *Ax.* the 2*d*.

As t, (ED=)<C : R : : t, AB : S. BC. That is,

As the Tangent of the < at the Perp. C=61.50.29 ——10.2714307

To the Radius ——————————10.0000000
So is the Tangent of the Baſe AB=50.30.30————10.0840242

To the Sine of the Perpendicular BC=40.30.20————9.8125935

Or,

According to *Napiers Theorem*, becauſe BC is the Middle Part and required, and AB and <C Extreams Adjunct, it will be,

As R : t, AB : : c t, <C : t, BC. That is,

As the Radius ———————————10.0000000

To the Tangent of the Baſe AB=50.30.30 ————10.0840242
So is the Co-tang. of the Angle at the Perp.,C=61.50.29 9.7285693

To the Sine of the Perpendicular BC=40.30.20 ——9.8125935

Caſe

Caſe 10.

Given one Side or Leg, and an Angle oppoſite, to find the Hypothenuſe.

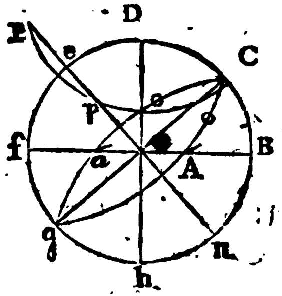
Or,

Given the Perpendicular and the Angle at the Baſe, to find the Hypothenuſe.

Example.

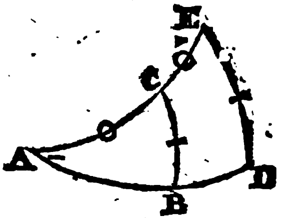
In the Spherical Triangle *ABC*, Right-angled at *B*,

are given, $\left\{ \begin{matrix} \angle A = 47.54.20 \\ BC = 40.30.20 \end{matrix} \right\}$ req. *AC*.



This *Caſe* is doubtful, and is projected like the *1ſt Example* of the *9th Caſe*, and *AC* or its Supplement *aC* may be meaſured by the *3d Caſe* of the *7th Problem*, of *Sect.* the *2d.*

Logarithmically, or by *Calculation*.



Produce the Sides *AB* and *AC* to *D* and *E*, till *AD* and *AE* are Quadrants, *&c.* then in the two Spherical Triangles *ADE* and *ABC*, Right-angled at *B* and *D*, *&c*, it will be,

As S. (*DE*=) $\angle A$: S. *BC* : : (S. *EA*=) *R* : S. *AC*. That is,

As the Sine of the Angle at the Baſe *A*=47.54.20 ——— 9.8704278

To the Sine of the Perpend. *BC*=40.30.20 ——— 9.8125937
So is the Radius ——— 10.0000000

To the Sine of the Hypothenuſe *AC*=61.04.56 ——— 9.9421659

Whoſe

Whoſe Supplement 118.55.04 is the Length of *a C.*

Or,

As the Radius ——————————— 10.0000000

To the Co-ſec. of the Angle at the Baſe *A*=47.54.20 10.1295722
So is the Sine of the Perpendicular *BC*=40.30.20 ——— 9.8125937

To the Sine of the Hypothenuſe *AC*=61.04.56 ——— 9.9421659

Or,

As the Secant of the Perpendicular *BC*=40.30.20 ——— 10.1874063

To the Radius ——————— 10.0000000
So is the Co-ſecant of the Angle at the Baſe *A*=47.54.20 10.1295722

To the Sine of the Hypothenuſe *AC*=61.04.56 ——— 9.9421659

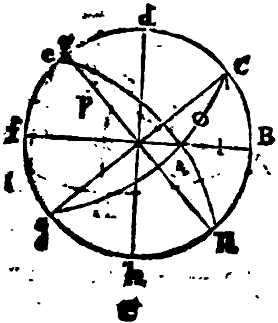
Caſe 10.

Example 2.

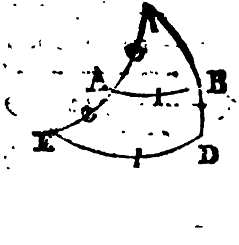
Given the Baſe and the Angle at the Perpendicular, to find the Hypothenuſe. That is,

In the Spherical Triangle *ABC*, Right-angled at *B*.

are given $\left\{\begin{matrix} AB=50.30.30 \\ <C=61.50.29 \end{matrix}\right\}$ req. *AC*.



This *Caſe* is doubtful, and is Conſtructed like the 2*d* *Example* of the 9*th* *Caſe*, and *AC* or its Supplement *A c* may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, *Section* the 2*d*.



Logarithmically, or by *Calculation*.

Produce the Sides *CA*. *CB*, &c. as in *Caſe* the 2*d*, then in the two Spherical Triangles *CDE* and *CBA*, &c are given, *AB* and *DE* the Meaſure of the Angle at *C*, whence to find *AC* it will be by *Ax.* the 2*d.*

S. (*DE*=) <*C* : *R* : : S. *BA* : S. *AC*. That is,
As the Sine of the Angle at the Perpend. *C*=61.04.56——9.9452936

To the Radius ————————————10.0000000
So is the Sine of the Baſe *AB*=50.30.30 ————9.8874582

To the Sine of the Hypothenuſe *AC*=61.04.56——9.9421646

Whoſe Supplement 118.55.04, is the Meaſure of the Hypthenuſe *A c*.

Which is the ſame Proportion with that deduced from *Napiers Theorem*.

Or,

As the Radius————————————10.0000000

To the Co-ſec. of the Angle at the Perp. *C*=61.04.56 10.0547064
So is the Sine of the Baſe *AB*=50.30.30 ————9.8874582

To the Sine of the Hypothenuſe *AC*=61.04.56——9.9421646

Or,

As the Co-ſecant of the Baſe *AB*=50.30.30 ————10.1125418

To the Radius ————————————10.0000000
So is the Co-ſec. of the Angle at the Perp. *C*=61.04.56 10.0547064

To the Sine of the Hypothenuſe *AC*=61.04.56——9.9421646

Caſe 11.

Given one Side and an Angle oppoſite, to find the other Angle.

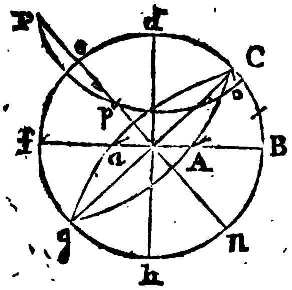
Or,

Given the Perpendicular and the Angle at the Baſe to find the Angle at the Perpendicular.

Example

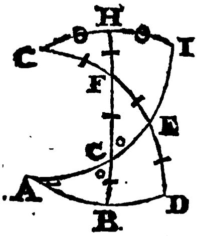
In the Spherical Triangle *ABC*, Right-angled at *B*.

are given $\left\{\begin{array}{l}\angle A=47.54.20\\ BC=40.30.20\end{array}\right\}$ req. <*C*.



This *Caſe* is doubtful, and is projected like the *9th Caſe*, and the Angle *ACB* the Supplement of *a CB* may be meaſured (by *Caſe* the *2d* of *Problem* the *10th*, *Sect.* the *2d*,

Logarithmically, or by *Calculation*.



Having produced the Sides *AB*, *AC*, &c. as in *Caſe* the *3d*; in the two Spherical Triangles *CEF* and *CIH*, Right-angled at *E* and *F*, and having the Angle at *C* common to both, are given *CF* equal to the Complement of *CB*, *FE* equal to the Complement of *ED*, the Meaſure of the Angle at *A*, whence to find *HI* the Meaſure of the Angle at *C*, it will be by *Ax.* the *1ſt*.

As S. (*CF*= Comp.) *CB* : *R* : : S. (*FE*=) Comp. (*ED*=) < *A* : S. (*IH*=) < *C*. That is,

As the Co-ſine of the Perpendicular *BC*=40.30.20——9.8810095

To the Radius——————10.0000000

So is the Co-ſine of the Angle at the Baſe *A*=47.54.20 9.8263046

To the Sine of the Angle at the Perp. *C*=61.50.29——9.9452951

Whoſe Supplement 118.09.31, is equal to the Angle *C* of the Triangle *a CB*, and is the ſame Proportion with that deduced from *Napiers Theorem*.

Or,

As the Radius——————10.0000000

To the Secant of the Perpendicular *BC*=40.30.20 —10.1189905

So is the Co-ſine of the Angle at the Baſe *A*=47.54.20—9.8263046

To the Sine of the Angle at the Perp. *C*=61.50.29——9.9452951

Or,

As the Secant of the Angle at the Baſe *A*=47.54.20 ——10.1736954

To the Radius———————————————10.0000000

So is the Secant of the Perpendicular *BC*=40.30.20——10.1189905

To the Sine of the Angle at the Perp. *C*=61.50.29 — 9.9452951

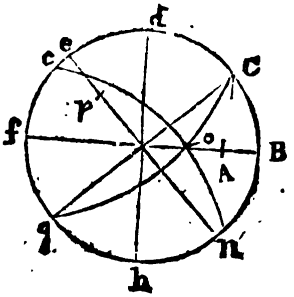
Caſe 11:

Example 2.

Given the Baſe and the Angle at the Perpendicular, to find the Angle at the Baſe. That is,
In the Spherical Triangle ABC, Right-angled at B,

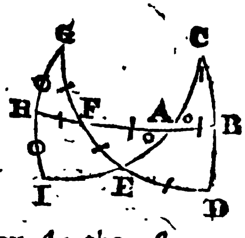
$$\text{are given} \left\{ \begin{array}{l} AB=50.30.30 \\ <C=61.50.29 \end{array} \right\} \text{req.} <A.$$

This *Caſe* is doubtful, and is Conſtructed like the 2*d* *Example* of the 9*th* *Caſe*, and the Angle *CAB* the Supplement of *c AB* may be meaſured (by the 3*d* *Caſe* of the 10*th* *Problem*, *Section* the 2*d*.)



Logarithmically, or by *Calculation*.

Having produced the Sides CB, CA, *&c.* as in *Caſe* the 3*d*, in the two Spherical Triangles AIH and AEF, Right-angled at E and I, and having the Acute Angle at A common, are given AF equal to the Complement of AB, FE equal to the Complement of *ED*, equal to the Angle at *C*, whence to find HI equal to the Angle at A, it will be by *Ax.* the 1*ſt*.



As S. (AF=) Comp. AB : (S. AH=) R : : S. (FE=) Comp. (ED=) <C. : : S. (HI=) <A. That is,

As the Co-ſine of the Baſe AB=50.30.30 —— 9.8034338

To the Radius —— 10.0000000
So is the Co-ſine of the Angle at the Perp. C=61.50.29 9.6738628

To the Sine of the Angle at the Baſe A=47.54.20 —— 9.8704290

Whoſe Supplement 132.05.40, is the Meaſure of the Angle c AB, the ſame as is deduced from *Napiers Theorem.*

Or,

As the Radius —— 10.0000000

To the Secant of the Baſe AB=50.30.30 —— 10.1965662
So is the Co-ſine of the Angle at the Perp. C=61.50.29 —— 9.6738628

To the Sine of the Angle at the Baſe A=47.54.20 —— 9.8704290

Or,

As the Secant of the Angle at the Perp. C=61.50.29 —— 10.3261372

To the Radius —— 10.0000000
So is the Secant of the Baſe AB=50.30.30 —— 10.1965662

To the Sine of the Angle at the Baſe A=47.54.20 —— 9.8704290

Caſe 12.

Given the Hypothenuſe and one of the Sides or Legs, to find the contained Angle.

Or,

Given the Hypothenuſe and the Perpendicular, to find the Angle at the Perpendicular.

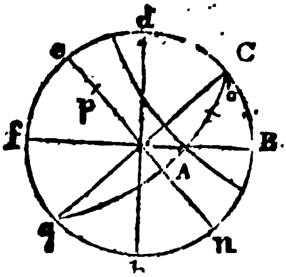
Example.

Example.

In the Spherical Triangle *ABC*, Right-angled at *B*.

are given $\left\{\begin{array}{l} AC=61.04.56 \\ BC=40^{\circ}\ 30'\ 20'' \end{array}\right\}$ req. $<C$.

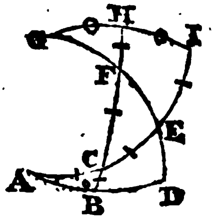
Stereographically.



1. Having drawn the Primitive Circle *f d B b*, the Diameter *f B*, and at Right-angles to it *d b*, ſet off the Chord of 40° 30′ 20″ from *B* to *C*, and draw the Diameter *c g*, and at Right-angles to it *e n*.

2. About the Point *C* as a Pole, deſcribe a Parallel of 61° 04′ 56″ the Length of the Hypothenuſe *AC* (by the *2d Caſe* of the *4th Prob. Section the 2d.*) and thro' the Interſection *A* and the Point *C*, draw a Great Circle (by *Caſe* the *2d* of *Problem* the *1ſt*, *Sect. 2d*) and the thing is done; and the Angle at *C* may be meaſured by *Caſe* the *2d* of *Problem* the *10th*, *Sect.* the *2d*.

Logarithmically, or by *Calculation.*



Having produced the Sides *AC*, *BC*, &c. as in the *3d Caſe*, in the two Spherical Triangles *EIG* and *FHG* are given, *EI* equal to the Complement of *EC*, equal to *AC*, *HF* equal to the Complement of *FC*, equal to *AB*, whence to find *GH* equal to the Complement of *HI*, equal to the Angle at *C*, it will be by *Axiom* the *2d*.

As t, (*EI*=Comp. of *EC*=) *AC* : t, (*FH*=Comp. of *FC*=) *CB* :: S. (*IG*=) *R* : S. (*GH*=) Comp. of (*HI*=) <*C*. That is,

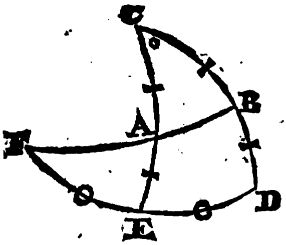
As the Tangent of the Hypothenuſe *AC*=61.04.56 —— 10.2577192

To the Radius———————————————— 10.0000000

So is the Tangent of the Perpend. *BC*=40.30.20 ——— 9.9315842

To the Co-ſine of the Angle at the Perp. C=61.50.29 9.6738650

Or,



Having produced the Sides CB, CA, *&c.* as in *Caſe* the 3*d*, in the two Spherical Triangles FDB, FEC, Right-angled at D and E, are given, DB equal to the Complement of CB, EC equal to the Complement of AC, whence to find FE equal to the Complement of *ED*, the Meaſure of the Angle at C, it will be by *Axiom* the 2*d*.

As t, (BD)=Comp. of CB : t, (EC=) Comp. AC : : (S. FD=) : S. (FE=) Comp. (ED=) <C. That is,

As the Co-tangent of the Perpendicular BC=40.30.20—10.0684158

To the Co-tangent of the Hypoth. AC=*61.04.56*——*9.7422808*
So is the Radius——————10.0000000

To the Co-ſine of the Angle at the Perp. C=*61.50.29* *9.67386*5*0*

Or,

According to *Napiers Theorem,* becauſe the Angle at C is the Middle Part and required, and AC and BC Extreams Adjunct, it will be

R : c t, AC : : t, BC : c t, < C. That is,

As the Radius——————10.0000000

To the Co-tangent of the Hypoth. AC=*61.04.56*—*9.7422808*
So is the Tangent of the Perpendicular BC=40.30.20—*9.9315842*

To the Co-ſine of the Angle at the Perp. C=*61.50.29* *9.6738650*

Caſe 12.

Example 2.

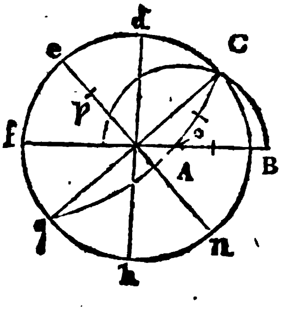
Given the Hypothenuſe and the Baſe, to find the Angle at the Baſe. That is,

In the Spherical Triangle ABC, Right-angled at B.

are given $\left\{\begin{array}{l}AC=61.0456\\AB=50.30.20\end{array}\right\}$ req. <A.

Con-

The Stereographic Solution.

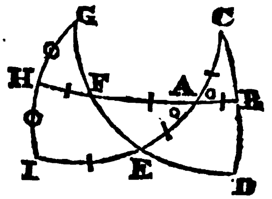


1. Having drawn the Primitive Circle *Bdfh*, the Diameter *fB*, and at Right-angles to it *dh*, ſet off *AB* 50 deg. 30 min. 30 ſec. from *B* to *A*, (by *Caſe* the 2*d* of *Problem* the 6*th*, *Section* the 2*d*.)

2. About the Point *A* as a Pole, deſcribe the Parallel *C B* of 61° 04′ 56″ the Length of the Hypothenuſe (by *Caſe* the 3*d* of *Prob.* the 4*th*, *Sect.* the 2*d*.) to interſect the Primitive Circle in the Point *C*.

3. Thro' the Points *C* and *A* draw the Great Circle *CAg* (by *Prob.* the 1*ſt* of *Caſe* the 2*d*, of *Sect.* the 2*d*.) and the thing is done; and the Angle at *A* may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th*, of *Section* the 2*d*.

Logarithmically, or by *Calculation*.



Having produced the Sides *CB*, *CA*, &c. as in *Caſe* the 3*d*; in the two Spherical Triangles *GIE* and *GHF*, Right-angled at *H* and *I*, and having the Angle at *G* common, are given *HF* equal to the Complement of *FA* equal to *AB*, *IE* equal to the Complement of *EA*, equal to *AC*, whence to find *GH* equal to the Complement of *HI*, equal to the Angle at *A*, it will be by *Ax.* the 2*d*.

As t, (IE= Comp. *AE*=) *AC* : t, (*HF*= Comp. of *FA*=) *AC* : : (S. *IG*=) *R* : S. (*GH*= Comp. of *HI*=) $\angle A$. That is,

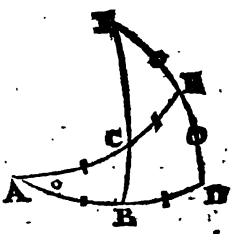
As the Tangent of the Hypothenuſe *AC*=61.04.56 10.2577192

To the Tangent of the Baſe *AB*=50.30.30 ——— 10.0840242

So is the Radius——————————10.0000000

To the Co-ſine of the Angle at the Baſe *A*=47.54.20—9.8263050

Or,



Having produced the Sides *CB*, *CA*, &c. as in *Caſe* the 1*ſt*, in the two Spherical Triangles *FDB*, *FEC*, Right-angled at *E* and *D*, and having the Angle at *F* common, are given *BD* equal to the Complement of *AB*, *EC* equal to the Complement of *AC*, whence to find *FE* equal to the Complement of *ED*,

the

the Meaſure of the Angle at *A*, it will be by *Axiom* the 2*d*:

As t, (*BD*=) Comp. *AB* : t, (*CE*=) Comp. *AC* : : (S. *DF*=) *R*
S. (*FE*=) Comp. of (*ED*=) <*A*. That is,

As the Co-tang. of the Baſe *AB*=50.30.30 ——— 9.9159758

To the Co-tangent of the Hypothenuſe *AC*=61.04.56 — 9.7422808

So is the Radius ——— 10.0000000

To the Co-ſine of the Angle at the Baſe *A*=47.54.20 9.8263050

Or,

According to *Napiers Theorem*, becauſe the Angle *A* is the Middle Part and required, and *AC* and *AB* Extreams Adjunct it will be,

As the *R*: c t, *AC* : : t, *AB* : c s, <*A*. That is,

As the Radius ——— 10.0000000

To the Co-tangent of the Hypothenuſe=61.50.29— 9.7422808

So is the Tangent of the Baſe *AB*=50.30.30 ——— 10.0840242

To the Co-ſine of the Angle at the Baſe *A*=47 54.20 9.8263050

Caſe 13.

Given the Hypothenuſe and one of the Legs, to find the Angle oppoſite.

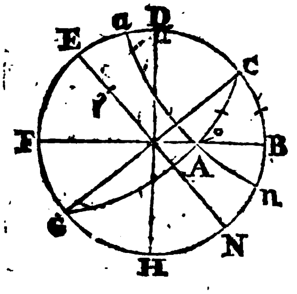
Or,

Given the Hypothenuſe and the Perpendicular, to find the Angle at the Baſe.

Example

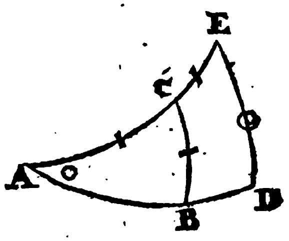
In the Spherical Triangle *ABC*, Right-angled at *B*.

are given { *AC*=61.04.56 / *BC*=40.30.20 } req. <*A*.



This Caſe is Conſtructed after the ſame manner as *Caſe* the 12*th*, and the Angle at *A* may be meaſured by *Caſe* the 3*d* of *Prob.* the 10*th*, of *Sect.* the 2*d*.

Logarithmically, or by *Calculation.*



Having produced the Sides *AB*, *AC*, &c. as in *Caſe* the 2*d*, in the two Spherical Triangles *ADE* and *ABC*, Right-angled at *B* and *D*, and having the ſame Acute Angle at the Baſe *A* common, are given *AC* and *BC*, whence to find *DE* the Meaſure of the Angle at *A*, it will be by *Ax.* the 1*ſt*.

As S. *AC* : S. *CB* : : (S. *AE*=) *R* : S. (*ED*=) < *A*. That is,

As the Sine of the Hypothenuſe AC=61.04.56 ——9.9421641

To the Sine of the Perpendicular BC=40.30.20 —— 9.8125937
So is the Radius —————————— 10.0000000

To the Sine of the Angle at the Baſe A=47.54.20 ——9.8704296

Or,

As the Radius ——————————10.0000000

To the Co-ſecant of the Hypothenuſe AC=61.04.56 — 10.0578359
So is the Sine of the Perpendicular BC=40.30.20 ——9.8125937

To the Sine of the Angle at the Baſe A=47.54.20 ——9.8704296

Or,

As the Co-ſecant of the Perpendicular BC=40.30.20 10.1874063

To the Radius —————————— 10.0000000
So is the Co-ſecant of the Hypoth. AC=61.04.56——10.0578359

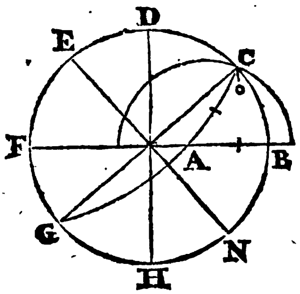
To the Sine of the Angle at the Baſe A=47.54.20——9.8704296

Caſe 13.

Example 2.

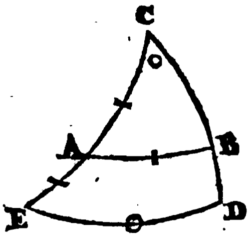
Given the Hypothenuſe and the Baſe, to find the Angle at the Perpendicular. That is, In the Spherical Triangle *ABC*, Right-angled at *B*,

are given $\left\{\begin{matrix}AC=61.04.56\\ AB=50.30.20\end{matrix}\right\}$ req. $<C$.



This *Caſe* is Conſtructed like the 2*d* *Example* of the 12*th* *Caſe*, and the Angle at C, may be meaſured by *Caſe* the 2*d*. of *Problem* the 10*th*, *Section* the 2*d*.

Logarithmically, or by *Calculation*.



Having produced the Sides CB, CA, to D and E, *&c.* as in *Caſe* the 2*d*, in the two Spherical Triangles CDE, CBA, Right-angled at B and D, and having the Angle at C common, are given AC, AB, whence to find ED the Meaſure of the Angle at C, it will be

As S. AC : S. AB : : (S. CE=) R : S. (ED=) <C. That is,

As the Sine of the Hypothenuſe AC=*61.04.56* ——*9.9421641*

To the Sine of the Baſe AB=50.30.30 ————*9.8874582*

So is the Radius ———————— 10.0000000

To the Sine of the Angle at the Perpend. C=*61.50.29*—*9.9452941*

Or,

As the Radius ———————————10.0000000

To the Co-ſecant of the Hypothenuſe CA=*61.04 56*—10.0578359

So is the Sine of the Baſe AB=50.30.30 ————*9.8874582*

To the Sine of the Angle at the Perpend. C=*61.50.29*—*9.9452941*

Or,

As the Co-ſecant of the Baſe AB=50.30.30 ——— 10.1125418

To the Radius ——————————— 10.0000000

So is the Co-ſecant of the Hypoth. AC=61.04.56—10.0578359

To the Sine of the Angle at the Perpend. C=61.50.29 9.9452941

Caſe 14.

Given the Hypothenuſe and one of the Legs, to find the other Leg.

Or,

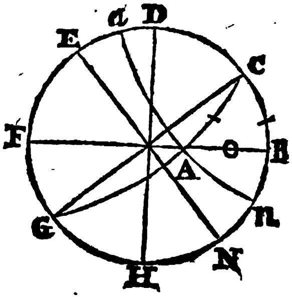
Given the Hypothenuſe and the Perpendicular, to find the Baſe

Example.

In the Spherical Triangle *ABC*, Right-angled at *B*,

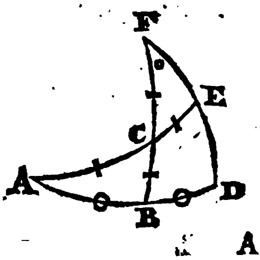
are given, $\left\{ \begin{array}{l} AC=61.04.56 \\ CB=40.30.20 \end{array} \right\}$ req. *AB.*

This *Caſe* is Conſtructed like the 12*th* *Caſe*, and the Baſe AB may be meaſured by the 2*d* *Caſe* of the 7*th* *Problem*, of *Sect.* the 2*d*.



Logarithmically, or by *Calculation*.

Produce the Sides AC, AB, &c. as in *Caſe* the 1*ſt*, then in the two Spherical Triangles FDB, FEC, Right-angled at D and E, &c. are given, FE equal to the Complement of CB, EC equal to the Complement of AC, whence to find BD the Complement of AB, it will be by *Axiom* the 1*ſt*.



As S. (FC= Comp.) CB : (S. FB=) R : : S. (CE=) Comp. AC : S. (BD=) Comp. AB. That is,

As the Co-ſine of the Perpendicular CB=40.30.20——9.8810095

To the Radius——————10.0000000

So is the Co-ſine of the Hypothenuſe AC=61.04.56 ——9.6844449

To the Co-Sine of the Baſe AB=50.30.30 —— 9.8034354

Or,

As the Radius—————— 10.0000000

To the Secant of the Perpendicular CB=40.30.20 —10.1189905

So is the Co-ſine of the Hypothenuſe AC=61.04.56——9.6844449

To the Co-ſine of the Baſe AB=50.30.30 ——9.8034354

Or,

As the Secant of the Hypothenuſe AC=61.04.56——10.3155551

To the Radius —————— 10.0000000

So is the Secant of the Perpendicular BC=40.30.20——10.1189905

To the Co-ſine of the Baſe AB=50.30.30 ——9.8034354

Caſe 14.

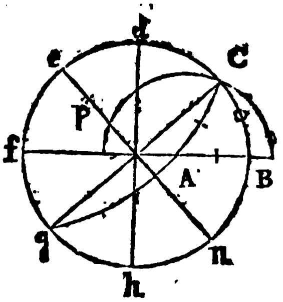
Example 2.

Given the Hypothenuſe and the Baſe, to find the Perpendicular That is,

In the Spherical Triangle ABC, Right-angled at B.

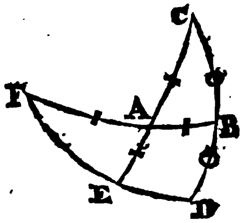
are given $\left\{\begin{matrix}AC=61.04.56\\AB=50.30.20\end{matrix}\right\}$ req. BC.

This *Caſe* is Conſtructed like the *2d Example* of the *12th Caſe*, and BC may be meaſured by the Line of *Chords.*



Logarithmically, or by *Calculation.*

Produce the Sides CB, BA, *&c.* as in the *3d Caſe*, then in the two Spherical Triangles FBD and FEA, Right-angled at D and E, *&c.* are given FA equal to the Complement of AB, AE equal to the Complement of AC, whence to find DB the Complement of BC, it will be by *Axiom* the *1ſt.*



S. (FA=) Comp. AB : S. (AE=) Comp. AC : : (S. FB=) R : S. (BD=) Comp. BC. That is,

As the Co-ſine of the Baſe AB=50.30.30 ——— 9.8034338

To the Co-ſine of the Hypothenuſe AC=61.04.56 ——— 9.6844449
So is the Radius ——— 10.0000000

To the Co-ſine of the Perpendicular BC=40.30.20 ——— 9.8810111

Or,

As the Radius ——— 10.0000000

To the Secant of the Baſe AB=50.30.30 ——— 10.1965662
So is the Co-ſine of the Hypothenuſe AC=61.04.56 ——— 9.6844449

To the Co-ſine of the Perpendicular BC=40.30.20 ——— 9.8810111

Or,

As the Secant of the Hypothenuſe AC=61.04.56———10.3155551

To the Radius ——————————————————10.0000000
So is the Secant of the Baſe AB=50.30.30 ————10.1965662

To the Co-ſine of the Perpendicular BC=40.30.20——9.8810111

Caſe 15.

Given the two Angles, to find the Hypothenuſe.

Or,

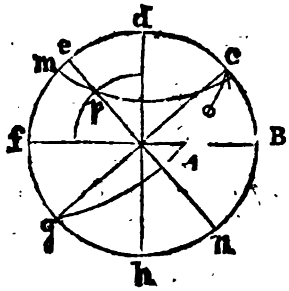
Given the Angle at the Baſe and the Angle at the Perpendicular, to find the Hypothenuſe.

Example.

In the Spherical Triangle ABC, Right-angled at B,

are given $\left\{ \begin{array}{l} \angle A = 47.54.20 \\ \angle C = 61.50.29 \end{array} \right\}$ req. AC.

The Stereographic Solution.



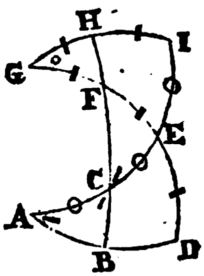
1. Having drawn the Primitive Circle *B d f h*, the Diameter *f B*, and at Right-angles to it *d h*, about the Point *d* as a Pole, deſcribe the Parallel *q p n*, at the diſtance of 47 deg. 54 min 20 ſec. equal to the Angle C, by *Caſe* the 2*d* of *Problem* the 4*th*, *Sect.* the 2*d*

2. About the Point *d* deſcribe the Parallel *m p n*, at the diſtance of 61 deg. 50 min. 29 ſec. equal to the Angle *A*, by *Caſe* the 1*ſt* of *Problem* the 4*th*, *Section* the 2*d*.

3. Thro' the Interſection *p* draw the Diameter *e n*, and at Right-angles to it *c g*.

4. About the Point *p* as a Pole, deſcribe the Great Circle *c A g*, and the thing is done; and *AC* may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, *Section* the 2*d*.

Logarithmically, or by *Calculation*.



Having produced the Sides *AB*, *AC*, &c. as in *Caſe* the 3*d* in the two Spherical Triangles *CIH* and *CEF*, Right-angled at *I* and *E*, &c. are given *HI* equal to the Angle at *C*, *FE* equal to the Complement of *ED*, equal to the Angle at *A*, whence to find *EC* equal to the Complement of *AC*, it will be by *Axiom* the 2*d*.

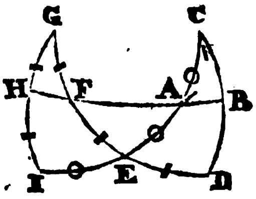
As t, $(HI=) < C$: t, $(FE=)$ Comp. $(ED=) < A$: : (S. $CI=$)R : S. $(CE=)$ Comp. *AC*. That is,

As the Tangent of the Angle at the Perp. *C*=61.50.29 10.2714307

To the Co-tang. of the Angle at the Baſe *A*=47.54.20 9.9558735
So is the Radius —————— 10.0000000

To the Co-ſine of the Hypothenuſe *AC*=61.04.56 — 9.6844428

Or,



Having produced the Sides *CB*, *CA*, &c. as in *Caſe* the 3*d*, in the two Spherical Triangles *AIH*, *AEF*, Right-angled at *E* and *I*, &c. are given *HI* equal to the Angle at *A*, *FE* equal to the Complement of *ED*, the Meaſure of the Angle at *C*, whence to find *EA* the Complement of *AC*, it will be by *Axiom* the 2*d*.

As t, $(HI=) < A$: t, $(FE=)$ Comp. $(ED=) < C$: : (S. $IA=$) R : S. $(EA=)$ Comp. *AC*. That is,

As the Tang. of the Angle at the Baſe *A*=47.54.20 10.0441265

To the Co-tang. of the Angle at the Perp. *C*=61.50.29 — 9.7285693
So is the Radius —————— 10.0000000

To the Co-ſine of the Hypothenuſe *AC*=61.04.56 — 9.6844428

Or,

According to *Napiers Theorem*, becauſe *AC* is the Middle Part and required, and the Angles *A* and *C* Extreams Adjunct, it will be

As $R : c\,t, <A :: c\,t, <C : c\,s, AC$. That is,

As the Radius —————————————— 10.0000000

To the Co-tang. of the Angle at the Baſe $A = 47.54.20$ 9.9558735

So is the Co-tang. of the Angle at the Perp. $C = 61.50.29$ 9.7285693

To the Co-ſine of the Hypothenuſe $AC = 61.04.56$ 9.6844428

Caſe 16.

Given the two Angles, to find one of the Legs.

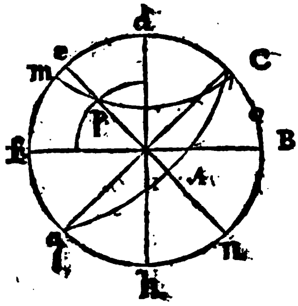
Or,

Given the Angle at the Baſe and the Angle at the Perpendicular, to find the Perpendicular.

Example.

In the Spherical Triangle *ABC*, Right-angled at *B*.

are given $\left\{ \begin{matrix} <A = 47.54.20 \\ <C = 61.50.29 \end{matrix} \right\}$ req. *BC*.

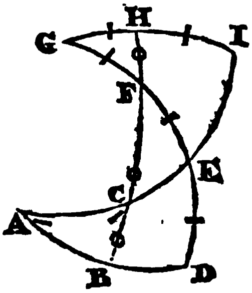


This Caſe is Conſtructed like the 15*th*, and *BC* may be meaſured by the Line of *Chords*.

Logar

Logarithmically, or by *Calculation*.

Having produced the Sides *AC*, *BC*, &c. as in *Caſe* the 3*d*, then in the two Spherical Triangles *CIH*, *CEF*, Right-angled at *I* and *E*, &c. are given, *HI* equal to the Angle at *C*, *FE* equal to the Complement of *ED*, equal to the Angle at *A*, whence to find *CF* equal to the Complement of *CB*, it will be by *Ax*. the 1*ſt*.



As S. $(IH=) \angle C$: (S. $HC=$) R :: S. $(EF=)$Comp.$(ED=) \angle A$: S. $(FC=)$ Comp. CB. That is,

As the Sine of the Angle at the Perp. $C=61.50.29$ —— 9.9452936

To the Radius —————— 10.0000000

So is the Co-ſine of the Angle at the Baſe $A=47.54.20$ — 9.8263046

To the Co-ſine of the Perpendicular $BC=40.30.20$ —— 9.8810110

Or,

As the Radius —————— 10.0000000

To the Co-ſecant of the Angle at the Perp. $C=61.50.29$ 10.0547064

So is the Co-ſine of the Angle at the Baſe $A=47.54.20$ 9.8263046

To the Co-ſine of the Perpendicular $BC=40.30.20$ —— 9.8810110

Or,

As the Secant of the Angle at the Baſe $A=47.54.20$ — 10.1736954

To the Radius —————— 10.0000000

So is the Co-ſec. of the Angle at the Perp. $C=61.50.29$ 10.0547064

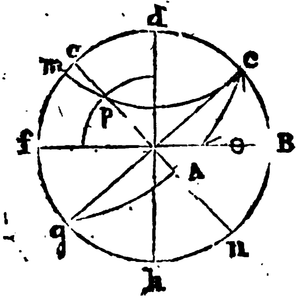
To the Co-ſine of the Perpendicular $BC=40.30.20$ — 9.8810110

Caſe 16.

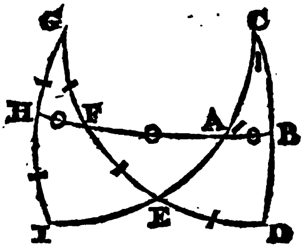
Example 2.

Given the Angle at the Baſe and the Angle at the Perpendicular, to find the Baſe. That is,
In the Spherical Triangle *ABC*, Right-angled at *B*.

are given $\left\{\begin{matrix} <A=47.54.20 \\ <C=61.50.29 \end{matrix}\right\}$ req. AB.



This *Caſe* is Conſtructed like the 15*th*, and *AB* may be meaſured by *Caſe* the 2*d* of *Problem* the 7*th*, *Section* the 2*d*.



Logarithmically, or by *Calculation*.
Produce the Sides *CB*, *CA*, &c as in *Caſe* the 3*d*, then in the two Spherical Triangles *AIH* and *AEF*, Right-angled at *E* and *I*, &c. are given *HI* equal to the Angle at *A*, *FE* equal to the Complement of *ED*, equal to the Angle at *C*, whence to find *FA* equal to the Complement of *AB*, it will be by *Ax*. the 1*ſt*.

As S.(*HI*=)$<A$: S. (*FE*=)Comp.(*ED*=)$<C$: : S. (*AH*=)*R* : S. (*FA*=)Comp. *AB* That is,

As the Sine of the Angle at the Baſe *A*=47.54.20——9.8704278

To the Co-ſine of the Angle at the Perp. *C*=61.50.29—9.6738628
So is the Radius——————————10.0000000

To the Co-ſine of the Baſe *AB*=50.30.30——————9.8034350

Or,

As the Radius———————————————10.0000000

To the Co-ſec. of the Angle at the Baſe *A*=47.54.20 10.1295722
So is the Co-ſine of the Angle at the Perp. *C*=61.50.29 9.6738628

To the Co-ſine of the Baſe *AB*=50.30.30 ——— —9.8034350

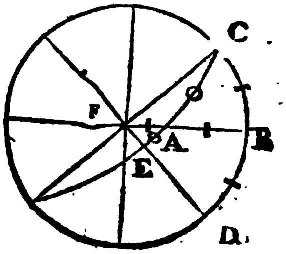
Or,

As the Secant of the Angle at the Perp. 61.50.29—10.3261372

To the Radius———————————————10.0000000
So is the Co-ſec. of the Angle at the Baſe *A*=47.54.20 10.1295722

To the Co-ſine of the Baſe *AB*=50 30.30 ————————9.8034350

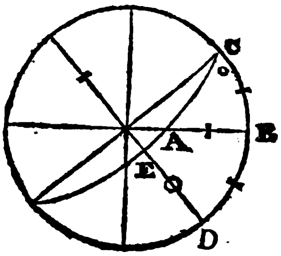
To avoid Confuſion as much as poſſible, and to render every thing as clear as may be, I have in the Application of the two firſt *Axioms* to the *Solution* of the former *Caſes*, made the Productions from whence the Proportions are deduced by themſelves, tho they are ready prepared to our Hands, in the *Conſtruction* of almoſt every *Caſe*; thus, in *Caſe* the 1*ſt*, where the two Legs *AB* and *BC*, are given, to find the Hypothenuſe, becauſe in the two Spherical Triangles *FBD*, and *FAE*, Right-angled at *D* and *E*, and having the ſame Acute Angle at the Baſe *AFE*, we have given *FA* the Complement of *AB*; and *BD* the Complement of *CB* whence to find *AE* the Complement of *AC*, it will be by *Axiom* the 1*ſt*.



As the S. *FB* : S. *FA* : : S. *BD* : S. *AE*. That is, becauſe *FB* is a Quadrant or 90 deg. 00 min.

As the *R* : c s, *AB* : : c s, *BC* : c s, *AC*.

Again,

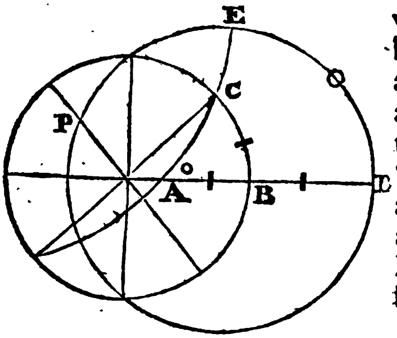


In *Caſe* the 2*d*, where the two Sides *AB* and *AC* are given, to find either of the Angles, ſuppoſe *C*; becauſe in the two Spherical Triangles *CBA* and *CDE*, Right-angled at *B* and *D*, and having the ſame Acute Angle at the Baſe *C*, are given *CB* and *BA*, whence to find *ED* the Meaſure of the Angle at *C*, it will be by *Axiom* the 2*d*.

As the S. *CB* : S. *CD* : : t, *AB* : t, *ED*. That is, becauſe *CD* is equal to a Quadrant,

As the S. *CB* : *R* : : t, *AB* : t, < *C*.

Again,



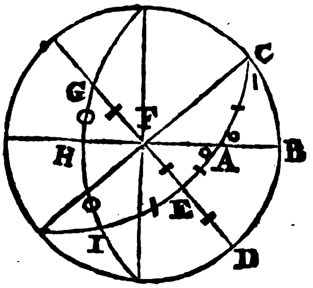
If the ſame two Sides AB, BC, were given to find the Angle at A, having deſcribed about the Point A as a Pole, the Great Circle ED, and produced the Sides AB and AC, to D and E, in the two Spherical Triangles ABC and ADE, Right-angled at B and D, are given AB and BC whence to find DE, the Meaſure of the Angle at A, it will be by *Axiom* the 2*d*.

As the S. AB : S. AD : : t, BC : t, DE. That is becauſe, AD is a Quadrant,

As the S. AB : R : : t, BC : t, < A.

Again

In *Caſe* the 3*d*, where the Hypothenuſe AC is given and one of the adjacent Angles, ſuppoſe C, to find the other adjacent Angle A.



Having about the Point A as a Pole, deſcribed the Great Circle GHI, in the two Spherical Triangles AEF and AIH, Right-angled at E and I, and having the ſame Acute Angle at the Baſe FAE common to both, are given, AE the Complement of AC, EF the Complement of ED, the Meaſure of the Angle at C, whence to find IH the Meaſure of the Angle HAI equal to CAB, it will be by *Axiom* the 2*d*.

As the S. AE : S. AI : : t, EF : t, IH. That is, becauſe AI is a Quadrant.

As the c s, AC : R : : c t, <C : t, <A.

After the ſame manner may the Proportions be deduced for every *Caſe*, from the *Conſtructions* themſelves by Inſpection.

Tho' I have given but three different *Solutions* to moſt of the *Caſes*, and which is ſufficient for my purpoſe, yet, he that underſtands the former *Section* may eaſily deduce ſeveral others.

To ſhew the Extenſive Uſe of the 2*d Section*, as well as to gratify the Curious, I have added a great Variety of different *Conſtructions* of the ſame *Caſe*, which being well underſtood, will contribute very much to the thorough Underſtanding of the Projection of the *Sphere*, the Conſtructing of *Spherical Triangles*, and *Aſtronomical Problems*.

Section VI.

Containing a Demonſtration of thoſe Theorems *commonly called* Axioms, *by which the ſeveral Caſes of* Oblique-angled Spherical Triangles *are reſolved.*

1. EVERY Oblique-angled Spherical Triangle may be reduced into two Right-angled Spherical Triangles, by letting fall a Perpendicular from any one of its Angles upon the oppoſite Side, produced if need be.

2. The Perpendicular when let fall, either divides the Oblique-angled Triangle into two Right-angled Triangles, or produces two by adding of one to it.

Fig. 1.

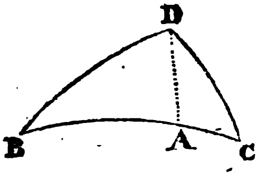
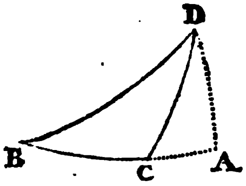


Fig. 2.



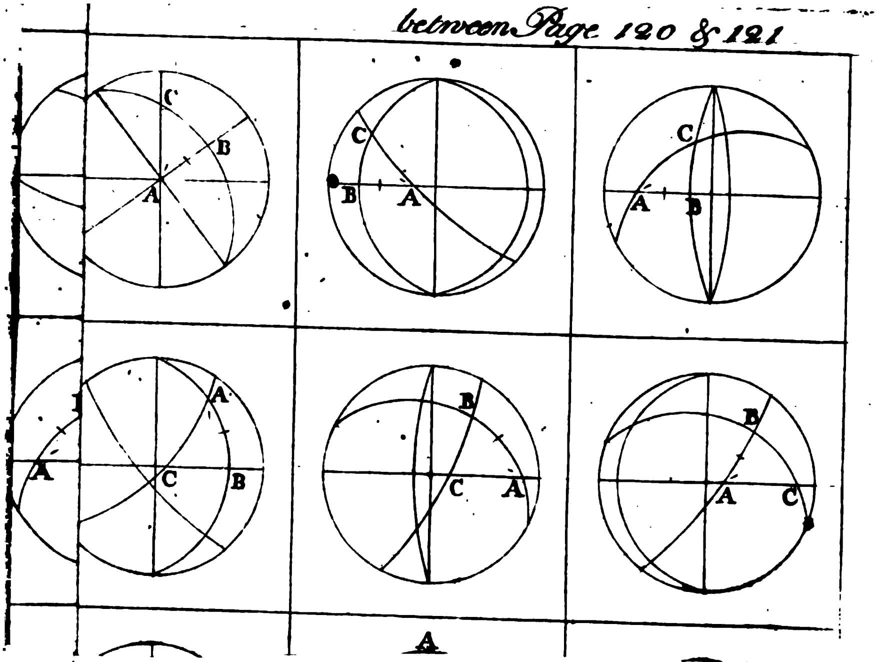
3. If the Perpendicular *DA* falls within (as in *Fig.* the 1ſt.) i divides the Oblique-angled Triangle *BDC* into two Right-angl Triangles *BAD* and *CAD*. But,

4. If the Perpendicular *DA* falls without (as in *Fig.* the 2*d.*) i produces two Right-angled Triangles *BAD* and *CAD*, by addin of one Right-angled Triangle *CAD*, to the Oblique-angled Tr angle *BDC* firſt given.

5. If the Angles *DBC* and *DCB*, at the Baſe *BC* of the Obliqu angled Triangle *BDC* are ſimilar, that is, both Acute or both Ob tuſe, (as in *Fig.* the 1ſt.) then the Perpendicular let fall from the An gle *D* oppoſite to the Baſe, falls within the Triangle.

For,

between Page 120 & 121



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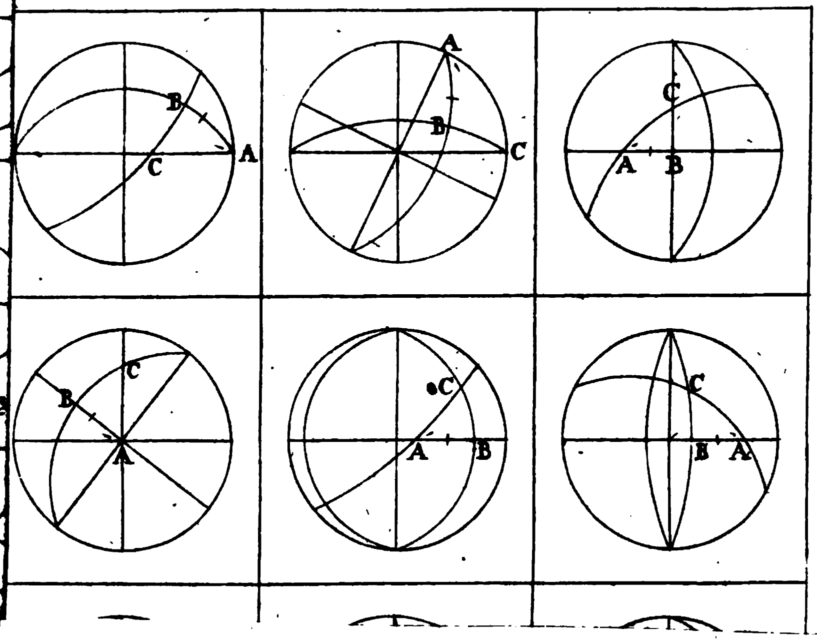


For if the Perpendicular fall without (as in *Fig.* the *2d*) then becauſe (by the *28th Ar.* of *Sect.* the *3d*) the Angle DCA of the Triangle DAC, is ſimilar to the Perpendicular DA, and the Angle DBA of the Triangle DAB, ſimilar to the Perpendicular DA by the ſame, it follows that the Angles DCA and DBC are ſimilar, that is, both Acute or both Obtuſe, and conſequently the Angles DCB and BCD are diſſimilar, that is, one Acute and the other Obtuſe, which is contrary to the Hypotheſis; wherefore the Perpendicular DA muſt fall within (as in *Fig.* the *1ſt*.) Again,

7. If the Angles DBC and DCB at the Baſe BC are diſſimilar, that is, one Acute and the other Obtuſe (as in *Fig.* the *2d*) then the Perpendicular let fall from the Angle oppoſite to the Baſe, will fall without the Triangle upon the Baſe produced.

For if the Perpendicular falls within (as in *Fig.* the *1ſt*.) then becauſe the Angle DBA of the Triangle DAB, is ſimilar to the Perpendicular DA (by the *28th Ar.* of *Sect.* the *3d*.) and the Angle DCA of the Triangle DAC ſimilar to the ſame Perpendicular DA: by the ſame it follows, that the Angles DBC and DCB at the Baſe are ſimilar, that is, both Acute or both Obtuſe, which is contrary to the Hypotheſis; wherefore in this *Caſe* the Perpendicular will fall without (as in *Fig* the *2d*.)

8. Wherefore, If a Perpendicular DA, let fall from any Angle D of the Oblique angled Spherical Triangle BDC, fall within the Triangle (as in *Fig.* the *1ſt*) the Angles DBA and DCA at the Baſe DC, ſhall be both ſimilar, that is both Acute or both Obtuſe, and on the contrary, if the Perdendicular DA fall without the Triangle (as in *Fig.* the *2d*.) the Angles DBA and DCA at the Baſe DC, ſhall be diſſimilar, that is, one Acute and the other Obtuſe, and the Perpendicular ſhall fall on that part of the Baſe produced, where the Angle is greater than a Right-angle.

For, If the Angles DBA and DBC at the Baſe DC are diſſimilar, that is, one Acute and the other Obtuſe, then the Perpendicular DA will fall without the Triangle (as in *Fig.* the *2d*) by the *7th* preceding *Ar.* which is contrary to the Hypotheſis; and on the contrary, if the Angles at the Baſe are ſimilar, that is, both Acute or both Obtuſe, (as in *Fig.* the *1ſt*) then the Perpendicular will fall within by the *6th* preceding *Ar.* which is alſo contrary to the Hypotheſis; wherefore if the Perpendicular fall within, the Angles at the Baſe will be ſimilar.

1. That

The Sides BD, DC, comprehending the Angle D, from whence the Perpendiculcar is drawn are called theHypothenuſes, BA, AC, the Baſes, the Angles B and C the Angles at the Baſe, and the Angles BDA, BDC, the Angles at the Vertex or Angles at the Perpendicular, of the two Right-angled Spherical Triangles BAD, DCA, having the ſame common Perpendicular DA; theſe things being premiſed, it follows,

1. That the Sines of the Hypothenuſes BD and DC, are reciprocally proportional to the Sines of the Angles at the Baſe B and C, and the contrary.

For in the Triangle BDA Right-angled at A, S. BD : R : : S. DA : S.<B (by the 2*d Cor.* of the 4*th Section*) and in the Trigle CDA Right-angled at A, S. DC : R : : S DA S. < C by the ſame; wherefore by the 12*th Cor.* of the 15*th Geom. Theor.* it will be, S. DC : S. BD : : S.<B : S. <C.

2. The Co-ſines of the Angles B and C at the Baſe, are directly proportional to the Sines of the Angles BDA, CDA at the Vertex, and the contrary;

For by the 5*th Cor.* of *Sect.* the 4*th*, in the Triangle BAD the R : c s, DA : : S. BDA : c s,<B, and in the Triangle CAD, R : c s, DA : : S. CDA : c s, <C by the ſame; therefore by the 11*th Cor.* of the 15*th Geom. Theor.* it will be, S. BDA : S. CDA : : c s,<B : c s,<C. That is, As c s,<B : c s, <C : : S. BDA : S. CDA.

3. The Co-ſines of the Hypothenuſes BD, DC, are directly proportional to the Co-ſines of the Baſes, and the contrary;

For by the 3*d Cor.* of the 4*th Sect.* in the Triangle BAD, the R : c s, DA : : c s, BD : c s, BA; and in the Triangle, CAD R : c s, DA : : c s, DC : c s, AC by the ſame, wherefore (by the 11*th Cor.* of the 15*th Geom. Theor.* 1*ſt*) cs, BD : c s, DC : : BA : c s, AC.

4. The Sines of the Baſes BA and AC are reciprocally proportional to the Tangents of the Angles at the Baſe, and the contrary;

For by the 2*d Cor.* of the 4*th Section* in the Triangle BAD it will be, S. BA : R : : t, DA : t, <B, and in the Triangle CAD, S. DA : R : : t, DA : t,<D by the ſame; wherefore by the 12*th Cor.* of the 15*th Geom. Theor.* it will be, S. DA : S. BA : : t,<B : t,< D.

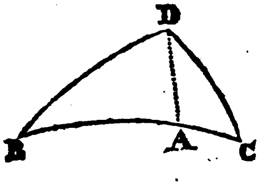
5. The Tangents of the Hypothenuſes BD and DC; are reciprocally proportional to the Co-ſines of the Angles *BDA* and ADC at the Vertex, and the contrary;

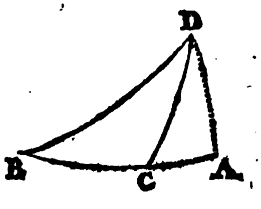
For by the *4th Cor.* of the *4th Section* in the Triangle ADB, t, BD : R : : t, DA : c s, BDA, and by the ſame in the Triangle CAD, t, DC : R : : t, DA : c s, ADC, wherefore by the *12th Cor.* of the *15th Geom. Theor.* t, BD : t, DC : : c s, ADC : c s, ADB.

6. The Tangents of the Baſes BA and AC, are directly proportional to the Tangents of the Angles BDA and CDA at the Vertex, and the contrary;

For by the *2d Cor.* of the *4th Section*, in the Triangle BAD, R : S, DA : : t, BDA : t, BA; and by the ſame *Cor.* in the Triangle CAD, R : c s, DA : : t, CDA : t, CA, wherefore by the *12th Cor.* of the *15th Geom. Theor.* t, BDA : t, CDA : : t, BA : t, CA; or which is the ſame, t, BA : t, CA : : t, BDA : t, CDA.

Hence, and by the two firſt Propoſitions of *Section* the *4th*, may any of the firſt Ten *Caſes* of Oblique-angled Spherical Triangles be readily reſolved, for becauſe in every Oblique-angled Spherical Triangle, there are always three things given, and in each of the firſt Ten *Caſes* either two Sides and one Angle or two Angles and one Side, if a Perpendicular be let fall from the end of that given Side which is adjacent to the given Angle, and ſo as to fall oppoſite to the ſame given Angle, we ſhall always have a Hypothenuſe (*viz.* the Side from whence the Perpendicular is drawn) and an Angle at the Baſe (*viz.* that Angle which is oppoſite to the Perpendicular when drawn) given, in one of the Right-angled Spherical Triangles thus produced, and in the other Triangle either a Hypothenuſe or Angle at the Baſe, or the Baſe and Angle at the Perpendicular may be readily found, by adding or ſubſtracting the Baſe or Angle at the Perpendicular in the firſt Triangle (inveſtigated by the help of the *1ſt* or *2d Axiom* or *Propoſition* of *Sect.* the *4th*, from the Hypothenuſe and Angle at the Baſe) from the Side in the Oblique-angled Triangle upon which the Perpendicular fell, or the Angle from which it was drawn, and then comparing the given parts in one with the ſimilar parts in the other, the part required will be eaſily found by the help of ſome of the former *Corollarys*.





Suppoſe for *Example*, in the Oblique-angled Spherical Triangle BDC there were given BD, the Angle B, and DC, to find the Angle C, if in this *Caſe* the Perpendicular DA be let fall from D, one end of the given Side BD, which Side is adjacent to the given Angle B, and ſo as to fall oppoſite to the ſame given Angle, we ſhall have given in one of the Right-angled Spherical Triangles BDA thus produced, BD the Hypothenuſe, and the Angle B the Angle at the Baſe, and in the other Triangle DAC, the Hypothenuſe DC, to find the Angle C the Angle at the Baſe, and becauſe (by the *1ſt Cor.* of this *Section*) the Sines of the Hypothenuſes are reciprocally proportional to the Sines of the Angles at the Baſe, it will be as the Sine of the Side DC (one of the Hypothenuſes) to the Sine of the Side DB (the other Hypothenuſe) ſo is the Sine of the Angle B (one of the Angles at the Baſe) to the Sine of the Angle C (the other Angle at the Baſe) which is ſimilar to the Angle B if the Perpendicular fall within as in the firſt Figure, but diſſimilar, if it fall without, as in the 2*d* Figure.

Again, in the ſame Triangle, if the ſame things were given as before, and inſtead of the Angle at C the Side BC were required, having let fall the Perpendicular DA as before, firſt in the Triangle BDA we have given, BD the Hypothenuſe and the Angle B the Angle at the Baſe, whence to find BA the Baſe it will be by *Caſe* the 4*th* of *Sect.* the 5*th*, As c s, <B : R : : c t, BD : c t, BA, and becauſe by the 3*d Cor.* of this *Section*, the Co-ſines of the Hypothenuſes are directly proportional to the Co-ſines of the Baſes, it will be As c s, BD : c s, DC : : c s, BA : c s, AC the Baſe of the Triangle DAC, this therefore added to BA before found, where the Perpendicular falls within as in *Fig.* the 1*ſt*, or ſubſtracted from BA, where the Perpendicular falls without as in *Fig.* the 2*d*, will give the Side BC required.

Again, in the ſame Triangle, the ſame things being given as before, If inſtead of BC the Angle at D were required, becauſe in the Triangle BDA the Hypothenuſe BD and Angle B at the Baſe are given, (it will be by *Caſe* the 3*d*, of *Section* the 5*th*) to find the Vertical Angle BDA, As the c s, BD : R : : c t, <B : t, <BDC, and becauſe by the 5*th* preceding *Cor.* the Tangents of the Hypothenuſes are reciprocally proportional to the Co-ſines of the Angles at the Vertex, it will be As t, DC : t, DB : : c s, < BDA : c s, < CDA, this therefore added to the Angle BDA before found, when the Perpendicular falls within, but ſubſtracted from the Angle BDA when the Perpendicular falls without, will give the Angle BDC required.

Again, in the ſame Triangle *BDC*, if *BD*, *BC*, and the Angle *B* were given, to find the third Side *DC*, becauſe in this *Caſe* each of the given Sides is adjacent to the given Angle, the Perpendicular may be let fall either from the Point *D* upon *BC* (produced if need be) or from the Point *C* upon *BD* (produced if need be) let it fall therefore from the Point *D* upon *BC* (produced it need be) for both Varieties are anſwered in this *Caſe*: becauſe the Angle *D* is Obtuſe in *Fig.* the 1ſt and Acute in *Fig.* the 2d, in the Right-angled Spherical Triangle *BAD* are given, *BD* the Hypothenuſe, and the Angle *B* the Angle at the Baſe, whence to find the Baſe *BA* it will be by *Caſe* the 4th of *Section* the 5th, As the $\text{c s}, \angle B : R :: \text{c t}, BD : \text{c t}, BA$; which in *Fig.* the 1ſt taken from *BC*, and in *Fig.* the 2d leſſen'd by *BC*, will give *AC* the Baſe of the Right-angled Triangle *CAD*; and becauſe by the 3d preceding *Cor.* the Co-ſines of the Baſes are directly proportional to the Co-ſines of the Hypothenuſes, it will be, As the $\text{c s}, BA : \text{c s}, CA :: \text{c s}, BD : \text{c s}, DC$ required. Again,

If the Angle at *C* in the ſame Oblique-angled Triangle BDC were to be found from the ſame Data, becauſe by the 4th preceding *Cor.* the Sines of the Baſes are reciprocally proportional to the Tangents of the Angles at the Baſe, it will be as the Sine of *CA* to the Sine of *BA*, ſo is the Sine of the Angle *B* to the Sine of the Angle at *C* required.

After the ſame manner and by the help of the ſame *Cor.* may any other *Caſe* in Oblique-angled Spherical Triangles, where one Side and one Angle are part of the Data, be eaſily reſolved; as the Intelligent *Reader* will readily perceive.

Propoſition III. *Or* Axiom III.

In all Spherical Triangles, the Sines of the Sides are directly proportional to the Sines of their oppoſite Angles, that is, in the Spherical Triangle *BDC* (ſee the *Figure* in *Page* the 123d) it will be, $\text{S}.\, BD : \text{S}.\, DC :: \text{S}. \angle C : \text{S}. \angle B$.
for having let fall the Perpendicular *DA* it will be (by *Cor.* the 1ſt of *Prop.* the 1ſt and 2d of *Sect.* the 4th, or *Caſe* the 13th of *Sect.* the 5th.).

$$\text{As S}.\, BD : R :: \text{S}.\, DA : \text{S}. \angle B.$$
$$\text{And S}.\, DC : R :: \text{S}.\, DA\ \text{S}. \angle C.$$

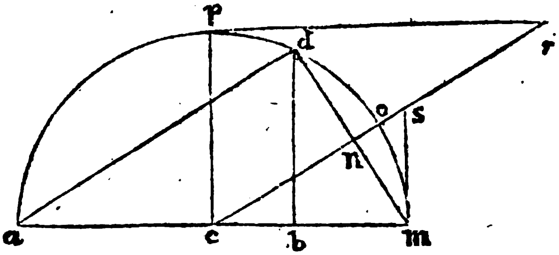
Wherefore, by *Cor.* the 12*th* of *Prop.* the 15*th* of *Part* the 1*ſt*, it will be,

$$S.BD : S.DC :: S.\angle C : S.\angle B.$$ W. W. D.

To render the Demonſtration of the remaining *Axioms* more eaſy, it is neceſſary to premiſe the following *Lemma's*.

Lemma 1.

The Verſed Sine of an Arch multiplied into half the Radius, is equal to the Square of the Sine of half that Arch, or to half the Sine of the Arch multiplied into the Tangent of half the ſame Arch.



Let the given Arch be md, then will bm be its Verſed Sine, and mn the Right Sine, and ms the Tangent of the Arch mc, the half of md.

And becauſe the Triangles mbd and ncm are ſimilar, (for the Angle at m is common, and the Angles at n and b are Right-angles) it will be (by *Prop.* the 18*th* of Part the 1*ſt*) as $bm : md :: mn : cm$, that is, (by *Cor.* the 18*th* of *Prop.* the 15*th Part* the 1*ſt*) as $bm : mn :: dm : cm$, and conſequently, as $bm : nm :: (\frac{1}{2} dm =) mn : \frac{1}{2} cm$, wherefore (by *Cor.* the 1*ſt* of *Prop.* the 15*th Part* the 1*ſt*) $bm \times \frac{1}{2} cm = mnq$, that is, the Verſed Sine bm of the Arch dm, multiplied into half the Radius cm, is equal to the Square of mn the Right Sine of mc, the half of the Arch mod. W. W. D.

2. The $\frac{1}{2} db : db :: (\frac{1}{2} dm) =) mn : dm$ (by the 5*th Cor.* of *Prop.* the 15*th Part* the 1*ſt*) and conſequently $\frac{1}{2} db : mn :: db : dm$ (by the 13*th Cor.* of *Prop* the 1*ſt Part* the 1*ſt*) and becauſe the Triangles dbm and smn are ſimilar, (for sm and db are parallel, and the Angles at n and b are Right-angles ($db : dm :: mn : ms$, wherefore, $\frac{1}{2} db : mn :: mn : mq$; and conſequently (by the 1*ſt Cor.* of *Prop.* the

15*th Part* the 1*ſt*) $\frac{1}{2}\,db\times ms = mnq$, that is, half the Sine db of the Arch dcm multiplied into ms the Tangent of mc, one half of the Arch md, is equal to the Square of mn the Sine of the half Arch mo.

Lemma 2.

The Verſed Sine of the Supplement of an Arch multiplied into half the Radius, is equal to the Square of the Co-ſine of half that Arch, or to the Product of half the Sine of that Arch multiplied into the Co-tangent of the ſame Arch.

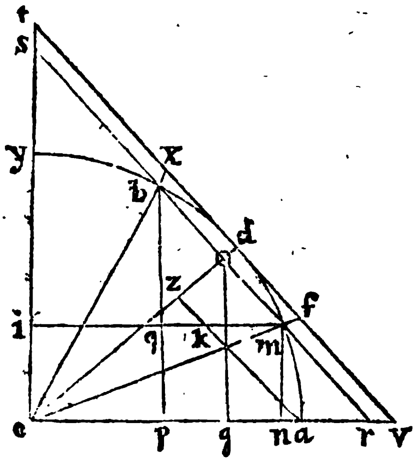
In the former Figure, let the given Arch be dm, then will ab be the Verſed Sine of its Supplement, and cn the Co-ſine of mo its one half, and pq the Co-tangent of the ſame.

And becauſe the Triangles adb and cmn are ſimilar, (for the Angles at m and d are equal, and the Angles at n and b Right-angles) it will be (by the 18*th Prop.* of *Part* the 1*ſt*) as $ab : ad :: cn : cm$; that is, by alternation, as $ab : cn :: ad : cm$; but becauſe am is the double of cm, ad is equal to twice cn, for the Triangles adm and cnm are ſimilar, (becauſe the Angles at d and n are Right-angles, and the Angle m common) wherefore, $ab : cn :: 2cn : cm$, and conſequently, $ab : cn :: cn : \frac{1}{2}cm$, wherefore, (by *Cor.* the 1*ſt* of *Prop.* the 15*th*, *Part* the 1*ſt*) $ab\times\frac{1}{2}cm = cn\times cn = cnq$, that is, the Verſed Sine ab of the Arch ad, the Supplement of dm multiplied into half the Radius cm, is equal to the Square of cn the Co-ſine of the Arch mo the half of the Arch dm. W. W. D.

2. Becauſe the Triangles cpr and dba are ſimilar, (for the Angles at d and p are Right-angles, and the Angles a and r are equal), it will be as $cp : pr :: db : ba$, and ſo is half db to half ba, wherefore, (by *Cor.* the 1*ſt* of *Prop.* the 15*th Part* the 1*ſt*) $cp\times\frac{1}{2}ba = pr\times\frac{1}{2}db$, but by the former *Part* $pr\times\frac{1}{2}db = cnq$. wherefore by the 1*ſt* *Geom. Axiom*, $\frac{1}{2}db\times pr = cnq$, that is, half the Right Sine db of the Arch dom, multplied into pr the Tangent of the Arch po, the Complement of the Arch mo, the half of md is equal to the Square of cn, the Co-ſine of the ſame half Arch mo. W. W. D.

Lemma 3.

The Difference of the Verſed Sines of two Arches multiplied into half the Radius, is equal to the Sine of half the Sum of thoſe Arches multiplied into the Sine of half their Difference.



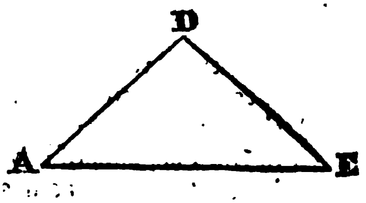
In the adjacent Figure, let ab and am be the two given Arches, then will mo the Sine of the Arch md, be the Sine of half their Difference, (for the Arch mb equal to twice md, is the Difference of the Arches ab and am) and az the Sine of the Arch ad, the Sine of half their Sum (for ab the greater Arch leſſened by bd the half Difference of the Arches, leaves ad the half Sum of the two Arches) alſo qm equal to pn, equal to the Difference of their Verſed Sines ap and an, (for ap is the Verſed Sine of the greater Arch ab, and an the Verſed Sine of the leſſer Arch am) and becauſe the Triangles bqm and caz are ſimilar (for the Angles q and z are Right-angles, and the Angles at a and m are equal, by reaſon the Lines qm, pa, and bm and az are parallel (by Conſtruction) it will be as $ac : az :: bm : qm = pn$ (for the Figure $pqmn$ is a Parallelogram) that is by Alternation, as $ac : (bm =)\ 2mo :: az : pn$, and conſequently, as $\frac{1}{2}ac : mo :: az : pn$, wherefore by *Cor.* the 1ſt of *Prop.* the 15th, *Part* the 1ſt, $\frac{1}{2}ac \times pn = mo \times oz$; that is, half the Radius ac multiplied into pn the difference of the Verſed Sines ap, an, of the Arches ab, am, is equal to mo the Sine of the half difference md, multiplied into az the Sine of ad the half Sum. *W. W. D.*

Lemma 4.

The Sum of the Sines of any two Arches, is to the Difference of the Sines of the ſame Arches; as the Tangent of half the Sum of thoſe two Arches, to the Tangent of half their Difference.

In the former Figure, let ab, and am, repreſent the two Arches, then will bp be the Sine of the greater Arch ab, and fn the Sine of the leſſer Arch am, and ok equal to half bp, the Difference of the Sines bp, and mn (for mo is equal to half mb, becauſe mo is equal to the half of mb) and conſequently og is equal to half the Sum of the Sines bp, and mn, alſo dv is the Tangent of the Arch da, half the Sum of the Arches ab, and am, and df the Tangent of the Arch dm equal to the half of the Arch mb, the Difference of the Arches ab, and am, and becauſe or is parallel to dv, one Side of the Triangle cdv) it will be (by the 17*th Prop.* of *Part* the 1*ſt*) as $vd : fd :: ro : mo$, but as $ro : mo :: og : ok$, (for the Triangles rog and mok are ſimilar, becauſe km and gr are parallel) and ſo is twice og equal to $bp+mn$, to twice ok equal to bq equal to $bp-mn$; wherefore, as $vd : df :: bp+mn : bp-mn$, or as $bp+mn : bp-mn :: vd : df$, that is, as the Sum of the Sines bp and mn, of the Arches ab and am, to the Difference of the Sines bp and mn, of the Arches ab and am, ſo is dv the Tangent of the Arch ad, the half Sum of the Arches ab and am, to df the Tangent of the Arch dm, the half Difference of the Arches ab and am. W. W. D.

Hence we are taught a very eaſy way of demonſtrating the third *Axiom* of *Plane Trigonometry*, for finding two Angles, from two Sides and the containing Angle being given.



For in the Triangle ADE, becauſe by the 2*d Axiom* $AD : DE :: S.\angle E : S.\angle A$, it will be (by *Cor.* the 18*th* of *Prop.* the 15*th Part* the 1*ſt*) $AD+DE : AD-DE :: S.\angle E+S.\angle A : S.\angle E-S.\angle A$, but by the preceding *Lemma*, $S.\angle E+S.\angle A : S.\angle E-S.\angle A :: t,\tfrac{1}{2}\angle s\, A+E : t,\tfrac{1}{2}\angle s\, \overline{E-A}$, wherefore, $AD+DE : AD-DE :: t,\tfrac{1}{2}\angle s\, \overline{E+A} : t,\tfrac{1}{2}\angle s\, \overline{E-A}$; that is, as the Sum of the Sides AD and AE to their Difference, ſo is the Tangent

gent of half the Sum of their oppoſite Angles, to the Tangent of half their Difference, *&c. W. W. D.*

Lemma 5.

The Sum of the Co-ſines of two Arches, is to the Difference of the Co-ſines of the ſame Arches, as the Co-tangent of half the Sum of thoſe Arches, to the Tangent of half their Difference: if the Arches are leſs than a Quadrant.

For in the ſame Figure, let ab and am repreſent the two Arches, then will cp be the Co-ſine of the greater Arch ab, and cn the Co-ſine of the leſſer Arch am, and km equal to gn, equal to the half of pn, the Difference of the Co-ſines cn and cp, (for mk is equal to half mq, becauſe mo is equal to half mb) and conſequently ik is equal to cg (becauſe the Figure $iqpc$ is a Parallelogram) is equal to half the Sum of the Co-ſines cn and cp, alſo dt is the Tangent of the Arch yd the Complement of the Arch ad, the half Sum of the Arches ab and am, and xt equal to df the Tangent of the Arch bd equal to dm, the half Difference of the Arches ab and am, and becauſe the Triangles cft and cms are ſimilar (for ms is parallel to ft, it will be (by *Prop.* the 18*th Part* the 1*ſt*) as $td : df :: so : om$, but as $so : om :: ik : km$ (for the Triangles mok and msi are ſimilar, becauſe ok and si are parallel) and ſo is twice ik equal to twice cg, equal to $cp+cn$ to twice km, equal to twice gn equal to $cn-cp$, wherefore as $td : df :: cn+cp : cn-cp$, or as $cn+cp : cn-cp :: td : df$; that is, as the Sum of the Co-ſines cn, cp, of the Arches ab, am, to the Difference of the Co-ſines cn, cp, of the ſame Arches ab, am, ſo is dt the Tangent of the Arch dy, the Complement of the Arch da, half the Sum of the Arches ab, am, to df the Tangent of the Arch dm, the half Difference of the ſame Arches. *W. W. D.*

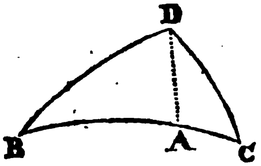
But if the Arches are greater than a Quadrant, the Sum of the Co-ſines will be to the Difference of the Co-ſines, as the Tangent of half the Sum of the Arches, to the Co-tangent of half their Difference.

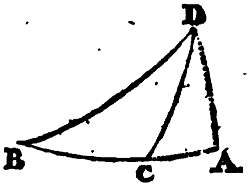
For the Tangents of any two Arches are reciprocally proportional to the Co-tangents, as has been proved in *Section* the 2*d* of *Part* the *Second.*

Lemma 6.

The Sum of the Tangents of any two Arches, is to the Difference of the Tangents of the ſame Arches, as the Sine of the Sum of thoſe Arches, to the Sine of their Difference.

In the Figure belonging to the 3*d Lemma*, let ad and db repreſent the two Arches, then will bp the Sine of the Arch ba be the Sine of the Sum of the Arches ad and db, and mn the Sine of the Arch am, the Sine of the Difference of the Arches ad and db, for the Arch dm is equal to the Arch db, alſo dv the Tangent of the greater Arch da, and dx equal to df the Tangent of the leſſer Arch dm equal to df, and becauſe br is parallel to xv, the Side xv of the Triangle cxv) it will be (by *Prop.* the 18*th* of *Part* the 1*ſt*) as $xv : fv :: br : mr$, but as $br : mr :: bp : mn$ (becauſe mn is parallel to the Side bp of the Triangle rbp) wherefore by Equality, as $xv : fv :: bp : mn$; that is, as xv the Sum of the Tangents xd, and dv, of the Arches bd and da to fv, the Difference of the ſame Tangents, ſo is bp the Sine of the Sum of the Arches ad and db, to mn the Sine of the Difference of the ſame Arches; whence it follows, that in any Spherical Triangle as *BDC*.





Cor. 1. The Sine of the Sum of any two Sides as *BD* and *DC*, is to the Sine of the Difference of the ſame two Sides *BD* and *DC*, as the Co-tangent of half the Vertical or contained Angle *D*, to the Tangent of half the Difference of the Vertical Angle *BDA*, and *CDA*, made by a Perpendicular let fall from the Vertical Angle *D*, upon the Side *BC* produced if need be; or ſo is the Co-tangent of the Difference of the Vertical Angles *BDA* and *CDA*, to the Tangent of half the Vertical Angle *BDC*.

For (by the 5*th Cor.* of the 6*th* preceding *Sect.*) $t,BD : t,CD :: cs,CDA : cs,BDA$, therefore, (by the 18*th Cor.* of *Prop.* the 15*th* of *Part* the 1*ſt*) $t,BD + t,CD : t,BD - t,CD :: cs,CDA + cs,BDA$

: c s, $CDA-$c s, BDA, but by the *6th* preceding *Lemma*, t, $BD+$ t, CD : t, $BD-$t, CD : : S. $\overline{BD + CD}$: S. $\overline{BD-CD}$, and by the *5th* preceding *Lemma*, c s, $CDA+$c s, BDA : c s, $CDA-$c s, BDA : : c t, $\overline{CDA+BDA}$ = c t, $\frac{1}{2} < D$: c t, $\frac{1}{2}\overline{CDA-BDA}$: : c t, $\frac{1}{2}\overline{CDA-BDA}$: t, $\frac{1}{2}\overline{CDA+BDA}$ = t, $\frac{1}{2} < BDC$, therefore by equality of Ratio, S. $\overline{BD+CD}$: S. $\overline{BD-CD}$: : c t, $\frac{1}{2}\overline{CDA+BDA}$ = c t, $< \overline{BDC}$: t, $\frac{1}{2}\overline{CDA-BDA}$: : c t, $\frac{1}{2}\overline{CDA-BDA}$: t, $\frac{1}{2}\overline{CDA+BDA}$ = t, $\frac{1}{2} < BDC$; that is, as the Sine of the Sum of the Sides *BD*, and *CD*, &c. as was to be Demonſtrated.

2. The Tangent of half the Sum of the Sides *BD* and *DC*, is to the Tangent of half the Difference of the Sides *BD*, and *DC*, as the Tangent of half the Sum of their oppoſites Angles *B* and *C*, to the Tangent of half the Difference of the ſame Angles *B* and *C*.

Or,

The Co-tangent of half the Sum of the Sides *BD* and *DC*, is to the Co-tangent of half the Difference of the Sides *BD* and *DC*, as the Tangent of half the Difference of their oppoſite Angles *C* and *D*, to the Tangent of half the Sum of the ſame Angles *C* and *D*.

For (by the *3d* preceding *Ax.*) the S. BD : S. CD : : S. $<C$: S. $<B$, therefore (by the *18th Cor.* of the *15th Prop. Part* the *1ſt*) S. BD + S. CD : S. BD — S. CD : : S. $<C$ + S. $<B$: S. $<C$ — S. $<B$, but (by the *4th* preceding *Lemma*) S. BD + S. CD : S. BD — S. CD : : t, $\frac{1}{2}\overline{BD+CD}$: t, $\frac{1}{2}\overline{BD-CD}$, and by the ſame *Lemma*, S. $<C$ + S. $<B$: S. $<C$ — S. $<B$: : t, $\frac{1}{2}\overline{<C+<B}$: t, $\frac{1}{2}\overline{<C-<B}$, therefore by Equality of Ratio, as t, $\frac{1}{2}\overline{BD+CD}$: t, $\frac{1}{2}\overline{BD-CD}$: : t, $\frac{1}{2}\overline{<C+<B}$: t, $\frac{1}{2}\overline{<C-<B}$, and becauſe the Tangents and Co-tangents of Arches are reciprocally proportional, c t, $\frac{1}{2}\overline{BD+CD}$: c t, $\overline{BD-CD}$: : t, $\frac{1}{2}\overline{<C-<B}$: t, $\frac{1}{2}\overline{<C+<B}$; that is, as the Tangent of half the Sum of the Sides *BD* and *CD*, to the *&c.* as was to be Demonſtrated.

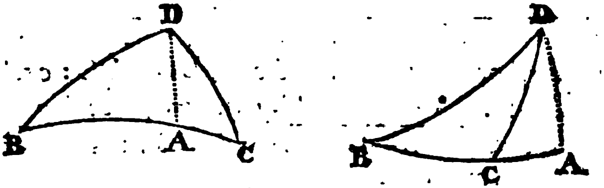
3. The Co-tangent of half the Vertical Angle *D*, is to the Tangent of half the Sum of the other two Angles at the Baſe *B* and *C*, as the Tangent of half the Difference of the Angles at the Baſe *B* and *C*, to the Tangent of half the Difference of the Angles at the Vertex *BDA* and *CDA* made by a Perpendicular let fall from the Vertical Angle *D* upon the Baſe *BC*, (produced if need be): For by the *2d Cor.* of the *6th* preceding *Lemma*) the S. BDA : S. CDA : : c s, $<B$: c s, $<C$, therefore (by the *18th Cor.* of *Prop.* the *15th Part* the *1ſt*) as the

S. *BDA*

$BDA + S,\ CDA : S.\ BDA - S.\ CDA :: cs, < B + cs, < C : cs, < C - cs, < B$, and by the *4th* preceding *Lemma*, $\overline{S.\ BDA + S.\ CDA} : S.\ BDA - S.\ CDA :: t, \overline{BDA + CDA} : t, \overline{BDA - CDA}$, and by the *5th* preceding *Lemma*, the $cs, < C + cs, < B : cs, < C - cs, < B :: ct, \tfrac{1}{2} <_s \overline{B + C} : t, \tfrac{1}{2} <_s \overline{C - B}$; therefore by equality of Ratio, the $t, <_s \overline{BDA + CDA} : t, <_s \overline{BDA - CDA} :: ct, \tfrac{1}{2} <_s \overline{C + B} : t, \tfrac{1}{2} <^s C - B$, and conſequently (by the *11th Cor.* of *Prop.* the *15th Part* the *1ſt*) $t, <^s \overline{BDA + CDA} : ct, \tfrac{1}{2} <^s \overline{B + C} :: ct, <^s \overline{BDA - CDA}, t, \tfrac{1}{2} <_s \overline{C - B}$, therefore, becauſe the Tangents, and co-tangents of Arches are reciprocally proportional, $ct, <^s \overline{BDA + CDA} = ct, < D : t, \tfrac{1}{2} <_s B + C :: t, \tfrac{1}{2} <^s \overline{C - B} : t, <^s \overline{BDA - CDA}$; that is, as the Co-tangent of half the Sum of the Vertical Angles, to the Tangent, *&c.* as was to be demonſtrated.

Propoſition IV. *Or* Axiom IV.

In every Oblique-angled Spherical Triangle it will be,
As the Sine of half the Sum of any two Sides,
Is to the Sine of half their Difference;
So is the Co-tangent of half the contained Angle,
To the Tangent of half the Difference of the oppoſite Angles.
And again,
As the Co-ſine of half the Sum of the two Sides,
To the Co-ſine of half their Difference;
So is the Co-tangent of half the contained Angle,
To the Tangent of half the Sum of the oppoſite Angles.



That is, in the Oblique-angled Spherical Triangle BDC, it will be.

1. As $S, \tfrac{1}{2} \overline{BD + DC} : S, \tfrac{1}{2} \overline{BD - DC} :: ct, \tfrac{1}{2} < \overline{BDC} : t, \tfrac{1}{2} <^s \overline{C - B}$.

2. As $\mathrm{cs},\frac{1}{2}\overline{BD+DC} : \mathrm{cs},\frac{1}{2}\overline{BD-DC} :: \mathrm{ct},\frac{1}{2}\angle BDC : \mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B}$.

For having let fall the Perpendicular DA from the contained Angle, upon the oppoſite ſide it will be,

As $\mathrm{S.}\,\overline{BD+DC} : \mathrm{S.}\,\overline{BD-DC} :: \mathrm{ct},\frac{1}{2}\angle D : \mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA}$ by the 1ſt preceding *Cor.* and $\mathrm{t},\frac{1}{2}\overline{BD+DC} : \mathrm{t},\frac{1}{2}\overline{BD-DC} :: \mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B} : \mathrm{t},\frac{1}{2}\angle^{s}\overline{C-B}$, by the 2d preceding *Cor.* and $\mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B} : \mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA} :: \mathrm{ct},\frac{1}{2}\angle D : \mathrm{t},\frac{1}{2}\angle^{s}\overline{C-B}$, by the 3d preceding *Cor.* multiplying therefore the Homologus Terms in each Proportion one into another, we ſhall have $\mathrm{S.}\,\overline{BD+DC}\times\mathrm{t},\frac{1}{2}\overline{BD+DC}\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B} : \mathrm{S.}\,\overline{BD-DC}\times\mathrm{t},\frac{1}{2}\overline{BD-DC}\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA} :: \mathrm{ct},\frac{1}{2}\angle D\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B}\times\mathrm{ct},\frac{1}{2}\angle D : \mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA}\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{C-B}\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{C-B}$; and dividing the 1ſt and 3d Terms by $\mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B}$, and the 2d and 4th Terms by $\mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA}$, we ſhall have $\mathrm{S.}\,\overline{BD+DC}\times\mathrm{t},\frac{1}{2}\overline{BD-DC} : \mathrm{S.}\,\overline{BD-DC}\times\mathrm{t},\frac{1}{2}\overline{BD-DC} :: \mathrm{ctq},\frac{1}{2}\angle D : \mathrm{tq},\frac{1}{2}\angle^{s}\overline{C-B}$, and by dividing the 1ſt and 3d again by 2 we ſhall have $\frac{1}{2}\mathrm{S.}\,\overline{BD+DC}\times\mathrm{t},\frac{1}{2}\overline{BD+DC} : \frac{1}{2}\mathrm{S.}\,\overline{BD-DC}\times\mathrm{t},\frac{1}{2}\overline{BD-DC} :: \mathrm{ctq},\frac{1}{2}\angle D : \mathrm{tq},\frac{1}{2}\angle^{s}\overline{C-B}$, but by the latter part of the 1ſt *Lemma*, $\frac{1}{2}\mathrm{S.}\,\overline{BD+DC}\times\mathrm{t},\overline{BD+DC}=\mathrm{Sq}\,\frac{1}{2}\overline{BD+DC}$, and by the ſame *Lemma*, $\frac{1}{2}\mathrm{S.}\,\overline{BD-DC}\times\mathrm{t},\frac{1}{2}\overline{BD-DC}=\mathrm{Sq}\,\frac{1}{2}\overline{BD-DC}$, wherefore $\mathrm{Sq}\,\frac{1}{2}\overline{BD+DC} : \mathrm{Sq}\,\frac{1}{2}\overline{BD-DC} :: \mathrm{ctq},\frac{1}{2}\angle D : \mathrm{tq},\frac{1}{2}\angle^{s}\overline{C-B}$, and conſequently by *Prop.* the 15th of *Part* the 1ſt, the Roots of theſe Terms will be proportional. That is,

$\mathrm{S.}\,\frac{1}{2}\overline{BD+DC} : \frac{1}{2}\overline{BD-DC} :: \mathrm{ct},\frac{1}{2}\angle D : \mathrm{t},\frac{1}{2}\angle^{s}\overline{C-B}$; which was one part to be demonſtrated.

Again, 2.

As $\mathrm{S.}\,\overline{BD+DC} : \mathrm{S.}\,\overline{BD-DC} :: \mathrm{ct},\frac{1}{2}\angle D : \mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA}$ by the 1ſt preceding *Cor.* and $\mathrm{ct},\frac{1}{2}\overline{BD+DC} : \mathrm{ct},\frac{1}{2}\overline{BD-DC} :: \mathrm{t},\frac{1}{2}\angle^{s}\overline{C-B} : \mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B}$ by the 2d preceding *Cor.* and $\mathrm{t},\frac{1}{2}\angle^{s}\overline{C-B} : \mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA} :: \mathrm{ct},\frac{1}{2}\angle D : \mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B}$, by the 2d preceding *Cor.* multiplying therefore the Homologus Terms in each Proportion one into another, we ſhall have $\mathrm{S.}\,\overline{BD+DC}\times\mathrm{ct},\frac{1}{2}\overline{BD+DC}\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{C-B} : \mathrm{S.}\,\overline{BD-DC}\times\mathrm{ct},\frac{1}{2}\overline{BD-DC}\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA} :: \mathrm{ct},\frac{1}{2}\angle D\times\mathrm{ct},\frac{1}{2}\angle^{s}\overline{C-B}\times\mathrm{ct},\frac{1}{2}\angle D : \mathrm{t},\frac{1}{2}\angle^{s}\overline{BDA-CDA}\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B}\times\mathrm{t},\frac{1}{2}\angle^{s}\overline{C+B}$; and dividing the 1ſt and 3d Terms by $t, \frac{1}{2} \angle^{s} \overline{C-B}$, and the 2d and 4th Terms by $t, \frac{1}{2} \angle^{s} \overline{BDA-CDA}$, we ſhall have $S.\, \overline{BD+DC} \times c\,t, \frac{1}{2} \overline{BD+DC} : S.\, \overline{BD-DC} \times c\,t, \overline{BD-DC} :: c\,t^{q}, \frac{1}{2} \angle D : t^{q}, \frac{1}{2} \angle^{s} \overline{B+C}$, and dividing the 1ſt and 2d Terms by 2 we ſhall have $\frac{1}{2} S.\, \overline{BD+DC} \times c\,t, \frac{1}{2} \overline{BD+DC} : \frac{1}{2} S.\, \overline{BD-DC} \times c\,t, \frac{1}{2} \overline{BD-DC} :: c\,t^{q}, \frac{1}{2} \angle D : t^{q}, \frac{1}{2} \angle^{s} \overline{B+C}$, but by the latter part of the 2d *Lemma*, $\frac{1}{2} S.\, \overline{BD+DC} \times c\,t, \frac{1}{2} \overline{BD+DC} = c\,s^{q}, \frac{1}{2} \overline{BD+DC}$; and by the ſame *Lemma* $\frac{1}{2} S.\, \overline{BD-DC} \times c\,t, \frac{1}{2} \overline{BD-DC} = c\,s^{q}, \frac{1}{2} \overline{BD-DC}$, wherefore by Subſtitution $c\,s^{q}, \frac{1}{2} \overline{BD+DC} : c\,s^{q}, \frac{1}{2} \overline{BD-DC} :: c\,t^{q}, \frac{1}{2} \angle D : t^{q}, \frac{1}{2} \angle^{s} \overline{B+C}$, and conſequently by *Prop.* the 15th of *Part* the 1ſt, their Roots will be proportional. That is,

As $c\,s, \frac{1}{2} \overline{BD+DC} : c\,s, \frac{1}{2} \overline{BD-DC} :: c\,t, \frac{1}{2} \angle D \,.\, t, \frac{1}{2} \angle^{s} \overline{B+C}$ which was the 2d *Part* to be demonſtrated. Wherefore,

As the Sine of half the Sum of any two Sides,
Is to the Sine of half their Difference, *&c.* as was to be demonſtrated.

Propoſition V. *Or* Axiom V.

In every Oblique-angled Spherical Triangle it will be,
As the Sine of half the Sum of any two Angles,
To the Sine of half their Difference;
So is the Tangent of half the interjacent Sides,
To the Tangent of half the Difference of the oppoſite Sides.
And again,
As the Co-ſine of half the Sum of the two Angles,
To the Co-ſine of half their Difference:
So is the Tangent of half the interjacent Side,
To the Tangent of half the Sum of the oppoſite Angles:

That is, in every Oblique-angled Spherical Triangle *BDC* it will be,

As $S. \frac{1}{2} \angle^{s} \overline{C+B} : S. \frac{1}{2} \angle^{s} \overline{C-B} :: t, \frac{1}{2} BC : t, \frac{1}{2} \overline{BD-DG}$, and $c\,s, \frac{1}{2} \angle^{s} \overline{C+B} : c\,s, \frac{1}{2} \angle^{s} \overline{C-B} :: t, \frac{1}{2} BC : t, \frac{1}{2} \overline{BD+DC}$.

For by the 17th *Ar.* of *Section* the 3d, the Side *a c* of the Triangle *d a c* (ſee the Figure in *Page* the 48*th*) conſtituted between the three nearer Poles *d*, *a*, *c*, of the Circles DC, BC, BD, which form the Spherical Triangle BDC) is equal to the Angle B of the given Triangle BDC, and the Side *d a* of the Triangle *d a c*, is equal

qual to the Angle C of the ſame Triangle BDC, alſo the Angle *a* of the one is equal to the Supplement of the Side BC of the other, and the Angles *d* and *c* equal to the Sides DC and DB, and becauſe by the preceding *Axiom* the $S.\ \frac{1}{2}\,\overline{da+ac}$ (in the Triangle *dac*) $:\ S.\ \frac{1}{2}\,\overline{da-ac}\ ::\ c\,t,\ \angle a\ :\ t,\ \frac{1}{2}\,\angle^{s}\,\overline{c-d}$ alſo the $c\,s,\ \frac{1}{2}\,\overline{da+ac}\ :\ c\,s,\ \frac{1}{2}\,\overline{da-ac}\ ::\ c\,t,\ \frac{1}{2}\,\angle a\ :\ t,\ \frac{1}{2}\,\angle^{s}\,\overline{c+d}$, it follows that the $S.\ \frac{1}{2}\,\angle^{s}\,\overline{C+B}$ of the Triangle BDC (equal to the $S.\ \frac{1}{2}\,\overline{da+ac}$ in the Triangle *dac*) is to $S.\ \frac{1}{2}\,\angle^{s}\,\overline{C-B}$ (equal to the $S.\ \frac{1}{2}\,\overline{da-ac}$) $::\ t,\ \frac{1}{2}\,BC$ (equal to the Complement of the Angle *a*) to the $t,\ \frac{1}{2}\,\overline{BD-DC}$ equal to the $t,\ \frac{1}{2}\,\overline{c-d}$; and again, as the $c\,s,\ \frac{1}{2}\,\angle^{s}\,\overline{C+B}$ to the $c\,s,\ \frac{1}{2}\,\angle^{s}\,\overline{C-B}$ ſo is the $t,\ \frac{1}{2}\,BC$ to the $t,\ \frac{1}{2}\,\overline{BD+DC}$; wherefore, as the Sine of half the Sum of the two Angles, to the Sine of half, *&c.* as was to be demonſtrated.

Theſe two *Axioms* are applicable to the *Solution* of the *7th* and *9th Caſes* of Oblique-angled Spherical Triangles, for having found the Sum and Difference of the two unknown Angles in *Caſe* the *7th*, the half Sum added to the half Difference will give the greater of the unknown Angles, but the half Difference taken from the half Sum will leave the leſſer of the two Angles.

After the ſame manner, having found the half Sum and half Difference of the two unknown Sides in *Caſe* the *9th*, the half Difference added to the half Sum will give the greater of the two unknown Sides, but ſubſtracted will leave the leſſer; as is abundantly manifeſt from what goes before.

Propoſition VI. *Or* Axiom VI.

In every Oblique-angled Spherical Triangle it will be,
As the Tangent of half the Baſe,
To the Tangent of half the Sum of the other two Sides;
So is the Tangent of half the Difference of the two Sides,
To the Tangent of half the Difference of the Segments of the Baſe; made by a Perpendicular let fall from the Angle oppoſite to the Baſe.

That is, in the Oblique-angled Spherical Triangle BDC it will be,

As the $t,\ \frac{1}{2}\,BC\ :\ t,\ \frac{1}{2}\,\overline{BD+DC}\ ::\ t,\ \frac{1}{2}\,\overline{BD-DC}\ :\ t,\ \frac{1}{2}\,\overline{BA-AC}$.

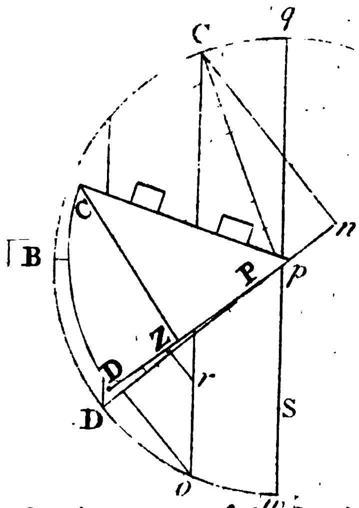
For by the *3d* preceding *Cor.* $cs, BD : cs, DC :: cs, BA : cs, CA$ (therefore by the *18th Cor.* of *Prop.* the *15th Part* the *1ſt*) $cs, BD + cs, DC : cs, BD - cs, DC :: cs, BA + cs, AC : cs, BA - cs, AC$; and by the *5th* preceding *Lemma*, $cs, BD + cs, DC : cs, BD - cs, DC :: ct, \frac{1}{2}\overline{BD+DC} : t, \frac{1}{2}\overline{BD-DC}$, and by the ſame *Lemma*, $cs, BA + cs, AC : cs, BA - cs, AC :: ct, \frac{1}{2}\overline{BA+AC} : t, \frac{1}{2}\overline{BA-AC}$, therefore by Equality of Ratio $ct, \frac{1}{2}\overline{BD+DC} : t, \frac{1}{2}\overline{BD-DC} :: ct, \frac{1}{2}\overline{BA+AC} : t, \frac{1}{2}\overline{BA-AC}$; and by Alternation, $ct, \frac{1}{2}\overline{BD+DC} : ct, \frac{1}{2}\overline{BA+AC} :: t, \frac{1}{2}\overline{BD-DC} : t, \frac{1}{2}\overline{BA-AC}$, but becauſe the Tangents and Co-tangents of Arches are reciprocally proportional, it will be $ct, \frac{1}{2}\overline{BD+DC} : ct, \frac{1}{2}\overline{BA+AC} :: t, \frac{1}{2}\overline{BA+AC} : t, \frac{1}{2}\overline{BD+DC}$, therefore by Equality of Ratio, $t, \frac{1}{2}\overline{BA+AC} : t, \frac{1}{2}\overline{BD+DC} :: t, \frac{1}{2}\overline{BD-DC} : t, \frac{1}{2}\overline{BA-AC}$, and becauſe $BA+AC=BC$ it will be, As $t, \frac{1}{2}BC : t, \frac{1}{2}\overline{BD+DC} :: t, \frac{1}{2}\overline{BD-DC} : t, \frac{1}{2}\overline{BA-AC}$; that is, as the Tangent of the Baſe, to the Tangent of half, *&c.* as was to be demonſtrated.

The half Difference of the Segments (thus found) added to the half Sum will give the greater Segment, for $\frac{1}{2}\overline{BA+AC} + \frac{1}{2}\overline{BA-AC} = BA$, and the half Difference taken from the half Sum will leave the leſſer of the two Segments, for $\frac{1}{2}\overline{BA+AC} - \frac{1}{2}\overline{BA-AC} = AC$.

But if the Perpendicular fall without the Triangle (as will always happen when the Angles at the Baſe are diſſimilar, that is, one Obtuſe and the other Acute) then the fourth Tangent found by the former Law, will be greater than the half of the Baſe *BC*; and therefore in this *Caſe*, being added to half the Baſe *BC*, will give the Baſe *BA* of the greater Triangle *BDA*, but the Sum leſſened by half the Baſe *BC* will leave the Baſe *CA* of the Supplemental Triangle *CDA*, as will be Exemplified at large in the next *Section*.

Propoſition VII. *Or* Axiom VII.

In every Oblique-angled Spherical Triangle, the Rectangle or Product of the Sines of any two Sides : is to the Square of the Radius : : as the Difference of the Verſed Sines of the Baſe and Difference of the two Sides : to the Verſed Sine of the Angle contained.



Let the upper Quadrant BCq of the Semicircle wBq, be turned about upon its Axis $Bamp$ (the Sector CDP being firſt erected obliquely above the Plane Bpw) till the Point C of the Semicircle, coincide with the Point C of the Sector, then will the Arches BC, CD, and DB, form an Oblique-angled Spherical Triangle.

Let CBD be the Angle required, then will CB and BD be the two containing Sides, and CD the Baſe or oppoſite Side, and becauſe Bo is equal to BC, oD will be equal to the Difference of the two containing Sides BC and BD.

From q draw qs a Perpendicular to pw, then will sw be the Verſed Sine of the Angle CBD, and from C draw Cz a Perpendicular to pD, then will zD be the Verſed Sine of the Baſe or oppoſite Side CD, alſo from o draw oy Perpendicular to pD, then will yD be the Verſed Sine of the Arch oD, the Difference of the two Sides, and conſequently zy the Difference of the Verſed Sines of the Baſe and Difference of the Sides, laſtly from C let fall Cr a Perpendicular to om, and joyn the Points z and r by the Right-line zr, which will be perpendicular to pD, and parallel to oy, and it will be,

As $Da \times mC : Rq :: yz : sw$.

For becauſe the Triangles Dap, tmp, tzr, and tyo are ſimilar, it will be $Da : Dp :: mp ; tp :: ty : to :: tz : tr :: ty - tz : to - tr$, but $ty - tz = zy$, and $to - tr = ro$; wherefore, $Da : Dp :: zy : ro$.

Again, becauſe the Triangles Cmr and qps are ſimilar it will be, $Cm:mr::qp:ps$, but $Cm=om$, and $qp=wp$, wherefore, $om:mr::wp:ps$, and conſequently $om:om-mr::wp:wp-ps$, that is, becauſe $om-mr=or$, $wp-ps=sw$, as $Cm\ (=om):or::wp:ps$; or as $Cm:wp::or:ws$, but it was before proved $Da:Dp::zy:or$, therefore $Cm\times Da:wp\times Dp::or\times zy:sw\times ro$, and conſequently $Cm\times Da:wp\times Dp::zy:sw$; but Cm is the Sine of one of the containing Sides *CB*, and Da the Sine of the other containing Side *BD*; alſo wp and Dp each equal to the Radius, wherefore $Cm\times Da:Rq::zy:sw$, that is, as the Rectangle or Product of the Sines of any two containing Sides, is to the Square of the Radius, *&c.* as was to be demonſtrated.

Again,

Cor. 1. Becauſe by the preceding Demonſtration $Cm\times Da:Rq::zy:sw$, if we multiply the two laſt Terms by $\frac{1}{2}R$ or half the Radius, we ſhall have $Cm\times Da:Rq::zy\times\frac{1}{2}R:sw\times\frac{1}{2}R$; that is, the Rectangle or Product of the Sines of any two containing Sides, is to the Square of the Radius, as the Rectangle or Product of half the Radius, multiplied into the Difference of the Verſed Sines of the Difference of the two Sides and Side oppoſite, to the Rectangle or Product of half the Radius, multiplied into the Verſed Sine of the contained Angle.

But by the 3*d Lemma*, the Difference of the Verſed Sines of two Arches multiplied into half the Radius, is equal to the Sine of half the Sum of thoſe Arches multiplied into the Sine of half their Difference.

That is, $zy\times\frac{1}{2}R=\text{S.}\frac{DC+BC-BD}{2}\times\text{S.}\frac{DC-BC+BD}{2}=$ $\text{S.}\frac{DC+BC+BD}{2}-BD\times\text{S.}\frac{DC+BC+BD}{2}-BC.$

And by *Lemma* the 1ſt, the Verſed Sine of an Arch multiplied into half the Radius, is equal to the Square of the Sine of half the Arch. That is,

$sw\times\frac{1}{2}R=\text{Sq}\ \frac{1}{2}$ Angle B, wherefore, As $\text{S. }BC\times\text{S. }BD:Rq::\text{S.}\frac{DC+BC+BD}{2}-BD\times\text{S.}\frac{DC+BC+BD}{2}-BC$: Sq. of half the Angle B. That is,

2. The Rectangle or Product of the Sines of any 2 containing Sides is to the Square of the Radius, as the Rectangle or Product of the

Sines of half the Sum of the three Sides, leſſened by one of the containing Sides multiplied into the Sine of half the Sum leſſened by the other containing Side, to the Square of the Sine of half the contained Angle.

Put $z=\frac{BC+BD+DC}{2}$ then, $S.BC\times S.BD : Rq :: S.\overline{Z-BC}\times S.\overline{Z-BD} : Sq.\frac{1}{2}\angle B$; wherefore, $\frac{Rq}{S.BC\times S.BD}=\frac{Sq\,\frac{1}{2}\angle B.}{S.\overline{Z-BC}\times S.\overline{Z-BD}}$;

That is, $\frac{Rq}{S.BC\times S.BD}\times S.\overline{Z-BC}\times S.\overline{Z-BD}=Sq\,\frac{1}{2}\angle B.$

Or, $\frac{R}{S.BC}\times\frac{R}{S.BD}\times S.\overline{Z-BC}\times S.\overline{Z-BD}=Sq\,\frac{1}{2}\angle B.$

That is, becauſe the Difference between the Logarithmic Radius and the Logarithmic Sine, is called the Arithmetical Complement of the Sine.

3. To the Arithmetical Complement of the Sine of one of the containing Sides,

Add the Arithmetical Complement of the Sine of the other containing Side,

Alſo the Sine of half the Sum of the three Sides, leſſened by one of the containing Sides,

And the Sine of half the Sum of the three Sides leſſened by the other containing Side,

Half the the Sum of theſe four Numbers will be the Sine of half the contained Angle.

Again, becauſe the Sine of $BC\times S.BD : Rq :: S.\overline{Z-BC}\times S.\overline{Z-BD} : Sq\,\frac{1}{2}\angle B$, the $S.BC\times S.BD\times S.\frac{1}{2}\angle B=S.\overline{Z-BC}\times S.\overline{Z-BD}\times SRq$; and conſequently $\frac{S.BC\times S.BD\times Sq\,\frac{1}{2}\angle B}{Rq}=S.\overline{Z-BC}\times S.\overline{Z-BD}$, that is, $\frac{S.BC\times S.BD}{R}\times\frac{Sq\,\frac{1}{2}\angle B}{R}=S.\overline{Z-BC}\times S.\overline{Z-BD}$, wherefore, $\frac{S.BC\times S.BD}{R} : S.\overline{Z-BC} :: S.\overline{Z-BD} : \frac{Sq\,\frac{1}{2}\angle B}{R}$; and putting $\frac{S.BC\times S.BD}{R}=F$, and $\frac{Sq\,\frac{1}{2}\angle B}{R}=G$ it will be $F : S.\overline{Z-BC} :: S.\overline{Z-BD} : G$. That is,

4. As the Radius,

To the Sine of one of the containing Sides;

So is the Sine of the other containing Side,
To a fourth Sine.

As that fourth Sine,
To the Sine of half the Sum of the three Sines leſſened by one of the containing Sides;
So is the Sine of the half Sum leſſened by the other containing Side,
To a ſeventh Sine equal to G.

But $G=\frac{Sq\,\frac{1}{2}<B,}{R}$ therefore $G\times R=Sq\,\frac{1}{2}<B$; wherefore if to the ſeventh Sine you add the Radius, half that Sum will be the Sine of half the contained Angle.

Again, becauſe $S.BC\times S.BD : Rq :: S.\overline{Z-BC}\times S.\overline{Z-BD} : Sq\,\frac{1}{2}<B$, the $S.BC\times S.BD : S.\overline{Z-BC}\times S.\overline{Z-BD} :: Rq : Sq\,\frac{1}{2}<B$, and conſequently, $\frac{S.BC\times S.BD}{S.\overline{Z-BC}} : S.\overline{Z-BD} :: R : \frac{Sq\,\frac{1}{2}<B}{R}$ putting therefore $\frac{S.BC\times S.BD}{S.\overline{Z-BC}}=F$, and $\frac{Sq\,\frac{1}{2}<B}{R}=G$, it will be $F : S.\overline{Z-BD} :: R : G$. That is,

5. As the Sine of half the Sum of the three Sides leſſened by any one of the Containing Sides,
To the Sine of the ſame containing Side;
So is the Sine of the other containing Side,
To a fourth Sine.

As this fourth Sine,
To the Sine of the half Sum leſſened by the other containing Side;
So is the Radius,
To a ſeventh Sine equal to G, and becauſe $G\times R=Sq\,\frac{1}{2}<B$, if to the ſeventh Sine you add the Radius, half that Sum, &c.

Again, becauſe $S.BC\times S.BD : S.\overline{Z-BC}\times S.\overline{Z-BD} :: Rq : Sq\,\frac{1}{2}<B$, the $S.BD : \frac{S.\overline{Z-BC}\times S.\overline{Z-BD}}{S.BC} :: R : \frac{Sq\,\frac{1}{2}<B}{R}$ putting therefore; $\frac{S.\overline{Z-BC}\times S.\overline{Z-BD}}{S.BC}=F$, and $\frac{Sq\,\frac{1}{2}<B}{R}=G$, it will be, $S.BD : F :: R : G$. That is,

6. As the Sine of any one of the containing Sides,
To the Sine of half the Sum of the three Sides leſſened by the ſame containing Side;

So is the Sine of half the Sum leſſened by the other containing Side,

To a fourth Sine. Then,

As the Sine of the other containing Side,

To the fourth Sine;

So is the Radius,

To a ſeventh Sine equal to G; and becauſe $G \times R = Sq\,\frac{1}{2} < B$, therefore to the ſeventh Sine, if you add the Radius, half that Sum, *&c.*

Again, putting V for the Verſed Sine of an Arch,

7. Becauſe $S.\,BD \times S.\,BC : Rq :: V,\,DC - V,\overline{BC - BD} : V, < B$, by the *7th Axiom.*

The $S.\,BD : \frac{Rq}{S\ BC} :: V,\,DC - V,\overline{BC - BD} : V, < B$; that is, becauſe the Square of the Radius divided by the Right Sine is equal to the Co-ſecant of the ſame Arch.

The $S.\,BD : c\,s^{t},BC :: V,\,DC - V,\overline{BC - BD} : V, < B$.

And becauſe the Difference of the Co-ſines of two Arches is the ſame with the Difference of the Verſed Sines of the ſame Arches.

The $S.\,BD : c\,s^{t},BC :: c\,s,\,DC - c\,s,\overline{BC - BD} : V, < B$. That is, the Sine of one of the containing Sides is to the Co-ſecant of the other, as the Difference between the Co-ſines of the oppoſite Side and Difference of the two Sides, to the Verſed Sine of the contained Angle.

8. But if in the room of BC be put its Complement, then the $S.\,BD : c\,s^{t},BC :: c\,s,\,DC - c\,s,\overline{BC + BD} : V, B$. That is, the Sine of one of the containing Sides is to the Co-ſecant of the other, as the Difference between the Co-ſine of the oppoſite Side, and the Sine of one of the Sides increaſed by the Complement of the other, to the Verſed Sine of the contained Angle.

For if *c* be put for the Complement of BC, then $90° - BC + BD =$ Complement of $BC + BD$, and putting *o* in the room of $90° - BC$ we ſhall have $c + BD$ for the Complement of the Co-ſine, that is for the Sine.

Prop.

Propoſition VIII. *Or* Axiom VIII.

In every Oblique-angled Spherical Triangle, the Rectangle or Product of the Sines of any two Sides, is to the Square of the Radius, as the Difference of the Verſed Sines of the Sum of the two Sides and the Baſe or Side oppoſite, to the Verſed Sine of the Supplement of the contained Angle.

See the *Scheme* in *Page* the 138*th*.

That is, As Da the Sine of DB multiplied into mC the Sine of BD, is to the Radius ſquared; ſo is zn the Difference of the Verſed Sines nd and zd, of the Sum of the Sides and Side oppoſite, to qs the Verſed Sine of the Supplement of the Angle B.

For becauſe the Triangles qsp and Crm are ſimilar, it will be, As $mC : mr :: pq : qs$.

Wherefore, $mC : mC + mr :: pq : pq + ps$. But $mC + mr = Cr$, and alſo $pq + ps = qs$, and pq equal to the Radius.

Wherefore, as $mC : Cr :: R : qs$.

That is, as $mC : R :: Cr : qs$.

Reduce the Quadrant Bpq to the ſame Plane with Bpw, ſo that both may make one Plane, and draw Cn perpendicular to Dp produced, then will Dn be the Verſed Sine of the Arch CD equal to the Sum of the Sides CB and BD, and zD the Verſed Sine of CD the Side oppoſite, and conſequently zn equal to the Difference of the Verſed Sines of the Sum of the Sides and Side oppoſite, and becauſe the Triangles Dap, tzr, and tnC are ſimilar, it will be $Da : Dp :: tz : tr :: tn : tC :: tz + tn : tr + tC$; wherefore, $Da : Dp :: tz + tn : tr + tC$, but $tz + tn = zn$, alſo $tr + tC = Cr$, and Dp equal to the Radius.

Wherefore, as $Da : R :: zn : Cr$.

But it was before proved, as $mC : R :: Cr : qs$.

Wherefore $Da \times mC : R \times R :: zn \times Cr : Cr \times qs$; and conſequently $Da \times mC : Rq :: zn : qs$, which was to be Demonſtrated.

Cor. 1. Again becauſe by the preceding *Theorem*

As S. $Da \times mC : Rq :: zn : qs$, if we multiply the two laſt Terms by $\frac{1}{2}R$ or half the Radius, it will be, $Da \times mC : Rq :: zn \times \frac{1}{2}R : qs \times \frac{1}{2}R$; that is, the Rectangle or Product of the Sines of any two containing Sides, is to the Square of the Radius, as the Rectangle or Product of half the Radius multiplied into the Difference of the Verſed Sines of the Sum of the Sides and Side oppoſite, to the Rectangle or Product of half the Radius, multiplied into the Verſed Sine of the Supplement of the contained Angle.

But by *Lemma* the 3*d*, the Difference of the Verſed Sines of two Arches multiplied into half their Radius, is equal to the Sine of half the Sum of thoſe Arches multiplied into the Sine of $\frac{1}{2}$ their Difference. That is, $z\,n \times \frac{1}{2} R = S.\frac{BC+BD+DC}{2} \times S.\frac{BC+BD-DC}{2}$

And by *Lemma* the 2*d*, the Verſed Sine of the Supplement of an Arch multiplied into half the Radius, is equal to the Square of the Co-ſine of half that Arch, that is, $q\,s \times \frac{1}{2} R = \text{csq}, \frac{1}{2} < B$. Wherefore,

$$S.\,BC \times S.\,BD : Rq :: S.\frac{BC+BD+DC}{2} \times S.\frac{BC+BD-DC}{2} : c\,sq, \tfrac{1}{2} < B.$$ That is,

2. The Rectangle or Product of the Sines of any two Sides, is to the Square of the Radius, as the Rectangle or Product of half the Sum of the two Sides and Side oppoſite, multiplied into half the Difference of the Sum of the two Sides and Side oppoſite, to the Square of the Co-ſine of half the contained Angle.

But becauſe $S.\frac{BC+BD+DC}{2} - DC = S.\frac{BC+BD-DC}{2}$ therefore, $S.\,BC \times S.\,BD : Rq :: S.\frac{BC+BD+DC}{2} \times S.\frac{BC+DB+DC}{2} - DC : csq, \frac{1}{2} < B$. That is,

3. The Rectangle or Product of the Sines of any two Sides, is to the Square of the Radius, as the Rectangle or Product of the Sine of half the Sum of the three Sides, multiplied into the Sine of half the Sum of the three Sides leſſened by the Side oppoſite, to the Square of the Co-ſine of half the contained Angle.

Put $Z = \frac{BC+BD+DC}{2}$ and $X = \frac{BC+BD+DC}{2} - DC$, then $S.\,BC \times S.\,BD : Rq :: S.\,Z \times S.\,X : csq, \frac{1}{2} < B$; wherefore, $\frac{Rq}{S.\,BC \times S.\,BD} = \frac{csq, \frac{1}{2} < B}{S.\,Z \times S.\,X}$ that is, $\frac{Rq}{S.\,BC \times S.\,BD} \times S.\,Z \times S.\,X = csq, \frac{1}{2} < B$. Or, $\frac{R}{S.\,BC} \times \frac{R}{S.\,BD} \times S.\,Z \times S.\,X = c\,sq, \frac{1}{2} < B$. That is, Becauſe the Difference between the Logarithmic Radius and the Logarithmic Sine of any Arch, is called the Arithmetical Complement of the Sine of that Arch.

4. To the Arithmetical Complement of the Sine of one of the containing Sides,
Add the Arithmetical Complement of the Sine of the other containing Side;
Alſo the Sine of half the Sum of the three Sides,
And the Sine of half the Sum leſſened by the Side oppoſite,
Half the Sum of theſe four Sines will be the Co-ſine of half the contained Angle.

5. Again, becauſe $\text{S. BC}\times\text{S. BD} : Rq :: \text{S. Z}\times\text{S. X} : csq,\ \frac{1}{2}<B.$ The $\text{S. BC}\times\text{S. BD}\times csq,\ \frac{1}{2}<B=\text{S. Z}\times\text{S. X}\times Rq.$

And conſequently, $\dfrac{\text{S. BC}\times\text{S. BD}\times csq,\ \frac{1}{2}<B}{Rq}=\text{S. Z}\times\text{S. X}.$

That is, $\dfrac{\text{S. BC}\times\text{S. BD}}{R}\times\dfrac{csq,\ \frac{1}{2}<B}{R}=\text{S. Z}\times\text{S. X},$

Wherefore, $\dfrac{\text{S. BC}\times\text{S. BD}}{R} : \text{S. Z} :: \text{S. X} : \dfrac{csq,\frac{1}{2}<B.}{R}$

And putting $\dfrac{\text{S. BC}\times\text{S. BD}}{R}=F$, and $\dfrac{csq,\ \frac{1}{2}<B}{R}=G$, it will be, As $F : \text{S. Z} :: \text{S. X} : G$. That is,

As the Radius,
To the Sine of one of the containing Sides;
So is the Sine of the other containing Side;
To a fourth Sine.
As the fourth Sine,
To the Sine of half the Sum of the three Sides;
So is the Sine of half the Sum of the three Sides, leſſened by the Side oppoſite,
To a ſeventh Sine equal to G.

But becauſe $G=\dfrac{csq,\ \frac{1}{2}<B}{R}$ therefore $G\times R=csq,\ \frac{1}{2}<B.$

Wherefore if to this ſeventh Sine you add the Radius; half that Sum will be the Co-ſine of half the contained Angle.

6. Again, becauſe the $\text{S. BC}\times\text{S. BD} : Rq :: \text{S. Z}\times\text{S. X} : csq,\ \frac{1}{2}<B$; the $\text{S. BC}\times\text{S. BD} : \text{S. Z}\times\text{S. X} :: Rq : csq,\ \frac{1}{2}<B.$

And conſequently, $\dfrac{\text{S. BC}\times\text{S. BD}}{\text{S. Z}}\ \text{S. X} :: R : \dfrac{csq,\ \frac{1}{2}<B;}{R}$

Put $\dfrac{\text{S. BC}\times\text{S. BD}}{\text{S. Z}}=F$, and $\dfrac{csq,\ \frac{1}{2}<B}{R}=G$, and it will be,

As $F : \text{S. X} :: R : G$. That is,

As the Sine of half the Sum of the three Sides,
To the Sine of one of the containing Sides;
So is the Sine of the other containing Side,
To a fourth Sine.
As the fourth Sine,
To the Sine of half the Sum of the three Sides, leſſened by the Side oppoſite;
So is the Radius,
To a ſeventh Sine equal to G, to which (becauſe $G \times R = csq, \frac{1}{2} < B$) add the Radius, and half that Sum will be the Co-ſine of half the contained Angle.

7. Again, becauſe $S.\,BC \times S.\,BD : S.\,Z \times S.\,X :: Rq : csq, \frac{1}{2} < B$, the $\frac{S.\,BC \times S.\,BD}{S.\,X} : S.\,Z :: R : \frac{csq, \frac{1}{2} < B}{R}$

Putting therefore $\frac{S.\,BC \times S.\,BD}{S.\,X} = F$, and $\frac{csq, \frac{1}{2} < B}{R} = G$,

it will be, $F : S.\,Z :: R : G$. That is,

As the Sine of half the Sum of the three Sides, leſſened by the Side oppoſite,
To the Sine of one of the containing Sides;
So is the Sine of the other containing Side,
To a fourth Sine.
As the fourth Sine,
To the Sine of half the Sum of the three Sides;
So is the Radius,
To a ſeventh Sine; to which add the Radius, half that Sum, &c.

8. Again, becauſe $S.\,BC \times S.\,BD :: S.\,Z \times S.\,X :: Rq : csq, \frac{1}{2} < B$, the $S.\,BC : \frac{S.\,Z \times S.\,X}{S.\,BD} :: R : \frac{csq, \frac{1}{2} < B}{R}$

put $\frac{S.\,Z \times S.\,X}{S.\,BD} = F$, and $\frac{csq, \frac{1}{2} < B}{R} = G$.

Then $S.\,BC : F :: R : G$. That is,

Again,

The $S.\,BD : \frac{S.\,Z \times S.\,X}{S.\,BC} :: R : \frac{csq, \frac{1}{2} < B}{R}$.

That is putting $F = \frac{S.\,Z \times S.\,X}{S.\,BC}$ and $\frac{csq, \frac{1}{2} < B}{R} = G$

it will be, $S.\,BD : F :: R : G$. That is in either Caſe,

As the Sine of any one of the containing Sides
To the Sine of the half Sum of the three Sides;
So is the Sine of the half Sum, leſſened by the Side oppoſite,
To a fourth Sine.
As the Sine of the other containing Side,
To the fourth Sine;
So is the Radius,
To a ſeventh Sine; to which if the Radius be added half that Sum, *&c.*

9. Again, becauſe by *Cor.* the *3d* of *Axiom* the *8th*, $S.\,BD \times S.\,BC : Rq :: S.\,Z \times S.\,X : csq,\ \frac{1}{2} \angle B$; and by *Cor.* the *2d* of *Ax.* the *7th*,

The $S.\,BD \times S.\,BC : Rq :: S.\,\overline{Z-BD} \times S.\,\overline{Z-BC} : Sq\ \frac{1}{2} \angle B$.

The $S.\,Z \times S.\,X : csq,\ \frac{1}{2} \angle B :: S.\,\overline{Z-BD} \times S.\,\overline{Z-BC} : Sq\ \frac{1}{2} \angle B$,

And conſequently $S.\,Z \times S.\,X : S.\,\overline{Z-BD} \times S.\,\overline{Z-BC} :: csq,\ \frac{1}{2} \angle B : Sq\ \frac{1}{2} \angle B$.

But it has been already ſhewn, that the Co-ſine of an Arch is to the Right-ſine, as the Radius, to the Tangent of the ſame Arch; and conſequently, the Square of the Co-ſine, is to the Square of the Right-ſine, as the Square of the Radius, to the Square of the Tangent. Wherefore, the

$S.\,Z \times S.\,X : S.\,\overline{Z-BD} \times S\,\overline{Z-BC} :: Rq : tq,\ \frac{1}{2} \angle B$. That is,

The Rectangle or Product of the Sines of half the Sum of the three Sides multiplied into the Sine of half the Sum of the three Sides, leſſened by the Side oppoſite, is to the Rectangle or Product of half the Sum of the three Sides leſſened by one of the containing Sides, multiplied into the half Sum of the three Sides, leſſened by the other containing Side, as the Square of the Radius, to the Square of the Tangent of half the contained Angle.

Put $A = S.\,\overline{Z-BD}$, and $E = S.\,\overline{Z-BC}$.

Then $S.\,Z \times S.\,X : S.\,A \times S.\,E :: Rq : tq,\ \frac{1}{2} \angle B$;

And, $\dfrac{Rq}{S.\,Z \times S.\,X} = \dfrac{tq,\ \frac{1}{2} \angle B}{S.\,A \times S.\,E}$

Alſo, $\dfrac{Rq}{S.\,Z \times S.\,X} \times S.\,A \times S.\,E = tq,\ \frac{1}{2} \angle B$.

Or, $\dfrac{R}{S.\,Z} \times \dfrac{R}{S.\,X} \times S.\,A \times S.\,E = tq,\ \frac{1}{2} \angle B$. That is,

10. To the Arithmetical Complement of half the Sum of the three Sides,

Add the Arithmetical Complement of half the Sum of the three Sides leſſened by the Side oppoſite; alſo the Sine of half the Sum of the three Sides leſſened by one of the containing Sides,

And the Sine of the half Sum leſſened by the other containing Side.

Half the Sum of theſe four Numbers will be the Tangent of half the contained Angle.

11. Again, becauſe $S.Z \times S.X : S.A \times S.E :: Rq : tq, \frac{1}{2} < B$,

The $S.Z \times S.X : Rq :: S.A \times S.E\ tq, \frac{1}{2} < B$,

And conſequently, $\frac{S.Z \times S.X \times tq, \frac{1}{2} < B}{Rq} = S.A \times S.E$,

That is, $\frac{S.Z \times S.X}{R} \times \frac{tq, \frac{1}{2} < B}{R} = S.A \times S.E$,

Wherefore $\frac{S.Z \times S.X}{R} : S.A :: S.E : \frac{tq, \frac{1}{2} < B}{R}$

And putting $\frac{S.Z \times S.X}{R} = F$, and $\frac{tq, \frac{1}{2} < B}{R} = G$.

it will be, $F : S.A :: S.E : G$. That is,

As the Radius,
To the Sine of the half Sum of the three Sides;
So is the Sine of the half Sum, leſſened by the Side oppoſite,
To a fourth Sine.

As the fourth Sine,
To the Sine of the half Sum leſſened by one of the containing Sides;
So is the Sine of half the Sum leſſened by the other containing Side,
To a ſeventh Sine; to which if the Radius be added, half that Sum will be the Tangent of half the contained Angle.

12. Again, becauſe the Tangent of an Arch is to the Radius, as the Radius to the Co-tangent of the ſame Arch, it follows from the *9th* preceding *Cor.*

That $S.\overline{Z-BD} \times S.\overline{Z-BC} : S.Z \times S.X :: Rq : c\,tq, \frac{1}{2} < B$. That is,

The Rectangle or Product of the Sines of half the Sum of the three Sides leſſened by one of the containing Sides, multiplied into the half Sum of the Sides leſſened by the other containing Side, is to the Rectangle or Product of the Sine of half the Sum of the Sides, multiplied into the Sine of the half Sum leſſened by the Side oppoſite; as the Square of the Radius, to the Square of the Co-tangent of half the contained Angle.

That is, putting $\overline{Z-BD}=A$, and $\overline{Z-BC}=E$,
$S.\,A\times S.\,E :: S.\,Z\times S.\,X :: Rq : c\,tq, \frac{1}{2} < B$;

Whence, $\dfrac{Rq}{S.\,A\times S.\,E}=\dfrac{c\,tq, \frac{1}{2} < B}{S.\,Z\times S.\,X}$

And $\dfrac{Rq}{S.\,A\times S.\,E}\times S.\,Z\times S.\,X = c\,tq, \frac{1}{2} < B$,

And conſequently, $\dfrac{R}{S.\,A}\times\dfrac{R}{S.\,E}\times S.\,Z\times S.\,X = c\,tq, \frac{1}{2} < B$. That is,

13. To the Arithmetical Complement of the Sine of half the Sum of the three Sides leſſened by one of the containing Sides,

Add the Arithmetical Complement of the Sine of half the Sum leſſened by the other containing Side;

Alſo the Sine of half the Sum of the three Sides

And the Sine of the half Sum leſſened by the Side oppoſite,

Half the Sum of theſe four Numbers will be the Co-tangent of half the contained Angle.

14. Again becauſe $S.\,A\times S.\,E : S.\,Z\times S.\,X :: Rq : c\,tq, \frac{1}{2} < B$,

The $S.\,A\times S.\,E : Rq :: S.\,Z\times S.\,X : c\,tq, \frac{1}{2} < B$;

And conſequently, $\dfrac{S.\,A\times S.\,E\times c\,tq, \frac{1}{2} < B}{Rq}=S.\,Z\times S.\,X$.

And $\dfrac{S.\,A\times S.\,E}{R}\times\dfrac{c\,tq, \frac{1}{2} < B}{R}=S.\,Z\times S.\,X$,

Putting therefore $\dfrac{S.\,A\times S.\,E}{R}=F$, and $\dfrac{c\,tq, \frac{1}{2} < B}{R}=G$
it will be, $F : S.\,Z :: S.\,X : G$. That is;

As the Radius,
To the Sine of half the Sum of the three Sides leſſened by one of the containing Sides;
So is the Sine of half the Sum leſſened by the other containing Side,
To a fourth Sine.

As the fourth Sine,
To the Sine of half the Sum of the three Sides;
So is the Sine of the half Sum leſſened by the Side oppoſite,
To a ſeventh Sine; to which if the Radius be added, half that Sum will be the Co-tangent of half the contained Angle.

Again, by inverting, changing, dividing, *&c.* the Terms of the two laſt *Cor.* ſeveral other Methods may be found.

15. Put $P=BC+BD$, $O=BC-BD$, and $V=$ Verſed Sine,
Then $S.BD\times S.BC : Rq :: VP-VDC : V$ Sup. $<B$ by the 8th *Ax.*
And $S.BD\times S.BC : Rq :: VDC-VO : V<B$, by the 7th *Ax.*
Therefore, $VP-VDC : VDC-VO :: V$ Sup. $<B : V<B$;
And conſequently $VP-VDC+VDC-VO : VDC-VO :: V$ Sup. $<B+V<B : V<B$. That is,

$VP-VO : VDC-VO :: 2R : V<B$; for the Verſed Sine of an Arch together with the Verſed Sine of its Supplement, is equal to the double Radius or whole Diameter.

Therefore, $VP-VO : 2R :: VDC-VO : V<B$. That is,

As the Verſed Sine of the Sum of any two Sides, leſſened by the Verſed Sine of their Difference, is to twice the Radius; ſo is the Verſed Sine of the oppoſite Side leſſened by the Verſed Sine of the Difference of the Sides, to the Verſed Sine of the contained Angle.

Again,

16. $VP-VDC+VDC-VO : VP-VDC :: V$ Sup. $<B+V,<B : V$ Sup. $<B$. That is,

$VP-VO : VP-VDC :: 2R : V$ Sup. $<B$.

Or $VP-VO : 2R :: VP-VDC : V$ Sup. $<B$. That is,

As the Verſed Sine of the Sum of any two Sides, leſſened by the Verſed Sine of their Difference, is to twice the Radius; ſo is the Verſed Sine of the Sum of the two Sides, leſſened by the Verſed Sine of the Side oppoſite, to the Verſed Sine of the Supplement of the contained Angle.

17. Again, becauſe the Radius leſſened by the Verſed Sine, is equal to the Co-ſine of the ſame Arch, it follows from the 15th *Cor.*

That $cs,P-cs,O : 2R :: cs,DC-cs,O : cs,<B$. That is,

The Difference of the Co-ſines of the Sum and the Difference of any two containing Sides, is to twice the Radius; as the Difference of the Co-ſines of the oppoſite Side and the Difference of the two Sides, to the Co-ſine of the containing Angle. Or,

18. $\frac{cs,P-cs,O}{2} : R :: cs,DC-cs,O : cs,<B$. That is,

Half the Difference of the Co-ſines of the Sum and the Difference of any two containing Sides, is to the Radius; as the Difference of the Co-ſines of the oppoſite Side, and the Difference of the two Sides, to the Co-ſine of the contained Angle.

19. From the 16th *Cor.* it follows, $cs,P-cs,O : 2R :: cs,P-cs,DC : cs,$ Sup. $<B$. That is,

The Difference of the Co-ſines of the Sum, and the Difference of any two containing Sides, is to twice the Radius; as the Difference of the Co-ſines of the Sum of the two Sides and Side oppoſite, to the Co-ſine of the Supplement of the contained Angle.

Or,

20. $\frac{c\,s, P - c\,s, O}{2} : R :: c\,s, P - c\,s, DC : c\,s, Sup. \angle B$. That is,

Half the Difference of the Co-ſines of the Sum and the Difference of any two containing Sides, is to the Radius; as the Difference of the Co-ſines of the Sum of the two Sides and the Side oppoſite, to the Co-ſine of the Supplement of the contained Angle.

By Comparing, Compounding, &c. the Terms of the ſeveral Analogies, a great variety of Rules may be inveſtigated; but theſe that are here deduced, are more than ſufficient for the preſent Purpoſe.

Propoſition IX. *Or* Axiom IX.

In every Oblique-angled Spherical Triangle, (taking the Supplements of each of the Obtuſe Angles, inſtead of the Angles themſelves) it will be,

As the Rectangle or Product of the Sines of any two adjacent Angles is to the Square of the Radius: : ſo is the Rectangle or Product of the Sines of half the Sum of the three Angles, leſſened by one of the adjacent Angles, multiplied into the half Sum of the three Angles, leſſened by the other adjacent Angle; to the Square of the Sine of half the interjacent Side, if the Angle oppoſite be Acute, but to the Square of the Co-ſine if the Angle oppoſite be Obtuſe.

That is, putting $Z = \frac{\angle C + \angle B + Sup. \angle D}{2}$ if the Angle D be Obtuſe,

The $Sup. \angle D \times S. \angle C : Rq :: S. \overline{Z - Sup. \angle D} \times S. \overline{Z - \angle C} : Sq, \frac{1}{2} CD$.

Becauſe the Quantity of each Angle of a Spherical Triangle, is meaſured by an Arch of a Great Circle deſcribed about the angular Point as a Pole, and intercepted between the Sides which form the Angle, and is therefore equal to the Arch of the ſame Great Circle, contained between the Poles of the two Great Circles which form the Angle, it follows; as has been ſhewn in *Art.* the *17th* of *Section* the 3d, that the Side *a d* of the Triangle *d a c* (ſee the Figure in Page the 48*th*) conſtituted between the three nearer Poles *d*, *a*, *c*, of the Arches *DC*, *BC*, *BD*, which form the Spherical Triangle *BDC*, is equal to the Angle *C* of the Triangle *BDC*; alſo the Side *a c* of the Triangle *d a c*, is equal to the Angle *B* of the Triangle *BDC*, the Side *d c* of the one, equal to the Supplement of the Angle *D* of the other, and the Angle *d* of the Triangle *d a c*, is equal to the Side *DC* of the Triangle *BDC*; and becauſe by *Ax.* the 7*th*, in the Triangle *d a c*

Putting $Z = \frac{dc + ad + ac}{2}$ the $S.\,dc \times S.\,ad : Rq :: S.\,\overline{Z - dc} \times S.\,\overline{Z - ad} : Sq\,\tfrac{1}{2} < d$ it follows, that

The $S.\,Sup. < D \times S. < C\ (= S.\,dc \times S.\,ad) : Rq :: S.\,\overline{Z - Sup. < D} \times S.\,\overline{Z - < C} = S.\,\overline{Z - dc} \times S.\,\overline{Z - ad} : Sq\,\tfrac{1}{2}\,CD\ (= Sq\,\tfrac{1}{2} < d)$ wherefore,

$S.\,Sup. < D \times S. < C : Rq :: S.\,\overline{Z - Sup. < D} \times S.\,\overline{Z - < C} : Sq\,\tfrac{1}{2}\,CD$; which was to be demonſtrated.

If the Angle *B* oppoſite to the interjacent Side *CD* had been Obtuſe, then the Side *a c* of the Triangle *d a c*, had been equal to the Supplement of the Angle *B*, and conſequently the Angle oppoſite to the Side *a c* had been Obtuſe and equal to the Supplement of the Side *DC*; and conſequently half the Square of the Co-ſine of the Angle *d* had been equal to the Side *CD*; that is, it would have been to the *csq*, ½ *CD*.

3. Again, becauſe by the 3*d Cor.* of the 7*th Axiom*,

The $\frac{R}{S.\,dc} \times \frac{R}{S.\,ad} \times S.\,\overline{Z - dc} \times S.\,\overline{Z - ad} = Sq\,\tfrac{1}{2} < d$ it follows,

That the $\frac{R}{S.\,Sup. < D} \times \frac{R}{S. < C} \times S.\,\overline{Z - Sup. < D} \times S.\,Z - < C =$ Sq of half *DC*. That is,

3. To the Arithmetical Complement of the Sine of one of the adjacent Angles,

Add the Arithmetcal Complement of the Sine of the other adjacent Angle,

Alſo the Sine of half the Sum of the three Angles leſſened by one of the adjacent Angles,

And the Sine of half the Sum leſſened by the other adjacent Angle,

Half the Sum of theſe four Sines will be the Sine of half the interjacent Side if the Angle oppoſite be Acute, but if the oppoſite Angle be Obtuſe, it will be the Co-ſine of half the interjacent Side.

4. Again, becauſe by the *4th Cor.* of the *7th Axiom*,

The $\frac{S.\,dc \times S.\,da}{R} : S.\overline{Z - dc} :: S.\,\overline{Z - ad} : \frac{Sq\,\frac{1}{2} < d}{R}$ it follows,

That the $\frac{S.\,\text{Sup.} < D \times S. < C}{R} : S.\,Z - \text{Sup.} < D :: S.\,\overline{Z - < C} : \frac{Sq\,\frac{1}{2}\,DC}{R}$

And putting $\frac{S.\,\text{Sup.} < D \times S. < C}{R} = F$, and $\frac{Sq\,\frac{1}{2}\,DC}{R} = Q$ it will be, $S.\,F : S.\overline{Z - \text{Sup.} < D} :: S.\overline{Z - < C} : Q$. That is,

As the Radius,
To the Sine of one adjacent Angle,
So is the Sine of the other adjacent Angle,
To a fourth Sine.

As the fourth Sine,
To the Sine of half the Sum of the three Angles, leſſened by one of the adjacent Angles;
So is the Sine of the half Sum leſſened by the other adjacent Angle,
To a ſeventh Sine; to which if you add the Radius, half that Sum will be the Co-ſine of half the interjacent Side if the Angle oppoſite be Acute, but the Co-ſine of half the interjacent Side if the Angle oppoſite be Obtuſe, *&c.*

Propoſition X. *Or* Axiom X.

In every Oblique-angled Spherical Triangle, if inſtead of the Obtuſe Angles you make uſe of their Supplements it will be,

As the Rectangle or Product of the Sines of any two adjacent Angles, is to the Square of the Radius; ſo is the Rectangle or Product of the Sines of half the Sum of the three Angles, multiplied into the Sine of half the Sum of the three Angles, leſſened by the third or oppoſite Angle, to the Square of the Co-ſine of half the interjacent Side if the Angle oppoſite be Acute, but to the Sine of half the interjacent Side if the Angle oppoſite be Obtuſe. Wherefore,

1. To the Arithmetical Complement of the Sine of one of the adjacent Angles,

Add the Arithmetical Complement of the Sine of the other adjacent Angle, Alſo the Sine of half the Sum of the three Angles,

And the Sine of the half Sum leſſened by the third or Angle oppoſite,

Half the Sum of theſe four Sines, will be the Co-ſine of half the interjacent Side if the Angle oppoſite be Acute, but the Sine of half the interjacent Side if the Angle oppoſite be Obtuſe. Or,

3. As the Radius,
To the Sine of one of the adjacent Angles;
So is the Sine of the other adjacent Angle,
To a fourth Sine.
As the fourth Sine,
To the Sine of half the Sum of the three Angles;
So is the Sine of the half Sum leſſened by the oppoſite Angle,
To a ſeventh Sine; to which if the Radius be added, half that Sum will be the Co-ſine of half the interjacent Side if the Angle oppoſite be Acute, but the Sine of half the interjacent Side if the Angle oppoſite be Obtuſe.

The Demonſtration of this *Axiom* and its *Conſequential Rules*, depends intirely upon the 8*th Axiom* and 17*th Art.* of *Section* the 3*d*, as the former did upon the 7*th Axiom* and the 17*th Art.* and is performed after the ſame manner; and conſequently by the ſame 17*th Art.* of *Section* the 3*d*, it follows, That

Whatſoever *Laws* have been laid down and demonſtrated, for finding of an Angle where the three Sides are given, will hold good for finding a Side when the three Angles are given *mutatis mutandis*, provided that if one or more of the given Angles be Obtuſe, you make uſe of their Supplements inſtead of the Angles themſelves, and then if the Side required be oppoſite to an Obtuſe Angle, the Quantity produced will be the Supplement of the Side required; as will more evidently appear in the Application of theſe *Rules* to Practice.

Section VII.

Containing the actual Solution of the Twelve Caſes of Oblique-angled Spherical Triangles.

Caſe 1.

GIven two Sides and an Angle oppoſite to one of them, to find the other oppoſite Angle.

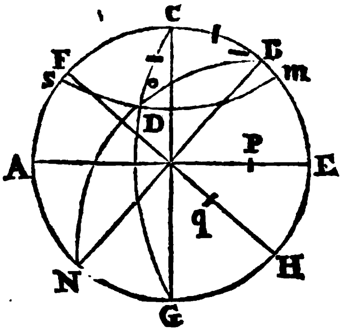
Example

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given,}\left\{\begin{array}{l} \qquad\;\; {}^{\circ}\;\;\;{}'\;\;\;{}'' \\ BC = 40.00.10 \\ <B = 42.15.13\tfrac{1}{4} \\ CD = 50.10.30 \end{array}\right\}\text{req.} <D.$$

The *Stereographical Solution.*

1. Having drawn the Primitive Circle *ACEG*, the Diameter *AE*, and at Right-angles to it *CG*, ſet off the Chord of 40° 00′ 10″ the Length of *CB* from *C* to *B*, and draw the Diameter *BN*, and at Right-angles to it *FH*.



2. Thro' the Point *B* draw the Great Circle *BDN*, forming an Angle *DBC* of 42° 15′ 13″ ¼ with the Primitive Circle, (by *Caſe* the 3*d* of *Prob.* the 9*th*, *Sect.* the 2*d.*

3. About the Point *C* as a Pole deſcribe the Parallel *Sm* of 50° 10′ 30″, equal to the Length of the Side *CD*, by *Caſe* the 2*d* of *Prob.* the 4*th*, *Sect.* the 2*d.*

4. Thro' the Interſection *D* of the Parallel *Sm*, with the Great Circle *BDN* and the Point *C*, draw the Great Circle *CDG* by *Problem* the 1ſt of *Section* the 2*d*, and the thing is done; and the Angle at *D* may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th*, of *Section* the 2*d*, but to find its Quantity by Calculation,

Becauſe by *Ax*. the 3*d* of *Sect*. 6*th*, the Sines of the Sides are directly proportional to the Sines of their oppoſite Angles, it will be

As S. DC : S. BC : : S. $< B$: S. $< D$. That is,

As the Sine of the Side $DC = 50^\circ\ 10'\ 30''$ ———— 9.8853635

To the Sine of the Side $BC = 40.00.10$ ———— 9.8080926

So is the Sine of the Angle at $B = 42.15.13\frac{1}{4}$ ———— 9.8276370

To the Sine of the Angle at $D = 34.15.02\frac{3}{4}$ ———— 9.7503661

Which becauſe a Perpendicular let fall from the end of the given Side *CB*, adjacent to the given Angle *B*, falls within the Triangle, is ſimilar to the given Angle *B*, and conſequently Acute.

Caſe I.

Example 2.

If inſtead of the Side *BC* the Side *BD* be given, that is if the Side oppoſite to the given Angle be the leſſer of the two given Sides, then the *Caſe* is doubtful, and the required Angle will be either Acute or Obtuſe; as is manifeſt from the following *Example*.

Suppoſe in the Oblique-angled Spherical Triangle *DCB*

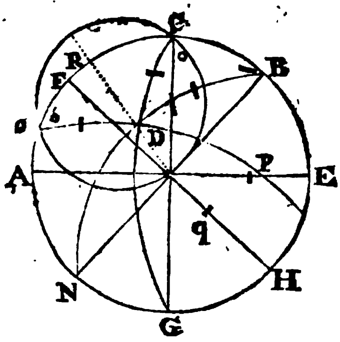
$$\text{there were given}\left\{\begin{array}{l} \ ^\circ\ \ '\ \ '' \\ DC = 50.10.30 \\ < B = 42.15.13\frac{1}{4} \\ BD = 76.35.36 \end{array}\right\}\text{req.} < C.$$

The *Stereographic Solution*.

1. Having drawn the Primitive Circle *ACEG*, the Diameter *BN*, and at Right-angles to it *FH*, thro' the Point *B* draw the Great Circle *BDN*, forming the Angle *CBD* of 42 deg. 15 min. 13 ſec. $\frac{1}{4}$ with the Primitive Circle *CBE*, (by *Caſe* the 3*d* of *Problem* the 9*th*, *Section* the 2*d*.)

2. Lay off 76° 35′ 36″ the length of *BD* from *B* to *D* (by *Caſe* the 3*d* of *Problem* the 6*th*, of *Section* the 2*d*.)

3. About the Point *D* as a Pole, deſcribe the Parallel *Cc* by *Caſe* the 3*d* of *Problem* the 4*th*, *Section* the 2*d*.



4. Thro' the Point *D* and the Interſection *c C*, of the Parallel laſt drawn, with the Primitive Circle draw the Great Circles *CDG*, *C D K*, and the thing is done; and the Angle *C* may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, *Section* the 2*d*.

From the *Conſtruction* it is evident that the *Caſe* is doubtful, for the Parallel *C c* cuts the Periphery in the Points *C* and *c* on the ſame Side of the Angle *B*, ſo that in the two Triangles *C B D*, *c B D*, the Side *BD* and the Angle *B* is common to both the Triangles, and the Side *DC* of the Triangle *DBC*, is equal to the Side *D c* of the Triangle *B D c*, but the required Angle *B c D* of the Triangle *B c D* (equal to the Angle *c CD* of the Triangle *c CD*, becauſe the Sides *c D* and *CD* are equal) is equal to the Supplement of the Angle *BCD* of the Triangle *BCD*, and to find each by *Calculation*, it will be by *Axiom* the 3*d*.

As S. *DC* : S. *BD* : : S. ∠*B* : S. ∠*C*. That

As the Sine of the Side *DC* = 50° 10′ 30″	9.8853635
To the Sine of the Side *BD* = 76.35.36	9.9880007
So is the Sine of the Angle at *B* = 42.15.13¼	9.8276370
To the Sine of the Angle *D c B* = 58.23.40	9.9302742

Equal to the Angle *D C c*, whoſe Supplement to a Semicircle 121 deg. 36 min. 20 ſec. is equal to the Angle *DCB* required.

Caſe II.

Given two Angles and a Side oppoſite to one of them, to find the other oppoſite Angle.

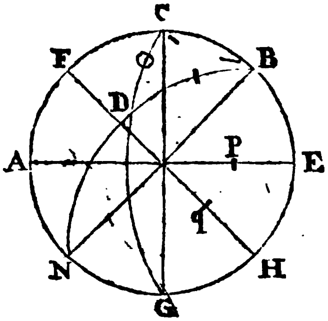
Example

Example.

In the Oblique-angled Spherical Triangle *BDC*

$$\text{Are given, } \left\{ \begin{array}{l} \angle C = 121.36.20 \\ DB = 76.35.36 \\ \angle B = 42.15.13\tfrac{3}{4} \end{array} \right\} \text{ req. } DC.$$

The *Stereographic Solution.*



1. Having drawn the Primitive Circle *ACEG*, the Diameter *EN*, and at Right-angles to it *FH*, draw the Great Circle *BDN*, forming an Angle of 42 deg. 15 min. 13 ſec. ¾ (equal to the Angle *B*) with the Primitive Circle (by *Caſe* the 3*d* of *Problem* the 9*th*, *Section* the 2*d*)

2. Lay off 76 deg. 35 min: 36 ſec. the length of *BD* from *B* to *D*, (by *Caſe* the 3*d* of *Problem* the 6*th*, of *Section* the 2*d*.)

3. Thro' the Point *D* draw the Great Circle *CDG*, forming the Angle *DCB* with the Primitive Circle, of 121 deg. 36 min. 20 ſec. (by *Caſe* the 6*th* of *Problem* the 9*th*, *Section* the 2*d*) and the thing is done; and *CD* may be meaſured (by *Caſe* the 3*d* of *Problem* the 7*th*, *Section* the 2*d*) but to determine its length by *Calculation*, it will be by *Axiom* the 3*d*,

As S. $\angle C$: S. $\angle B$: : S. *BD* : S. *DC*. That is,

As the Sine of the Angle at $C = 121^\circ\ 36'\ 20''$	9.9302744
To the Sine of the Angle at $B = 42.15.13\tfrac{3}{4}$	9.8276370
So is the Sine of the Side $BD = 76.35.36$	9.9880007
To the Sine of the Side $DC = 50.10.30$	9.8853633

Caſe

Caſe II.

Example 2.

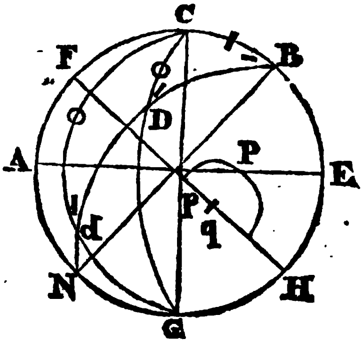
If inſtead of the Angle *C* the Angle *D* be given, that is, if the Angle oppoſite to the given Side be the leſſer of the two given Angels, the *Caſe* is doubtful, as will be manifeſt from the following *Example.*

Suppoſe in the Oblique-angled Spherical Triangle *DCB*

There are given $\left\{\begin{array}{l} \angle B = 42.15.13\frac{1}{4} \\ BC = 40.00.10 \\ \angle D = 34.15.02\frac{3}{4} \end{array}\right\}$ and it be required to find the Side *DC*.

The *Stereographic Solution.*

1. Having drawn the Primitive Circle *ACEG*, the Diameter *AE*, and at Right-angles to it *CG*, ſet off the Chord of 40 deg. 00 min. 10 ſec. the length of *CB* from *C* to *B*, and draw the Diameter *BN*, and at Right-angles to it *FH*.



2. Thro' the given Point *B* draw the Great Circle *B D d N*, forming an Angle of 42 deg. 15 min. 13 ſec. ¼ with the Primitive Circle, (by *Caſe* the 3*d* *Problem* the 9*th*, *Section* the 2*d*) and finds its Pole *q* (by *Caſe* the 3*d* *Problem* the 3*d*, *Section* the 2*d*.)

3. About the Point *q* deſcribe the Small Circle *p p q*, at the diſtance of 34 deg. 15 min. 02 ſec. ¾, equal to the Quantity of the Angle *D*, (by *Caſe* the 3*d* of *Problem* the 4*th*, *Section* the 2*d*.)

4. About the Points *p*, *p*, the Interſection of the Small Circle, with the Right Circle *AE* deſcribe the Great Circles *CDG*, *C d G*, (by *Caſe* the 3*d* of *Problem* the 2*d*, *Section* the 2*d*) and the thing is done; and the Sides *CD*, *C d*, may be meaſured (by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*.)

That this *Caſe* is Ambiguous appears from the *Conſtruction*, for the Small Circle *ppq*, cuts the Right Circle *AE* in two Points *p* and *p*, (the Poles of the Circles *CDG*, *C d G*) on the ſame ſide of the Center, whence is formed two Spherical Triangles *BCD* and *B C d*, to each of which the Side *CB* and Angle *B* are common, and the Angle *CDB* of the one is equal to the Side *C d B* of the other; but the required Side *CD* of the Triangle *CBD*, is equal to the Supplement of the required Side *C d* of the Triangle *C d B*, and to determine each length by *Calculation*, it will be by *Axiom* the 3*d* of *Section* the 6*th*.

As S. <*D* : S. <*B* : : S. *CB* : S. *CD*. That is,

As the Sine of the Angle at $D=34.15.02\frac{3}{4}$ —————— 9.7503665

To the Sine of the Angle at $B=42.15.13\frac{1}{4}$ —————— 9.8276370

So is the Sine of the Side $BC=40.00.10$ —————— 9.8080926

To the Sine of the Side $DC=50.10.30$ —————— 9.8853631

Whoſe Supplement to a Semicircle 129 deg. 49 min. 30 ſec. is the length of the Side *C d* of the Triangle *B C d*.

Caſe III.

Given two Sides and an Angle oppoſite to one of them, to find the other Side.

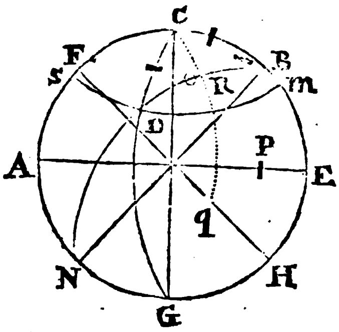
Example.

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given}\left\{\begin{array}{l} CB=40.00.10 \\ <B=42.15.13\frac{1}{4} \\ CD=50.10.30 \end{array}\right\}\text{req. } DB.$$

This *Caſe* is Conſtructed after the ſame manner as the 1*ſt Example* of the 1*ſt Caſe*, and *DB* may be meaſured (by *Caſe* the 3*d* of *Problem* the 7*th*, *Section* the 2*d*) but to find the length by *Calculation*,

From



From *C* the end of the given Side *CB*, which Side *CB* is adjacent to the given Angle *B*, let fall the Perpendicular *CR* upon the Baſe *DB* (by *Caſe* the 3*d* of *Problem* the 11*th*, *Section* the 2*d*) according to the *Rules* laid down in the former part of the laſt *Section*, which *Rules* ought to be well underſtood, and to which I refer the *Reader*) this Perpendicular becauſe it falls within the Triangle, will divide the Oblique-angled Triangle *DCB* into two Right-angled Triangles *BCR* and *DCR*, in one of which Triangles *BCR* which Triangle I call the 1*ſt* Triangle, are given, the Hypothenuſe *CB* and Angle at the Baſe *B*, whence to find the Baſe *BR*, it will be by the 2*d Example* of the 4*th Caſe*, of *Section* the 5*th*,

As R : cs, <B : : t, CB t, BR. That is,

As the Radius ——————————— 10.0000000

To the Co-ſine of the Angle at the Baſe B=42.15.13¼ 9.8693344

So is the Tangent of the Hyp. or Side CB=40.00.10 —— 9.9238563

To the Tangent of the Baſe BR=31.50.46 ——— 9.7931907

2. To find the Perpendicular CR, it will be by the 2*d Example* of the 5*th Caſe* of *Section* the 5*th*,

As the R : S. CB : : S. <B : S. CR. That is,

As the Radius ——————————— 10.0000000

To the Sine of the Hyp. or Side CB=40.00.10 ——— 9.8080926

So is the Sine of the Angle at the Baſe B=42.15,13¼ — 9.8276370

To the Sine of the Perpendicular CR=25.36.36¼ ——— 9.6357296

3. In the Triangle DRC, which Triangle I call the 2*d* Triangle, there are given, the Side CD or Hypothenuſe, and the Perpendicular CR laſt found, whence to find the Baſe DR, it will be by *Caſe* the 14*th* of *Section* the 5*th*,

As cs, CR : R : : cs, CD : cs, DR. That is,

As the Co-ſine of the Perpendicular CR=25.36.36¼ —— 9.9550889

To the Radius ——————————— 10.0000000

So is the Co-ſine of the Hypoth. or Side CD=50.10.30 9.8064817

To the Co-ſine of the Baſe DR=44.44.50 ——— 9.8513928

This Baſe DR equal to 44 deg. 44 min. 50 ſec. of the 2*d* Triangle, added to the Baſe BR of the 1*ſt* Triangle, equal to 31 deg. 50 min. 46 ſec. before found, becauſe the Perpendicular CR falls within the Triangle, gives the Side DB equal to 76 deg. 35 min. 36 ſec.

By comparing the ſimilar parts of the two Triangles BRC and CRD together, with the help of the 11*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt*, may the Segment DR be directly inveſtigated, without finding of the Perpendicular CR.

For becauſe the Perpendicular CR is common to both the Triangles, let it be ſuppoſed to be known or given, and becauſe in the Triangle DCR the Hypothenuſe DC is given and the Baſe DR required, let the Proportion be made for finding the Baſe RB of the Triangle CRB, from the Hypothenuſe CB and Perpendicular CR; alſo the Proportion for finding the Baſe DR of the Triangle DRC, from the Hypothenuſe DC and common Perpendicular CR; then by comparing theſe two Proportions together and expunging the Terms common to both, (according to the 11*th Cor.* of the 15*th Geom. Prop.*) a new Proportion will ariſe for finding the part required directly; thus in the Triangle CRB to find the length of RB, it will be by the 14*th Caſe* of *Right-angled Spheric* Triangles,

As c s, CR : R : : c s, BC : c s, BR.

And in the Triangle DRC to find the length of DR, it will be by the 14*th Caſe* of *Right-angled*, &c.

As c s,R : R : : c s, DC : c s, DR.

But becauſe in theſe two Sets of Proportionals the two firſt Terms each, *viz.* c s,CR and the Radius are both the ſame, the others will be directly proportional by the 11*th Cor.* of *Prop.* the 15*th*, *Sect.* the 1*ſt* of *Part* the 1*ſt*; that is,

As c s, BC : c s, DC : : c s, BR : c s, DR.

As the Co-ſine of the Side BC or Hypothenuſe of the 1*ſt* Triangle BC=40.00.10	9.8842363
To the Co-ſine of the Side DC or Hypothenuſe of the 2*d* Triangle=50.10.30	9.8064817
So is the Co-ſine of the Baſe BR of the 2*d* Triangle =31.50.46	9.9291472
To the Co-ſine of the Baſe of the 2*d* Tri. DR=44.44.50	9.8513926

Which added to the Baſe BR equal to 31 deg. 50 min. 46 ſec. before found, becauſe the Perpendicular falls within makes 76 deg. 35 min. 36 ſec. for the length BC required.

And univerſally, having drawn a Perpendicular from the end of that given Side which is adjacent to the given Angle, and ſo that it may fall oppoſite to the given Angle, we ſhall always have an Hypothenuſe and an Angle at the Baſe given in one of the two Triangles, which is called the *1ſt* Triangle, to find either the Baſe as in the *3d*, *6th*, and *9th* and *10th Caſes*, or the Angle at the Perpendicular, as in the *4th*, *5th* *7th* and *8th Caſes*, according as the *Caſe* requires; and in the other Triangle which is called the *2d* Triangle, there is given, either the Hypothenuſe as in the *3d* or *4th Caſes*, or the Angle at the Baſe, as in the *5th* or *6th Caſes*, or the Baſe which is eaſily had by adding or ſubſtracting, (according as the Perpendicular falls within or without the Triangle) the Baſe as in the *9th* and *10th Caſes* in the *1ſt* Triangle, to or from the Side of the Oblique-angled Triangle firſt given, upon which the Perpendicular falls; alſo the Angle at the Perpendicular in the *2d* Triangle is in like manner readily found, by adding or ſubſtracting (according as the Perpendicular falls within or without the Triangle) the Angle at the Perpendicular as in the *7th* and *8th Caſes*, to or from the Angle of the Oblique-angled Triangle firſt given, from whence the Perpendicular is drawn.

In the *2d* Triangle a Side or Angle being thus known, the Side or Angle required may be found by comparing the ſimilar part of the two Triangles together, for the two Right-angled Triangles having a common Perpendicular their parts have an Analogy one to another; and by taking the common Perpendicular with the Side or Angle given and the Side or Angle required, the Proportion of the known to the unknown will be readily deduced by the *1ſt Axiom*, and by comparing theſe with the ſimilar parts of the firſt Triangle, by the help of the known Laws of Proportion, the unknown parts of the *2d* Triangle will be readily diſcovered; thus if an Hypothenuſe as in the *8th* or *9th Caſes*, or an Angle at the Baſe as in the *7th* and *10th Caſes* be required, the Proportion gives the Side or Angle required of the Oblique-angled Triangle; but if a Baſe as in the *3d* or *6th Caſes*, or an Angle at the Perpendicular as in the *4th* and *5th Caſes* be required, then this Baſe or this Angle added to or ſubſtracted from (according as the Perpendicular falls within or with-the Triangle) the Baſe or Vertical Angle of the *1ſt* Triangle will give the Side or Angle required of the Oblique-angled Triangle firſt given, as will be Exemplified at large in the following Caſes.

And altho there are ſeveral *Rules* laid down and demonſtrated in the former *Section*, for the ready *Solution* of any of the *1ſt* 10 Caſes, yet this way of reſolving of them by letting fall the Perpendicular is preferable to any of them, as being univerſal and leſs burthenſome to the Memory.

The Method of finding the Perpendicular firſt, in order to find the thing required, of which I have given here an Inſtance (but ſhall omit to do it in the following *Caſes* for brevitys ſake) as it is very eaſy to be underſtood, and ſhews the Extenſive Uſe of the *1ſt Axiom*, ſo it ought to be well known firſt, and tho' it be generally eſteemed unartificial and ſeldom made Uſe of in common Practice, becauſe it augments the Number of Proportions by one Proportion, yet by rendering the Operations themſelves much more eaſy it has its Advantages, as is well known to thoſe who are well verſed in Calculations.

Again, the Side DB may be directly inveſtigated by the help of the *3d* and *4th Axioms* of *Section* the *6th*, without letting fall the Perpendicular; for by *Axiom* the *3d* it will be,

S. DC : S. BC : : S. <B : S. <D; that is

As the Sine of the Side DC=50.10.30 ———————— 9.8853635

To the Sine of the Side BC=40.00.10 ———————— 9.8080926
So is the Sine of the Angle B=42.15.13¼ ———————— 9.8276370

To the Sine of the Angle D=34.15 02¾ ———————— 9.7503661

And by the *1ſt* part of the *5th Axiom* of *Section* the *6th* it will be,

As $\mathrm{S}.\ \tfrac{1}{2}{<}s\,\overline{B-D} : \mathrm{S}.\ \tfrac{1}{2}{<}s\,\overline{B+D} :: t,\ \tfrac{1}{2}\,\overline{DC-BC} : t,\ \tfrac{1}{2}\,DB$; that is,

As the Sine of the half Difference of the two adjacent Angles B and D=4.00.5¼ ———————— } 8.8437423

To the S. of half the Sum of thoſe 2 Angles=38.15 08 9.7917780
So is the Tangent of half the Difference of the oppoſite Sides DC and BC=5.5.10 ———————— } 8.9494058

To the Tang. of ½ the interjacent Side DB=38.17.48 9.8974415

Which being doubled, gives 76 deg. 35 min. 36 ſec. for the length of the Side DB.

Caſe III.

Example 2.

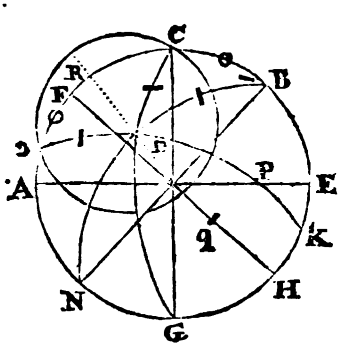
Given two Sides and an Angle oppoſite to one of them to find the third Side; that is,

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given, }\left\{\begin{array}{l} DB = 76.35.36 \\ < B = 42.15.13\tfrac{1}{4} \\ DC = 50.10.30 \end{array}\right\}\text{ req. } BC.$$

The *Stereographic Solution.*

This Caſe is Conſtructed after the ſame manner as the 2*d Example* of the 1*ſt Caſe*, and BC may be meaſured by the *Line of Chords*, but to find its length by Calculation,



From D one of the Extremitys of the given Side DB, let fall the Perpendicular DR (by *Caſe* the 3*d* of *Prob.* the 11*th*, *Sect.* the 2*d*) this Perpendicular becauſe it falls without the Triangle DBC, upon the Baſe BC produced, will form two Right-angled Spherical Triangles DRB and DRC, in the firſt of which Triangles DRB Right-angled at R are given, DB the Hypothenuſe equal to 76 deg. 35 min. 36 ſec. and the Angle at the Baſe DBR equal to 42 deg. 15 min. 13 ſec. ¼, whence to find the Baſe BR it will be (by the 2*d Example* of the 4*th Caſe* of Right-angled Spherical Triangles.

As R : c s, <B : : t, DB : t, BR ; that is,

As the Radius ———————— 10.0000000

To the Co-ſine of the Angle at the Baſe B=42.15.13¼——9.8693344

So is the Tang. of the Side or Hyp. DB=76.35.36——10.6227730

To the Tangent of the Baſe BR=72.09.00———10.4921074

Again, to find RC the Baſe of the 2*d* Triangle DRC, becauſe the Hypothenuſe DC is given, it will be (by the 14*th Caſe* of Right-angled Spherical Triangles,

In the 1*ſt* Triangle DBR, As c s, DR : R : : c s, BD : c s, BR.

In the 2*d* Triangle DRC, As c s, DR : R : : c s, CD : c s, CR.

Wherefore, becauſe the two firſt Terms in each Proportion are the ſame the others will be directly proportional (by the 11*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt.*) That is,

As c s, BD : c s, CD : : c s, BR c s, CR ; that is,

As the Co-ſine of the Side BD or Hypothenuſe of the 1*ſt* Triangle=76.35.36	9.3652280
To the Co-ſine of the Side DC or Hypothenuſe of the 2*d* Triangle=50.10.30	9.8064817
So is the Co-ſine of BR the Baſe of the 1*ſt* Tri.=72.9.0°	9.4864674
To the Co-ſine of the *Baſe* CR of the 2*d* Tri.=32.08.50	9.9277211

And becauſe the Perpendicular falls without, if the Baſe CR of the 2*d* Triangle equal to 32 deg. 08 min. 50 ſec. be taken from the Baſe DR of the 1*ſt* Triangle equal to 72 deg. 09 min. 00 ſec. there will remain the Side BC equal to 40 deg. 00 min. 10 ſec.

Again, becauſe the Parallel *C c* cuts the Periphery in the Points *C c*, it ſhews the *Caſe* is doubtful, and to find the greater Side *B c*, becauſe the Triangles *CDR c DR* are equal (for the Sides *CR*, *c R*, are equal, the Angles *CRD* and *c RD* are Right, and the Side *DR* common) the Side *BC* will be equal to the Side *B c*; wherefore, if to *BR* equal to 72 deg. 09 min. 00 ſec. be added *c R* equal to *CR*, equal to 32 deg. 08 min. 50 ſec. the Sum 104 deg. 17 min. 50 ſec. will be equal to *B c*, whence it is obvious, that if the

$$\text{Angle } C \text{ be } \left\{\begin{matrix}\text{Obtuſe}\\ \text{Acute}\end{matrix}\right\} \text{ the required Side } BC \text{ is } \left\{\begin{matrix}\overset{\circ\quad\prime\quad\prime\prime}{40.00.10}\\ 104.17.50\end{matrix}\right\}$$

and by comparing this *Example* with the former it is manifeſt, that

$$\text{if the Perpendicular falls } \left\{\begin{matrix}\text{within}\\ \text{without}\end{matrix}\right\} \text{ then the Baſe of the 1ſt Triangle } \left\{\begin{matrix}\text{added to}\\ \text{ſubſtracted from}\end{matrix}\right\}$$

the Baſe of the 2*d* Triangle, gives the Side required.

Caſe

Caſe IV.

Given two Sides and an Angle oppoſite to one of them, to find the contained Angle.

Example.

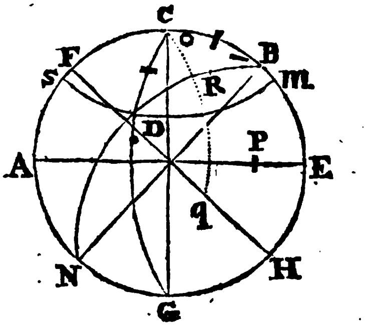
In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given}\left\{\begin{array}{l} BC = 40.00.10 \\ < B = 42.15.13\tfrac{1}{4} \\ CD = 50.10.30 \end{array}\right\}\text{req.} < C.$$

This *Caſe* is Conſtructed after the ſame manner as the *1ſt Example* of the *1ſt Caſe*, and the Angle at *C* may be meaſured (by *Caſe* the *2d* of *Prob.* the 10*th*, *Section* the *2d.*)

And to find the Quantity by *Calculation*,

From C the end of the given Side *CB*, let fall the Perpendicular *CR* (by *Caſe* the *3d* of *Problem* the 11*th*, *Section* the *2d*) this Perpendicular becauſe it falls within will divide the Oblique angled Spherical Triangle *BCD*, into two Right-angled Spherical Triangles *BRC* and *DRC* Right-angled at *R*, in the *1ſt* of which Triangles *BRC* Right-angled at *R*, are given, *CB* the Hypothenuſe equal to 40 deg. 00 min. 10 ſec. the Angle at the Baſe *B*, equal to 42 deg. 15 min. 13 ſec. $\frac{1}{4}$, whence to find the Angle at the Perpendicular *BCR*, it will be by the *2d Example* of the *3d Caſe*,



As the $\dot{R}$: c s, *BC* : : t. $<B$: c t, $<C$. That is,

As the Radius———————————10.0000000

To the Co-ſine of the Hyp. or Side *BC*=40.00.10——9.8842363
So is the Tangent of the Angle at the Baſe *B*=42.15.13¼ 9.9583029

To the Co-tang. of the Angle at Perp. *BCR*=55.09.59 9.8427392

Again, becauſe the Hypothenuſe *DC* is given in the 2*d* Triangle, to find the Vertical Angle or Angle at the Perpendicular, *DCR*, it will be (by the 12*th Caſe* of Right-angled Spherical Triangles.

In the 1*ſt* Triangle *BRC*, As t, *BC* : *R* : : t, *CR* : c s, <*BCR*.
In the 2*d* Triangle *DRC*, As t, *DC* : *R* : : t, *CR* : c s, <*DCR*.

Wherefore, becauſe the two middle Terms in each Proportion are the ſame, the others will be reciprocally proportional, by the 12*th Cor.* of the 15*th Geom. Theor.* of *Part* the 1*ſt*. That is,

As t, *DC* : t, *BC* : : c s, <*BCR* : c s, <*DCR*. Wherefore,

As the Tangent of the Side BC or Hypothenuſe in the 2*d* Triangle=50.10.30———— 10.0788818

To the Tangent of the Side DC or Hypothenuſe in the firſt Triangle=40.00.10 ———— 9.9238563
So is the Co-ſine of the Angle at the Perpendicular *DCR* in the 1*ſt* Triangle=58.09.55 ———— 9.7567846

To the Co-ſine of the Angle at the Perpendicular *DCR* in the 2*d* Triangle=66.26.21———— 9.6017591

This Angle at the Perpendicular *DCR*, in the 2*d* Triangle equal to 66 deg. 26 min. 21 ſec. added to the Angle at the Perpendicular *BCR* in the 1*ſt* Triangle before found, equal to 55 deg. 09 min. 59 ſec. (becauſe the Perpendicular falls within the Triangle) makes 121 deg. 36 min. 20 ſec. for the Angle *DCB* required.

Again, the Angle at *C* may be directly inveſtigated, by the help of the 3*d* and 4*th Axioms* of *Section* the 6*th*, without letting fall the Perpendicular; for by the 3*d Axiom* it will be

As S. *DC* : S. *BC* : : S. <*B* : <*D*. That is,

As the Sine of the [illegible]e DC=50.10.30 ———— =9.8853635
To the Sine of the Side BC=40.00.10 ———— 9.8080926
So is the Sine of the Angle at B=42.15.13¼ ———— 9.8276370
To the Sine of the Angle at D=34.15.02¾ ———— 9.7503661

And by the firſt part of the *4th Axiom* it will be,
As $S.\tfrac{1}{2}\overline{DC-BC}:S.\tfrac{1}{2}\overline{DC+BC}::ct,\tfrac{1}{2}<s\overline{B-D}:t,\tfrac{1}{2}<C$; that is,
As the Sine of ½ the diff. of Sides DC and BC=05.05.10 8.9476924

To the Sine of half their Sum 45.05.20 ———— 9.8501577
So is the Tangent of half the Difference of the Angles B and D=04.00.05¼ ———— 8.8448022

To the Co-tangent of half the Angle C=60.48.10½—9.7472675

Which being doubled gives 121 deg. 36 min. 21 ſec. for the Angle at C required.

Caſe IV.

Example 2.

Given two Sides and an Angle oppoſite to one of them, to find the contained Angle; that is,

In the Oblique-angled Spherical Triangle *DCB*

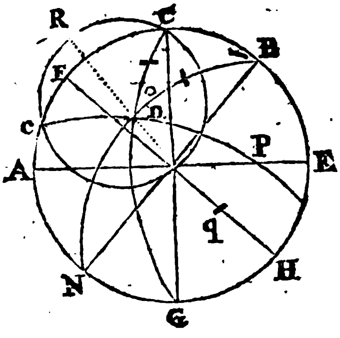
$$\text{Are given}\begin{cases} DC=50.10.30 \\ <B=42.15.13\tfrac{1}{4} \\ BD=76.35.36 \end{cases}\text{req.} <D.$$

The *Stereographic Solution.*

This *Caſe* is Conſtructed after the ſame manner as the *2d Example* of the *1ſt Caſe*, and the Angle at D may be meaſured by *Caſe* the *3d* of *Problem* the 10*th*, *Section* the 2*d*.

But to find its length by Calculation,

From D one end of the given Side DB, let fall the Perpendicular DR, by *Caſe* the *3d* of *Problem* the 11*th*, of *Section* the 2*d*, this Perpendicular becauſe it falls without the Triangle DBC, upon the Baſe BC produced to R, will form two Right-angled Spherical Triangles DRB and DRC Right-angled at R, in the *1ſt* of which Triangles DRB Right-angled at R, are given, the Hypothenuſe



DB equal to 76 deg. 35 min. 36 ſec. the Angle at the Baſe B equal to 42 deg. 15 min. 13 ſec. $\frac{2}{4}$, whence to find the Angle at the Perpendicular BDR, it will be by the *2d Example* of the *3d Caſe* of Right-angled Spherical Triangles,

As the R : c s, DB : : t, <B : ct, <CDR ; that is,

As the Radius	10.0000000
To the Co-ſine of the Side DB or Hyp.=76.35.36	9.3652280
So is the Tang. of the Angle at the Baſe B=42.15.13¼	9.9583029
To the Co-tang. of the Angle at Perp. BDR=78.06.19¼	9.3235309

Again, Becauſe the Hypothenuſe DC is given in the *2d* Triangle DRC, to find the Angle at the Perpendicular CDR, it will be by the *12th Caſe* of *Right-angled Spherical Triangles,*

In the *1ſt* Triangle DRB, As t, DB : R : : t, DR : c s, BDR. And In the *2d* Triangle DRC, As t, CD : R : : t, DR : c s, CDR. Wherefore, becauſe the two middle Terms in each Proportion are the ſame, the others will be reciprocally proportional (by the *12th Cor.* of the *15th Geom. Prop.* of *Part* the *1ſt.*) That is,

As the t, CD : t, BD : : c s, BDR : c s, CDR.

As the Tangent of the Side CD or Hypothenuſe in the *2d* Triangle=50.10.30	10.0788818
To the Tangent of the Side BD or Hypothenuſe in the *1ſt* Triangle=76.35.36	10.6227730
So is the Co-ſine of the Angle at the Perpendicular BDR in the *1ſt* Triangle=78.06.19¼	9.3141050
To the Co-ſine of the Angle at the Perpendicular CDR in the *2d* Triangle=43.51.16½	9.8579962

The Angle at the Perpendicular CDR in the 2*d* Triangle, equal to 43 deg. 51 min. 16 [illegible] taken from the Angle at the Perpendicular BDR in the 1*ſt* Triangle, equal to 78 deg. 06 min 19 ſec. ¼, (becauſe the Perpendicular falls without the Triangle) will leave the Angle BDC required, equal to 34 deg. 15 min. 2 ſec. ¾.

Again, becauſe the Parallel *C c* cuts the Periphery in the Points C and *c*, it ſhews the Caſe is doubtful; and to find the greater Angle *BD c*, becauſe the Triangles *CRD* and *c RD* are equal (for the Sides *CR*, *c R*, are equal, the Angles at *R* Right, and the Side *DR* common) the Angle *CDR* will be equal to the Angle *c DR*, wherefore if to the Angle *BDR* equal to 78 deg. 06 min. 19 ſec. ¼, be added the Angle *c DR* equal to 43 deg. 51 min. 16 ſec. ½, the Sum 121 deg. 57 min. 35 ſec. ¾, will be equal to the Angle *BDc*; whence it is manifeſt, that if the Angle *C* be

$\left\{\begin{matrix}\text{Obtuſe}\\ \text{Acute}\end{matrix}\right\}$ the required Angle D is $\left\{\begin{matrix}43^\circ & 51' & 16''\,\tfrac{1}{2}\\ 121 & 57 & 35\,\tfrac{3}{4}\end{matrix}\right\}$ and by comparing this *Example* with the former, it is evident, that If the Perpendicular fall $\left\{\begin{matrix}\text{within}\\ \text{without}\end{matrix}\right\}$ the Triangle, then the Vertical Angle of the 1*ſt* Triangle $\left\{\begin{matrix}\text{added to}\\ \text{ſubſtr. from}\end{matrix}\right\}$ the Vertical Angle of the 2*d* Triangle will give the Angle required.

Caſe V.

Given two Angles and a Side oppoſite to one of them, to find the other Angle.

Example.

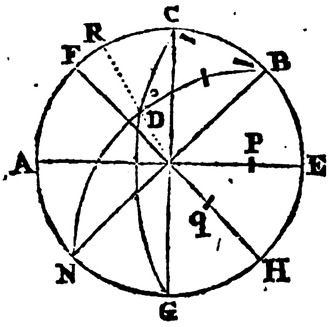
In the Oblique-angled Spherical Triangle *DCB*,

$$\text{Are given}\left\{\begin{matrix}<C = 121.36.20\\ DB = 76.35.36\\ <B = 42.15.13\tfrac{1}{4}\end{matrix}\right\}\text{req.}<D.$$

The *Stereographic Solution.*

This *Caſe* is conſtructed after the ſame manner as the 2*d Caſe*, and the Angle at *D* may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th*, *Section* the 2*d*.

But to determine its leng[illegible] Calculation.



From *D* one Extremity of the given Side *DB* adjacent to the given Angle *B*, let fall the Perpendicular *DR*, by *Caſe* the 3*d* of *Prob*. the 11*th*, of *Sect*. the 2*d*; this Perpendicular becauſe the Angles at the Baſe *C* and *B* are diſſimilar, will fall without the Triangle *DBC* upon the Baſe *BC* produced to *R*, wherefore in the Right-angled Spherical Triangle *DRB* Right-angled at *R*, are given, the Hypothenuſe *DB* equal to 76 deg. 35 min. 36 ſec. the Angle at the Baſe *B* equal to 42 deg. 15 min. 13 ſec. $\frac{1}{4}$, whence to find the Angle at the Perpendicular *BDR*, it will be by the 2*d Example* of the 3*d Caſe* of *Right-angled Spherical Triangles*.

As the *R* : c s, *DB* : : t, <*B* : c t, *CDR*; that is,

As the Radius ———————————— 10.0000000

To the Co-ſine of the Hypothenuſe or Side *DB* = 76.35.36 9.3652280
So is the Tang. of the Angle at the Baſe *B* = 42.15.13$\frac{1}{4}$—9.9583029

To the Tang. of the Angle at the Perp. *BDR* = 78.06.19$\frac{1}{4}$ 9.3235309

Again, to find the Angle at the Perpendicular *CDR* in the 2*d* Triangle *CRD*, becauſe the Angle at the Baſe *DCR*, the Supplement of the Angle *DCB* is given, it will be by the 11*th Caſe* of *Right-angled Spherical Triangles*,

In the 1*ſt* Triangle *DBR*, As c s, *DR* : *R* : : c s, <*B* : S. <*BDR*. And
In the 2*d* Triangle *DCR*, As c s, *DR* : *R* : : c s, <*C* : S. < *CDR*.

But becauſe the two firſt Terms in each Proportion are the ſame they may be rejected, and the others will be directly proportional by the 11*th Cor*. of the 15*th Geom. Prop*. of *Part* the 1*ſt*; that is,

As c s, <*B* : c s, <*C* : : S. *BDR* : S. *CDR*.

As the Co-ſine of the Angle at the Baſe *B* in the 1ſt Triangle 42.15.13¼ —— 9.8693344

To the Co-ſine of the Angle at the Baſe C in the 2*d* Triangle 121.36,20 —— 9.7193880

So is the Sine of the Angle at the Perpendicular *BDR* in 1ſt Triangle 78.06.19¼ —— 9.9905734

To the Sine of the Angle at the Perpendicular *CDR* in the 2*d* Triangle 43.51.16½ —— 9.8406270

But the Angle at the Perpendicular *CDR* in the 2*d* Triangle equal to 43 deg. 51 min. 16 ſec. ½, taken from the Angle at the Perpendicular *BDR* in the 2*d* Triangle, equal to 78 deg. 06 min. 19 ſec. ¼, will leave 34 deg. 15 min. 02 ſec. ¾, for the Quantity of the Angle *BCD* required.

By the help of the 3*d* and 4*th Axioms*, may the Angle at *D* in this *Caſe* be inveſtigated without letting fall the Perpendicular *DR*; for by the 3*d Axiom* it will be,

$\text{S.} < C\ \text{S.} : \text{S.} < B :: \text{S.}\, DB : \text{S.}\, DC :$ that is,

As the Sine of the Angle at $C = 121.36.20$ —— 9.9302744

To the Sine of the Angle at $B = 42.15.13\frac{1}{4}$ —— 9.8276370

So is the Sine of the Side $DB = 76.35.36$ —— 9.9880007

To the Sine of the Side $DC = 50.10.36$ —— 9.8853633

And by the 1ſt *Part* of the 4*th Axiom* it will be,

$\text{As S.}\,\frac{1}{2}\,\overline{DB - DC} : \text{S.}\,\frac{1}{2}\,\overline{DB + DC} :: \text{t},\ \frac{1}{2} <^{s} \overline{C - B} : \text{ct},\ \frac{1}{2} < D.$

As the S. of ½ the Diff. of the Sides *DB* & $DC = 13.12.33$ 9.3588987

To the Sine of half their $\text{Sum} = 63.23\ 03$ —— 9.9513524

So is the T. of ½ the Diff. of the $<^{s}$ C and $B = 39.40.33\frac{3}{8}$ 9.9188199

To the Co-tang. of ½ the contained $< D = 17.07.31\frac{1}{4}$ 10.5112736

Which being doubled gives 34 deg. 15 min. 02 ſec. ½, for the required Angle *D*.

Caſe

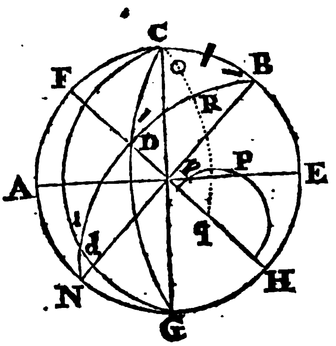
Caſe V.

Example 2.

Given two Angles and a Side oppoſite to one of them, to find the other Angle; that is,

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given}\left\{\begin{array}{l} <B=42\ 15.13\tfrac{1}{4} \\ BC=40.00.10 \\ <D=34.15.02\tfrac{3}{4} \end{array}\right\}\ \text{req.} <C.$$



The *Stereographic Solution.*

This *Caſe* is doubtful, and is projected after the ſame manner as the 2*d Example* of the 1*ſt Caſe*, and the Angle at *C* may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*.

But to find the length by Calculation,

From *C* one Extremity of the given Side *CB* adjacent to the given Angle *B*, let fall the Perpendicular *CR* by *Caſe* the 3*d* of *Prob.* the 11*th*, of *Sect.* the 2*d*, this Perpendicular becauſe the Angles at the Baſe are ſimilar, will fall within the Triangle, and will divide the Oblique-angled Spherical Triangle firſt given, into two Right-angled Spherical Triangles *CBR* and *CRD* Right-angled at *R*, in the firſt of which Triangles *BRC* are given, *CB* the Hypothenuſe equal to 40 deg. 00 min. 10 ſec. the Angle *B* at the Baſe equal to 42 deg. 15 min. 13 ſec. ¼, whence to find the Angle at the Perpendicular *BCR*, it will be by the 2*d Example* of the 3*d Caſe* of *Right-angled Spherical Triangles,*

As *R* : cs, *CB* : : t, <*B* : c t, *BCR* ; that is,

As the Radius ———————————————— 10.0000000

To the Co-ſine of the Side or Hyp. *CB*=40.00.10 —— 9.8842363

So is the Tang. of the Angle at the Baſe *B*=42.15.13¼ — 9.9583029

To the Co-tangent of the Angle at the Perpendicular *BCR*=55.09.59 ———————— 9.8425392

Again, becauſe in the 2*d* Triangle *DCR* the Angle at the Baſe is given, to find the Angle at the Perpendicular *DCR*, it will be by the 11*th Caſe* of *Right-angled Spherical Triangles Sect.* the 5*th*,

In the 1*ſt* Triangle *BRC*, As c s, *CR* : *R* : : c s, < *B* : S. *BCR*. And
In the 2*d* Triangle *DRC*, As c s, *CR* : *R* : : c s, < *D* : S. *DCR*.

But becauſe the two firſt Terms in each, *viz.* c s, *CR* and Radius are the ſame, they may be rejected, and the other Terms will be directly proportional, by the 11*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt*; that is,

As c s, < *B* : c s, < *D* : : S. *BCR* : S. *DCR*; that is;

As the Co-ſine of the Angle at the Baſe *B* in the 1*ſt* Triangle=42.15.13¼	9 8693344
To the Co-ſine of the Angle at the Baſe *D* in the 2*d* Triangle=34.15.02¾	9.9172861
So is the Sine of the Angle at the Perpendicular *BCR* in the 1*ſt* Triangle=55.09.59	9.9142450
To the Sine of the Angle at the Perpendicular *DCR* in the 2*d* Triangle=*66.26.21*	9.9621967

The Angle at the Perpendicular *BCR* in the 1*ſt* Triangle equal to 55 deg. 09 min. 59 ſec. added to the Angle at the Perpendicular *DCR* in the 2*d* Triangle, equal to *66* deg. 26 min. 21 ſec. becauſe the Perpendicular falls within, gives 121 deg. 36 min. 20 ſec. for the Angle *DCB*, but added to the Supplement of the Angle *DCR*, equal to 113 deg. 33 min. 39 ſec. will give the great Angle *d CB*, equal to 168 deg. 43 min. 38 ſec. by the 11*th Caſe* of *Right-angled Spherical Triangles.*

For that the *Caſe* is doubtful is manifeſt from this, that the Parallel *p p* deſcribed about *q*, the Pole of the Circle *B D d N*, cuts the Great Circle *AE* in two places *p* and *p*, on the ſame Side of the Center the Poles of the Circles *CDG* and *C d G* are.

By comparing the two *Examples* together, it is evident, that if the Perpendicular fall $\left\{\begin{matrix}\text{within}\\\text{without}\end{matrix}\right\}$ the Triangle, then the Vertical Angle of the 2*d* Triangle $\left\{\begin{matrix}\text{added to}\\\text{ſubſtr. from}\end{matrix}\right\}$ the Vertical Angle of the firſt Triangle will give the Angle required.

Caſe

Caſe VI.

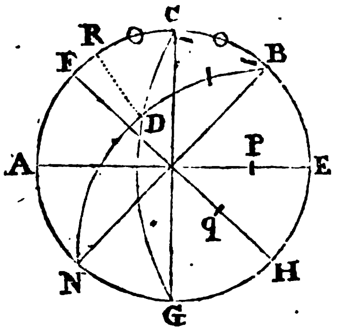
Given two Angles and a Side oppoſite to one of them, to find the interjacent Side.

Example

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given, }\left\{\begin{array}{l} <C = 121.36.20 \\ DB = 76.35.36 \\ <B = 42.15.13\tfrac{1}{4} \end{array}\right\}\text{ req. } BC.$$

The *Stereographic Solution.*



This *Caſe* is Conſtructed after the ſame manner as the *2d Caſe*, and the Side *BC* may be meaſured by the *Line of Chords*, but to determine the length by *Calculation*,

From *D* the end of the given Side *DB*, let fall the Perpendicular *DR* by *Caſe* the 3*d* of *Problem* the 11*th*, *Section* the 2*d*, this Perpendicular becauſe the Angles at the Baſe *B* and *C* are diſſimilar, will fall without the Triangle, and form two Spherical Triangles *DRB* and *DRC*, Right-angled at *R*, in the firſt of which *DRB* are given, *DB* the Hypothenuſe, equal to 76 deg. 35 min. 36 ſec. the Angle at the Baſe *B*, equal to 42 deg. 15 min. 13 ſec. ¼, whence to find the Baſe *BR*, it will be by the *2d Example* of the *4th Caſe* of *Right-angled Spherical Triangles*,

As *R* : c s, <*B* :: t, *BD* t, *BR* ; that is,

As the Radius————————————10.0000000

To the Co-ſine of the Angle at the Baſe *B*=42.15.13¼ 9.8693344
So is the Tangent of the Hyp. or Side *BD*=76.35.36—10.6227730

To the Tangent of the Baſe *BR*=72.09.00 ———10.4921074

Again, in the 2*d* Triangle DRC, becauſe the Angle at the Baſe DCR is given, to find the Baſe CR, it will be by *Caſe* the *9th* of *Right-angled Spherical Triangles*

In the 1*ſt* Triangle DRB, As t, <B : R : : t, DR : S. BR. And

In the 2*d* Triangle DRC, As t, <C : R : : t, CR : S. CR; that is,

Becauſe the two middle Terms in each Proportion are the ſame, they may be expunged, and the others will be reciprocally Proportional, by the 13*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt*. Wherefore,

As t, <C : t, <B : : S. BR : S. CR; that is,

As the Tangent of the Angle at the Baſe C in the 1*ſt* Triangle =121.36.20 ———————— 10.2108865

To the Tangent of the Angle at the Baſe B in the 1*ſt* Triangle=42.15.13¼ ———————— 9.9583029

So is the Sine of the Baſe BR in the 2*d* Triangle =72.09.00 ———————— 9.9785741

To the Sine of the Baſe CR in the 2*d* Triangle =32.08.50 ———————— 9.7259905

The Baſe CR equal to 32 deg. 08 min. 50 ſec. taken from the Baſe BR before found, (becauſe the Perpendicular falls without) equal to 72 deg. 09 min. 00 ſec. leaves the Side BC required 40 deg. 00 min. 10 ſec.

But the Side DC may be readily found without letting fall the Perpendicular, by the help of the 3*d* and 5*th Axioms*, of *Section* the 6*th*.

For by the 3*d Axiom*, to find the Side DC, it will be

As the S. <C : S. <B : : S. DB : S. DC; that is,

As the Sine of the Angle at C=121.36.20 ———————— 9.9307744

To the Sine of the Angle at B=42.15.13¼ ———————— 9.8276370

So is the Sine of the Side DB=76.35.36 ———————— 9.9880007

To the Sine of the Side DC=50.10.30 ———————— 9.8853653

Zzz   And

And by the former part of the *5th Axiom* it will be,

As the $S.\tfrac{1}{2}<^{s}\overline{C-B} : S.\tfrac{1}{2}<^{s}\overline{C+B} :: t,\tfrac{1}{2}\overline{DC-CB} : t,\tfrac{1}{2}BC.$

As the Sine of ½ the diff. of the Angles C & B=39.40.33½ 9.8051238

To the Sine of half the Sum=81.55.46¼———————9.9956775

So is the Tang. of ½ the diff. of DB and DC=13.12.33 9.3705442

To the Tang. of half the Side BC=20.00 05 ————9.5610979

The double of which, *viz.* 40 deg. 00 min. 10 ſec. will give the length of the Side BC, as before found.

Caſe VI.

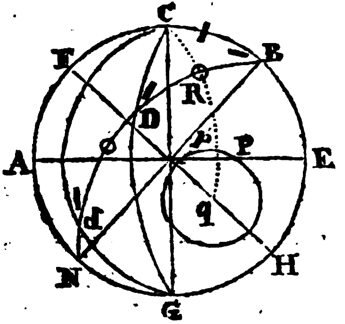
Example 2.

Given two Sides and an Angle oppoſite to one of them, to find the interjacent Side; that is,

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given}\left\{\begin{array}{l} <B=42.15.13\tfrac{3}{4} \\ BC=40.00.10 \\ <D=34.15.02\tfrac{3}{4} \end{array}\right\}\text{ req. } BD.$$

The *Stereographic Solution.*



This *Caſe* is doubtful, and is Conſtructed after the ſame manner as the *2d Example* of the *2d Caſe*; and the Side *BD* may be meaſured by the *3d Caſe* of *Problem* the *7th*, of *Section* the *2d*.

But to find the length by Calculation,

From *C* the end of the given Side *CB*, let fall the Perpendicular *CR*, by *Caſe* the *3d* of *Prob.* the *11th* of *Section* the *2d*, this Perpendicular becauſe the Angles at the Baſe are ſimilar, will fall within the Triangle, and will divide the Oblique-angled Spherical Triangle *BCD* firſt given, into two Right-angled Spherical Triangles *BCR* and *DCR* Right-angled at *R*, in the firſt of which

Triangles *BCR* are given, the Hypothenuſe *CB* equal to 40 deg. 00 min. 10 ſec. the Angle at the Baſe *B* equal to 42 deg. 15 min. 13 ſec. ¼, whence to find the Baſe *BR*, it will be by the 2*d Example* of the 4*th Caſe* of *Right-angled Spherical Triangles*,

As *R* : cs, < *B* : : t, *BC* : t, *BR* ; that is,

As the Radius—————————————10.0000000

To the Co-ſine of the Angle at the Baſe *B*=42.15.13¼ 9.8693344
So is the Tangent of the Side or Hypoth. *BC*=40.00.10 9.9238563

To the Tangent of the Baſe *BR*=31.50.46————9.7931907

Again, in the 2*d* Triangle *DRC*, becauſe the Angle at the Baſe *D* is given, to find the Baſe *DR*, it will be by the 9*th Caſe* of *Right-angled Spherical Triangles.*

In the 1*ſt* Triangle *BRC*, As t, < *B* : *R* : : t, *CR* : S. *RB*. And
In the 2*d* Triangle *DRC*, As t, < *D* : *R* : : t, *CR* : S. *CR* ; that is,
Becauſe the two middle Terms in each Sett of Proportionals are the ſame, the others will be reciprocally proportional, by the 12*th* Cor. of the 15*th Geom. Prop.* of *Part* the 1*ſt*. Wherefore,

As the t, < *D* t, : < *B* : : S. *BR* : S. *CR* ; that is,

As the Tangent of the Angle at the Baſe *D* in the 2*d* Triangle =34.15.02¾ ———————— } 9.8330805

To the Tangent of the Angle at the Baſe *B* in the 1*ſt* Triangle=42.15.13¾ ———————— } 9.9583029
So is the Sine of the Baſe *BR* in the 1*ſt* Triangle =31.50.46 ———————— } 9.7223373

To the Sine of the Baſe *CR* in the 2*d* Tri.=44.44.50 9.8275597

The Baſe *BR* of the 1*ſt* Triangle equal to 31 deg. 50 min. 46 ſec. added (becauſe the Perpendicular falls within) to the Baſe *CR* of the 2*d* Triangle equal to 44 deg. 44 min. 50 ſec. gives the Side *BD*, equal to 76 deg. 35 min. 46 ſec. but added to the Supplement of *CR*, equal to 135 deg. 15 min. 10 ſec. becauſe the Caſe is doubtful, will give the great Side *d B*, equal to 167 deg. 05 min. 56 ſec.

And by comparing the two *Examples* together, it is obvious, that If the Perpendicular fall $\left\{\begin{array}{l}\text{within}\\ \text{without}\end{array}\right\}$ the Triangle, the Baſe of the 2*d* Triangle $\left\{\begin{array}{l}\text{added to}\\ \text{ſubſtr. from}\end{array}\right\}$ the Baſe of the 1*ſt* Triangle, will give the Side requ^ir^ed.

Caſe VII.

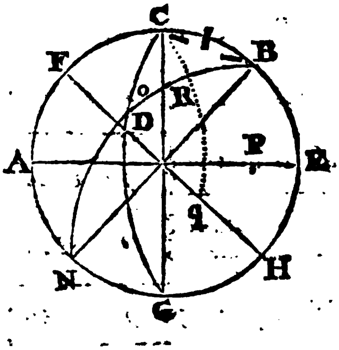
Given two Angles and the interjacent Side, to find the other Angle.

Example.

In the Oblique-angled Spherical Triangle *DCB*,

$$\text{Are given}\left\{\begin{array}{l}\angle C = 121.36.20\\ CB = \ \ 40.00.10\\ \angle B = \ \ 42.15.13\tfrac{1}{4}\end{array}\right\}\ \text{req. } \angle D$$

The *Stereographic Solution.*



1. Having drawn the Primitive Circle *ACEG*, the Diameter *AE*, and at Right-angles to it *CG*, ſet off the Chord of *CB*, equal to 40 deg. 00 min. 10 ſec. from *C* to *B*, and draw the Diameter *BN*, and at Right-angles to it *PH*.

2. Thro' the Point *B* draw the Great Circle *BDN*, forming an Angle of 42 deg. 15 min. 13 ſec. ¼, equal to the Angle *B*, with the Primitive Circle *CB*, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*.

3. Thro' the Point *C* draw the Great Circle *CDG*, forming an Angle of 121 deg. 36 min. 20 ſec. equal to the Angle *C* with the Primitive Circle *CB*, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*, then is the Triangle *DCB* form'd, and the Angle at *D* may be meaſured (by *Caſe* the 3*d* of *Problem* the 10*th*, of *Section* the 2.)

But to determine its length by Calculation.

From *C* one end of the given Side *CB*, let fall the Perpendicular *CR*, by *Caſe* the 3*d* of *Problem* the 11*th*, of *Section* the 2*d*, this Perpendipendicular becauſe it falls within, will divide the Triangle *DCB* firſt given, into two Right-angled Spherical Triangles *CBR* and *CDR* Right-angled at *R*, in the firſt of which *CBR* are given, *CB* the Hypothenuſe equal to 40 deg. 00 min. 10 ſec. the Angle *B* at the Baſe equal to 42 deg. 15 min. 13 ſec. ¾, whence to find the Angle at the Perpendicular *BCR*, it will be by *Caſe* the 3*d* of *Example* the 2*d*, of *Right-angled Spherical Triangles*

As *R* : c s, *BC* : : t, <*B* : c t, <*C* ; that is,

As the Radius ———————————— 10.0000000

To the Co-ſine of the Side *BC*=40.00.10 ———— 9.8842363
So is the Tangent of the Angle at the Baſe *B*=42.15.13¾ 9.9583029

To the Co-tang. of the Angle at Perp. *BCR*=55.09.59 9.8425392

The Angle at the Perpendicular *BCR* in the 1*ſt* Triangle, equal to 55 deg. 09 min. 59 ſec. taken from the Angle *DCB*, equal to 121 deg. 36 min. 20 ſec. (becauſe the Perpendicular falls within the Trriangle) will leave the Angle at the Perpendicular *DCR* in the 2*d* Triangle, equal to 66 deg. 26 min. 21 ſec.

Again, in the 2*d* Triangle *DRC*, becauſe the Angle at the Perpendicular *DCR*, is given to find the Angle at the Baſe *CDR*, it will be by *Caſe* the 7*th* of *Right-angled Spherical Triangles*.

In the 1*ſt* Triangle *BCR*, As *R* : c s, *CR* : : S. <*BCR* : c s, <*B*.
In the 2*d* Triangle *DCR*, As *R* : c s, *CR* : : S. <*DCR* : c s, <*D*.

Wherefore, becauſe the two firſt Terms in each Proportion are the ſame, they may be rejected; and the others will be directly proportional, by the 11*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt*.

As the S. *BCR* : S. *DCR* : : c s, < *B* : c s, <*D*; that is,

As the Sine of the Angle at the Perpendicular *BCR* in the 1*ſt* Triangle=55.09.59 ———— 9.9142450

To the Sine of the Angle at the Perpendicular *DCR* in the 2*d* Triangle=66.26.21 ———— 9.9621970
So is the Co-ſine of the Angle at the Baſe *B* in the 1*ſt* Triangle=42.15.13¾ ———— 9.8693344

To the Co-ſine of the Angle at the Baſe *D* in the 2*d* Triangle=34.15.[illegible] ———— 9.9172860

Or the Angle *BDC* of the 1*ſt* Triangle *BDC*, which Angle at *D* becauſe the Perpendicular falls within the Triangle, is ſimilar to the Angle *B*, and conſequently Acute.

Or,

Having found the two oppoſite Sides by the 5*th Axiom*, the contained or third Angle may be ſound by the 3*d Axiom*, without letting fall the Perpendicular.

Caſe VII.

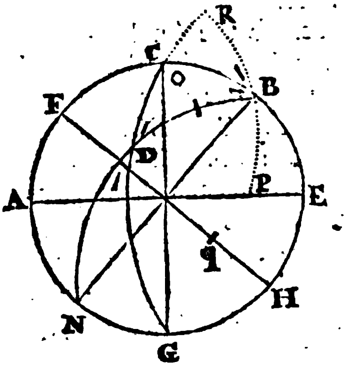
Example 2.

Given two Angles and the interjacent Side to find the other Angle; that is,

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given, }\left\{\begin{array}{l} \angle B = 42.15.13\tfrac{3}{4} \\ DB = 76.35.36 \\ \angle D = 34.15.02\tfrac{3}{4} \end{array}\right\}\text{ req. }\angle C.$$

The *Stereographic Solution.*



1. Having drawn the Primitive Circle *CAGE*, the Diameter *BN*, and at Right-angles to it the Diameter *FH*.

2. Thro' the Point *B* draw the Great Circle *BDN*, forming an Angle with the Primitive Circle of 42 deg. 15 min. 13 ſec. ¾, equal to the Angle at *B*, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*.

3. Lay off 76 deg. 35 min. 36 ſec. the length of *DB*, in the Great Circle *BDN* from *B* to *D*, by *Caſe* the 3*d* of *Problem* the 6*th*, of *Section* the 2*d*.

4. Thro' the given Point *D* draw the Great Circle *CDG*, forming an Angle of 34 deg. 15 min. 02 ſec. ¾, the Quantity of the Angle at *D*, with the Great Circle *BDG*, by *Caſe* the 4*th* of *Problem*

the *9th*, of *Section* the *2d* and the thing is done, and the Side *BD* may be meaſured by the *Line of Chords*.

But to determine its length by Calculation.

Thro' the Extremity *B* of the given Side *BD*, draw the Perpendicular *BR*, by *Caſe* the *3d* of *Problem* the *11th*, of *Section* the *2d*; this Perpendicular becauſe it falls without the Triangle upon the Side *DC* produced to *R*, will form two Right-angled Spherical Triangles *DRB* and *CRB*, Right-angled at *R*, in the firſt of which Triangles *DRB* are given, *DB* the Hypothenuſe equal to 76 deg. 35 min. 36 ſec. and the Angle *D* at the Baſe equal to 34 deg. 15 min. 02 ſec. ¾, whence to find the Angle at the Perpendicular *DBR*, it will be by the *2d Example* of the *3d Caſe* of *Right-angled Spherical Triangles*.

As the *R* : cs, *BD* : : t, < *D* : ct, < *DBR*; that is,

As the Radius ———————————— 10.0000000

To the Co-ſine of the Side or Hyp. *BD*=76.35.36 —— 9.3652280

So is the Tang. of the Angle at the Baſe *D*=34.15.02¾ 9.8330805

To the Co-tangent of the Angle at Perp. *DBR*=81.01.42¾ 9.1983085

If from the Angle *DBR*, equal to 81 deg. 01 min. 42 ſec. ¾, the Angle at the Perpendicular in the firſt Triangle *DBR*, be taken the Angle *DBC* equal to 42 deg. 15 min. 13 ſec. ¼, becauſe the Perpendicular falls without the Triangle, there will remain the Angle at the Perpendicular *CBR* equal to 38 deg. 46 min. 29 ſec. ½ in the *2d* Triangle, wherefore, becauſe in the *2d* Triangle the Angle at the Perpendicular is given, to find the Angle at the Baſe, it will be by the *7th Caſe* of *Right-angled Spherical Triangles*,

In the *1ſt* Triangle *DBR*, As *R* : cs, *BR* : : S. *CBR* : cs, *BDR*.

In the *2d* Triangle *CBR*, As *R* : cs, *BR* : : t, *CBR* : cs, *BCR*.

But becauſe the two firſt Terms in each are the ſame, they may be rejected, and the others will be directly proportional, by the *11th Cor* of the *15th Geom. Prop.* of *Part* the *1ſt*; that is, As the S. *DBR* : S. *CBR* : : cs. *BDR* : cs. *BCR*.

As the Sine of the Angle at the Perpendicular DBR in the 1ſt Triangle=81.01.42$\frac{3}{4}$	9.9946541
To the Sine of the Angle at the Perpendicular CBR in the 2*d* Triangle=38.46.29$\frac{1}{2}$	9.7967560
So is the Co-ſine of the Angle at the Baſe D in the 1ſt Triangle=34.15.02$\frac{3}{4}$	9.9172861
To the Co-ſine of the Angle at the Perpendicular BCR in the 2*d* Triangle=58.23.40	9.7193880

Whoſe Supplement to a Semicircle gives the Angle BCD, equal to 121 deg. 36 min. 20 ſec. becauſe the Perpendicular falls without, and by comparing theſe two *Examples* together, it is evident, that

If the Angle at the Perpendicular in the 1ſt Triangle be greater than the Angle at the Baſe in the ſame Triangle, and the Angle at the Baſe be $\left\{\begin{array}{c}\text{Acute}\\ \text{Obtuſe}\end{array}\right\}$ then the required Angle ſhall be $\left\{\begin{array}{c}\text{Obtuſe}\\ \text{Acute}\end{array}\right\}$

If the Angle at the Perpendicular in the 1ſt Triangle be leſſer than the Angle at the Baſe in the ſame Triangle, and the Angle at the Baſe be $\left\{\begin{array}{c}\text{Acute}\\ \text{Obtuſe}\end{array}\right\}$ then the required Angle ſhall be $\left\{\begin{array}{c}\text{Acute}\\ \text{Obtuſe}\end{array}\right\}$.

Caſe VIII.

Given two Angles and the interjacent Side, to find either of the other oppoſite Sides.

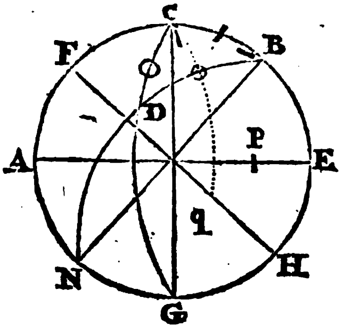
Example

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given}\left\{\begin{array}{l} <C=121.36.20\\ CB=40.00.10\\ <B=42.15.13\tfrac{1}{4}\end{array}\right\}\text{req. either } DC \text{ or } BD.$$

The *Stereographic Solution.*

This *Caſe* is conſtructed after the ſame manner as the 1ſt *Example* of the 7*th Caſe*, and the Side *BD* or *DC*, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th* of *Section* the 2*d*.



But to find the length of CD by Calculation,

From C one end of the given Side CB, let fall the Perpendicular CR, this Perpendicular becauſe it falls within, will divide the Triangle DCB into two Right-angle Spherical Triangles BCR and DCR, Right-angled at R, in the firſt of which CBR are given, CB the Hypothenuſe equal to 40 deg. 00 min. 10 ſec. the Angle B at the Baſe equal to 42 deg. 15 min. 13 ſec. ½, whence to find the Angle at the Perpendicular BCR, it will be by the 2*d Example* of the 3*d Caſe* of *Right-angled Spherical Triangles*

As R : c s,BC : : t,< B : t,< BCR ; that is,

As the Radius——————————————10.0000000

To the Co-ſine of the Hyp. or Side BC=40.00.10——9.8842363
So is the Tangent of the Angle at the Baſe B=42.15.13½ 9,9583029

To the Tangent of the Angle BCR=55.09.59————9.8425392

The Angle at the Perpendicular BCR in the firſt Triangle equal to 55 deg. 09 min. 59 ſec. taken from the Angle DCB, equal to 121 deg. 36 min. 20 ſec. (becauſe the Perpendicular falls within the Triangle) will leave the Angle at the Perpendicular DCR in the 2*d* Triangle, equal to 66 deg. 26 min. 21 ſec.

Again, in the 2*d* Triangle DRC, becauſe the Angle at the Perpendicular DCR is given, to find the Hypothenuſe DC, it will be by *Caſe* the 6*th* of *Right-angled Spherical Triangles.*

In the 1*ſt* Triangle BRC, As cs,<BCR : R : : t, CR : t,BC ; and
In the 2*d* Triangle DRC, As cs,<DCR : R : : t, CR : t, DC.

But becauſe the two middle Terms in each are the ſame, they may be rejected, and the other two will be reciprocally proportional, by the 12*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt* ; that is, As the c s, DCR : c s, BCR : : t, BC : t, DC; that is,

As the Co-ſine of the Angle at the Perpendicular DCR of the 2*d* Triangle=66.26.21	9.6017586
To the Co-ſine of the Angle at the Perpendicular BCR in the 1*ſt* Triangle=55.09.59	9.7567846
So is the Tangent of the Side BC or Hypothenuſe of the 1*ſt* Triangle=40.00.10	9.9238563
To the Tangent of the Side DC or Hypothenuſe of the 2*d* Triangle=50.10.30	10.0788823

Or,

The Sides DC and DB may be directly inveſtigated by the help of the 5*th Axiom*, without letting fall the Perpendicular. For,

As the S. ½ <s $\overline{C+B}$: S. ½ <s $\overline{C-B}$: : t, ½ BC : t, ½ $\overline{BD-DC}$; that is,

As the Sine of ½ the Sum of the <s C and B=81.55.46½	9.9956775
To the Sine of half their Difference=39.40.33½	9.8051238
So is the Tangent of half the cont. Side BC=20.00.5	95610979
To the Tangent of half the Difference of the two oppoſite Sides BD and DC=13.12.33	9.3705442

And again,

As c s, ½ <s $\overline{C+B}$: c s, ½ <s $\overline{C-B}$: : t, ½ BC : t, ½ $\overline{DB+DC}$.

As the Co-ſine of ½ Sum of the <s C and B=81,55.46½	9.1473342
To the Co-ſine of half their Difference=39.40.33½	9.8863032
So is the Tangent of half the cont. Side BC=20.00.05	9.5610979
To the Tangent of half the Sum of the two oppoſite Sides BD and DC=63,23.03	10.3080669

But becauſe $\frac{1}{2}\overline{BD+DC}+\frac{1}{2}\overline{BD-DC}=BD$,

	°	′	″
If to the half Sum of the Sides BD and DC ——— =	63	23	03
Be added the half Difference ——— =	13	12	33
The Sum will be the greater Side BD ——— =	76	35	36

And becauſe $\frac{1}{2}\overline{BD+DC}-\frac{1}{2}\overline{BD-DC}=DC$.

	°	′	″
If from half the Sum of the Sides BD and DC ——— =	63	23	03
Be taken the half Difference ——— =	13	12	33
The Remainder will be the leſſer Side DC ——— =	50	10	30

Caſe VIII.

Example 2.

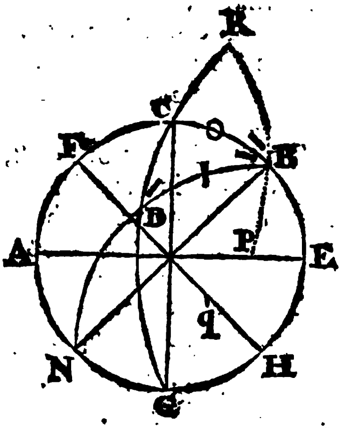
Given two Angles and the interjacent Side, to find either of the oppoſite Sides; that is,

In the Oblique-angled Spherical Triangle *DCB*,

$$\text{Are given}\left\{\begin{array}{l} \angle B = 42.15.13\tfrac{1}{4} \\ DB = 76.35.36 \\ \angle D = 34.15.02\tfrac{1}{4} \end{array}\right\}\text{req. } BC.$$

The *Stereographic Solution*:

This Caſe is Conſtructed after the ſame manner as the 2d *Example* of the 7th *Caſe*, and the Side *BC* may be meaſured by the *Line of Chords*.



But to determine its length by Calculation,

Thro' the Extremity *B* of the given Side *BD*, let fall the Perpendicular *BR*, this Perpendicular becauſe it falls without the Triangle, will form two Right-angled Spherical Triangles *DBR* and *CBR* Right-angled at *R*, in the firſt of which *DBR* are given, the Hypothenuſe *BD* equal to 76 deg. 35 min. 36 ſec. and the Angle *B* at the Baſe equal to 34 deg. 15 min. 02 ſec. ¾, whence to find the Angle at the Perpendicular *DBR*, it will be by the 2*d Example* of the 3*d Caſe* of *Right-angled Spherical Triangles*,

As *R* : c s, *BD* : : t, <*D* : c t, <*DBR* ; that is ;

As the Radius —————————— 10.0000000

To the Co-ſine of the Side or Hyp. *BD*=76.35.36 —— 9.3652280
So is the Tang. of the Angle at the Baſe *D*=34.15.02¾ 9.8330895

To the Co-tang. of the Angle at Baſe *DBR*=81.01.42¾ — 9.1983085

If from the Angle *DBR* in the 1*ſt* Triangle equal to 81 deg. 01 min. 42 ſec. ¾, be taken the Angle *DBC* equal to 42 deg. 15 min. 13 ſec. ¼, becauſe the Perpendicular falls without the Triangle, there will remain the Angle at the Perpendicular *CBR* in the ſecond Triangle, equal to 38 deg. 46 min. 29 ſec. ½.

Wherefore, becauſe the Angle at the Perpendicular in the ſecond Triangle, is given to find the Hypothenuſe, it will be by the 6*th Caſe* of *Right-angled Spherical Triangles*,

In the 1*ſt* Triangle *DBR*, As c s, <*DBR* : *R* : : t, *BR* : t, *BD*, And
In the 2*d* Triangle *CBR*, As c s, *CBR* : *R* : : t, *BR* : t, *CB*.

But becauſe the two middle Terms in each Propoportion are the ſame, let them be expunged, and the other two will be reciprocally proportional, by the 12*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt* ; that is,

As c s, *CBR* : c s, *DBR* : : t, *BD* : t, *CB* ; wherefore,

As the Co-ſine of *CBR* the Angle at the Perpendicular in the 2*d* Triangle=38.46.29½ ——— } 9.8918789

To the Co-ſine of *DBR*, the Angle at the Perpendicular in the 1*ſt* Triangle=81.01.42¾ ——— } 9.1929643
So is the Tangent of the Side *BD* or Hypothenuſe in the 1*ſt* Triangle=76.35.36 ——— } 10.6227730

To the Tangent of the Side *BC* or Hypothenuſe in the 2*d* Triangle=40.00.10½ ——— } 9.9238584

Or the Sides *BC* and *DC* may be inveſtigated directly, without letting fall the Perpendicular, by the help of the 5*th Ax.* as in the former *Example*.

By comparing the two *Examples* together, it is evident, that if the Perpendicular fall within the Triangle, the Angle at the Perpendicular of the *1ſt* Triangle, taken from the given Angle of the Triangle firſt given, will leave the Angle at the Perpendicular of the *2d* Triangle.

But if the Perpendicular falls without, then the Angle of the given Triangle, taken from the Angle at the Perpendicular of the *1ſt* Triangle, will leave the Angle at the Perpendicular of the *2d* Triangle.

If the Angle at the Perpendicular of the ſecond Triangle be $\left\{\begin{array}{l}\text{ſimilar}\\ \text{diſſimilar}\end{array}\right\}$ to the Angle at the *Baſe* of the *1ſt* Triangle, the required Side will be $\left\{\begin{array}{l}\text{leſs}\\ \text{greater}\end{array}\right\}$ than a Quadrant.

Caſe IX.

Given two Sides and the contained Angle; to find either of the oppoſite Angles.

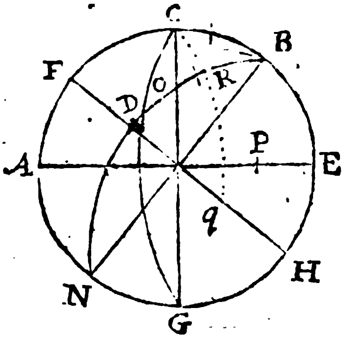
Example

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given, }\left\{\begin{array}{l} BD = 76.35.36\\ \angle B = 42.15.13\tfrac{1}{4}\\ BC = 40.00.10 \end{array}\right\}\text{ req. }\angle D.$$

The *Stereographic Solution.*

1. Having drawn the Primitive Circle *ACEG*, the Diameter *AE*, and at Right-angles to it *CG*; ſet off the Chord of 40 deg. 00 min 10 ſec. the length of *CB* from *C* to *B*, and draw the Diameter *BN* and at Right-angles to it *FH*.



2. Thro' the Point *B* draw the Great Circle *BDN*, forming an Angle *DBC* of 42 deg. 15 min. 13 ſec. ¼, equal to the Angle *B*, with the Primitive Circle *CB*, by *Caſe* the *3d* of *Problem* the *9th*, of *Section* the *2d*.

3. Lay off 76 deg. 35 min. 36 ſec. the length of *BD* from *B* to *D*, in the Great Circle *BD*, by *Caſe* the 3*d* of *Problem* the 6*th*, of *Section* the 2*d*.

4. Thro' the Points *C* and *D* draw the Great Circle *CDG*, by *Problem* the 1*ſt* of *Section* the 2*d*, and the thing is done; and the Angle at *D* may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th*, of *Section* the 2*d*.

But to determine its Quantity by Calculation.

From *C* the Extremity of the Side *CB*, let fall the Perpendicular *CR*, this Perpendicular, becauſe it falls without the Triangle, will divide the Triangle *DCB* firſt given, into two Right-angled Spherical Tiangles *DRB* and *BRC*, Right-angled at *R*; in the firſt of which Triangles *CRB* are given, *CB* the Hypothenuſe equal to 40 deg. 00 min. 10 ſec. and the Angle at the Baſe *B* equal to 42 deg. 15 min. 13 ſec. ¼, whence to find the Baſe *BR*, it will be by the 2*d Example* of the 4*th Caſe* of *Right-angled Spherical Triangles*.

As the *R* : cs, ∠*B* : : t, *BC* : t, *BR*; that is,

As the Radius ——————————— 10.0000000

To the Co-ſine of the Angle at the Baſe *B*=42.15.13¼ — 9.8693544
So is the Tangent of the Hypothenuſe *BC*=40.00.10 9.9238563

To the Tangent of the Baſe *BR*=31.50.46 ———— 9.7931907

If from the Side *DB* of the Triangle *DCB*, equal to 76 deg. 35 min. 36 ſec. there be taken *BR* the Baſe of the firſt Triangle, equal to 31 deg. 50 min. 46 ſec. becauſe the Perpendicular falls within the Triangle, there will remain the Baſe *DR* of the ſecond Triangle, equal to 44 deg. 44 min. 50 ſec.

Wherefore becauſe the Baſe of the ſecond Triangle is given, to find the Angle at the Baſe, it will be by the 2*d Caſe* of *Right-angled Spherical Triangles*.

In the 1*ſt* Triangle *BRC*, As S.*BR* : *R* : : t, *CR* : t, ∠*B*; And

In the 2*d* Triangle *DRC*, As, S. *DR* : *R* : : t, *CR* : t, ∠*D*.

But becauſe the two middle Terms in each Proportion are the ſame, they may be rejected, and the other two will be reciprocally proportional, by the 12*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt*; that is, As the S. DR : S. BR : : t, $<B$: t, $<D$. Wherefore,

As the Sine of DR the Baſe of the ſecond Triangle =44.44.50 ——— 9.8475605

To the S. of BR the Baſe of the 1ſt Triangle=31.50.46 9.7223373

So is the Tangent of the Angle at the Baſe B of the 1ſt Triangle=42.15.13 ——— 9.9583029

To the Tangent of the Angle at the Baſe D in the ſecond Triangle=34.15.02¾ ——— 9.8330797

The ſame with the Angle CDB of the Triangle CDB required.

Or,

The Angles at D and C may be directly inveſtigated, by the help of the *4th Axiom* of *Section* the *6th*, without letting fall the Perpendicular CR. For,

As S. $\frac{1}{2}\overline{BD+BC}$: S. $\frac{1}{2}\overline{BD-DC}$: : ct, $\frac{1}{2}<B$: t, $\frac{1}{2}<s\ \overline{C-D}$;
that is,

As the Sine of half the Sum of the two Sides BD and BC =58.17.53 ——— 9.9298241

To the Sine of half their Difference=18.17.43 ——— 9.4968169

So is the Co-tangent of half the Angle B=21.07.36¼ 10.4129561

To the Tangent of half the Difference of the Angles C and D=43.40.38¾ ——— 9.9799429

And again,

As cs, $\frac{1}{2}\overline{BD+DC}$: cs, $\frac{1}{2}\overline{BD-DC}$: : ct, $\frac{1}{2}<B$: t, $\frac{1}{2}<s\ \overline{C+D}$;
that is,

As the Co-ſine of half the Sum of the Sides BD and BC=58.17.53 ——— 9.7205761

To the Co-ſine of half their Difference=18.17.43 ——— 9.9774737

So is the Co-tang. of half the cont. $<B$=21.07.36¼ 10.4129561

To the Tangent of half the Sum of the oppoſite Angles C and D=77.55.41¼ ——— 10.6698537

But becauſe $\frac{1}{2}<s\,\overline{C+D}+\frac{1}{2}<s\,\overline{C-D}=<C$.

	°	′	″
If to half the Sum of the Angles *C* and *D*	77	55	41¼
Be added their half Difference	43	40	38¾
The Sum will be the greater Angle *BCD* =	121	36	20

And becauſe $\frac{1}{2}<s\,\overline{C+D}-\frac{1}{2}<s\,\overline{C-D}=<D$.

	°	′	″
If from the half Sum of the Angles *C* and *D*	77	55	41¼
Be taken their half Difference	43	40	38¾
The Remainder will be the leſſer Angle *CDB* =	34	15	02½

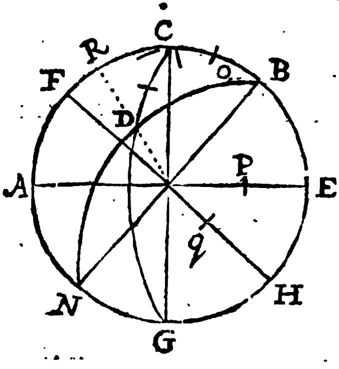
Caſe IX.

Example. 2.

Given two Sides and the contained Angle, to find either of the oppoſite Angles; that is,

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given}\left\{\begin{array}{l} DC = 50.10.30 \\ <C = 121.36.20 \\ CB = 40.00.10 \end{array}\right\}\text{req. }<B.$$



The *Stereographic Solution.*

1. Having drawn the Primitive Circle *CEGA*, the Diameter *AE*, and at Right-angles to it *CG*, ſet off the Chord of 40 deg. 00 min. 10 ſec. the length of *CB*, from *C* to *B*, and draw the Diameter *BN*, and at Right-angles to it the Diameter *AE*.

2. Thro' the Point *C* draw the Great Circle *CDG* forming an Angle of 121 deg. 36 min. 20 ſec. equal to the Angle at *C*, with the Primitive

Circle by *Caſe* the third of *Problem* the ninth, of *Section* the ſecond.

3. Lay off 50 deg. 10 min. 30 ſec. the length of DC, from C to D, by *Caſe* the 3*d* of *Problem* the 6*th* of *Section* the 2*d*.

4. Thro' the Points B and D draw the Great Circle BDN, by *Caſe* the 3*d* of *Problem* the 1*ſt* of *Section* the 2*d*, and the thing is done; and the Angle at B may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*.

But to determine the Quantity by Calculation,

Thro' one Extremity D of the given Side DC, let fall the Perpendicular DR, this Perpendicular, becauſe the Angles at the Baſe are diſſimilar, will fall without the Triangle, upon the Baſe BC produced to R, and will form two Right-angled Spherical Triangles DRB and CRD, Right-angled at R, in the firſt of which CRD are given, CD the Hypothenuſe, equal to 50 deg. 10 min. 30 ſec. the Angle at the Baſe C, equal to 58 deg. 23 min. 40 ſec. the Supplement of the Angle DCB, whence to find the Baſe CR, it will be by *Caſe* the 4*th Example* the 2*d*, of *Right-angled Spherical Triangles*,

As R : c s, < C : : t, CD : t, CR; that is,

As the Radius ———————————— 10.0000000

To the Co-ſine of the Angle at the Baſe C=58.23.40 — 9.7193880

So is the Tang. of the Hyp. or Side CD=50.10.30 10.0788818

To the Tangent of the Baſe CR=32.08.50 ———— 9.7982698

The Baſe CR of the 1*ſt* Triangle CRD, equal to 32 deg. 08 min. 50 ſec. added to 40 deg. 00 min. 10 ſec. the Side CB of the Triangle DCB (becauſe the Perpendicular falls without the Triangle) will give 72 deg. 09 min. 00 ſec. for the Baſe BR of the 2*d* Triangle BRD.

Again, becauſe the Baſe BR of the 2*d* Triangle is given, to find the Angle at the Baſe B, it will be by the 2*d Caſe* of *Right-angled Spherical Triangles*,

In the 1*ſt* Triangle CRD, As S. CR : R : : t, DR : t, < C. And
In the 2*d* Triangle BRD, As S. BR : R : : t, DR : t, < B.

But becauſe the two middle Terms in each Proportion are the ſame, they may be rejected; and the other two will be reciprocally proportional, by the 12*th Cor.* of the 15*th Geom. Prop.* of *Part* the 1*ſt*.

As S. *BR* : S. *CR* : : t. <*C* : t, <*B*; that is,

As the Sine of the Baſe *BR* of the firſt Triangle =72.09.00 —— 9.9785742

To the Sine of the Baſe *CR* of the 2*d* Triangle 32.08.50 —— 9.7259905
So is the Tangent of the Angle at the Baſe *C* in the 1*ſt* Triangle=58.23.40 —— 10.2108865

To the Tangent of the Angle *B* at the Baſe in the ſecond Triangle=42 deg. 15 min. 13 ſec. ¼ —— 9.9583029

Or the Angle *CBD* of the Triangle *DCB*; which Angle at *B* becauſe the Baſe *BR* is leſs than a Quadrant, is ſimilar to the Angle *DCR*, and conſequently Acute.

By comparing the two *Examples* together, it is evident, that if the Perpendicular falls within the Triangle as in the 1*ſt Example*, the Baſe of the firſt Triangle taken from the Side of the Triangle firſt given, upon which the Perpendicular falls gives the Baſe of the ſecond Triangle.

For *BD*—*BR*=*DR*, but if the Perpendicular falls without the Triangle as in the 2*d Example*, then the Side of the given Triangle upon which the Perpendicular falls, taken from the Baſe of the firſt Triangle, will give the Baſe of the ſecond Triangle; for *BC*+*CR* =*BR*.

If the Baſe of the firſt Triangle be $\left\{\begin{matrix}\text{leſs}\\ \text{greater}\end{matrix}\right\}$ than the Side of the given Triangle upon which the Perpendicular falls, then the Angle oppoſite to the Perpendicular ſhall be $\left\{\begin{matrix}\text{ſimilar}\\ \text{diſſimilar}\end{matrix}\right\}$ to the Angle given; that is, if the Side *BD* as in the 1*ſt Example*, be greater than the Baſe *BR*, the Angle at *D* will be ſimilar to the Angle at *B*, but if the Baſe *BR* be greater than the given Side *BC* as in the ſecond *Example*, then the Angle at *D* will be diſſimilar to the Angle at *B*.

Caſe X.

Given two Sides and the contained Angle; to find the third Side.

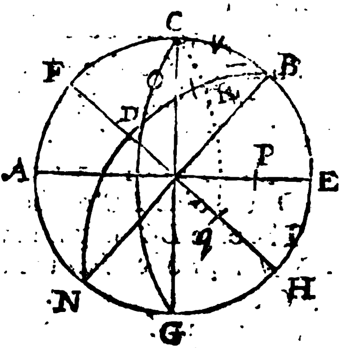
Example

Example

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given, }\left\{\begin{array}{l} BD = 76.35.36 \\ \angle B = 42.15.13\tfrac{3}{4} \\ BC = 40.00.10 \end{array}\right\}\text{ req. } DC.$$

The *Stereographic Solution.*



This *Caſe* is Conſtructed after the ſame manner as the *9th Caſe*, and the Side *DC* may be meaſured by *Caſe* the 3*d*, of *Prob*. the 7*th*, of *Sect*. the 2*d*.

But to determine the Length by Calculation,

From *C* the End of the given Side *CB*, let fall the Perpendicular *CR*; this Perpendicular becauſe it falls within, will divide the Oblique-angled Triangle firſt given, into two Right-angled Spherical Triangles *BRC* and *DRC* Right-angled at *R*, in the firſt of which *BRC* are given, *CB* the Hypothenuſe, equal to 40 deg. 00 min. 10 ſec. the Angle *B* the Angle at the Baſe, equal to 42 deg. 15 min: 13 ſec. ¾, whence to find the Baſe *BR*, it will be by the 2*d Example* of the 4*th Caſe* of *Right-angled Spherical Triangles.*

As *R* : : c s, < *B* : : t, *BC* : t, *BR*; that is,

As the Radius ———————————— 10.0000000

To the Co-ſine of the Angle at the Baſe *B*=42.15.13¾ 9.8693344

So is the Tangent of the Hypothenuſe *BC*=40.00.10 9.9238563

To the Tangent of the Baſe *BR*=31.50.46 ———— 9.7931907

If from *DB* the Side of the given Triangle equal to 76 deg. 35 min. 36 ſec. upon which the Perpendicular falls, be taken the Baſe *BR* of the firſt Triangle, equal to 31 deg. 50 min. 46 ſec. there will remain the Baſe *DR* of the 2*d* Triangle, equal to 44 deg. 44 min 50 ſec.

Wherefore, becauſe the Baſe of the 2*d* Triangle *DRC* is given to find the Hypothenuſe, it will be by *Caſe* the 1*ſt* of *Right-angled Spherical Triangles*,

In the 1*ſt* Triangle *BRC*, As *R* : c s, *CR* : : c s, *BR* : c s, *BC*, And

In the 2*d* Triangle *DRC*, As *R* : c s, *CR* : : c s, *DR* : c s, *DC*.

Wherefore becauſe the two firſt Terms in each Proportion are the ſame, the other two will be directly proportional, by the 11*th Cor.* of the 15*th Geom. Prob.* of *Section* the 2*d*; that is,

As c s, *BR* : c s, *DR* : : c s, *BC* : c s, *DC*; that is

As the Co-ſine of the Baſe *BR* of the 1*ſt* Triangle =31.50.46	9.9291472
To the Co-ſine of the Baſe *DR* of the 2*d* Triangle =44.44.50	9.8513926
So is the Co-ſine of the Hypothenuſe *BC* of the 1*ſt* Triangle=40.00.10	9.8842363
To the Co-ſine of the Hypothenuſe *DC* of the 2*d* Triangle=50.10.30	9.8064817

Or the Side *DC* of the Triangle *DCB*.

The Side *DC* might have been found by the help of the 3*d* and 4*th Axioms* of *Section* the 6*th*, without letting fall the Perpendicular *CR*.

For having found the Angle at *D* or *C* by the 4*th Axiom*, the Side *DC* might have been found by the 3*d Axiom*.

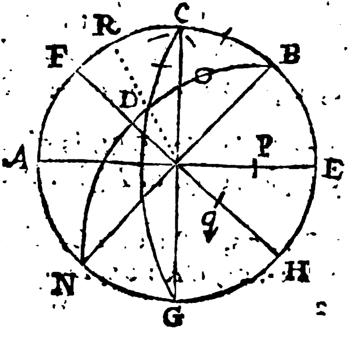
Caſe X.

Example 2.

Given two Sides and the Angle contained, to find the third Side; that is,

In the Oblique-angled Spherical Triangle *DCB*

Are given { *DC*= 50.10.30
<*C*=121.36.20
BC= 40.00.10 } req. *DB*.



The *Stereographic Solution.*

This *Caſe* is Conſtructed after the ſame manner as the 2*d Example* of the 9*th Caſe*, and the Side *DB* may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*.

But to determine the Length by Calculation,

From *B* the Extremity of the given Side *BC*, let fall the Perpendicular *BR*, this Perpendicular becauſe it falls without the Triangle, will form two Right-angled Spherical Triangles *DBR* and *CRD* Right-angled at *R*, in the firſt of which *BRC* are given, *BC* the Hypothenuſe, equal to 40 deg. 00 min. 10 ſec. the Angle *C* at the Baſe equal to 58 deg. 23 min. 40 ſec. the Supplement of the Angle *DCB*, whence to find the Baſe *CR*, it will be by *Caſe* the *4th Example* the *2d*, of *Section* the *2d* of *Right-angled Spherical Triangles*,

As R : c s, < *C* : : t, *CB* : t, *CR* ; that is,

As the Radius —————————— 10.0000000

To the Co-ſine of the Angle at the Baſe *C*=58.23.40 9.7193880
So is the Tang. of the Hypoth. Side *CB*=40.00.10 ——— 9.9238563

To the Tangent of the Baſe *CR*=23.44.21$\frac{3}{4}$ ——— 9.6432443

The Baſe *CR* equal to 23 deg. 44 min. 21 ſec. $\frac{3}{4}$, added to the Side *DC* equal to 50 deg. 10 min. 30 ſec. gives 73 deg. 54 min. 51 ſec. $\frac{3}{4}$, for the Length of the Baſe *DC* of the *2d* Triangle, (becauſe the Perpendicular falls within the Triangle.)

And becauſe the Baſe *BC* of the *2d* Triangle is given, to find the Hypothenuſe, it will be by *Caſe* the *1ſt* of *Right-angled Spherical Triangles*,

In the *1ſt* Triangle *BRC*, As *R* : c s, *BR* : : c s, *CR* : c s, *CB*, And
In the *2d* Triangle *DBR*, As *R* : c s *BR* : : c s, *DR* : c s, *DB*.

But becauſe the two firſt Terms in each Proportion are the ſame, they may be rejected ; and the others will be directly proportional, by the *11th Cor.* of the *15th Geom. Prop.* of *Part* the *1ſt* ; that is,

As c s, *CR* : c s, *DR* : : c s, *CB* : c s, *DB* ; wherefore,

As the Co-ſine of the Baſe *CR* of the *1ſt* Triangle =23.44.21$\frac{3}{4}$ —————— 9.9616044

To the Co-ſine of the Baſe *DR* of the *2d* Triangle =73.54.51$\frac{3}{4}$ —————— 9.4425952
So is the Co-ſine of the Hypothenuſe *CB* of the firſt Triangle=40.00.10 —————— 9.8842363

To the Co-ſine of the Hyp. *DB* of the *2d* Tri.=76.35.36 9.3652271

Or the Side *DB*, required of the Triangle *DCB*.

The Perpendicular *BR* might have been let fall from the Point *D*, and the Operation would have been the ſame, only changing *CB* for *CD*.

By comparing the two *Examples* together, it is evident, that if the Perpendicular falls within the Triangle as in the *1ſt Example*, the Baſe of the *1ſt* Triangle taken from the Side of the Triangle *DCB*, upon which the Perpendicular falls, will give the Baſe of the *2d* Triangle; for $DB-BR=DR$.

But if the Perpendicular fall without the Triangle, as in the *2d Example*, then the Side of the Triangle *DCB* upon which the Perpendicular falls, added to the Baſe of the firſt Triangle, will give the Baſe of the ſecond Triangle. For $DC+CR=DR$.

If the Baſe of the *2d* Triangle be $\left\{\begin{array}{l}\text{ſimilar}\\ \text{diſſimilar}\end{array}\right\}$ to the Perpendicular, or its oppoſite given Angle, then ſhall the required Side be accordingly $\left\{\begin{array}{l}\text{leſs}\\ \text{greater}\end{array}\right\}$ than a Quadrant.

Caſe XI.

Given the three Sides, to find either of the Angles.

Example.

In the Oblique-angled Spherical Triangle *DCB*,

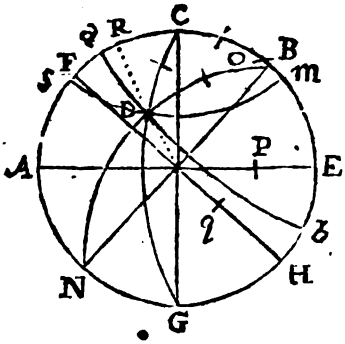
$$\text{Are given}\left\{\begin{array}{l}BC=40.00.10\\ DC=50.10.30\\ DB=76.35.36\end{array}\right\}\text{req.}\ \angle B.$$

The *Stereographic Solution:*

1. Having drawn the Primitive Circle *ACEG*, the Diameter *AE*, and at Right-angles to it *CG*, ſet off the Chord of 40 deg. 00 min. 10 ſec. the length of *BC* from *B* to *C*, and draw the Diameter *BN*, and at Right-angles to it *FH*.

2. About the Point *B* as a Pole, deſcribe a Small Circle *a b*, at the diſtance of 76 deg. 35 min. 36 ſec. the length of the Side *BD*, by *Caſe* the *2d* of *Problem* the *4th*, of *Section* the *2d*.

3. About the Point *C* as a Pole, deſcribe a Small Circle as *s m*, at the diſtance of 50 deg. 10 min. 30 ſec. the length of the Side *CD*, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*.



4. Thro' the Interſection *D* of the Parallels, *a b* and *s m* and the Points *B* and *C* draw two Great Circles *BDN* and *CDG*, by *Caſe* 3*d* of *Prob*. the 1*ſt* of *Section* the 2*d*, and the thing is done; and the Angle at *B* may be meaſured by *Caſe* the 2*d*, of *Prob*. the 10*th*, of *Section* the 2*d*.

But to determine its Length by Calculation.

Having let fall the Perpendicular *CR* it will be,

1. By *Axiom* the 6*th* of *Sect*. the 6*th*, As $t, \frac{1}{2} BD : t, \frac{1}{2} \overline{DC+CB} : : t, \frac{1}{2} \overline{DC-CB} : t, \frac{1}{2} \overline{DR-RC}$; that is,

As the Tangent of the half Baſe *BD*=38.17.48 ——— 9.8974390

To the Tangent of half the Sum of the two Sides *DC* and *CB*=45.05.20 ——— } 10.0013475

So is the Tangent of half their Difference=05.05.10 ——— 8.9494058

To the Tangent of half the Difference of the Segments *DR* and *RC*=6.27.02 ——— } 9.0533143

This half Difference of the Segments equal to 62 deg. 27 min. 2 ſec. ſubſtracted from the half Sum, equal to half the Side *DB*, equal to 38 deg. 17 min. 48 ſec. (becauſe the Perpendicular *CR* falls without the Triangle) will leave 31 deg. 50 min. 46 ſec. for the leſſer Segment *CR*, or Baſe of the Triangle *CRB*.

And becauſe in the Right-angled Spherical Triangle *CRB* there are given, the Hypothenuſe *CB* equal to 40 deg. 00 min. 10 ſec. and the Baſe *CR* equal to 31 deg. 50 min. 46 ſec. it will be to find the Angle at the Baſe *B*, by the 2*d Example* of the 12*th Caſe* of *Right-angled Spherical Triangles*, As R : c t, BC : : t, CR : c s, <B ; that is,

As the Radius ———————————— 10.0000000

To the Co tang. of the Hyp. or Side BC=40.00.10 —— 10.0761437

So is the Tangent of the Baſe CR=31.50.46 ——— 9.7931907

To the Co-ſine of the Angle B at the Baſe=42.15.13$\frac{3}{4}$ —— 9.8693344

Or the Angle B of the Triangle DCB.

But the half Difference of the Segments DR and RB, equal to 6 deg. 27 min. 02 ſec. added to half their Sum equal to half DB, equal to 38 deg. 17 min. 48 ſec. will give the greater Segment DR 44 dg. 44 min. 50 ſec. whence to find the Angle at D, becauſe the Hypothenuſe DC equal to 50 deg. 10 min. 30 ſec. and the Baſe BR, equal to 44 deg. 44 min. 50 ſec. in the Right-angled Spherical Triangle DCR are given, it will be by the *2d Example* of the *12th Caſe* of *Right-angled Spherical Triangles,*

As R : c t, DC : : t, DR : c s, <D ; that is,

As the Radius ———————————— 10.0000000

To the Co-tang. of the Hyp. or Side DC=50.10.30 ——— 9.9211182

So is the Tang. of the Baſe or Segment BR=44.44.50 9.9961679

To the Co-ſine of the Angle at the Baſe D=34.15.02$\frac{3}{4}$ — 9.9172861

Or the Angle D of the Triangle DCB.

Again,

2. If from D one Extremity of the Side DB, a Perpendicular as DR, be let fall upon the Side BC produced to R, it will be by *Ax.* the *6th* of *Sect.* the *6th*.

As t, $\frac{1}{2}$ BC : t, $\frac{1}{2}$ $\overline{BD+DC}$: : t, $\overline{DB-DC}$: t, $\overline{BR+RC}$; that is,

As the Tangent of half the Baſe BC=20.00.05 ——— 9.5610979

To the T. of the $\frac{1}{2}$ Sum of the Sides DB & DC=63.23.3 10.3000694

So is the T. of $\frac{1}{2}$ Dif. of the Sides DB & DC=13.12.33 9.3705442

To the T. of $\frac{1}{2}$ the Baſes or Seg. BR and RC=52.8.55 10.1095157

If to half the Sum of the Baſes or Segments BR and RC equal to 52 deg. 08 min. 55 ſec. be added half the Baſe BC equal to 20 deg. 00 min. 05 ſec. the Sum 72 deg. 09 min. 00 ſec. will give the greater Baſe BR, whence to find the Angle B, becauſe in the Right-angled Spherical Triangle DRB Right-angled at R, are given DB the Hypothenuſe, equal to 76 deg. 35 min. 36 ſec. and the Baſe BR equal to 72 deg. 09 min. 00 ſec. it will be by the 2*d Example* of the 12*th Caſe* of *Right-angled Spherical Triangles*,

As R : c t, DB : : t, BR : c s, $<$B; that is,

As the Radius —————— 10.0000000

To the Co-tang. of the Hyp. or Side DB=76.35.36 —— 9.3772270

So is the Tangent of the Baſe BR=72.09.00 —— 10.4921067

To the Co-ſine of the Angle at the Baſe B=34.15.13½—9.8693337

The ſame with the Angle B of the Triangle DCB.

Again, if from half the Sum of the Baſes or Segments BR and CR, equal to 52 deg. 08 min. 55 ſec. be taken the half of the Baſe BC, equal to 20 deg. 00 min. 05 ſec. there will remain 32 deg. 08 min. 50 ſec. for the length of the Baſe CR of the Triangle CRD, whence to find the Angle at C, it will be (becauſe the Hypothenuſe DC is equal to 50 deg. 10 min. 30 ſec. and the Baſe CR equal to 32 deg. 08 min. 50 ſec. by the 2*d Example* of the 12*th Caſe* of *Right-angled Spherical Triangles*,

As R : c t, CD : : t, CR : c s, $<$C; that is,

As the Radius —————— 10.0000000

To the Co-tangent of the Hypothenuſe or Side CD =50.00.10 —————— } 9.9211182

So is the Tang. of the Baſe or Segment CR=32.08.50—9.7982698

To the Co-ſine of the Angle at the Baſe C=58.23.40 9.7193880

Whoſe Supplement to a Semicircle 121 deg. 36 min. 20 ſec. is the Angle C of the Triangle DCB required.

Again,

3. Becauſe by the 2*d Cor.* of the 7*th Axiom*, of *Section* the 6*th.* the $S.BC \times S.BD : Rq : : S.\frac{DC+BC+CD}{2} - RD \times S.\frac{DC+BC+BD}{2}$ BC : Sq, ½ $<$B, it will be,

As the Product of the Sine of the Sides BC & BD —— 19.7960933

To the Square of the Radius —— 20.0000000
So is the Product of the S. of the $\frac{1}{2}$ Sum of the three Sides leſſened by each of the containing Sides —— } 18.9097452

To the Square of the Sine of half the cont. $<B=$ —— 19.1136519

The half of which Square will be —— 9.5568259

The Sine of 21 deg. 07 min 36 ſec. $\frac{7}{8}$, which being doubled gives 42 deg. 15 min. 13 ſec. $\frac{3}{4}$, for the Angle DBC required.

Again,

4. Becauſe by the 3*d Cor.* of the 7*th Axiom*, of the 6*th Section.*

$$\frac{R}{S.\,BC}\times\frac{R}{S.\,BD}\times S.\,\overline{Z-BC}\times S.\,\overline{Z-BD}=Sq\ \tfrac{1}{2}<B.$$

To the Arith. Complement of the Sine of BC=40.00.10 0.1919074
Add the Arith. Compl. of the Sine of BD=76.35.36 —— 0.0119993
Alſo the Sine of the half Sum leſſened by BC=43.22.58 9.8368740
And the Sine of the half Sum leſſened by BD=6.47.42 —— 9.0728712

Half the Sum of theſe four Sines —— 19.1136519
Will be —— 9.5568252

The Sine of 21 deg. 07 min. 36 ſec. $\frac{7}{8}$, which being doubled gives 42 deg. 15 min. 13 ſec. $\frac{3}{4}$, for the Angle DBC required.

Again,

5. Becauſe by the 4*th Cor.* of the 7*th Axiom*, of *Section* the 6*th*, the $\frac{S.\,BC\times S.\,BD}{R} : S.\,\overline{Z-BC} :: S.\,\overline{Z-BD} : \frac{Sq\ \frac{1}{2}<B}{R}$; it will be,

As the Radius —— 10.0000000

To the Sine of one of the cont. Sides BC=40.00.10 —— 9.8080926
So is the Sine of the other cont. Side BD=76.35.36 —— 9.9880007

To a fourth Sine —— 9.7960933

As the fourth Sine ———————————— 9.7960933

To the Sine of the half Sum leſſened by BC=43.22.58 9.8368740
So is the Sine of the half Sum leſſened by BD=6.47.32—9.0728712

To a Seventh Sine ———————————— 9.1136519
To which add the Radius ———————————— 10.0000000

19.1136519

Half that Sum will be ———————————— 9.5568259

The Sine of 21 deg. 07 min. 36 ſec. $\frac{5}{8}$ the half of the Angle B, which being therefore doubled gives 42 deg. 15 min. 13 ſec. $\frac{1}{4}$, for the Angle at B required.

Again,

6: Becauſe by the *3d Cor.* of the *8th Axiom* of *Section* the *6th*, the $\text{S.}\,BC\times\text{S.}\,BD : Rq :: \text{S.}\,\frac{BC+BD+DC}{2}\times\text{S.}\,\frac{BC+BD+DC}{2}-DC : csq,\ \frac{1}{2}\angle B$; it will be,

As the Product of the Sines of the two Sides BC & BD 19.7960933

To the Square of the Radius ———————————— 20.0000000
So is the Prod. of the Sines of the ½ Sum of the three Sides and their ½ Sum leſſened by the Side oppoſite } 19.7356562

To the Square of the Co-ſine of half the Angle B — 19.9395629

The half of which Square will be ———————————— 9.9697814

The Co-ſine of 21 deg. 07 min. 36 ſec. $\frac{5}{8}$, which being doubled gives 42 deg. 15 min. 13 $\frac{1}{4}$, for the Angle DBC required.

Again,

7. Becauſe by the *4th Cor.* of the *8th Axiom*, of *Section* the *6th*, $\frac{R}{\text{S.}\,BC}\times\frac{R}{\text{S.}\,BD}\times\text{S.}\,\frac{DC+DB+BC}{2}\times\text{S.}\,\frac{DC+DB+BC}{2}-DC= csq,\ \frac{1}{2}\angle B.$

To the Arith. Compl. of the Sine of *BC*=40.00.10——0.1919074
Add the Arith. Comp. of the Sine *BD*=76.35.36——0 0119993
Alſo the Sine of the half Sum=83.23.08————9.9970996
And the Sine of the half Sum leſſened by *DC*=33.12.38 9.7385566

Half the Sum of theſe four Sines————19.9395629

Will be————9.9697814

The Co-ſine of 21 deg. 07 min. 36 ſec. ⅛, which being doubled gives 42 deg. 15 min. 13 ſec. ¼, for the Angle *DBC* required.

Again,

8. Becauſe by the *5th Cor.* of the *8th Axiom*, of *Section* the *6th*, the $\frac{\text{S. }BC\times\text{S. }BD}{\text{R}}$: S. $\frac{DC+DB+BC}{2}$: : S. $\frac{DC+DB+BC}{2}-DC$: $\frac{\text{c sq, }\frac{1}{2}<B}{\text{R}}$; it will be

As the Radius————10.0000000

To the Sine of one of the cont. Sides *BC*=40.00.10——9.8080926
So is the Sine of the other cont. Side *BD*=76.35.36——9.9880007

To a fourth Sine equal to————9.7960933

As the fourth Sine————9.7960933

To the S. of ½ the Sum of the three Sides=83.23.08—9.9970996
So is the S. of the ½ Sum leſſened by *DC*=33.12.38——9.7385566

To a ſeventh Sine————19.9386629
To which add the Radius————10.0000000

Half that Sum————19.9386629

Will give———— 9.9693314

The Co-ſine of 21 deg. 07 min. 36 ſec. ⅛, which being doubled gives 42 deg. 15 min. 13 ſec. ¼, for the Quantity of the Angle *DBC* required.

Again,

9. Becauſe by the 10*th Cor.* of the 8*th Axiom* of *Section* the 6*th*,
$\frac{R}{S.\,Z}\times\frac{R}{S.\,X}\times S.\,A\times S.\,E = tq,\,\frac{1}{2} < B.$

To the Arith. Comp. of the Sine of half the Sum of the three Sides=83.23.08 ———— 0.0029004

Add the Arith. Compl. of half the Sum of the three Sides leſſened by the Side oppoſite=33.12.38 —— 0.2614434

Alſo the Sine of the half Sum of the three Sides leſſened by one of the containing Sides=43.22.58 —— 9.8368740

And the Sine of the half Sum leſſened by the other containing Side=6.47.32 ———— 9.0728712

Half the Sum of theſe four Sines ———— 19.1740890

Will be ———— 9.5870445

The Tangent of 21 deg. 07 min. 36 ſec. $\frac{5}{8}$, which being doubled will give the Angle at *B* equal to 42 deg. 15 min. 13 ſec. $\frac{1}{4}$.

Again,

10. Becauſe by the 11*th Cor.* of the 8*th Axiom*, of *Section* the 6*th*, the $\frac{S.\,Z\times S.\,X}{R} : S.\,A :: S.\,E : \frac{tq,\,\frac{1}{2} < B}{R}$ it will be

As the Radius ———— 10.0000000

To the S. of half the Sum of the three Sides 83.23.08 9.9970996

So is the Sine of the half Sum leſſened by *DC*=33.12.38 9.7385566

To a fourth Sine ———— 9.7356562

As the fourth Sine ———— 9.7356562

To the Sine of the half Sum leſſened by one of the containing Sides *BC*=43.22.58 ———— 9.8368740

So is the Sine of the half Sum leſſened by the other containing Side *DB*=6.47.32 ———— 9.0728712

To a ſeventh Sine ———— 9.1740890

To which add the Radius ——————— 10.0000000

Half that Sum ——————— 19.1740890
Will be ——————— 9.5870445

The Tangent of 21 deg. 07 min. 36 ſec. $\frac{5}{8}$, the half of the required Angle *B* equal to 42 deg. 15 min. 13 ſec. $\frac{1}{4}$.

Again,

11. Becauſe by the 13th *Cor.* of the 8*th Axiom*, of *Section* the 6*th*, the $\frac{R}{S.\,A} \times \frac{R}{S.\,E} \times S.\,Z \times S.\,X = c\,tq,\ \frac{1}{2} < B$.

To the Arith. Comp. of the S. of $\frac{1}{2}$ the Sum of the three Sides leſſened by one of the cont. Sides *BC*=43.22.58 } 0.1631260
Add the Arith. Comp. of the S. of the $\frac{1}{2}$ Sum of the three Sides leſſened by the other cont. Side *BD*=6.47.42 — } 0.9271288
Alſo the S. of the half Sum of the three Sides=83.23.08 9.9970996
And the S. of the half Sum leſſened by *CD*=33.12.38 — 9.7385566

Half the Sum of theſe four Sines ——————— 20.8259110

Will be ——————— 10.4129555

The Co-tangent of 21.7.36$\frac{5}{8}$ half the contained Angle *B* required.

Again,

12. Becauſe by the 14*th Cor.* of the 8*th Axiom* of *Section*, the 6*th*, the $\frac{S.\,A \times S.\,E}{R} : S.\,Z :: S.\,X : \frac{c\,tq,\ \frac{1}{2} < B}{R}$ it will be

As the Radius ——————— 10.0000000

To the S. of $\frac{1}{2}$ the Sum of the three Sides leſſened by *BC*=43.22.58 ——————— } 9.8368740
So is the Sine of the $\frac{1}{2}$ Sum leſſened by the Side *BD* =06.47.32 ——————— } 9.0728712

To a fourth Sine ——————— 8.9097452

As the fourth Sine ——————————— 8.9097452

To the S. of half the Sum of the three Sides 83.23.08 9.9970996
So is the Sine of the half Sum leſſened by *DC*=33.12.38 9.7385566

To a ſeventh Sine ——————————— 10.8259110
To which add the Radius ——————————— 10.0000000

Half that Sum ——————————— 20.8259110

Will be ——————————— 10.4129555

The Co-ſine of 21 deg. 07 min. 36 ſec. ⅝, the half of the Angle *DBC* equal to 42 deg. 15 min. 13 ſec. ¼ required.

Caſe XII.

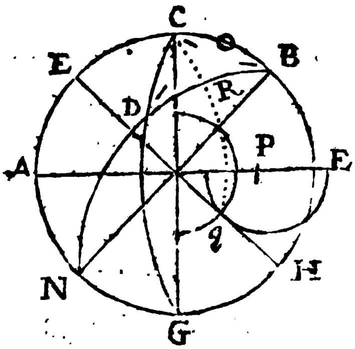
Given the three Angles, to find either of the Sides.

Example

In the Oblique-angled Spherical Triangle *DCB*

$$\text{Are given}\left\{\begin{array}{l} \angle B = 42.15.13\tfrac{1}{4} \\ \angle C = 121.36.20 \\ \angle D = 34.15.02\tfrac{3}{4} \end{array}\right\}\text{ req. } BC.$$

The *Stereographic Solution.*



1. Having drawn the Primitive Circle *CAGE*, the Diameter *AE*, and at Right-angles to it the Diameter *CG*; thro' the Point *C* draw the Great Circle *CDG*, forming an Angle of 121 deg. 36 min. 20 ſec. equal to the Angle *C* with the Primitive Circle, by *Caſe* the 3*d* of *Prob.* the 9*th*, of *Sect.* the 2*d*, and find the Pole *P* by *Caſe* the 3*d* of *Prob.* 3*d* of *Sect.* the 2*d*.

2. About the Pole *p* deſcribe a Small Circle *pq*, at the diſtance of 34 deg. 15 min. 02 ſec. ¾, equal to the Quantity of the Angle *D*, by *Caſe* the 3*d* of *Problem* the 4*th*, of *Section* the 2*d*.

3. About the Center of the Primitive Circle deſcribe a Small Circle, at the diſtance of 58 deg. 23 min. 40 ſec. the Supplement of the Angle at C, by *Caſe* the 1*ſt* of *Prob.* the 4*th*, of *Sect.* the 2*d*, where this interſects the former Parallel as in *q*, will give the Center of the Circle to be drawn. Wherefore,

4. Thro' the Interſection *q* having drawn the Diameter *FH*, and at Right-angles to it *BN*, about the Point *q* as a Pole, deſcribe the Great Circle *BDN*, by *Caſe* the 3*d* of *Problem* the 2*d* of *Section* the 2*d*; or thro' the Point *B* draw the Great Circle *BDN*, forming an Angle of 42 deg. 15 min. 13 ſec. ½, equal to the Angle *B* with the Primitive Circle *CBE*, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*, and the thing is done; and the Side *BC* may be meaſured by the *Line of Chords*.

It has been ſhewn in the 3*d* and 6*th Sections*, that if the Poles *o*, *p*, and *q*, of the Circles *CB*, *CD*, and *DB*, be connected together by Arches of Great Circles *op*, *pq*, and *qo*, that the Triangle *opq* which is called the Supplemental Triangle, will have its Sides *oq*, *pq* equal to the Angles *B* and *D*, and the third Side *op* equal to the Supplement of the Angle *C* of the Triangle *BCD*; alſo the Angles *qop*, and *qpo*, equal to the Sides *BD* and *DC*, and the third Angle *oqp*, equal to the Sup. of the Side *BD*. Wherefore,

Having let fall the Perpendicular *qrRC*, it will be by *Axiom* the 6*th* of *Section* the 6*th*,

As $(\mathrm{t}, \tfrac{1}{2}\, op =)\ \tfrac{1}{2}\,\mathrm{Sup.} \angle C : \mathrm{t}, (\tfrac{1}{2}\,\overline{oq + qp} =)\ \tfrac{1}{2} \angle \mathrm{s}\, \overline{B + D} : \mathrm{t}, (\tfrac{1}{2}\,\overline{oq - qp} = \tfrac{1}{2} \angle \mathrm{s}\, \overline{B - D} : \mathrm{t}, (\tfrac{1}{2}\,\overline{or - rp} =)\ \tfrac{1}{2}\,\mathrm{c}, \angle BCR - \tfrac{1}{2}\,\mathrm{c}, \angle DCR = \tfrac{1}{2} \angle \mathrm{s}\, \overline{DCR - BCR}$; that is,

As the Co-tangent of half the Angle C=60.48.10 —— 9.7472700

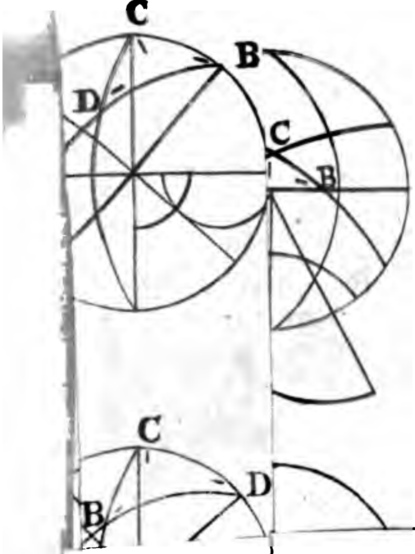
To the T. of ½ the Sum of the Angles *D* and *B*=38.15.8 9.8967463
So is the Tangent of ½ the Difference of the Angles *B* and *D*=04.00.05½ —— —— —— } 8.8448022

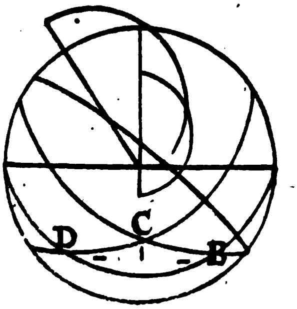
To the Tang. of ½ $\overline{or - rp} = \tfrac{1}{2} \angle \mathrm{s}\, \overline{DCR - BCR}$, or a 4*th* Arch=5.38.11 —— —— —— —— } 8.9942785

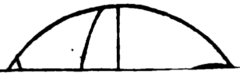
The 4*th* Arch 5 deg. 38 min. 11 ſec. added to half *op*, equal to half the Supplement of the Angle at *C*, equal to 29 deg. 11 min. 50 ſec. will give 34 deg. 50 min. 01 ſec. the Complement of 55 deg. 09 min. 59 ſec. the Angle *BCR*, but taken from half *op* equal to 29 deg.

Geo between 208 & 209

∠C = 121 36 20.







d As

ec. the Angle *BCR*, but t

29 deg. 11 min. 50 ſec. will leave 23 deg. 33 min. 39 ſec. the Complement of *66* deg. *26* min. 21 ſec. for the Angle *DCR*.

Or,

The ſame *4th* Arch added to 60 deg. 48 min. 10 ſec. the half of the Angle at *C*, will give *66* deg. *26* min. 21 ſec. for the Angle *DCR*.

But taken from *60* deg. 48 min. 10 ſec. will leave 55 deg. *9* min. 59 ſec. for the Meaſure of the Angle *BCR*; whence to find the Side *BC*.

Becauſe in the Spherical Triangle *BRC* Right-angled at *R*, are given, the Angle at the Baſe *B* equal to 42 deg. 15 min. 13 ſec. ¼, and the Angle at the Perpendicular *BCR* equal to 55 deg. 09 min. 59 ſec. it will be to find the Hypothenuſe *BC*, by *Caſe* the 15*th* of *Right-angled Spherical Triangles*,

As *R* : ct, <*B* : : ct, *BCR* : cs, *BC*; that is,

As the Radius —————————— 10.0000000

To the Co-tang. of the Angle at the Baſe *B*=42.15.13¼ 10.0416971

So is the Co-tangent of the Angle at the Perpendicular *BCR*=55.09.59 —————— 9.8425392

To the Co-ſine of the Hyp. or Side *BC*=40.00.10 —— 9.8842363

Again,

If the Side *DC* were required, becauſe in the Spherical Triangle *DCR* Right-angled at *R*, are given, the Angle at the Baſe *D* equal to 34 deg. 15 min. 02 ſec. ¾, and the Angle at the Perpendicular *DCR* equal to *66* deg. *26* min. 21 ſec. it will be by *Caſe* the 15*th* of *Right-angled Spherical Triangles*, to find the Hypothenuſe *DC*.

As *R* : ct, <*D* : : ct, *DCR* : cs, *DC*; that is,

As the Radius —————————— 10.0000000

To the Co-tang. of the Angle at the Baſe *D*=34.15.02¾ 10.1669195

So is the Co-tang. of the Ang. at Perp. *DCR*=*66.26.21* *9.6395617*

To the Co-ſine of the Hyp. or Side *DC*=50.10.30 —— 9.8064812

Again,

2: Having drawn the Perpendicular *r o D R* it will be by *Axiom* the *6th* of *Section* the *6th*, As $(t, \frac{1}{2}qp=)\ t, \frac{1}{2} < CDB : (t, \frac{1}{2}\overline{op+oq}=)\ t, \frac{1}{2}\text{Sup.} < \overline{C+<B} : : (t, \frac{1}{2}op-oq=)\ \frac{1}{2}\text{ Sup.} < C-<B : (t, \frac{1}{2}pr+rq=)$ Comp. $\frac{1}{2} <s$ $RDB+RDC$; that is,

As the Tang. of half the Angle $CDB=17{\cdot}07{\cdot}31\frac{3}{8}$ ——— 9.4887274

To the T. of half the Sup. of $<C+<B=50.19.26\frac{1}{4}$ 10.0811801

So is the Tang. of half the Sup. of $<C-<B=8.04.13\frac{3}{4}$ 9.1516568

To the Tangent of half $pr+rq=$ the Comp. of half the Angles RDB and $RDC=29.01.12\frac{1}{8}$ ——— 9.7441095

This therefore added to 17 deg. 07 min. 31 ſec. $\frac{3}{8}$ the half of $\overline{qp}$ will give 46 deg. 08 min. 43 ſec. $\frac{1}{2}$, the Complement of 43 deg. 51 min. 16 ſec. $\frac{1}{2}$ the Length of pz, the Meaſure of the Angle RDC; but leſſened by 17 deg. 07 min. 31 ſec. $\frac{3}{8}$, will leave 11 deg. 53 min. 40 ſec. $\frac{1}{2}$, the Complement of 78 deg. 06 min. 19 ſec. $\frac{1}{4}$, equal to the Length of qz the Meaſure of the Angle RDB. And

If to cr the half Sum of the Angles RDB and RDC 60 deg. 58 min. 47 ſec. $\frac{7}{8}$, the Complement of 29 deg. 01 min. 12 ſec. $\frac{1}{8}$, be added the half of the Angle CDB equal to 17 deg. 07 min. 31 ſec. $\frac{3}{8}$, the Sum 78 deg. 06 min. 19 min. $\frac{1}{4}$ will be the Angle BDR; but the half of the Angle CDB equal to 17 deg. 07 min. 31 ſec. $\frac{3}{8}$, taken from 60 deg. 58 min. 47 ſec. $\frac{7}{8}$, will leave 43 deg. 51 min. 16 ſec. $\frac{1}{2}$ for the Angle CDR; whence to find DB.

Becauſe in the Spherical Triangle RDB Right-angled at R are given, the Angle at the Baſe B equal to 42 deg. 15 min. 13 ſec. $\frac{3}{4}$, the Angle at the Perpendicular BDR equal to 78 deg. 06 min. 19 ſec. $\frac{1}{4}$, it will be by the 15th *Caſe of Right-angled Spherical Triangles*, to find the Hypothenuſe DB.

As $R : ct, <B : : ct, DRB : cs, DB$; that is

As the Radius ——— 10.0000000

To the Co-tang. of the Ang. at the Baſe $B=42{\cdot}15{\cdot}13\frac{3}{4}$ 10.0416971

So is the Co-tang. of the Ang. at the Perp. $BDR=78{\cdot}6{\cdot}19\frac{1}{4}$ 9.3235316

To the Co-ſine of the Hyp. or Side $BD=76{\cdot}35{\cdot}36$ ——— 9.3652287

Again,

To find the Side DC, becauſe in the Right-angled Spherical Triangle CDR Right-angled at R, are given, the Angle at the Baſe C

equal to 58 deg. 23 min. 40 ſec. the Supplement of the Angle *BCD*, and the Angle at the Perpendicular *CDR* equal to 43 deg. 51 min. 16 ſec. ½, it will be by the *15th Caſe* of *Right-Angled Spherical Triangles*,

$$\text{As } R : \text{c t}, \angle C : : \text{c t}, \angle CDR : \text{c s}, CD;$$ that is,

As the Radius ——————— 10.0000000

To the Co-tang. of the Ang. at the Baſe $C=58.23.40$ — 9.7891135

So is the Co-tang. of the Ang. at Perp. $CDR=43.51.16\frac{1}{2}$ 10.0173682

To the Co-ſine of the Hyp. or Side $DC=50.10.30$ — 9.8064817

Again,

3. Becauſe by the *9th Axiom* of *Section* the *6th*, the $\text{S. Sup.} \angle C \times \text{S.} \angle B : Rq : : \text{S.} \overline{\frac{\text{Sup.} \angle C + \angle D + \angle B}{2} - \text{Sup.} \angle C} \times \text{S.} \overline{\frac{\text{Sup.} \angle C + \angle D + \angle B}{2} - \angle B} : \text{Sq} \tfrac{1}{2} BC$; it will be,

As the Product of the Sines of the Angles *C* and *B* —— 19.7579114

To the Square of the Radius ——————— 20.0000000

So is the Prod. of the Sines of the half Sum of the three Angles leſſened by each of the adjacent Sides —— } 18.8260733

To the Sq. of the Sine of half the interjacent Side *BC* — 19.0681619

The half of which Square will be. ——————— 9.5340809

The Sine of 20 deg. 00 min. 05 ſec. which being doubled gives 40 deg. 00 min. 10 for the Length of the Side *BC*.

Again,

4. Becauſe by the *3d Cor.* of *Axiom* the *9th* of *Section* the *6th*, the $\frac{R}{\text{S. Sup.} \angle C} \times \text{S} \frac{R}{\angle B} \times \text{S.} \overline{Z - \text{Sup.} \angle C} \times \text{S.} \overline{Z - \angle B} = \text{Sq} \tfrac{1}{2} BC.$

To the Arith. Comp. of the S. of the Ang. $C=121.36.20$ 0.0697256

Add the Arith. Comp. of the S. of the Ang. $B=42.15.13\frac{1}{4}$ 0.1723630

Alſo the S. of the half Sum of the three Angles leſſened by one of the adjacent Angles ſuppoſe $C=9.03.18$ — } 9.1969563

And the S. of the ½ Sum leſſened by the other adjacent Angle $B=25.11.44\frac{3}{4}$ ——————— } 9.6291170

Half the Sum of theſe four Sines ———————— 19.0681619

Will be ———————————————— 9.5340809

The Sine of 20 deg. 00 min. 05 ſec. which being doubled gives 40 deg. 00 min. 10 ſec. for the Side *BC* required.

Again,

5. Becauſe by the *4th Cor.* of *Axiom* the *10th*, of *Section* the *6th*, The $\frac{\text{S.Sup.} < \text{C} \times \text{S.} < \text{B}}{\text{R}}$: S. $\overline{\text{Z} - \text{Sup.} < \text{C}}$: : S.Z — < B : $\frac{\text{Sq.} \frac{1}{2} \text{BC.}}{\text{R}}$

As the Radius ———————————— 10.0000000

To the Sine of the Angle C = 121.36.20 ———— 9.9302744

So is the Sine of the Angle at B = 42.15.13$\frac{1}{4}$ ———— 9.8276370

To a fourth Sine ———————————— 9.7579114

As the fourth Sine ———————————— 9.7579114

To the S. of the half Sum leſſened by Sup. Ang. C = 9.3.18 9.1969563

So is the Sine of the half Sum leſſened by the Angle *B* = 25.11.44$\frac{3}{4}$ ———————— 9.6291170

To a ſeventh Sine ———————————— 9.0681619

To which add the Radius ———————— 10.0000000

Half that Sum ———————————— 19.0681619

Will give ———————————————— 9.5340809

The Sine of 20 deg. 00 min. 05 ſec. the half of the Side *BC*, which being therefore doubled will give the Side *BC* 40.00.10.

Again,

6. Becauſe by the *10th Axiom* of *Section* the *6th*, the S. Sup. < C × S. < *B* : *Rq* : : S. $\frac{1}{2}$ < s $\frac{B + D + \text{Sup.} C}{2}$ × S. $\frac{1}{2}$ < s $\frac{B + D + \text{Sup.} < C}{2}$ — < *D* : c.sq. $\frac{1}{2}$ *BC*; it will be,

As the Prod. of the Sines of the Ang. C & B —— 19.7579114

To the Square of the Radius —— —— —— —— —— 20.0000000

So is the Prod. of the S. of ½ the Sum of the three <s mult. into the ½ Sum leſſened by the Side oppoſite } 19.7038754

To the Sq: of the Co-ſine of half the Side BC —— 19.9459640

The half of which Square will be —— —— —— 9.9729820

The Co-ſine of 20 deg. 00 min. 05 ſec. which being doubled gives 40 deg. 00 min. 10 ſec. for the Side BC.

7. Again, Becauſe by the 10*th Axiom* of *Section* the 6*th*,

The $\frac{R}{S.\ Sup.<C}\times\frac{R}{S.<B}\times S.\tfrac{1}{2}<s\ \frac{B+D+Sup.<C}{2}\times S.\tfrac{1}{2}<s\ \frac{B+D+Sup.<C}{2}-<D=c\,sq;\ \tfrac{1}{2}\,BC.$

To the Arith.Comp. of the S. of the Ang. C=121.36.20 0.0697256
Add the Arith.Comp. of the S. of the Ang. B=42.15.13¼ 0.1723630
Alſo the S. of the half Sum of the Angles=67.26.58 —— 9.9654564
And the S. of the ½ Sum leſſened by the Ang. D=33.11.55¼ 9.7384190

Half the Sum of theſe four Sines —— —— —— 19.9459640

Will be —— —— —— 9.9729820

The Co-ſine of 20 deg. 00 min. 05 ſec. which being doubled gives 40 deg. 00 min. 10 ſec. for the Side BC required.

Again,

8. Becauſe by the 10*th Axiom* of *Section* the 6*th*,

The $\frac{S.\ Sup.<C\times S.<B}{R} : S.\frac{Sup.<C+<D+<B}{2} :: S.\frac{Sup.<C+<D+<B}{2}-<D : \frac{c\,sq,\ \frac{1}{2}\,CB}{R}$ it will be,

As the Radius —— —— —— —— —— —— 10.0000000

To the Sine of one adjacent Angle C=121.36.20 —— 9.9302744
So is the S. of the other adjacent Angle. B=42.15.13¼ —— 9.8276370

To a fourth Sine —— —— —— —— —— 9.7579114

As

As the fourth Sine	9.7579114
To the Sine of the half Sum=67.26.58	9.9654564
So is the S. of the ½ Sum leſſened by Ang. D=33.11.55¼	9.7384190
To a ſeventh Sine	9.9459640
To which add the Radius	10.0000000
Half that Sum	19.9459640
Will give	9.9729820

The Co-ſine of 20 deg. 00 min. 05 ſec. which being doubled gives 40 deg. 00 min. 10 ſec. for the Side BC required.

Again,

9. Becauſe by the 10*th Axiom* of *Section* the *6th*,

$$\frac{R}{S.Z}\times\frac{R}{S.X}\times S.A\times S.E=tq,\tfrac{1}{2}BC.$$

To the Arith. Comp. of the S. of ½ the Sum, &c.=67.26.58	0.0345436
Add Arith. Co. of the S. of ½ the Sum leſſened, &c. 33.11.55¼	0.2615810
Alſo the Sine of half the Sum leſſened by one of the adjacent Angles C=09.03.18	9.1969563
And the Sine of the half Sum leſſened by the other adjacent Angle B=25.11.44¾	9.6291170
Half the Sum of theſe four Sines	19.1221979
Will be	9.5610989

The Tangent of 20 deg. 00 min. 05 ſec. which being doubled gives the Side BC equal to 40 deg. 00 min. 10 ſec.

Again,

10. Becauſe by the 10*th Axiom* of *Section* the *6th*,

The $\frac{S.Z\times S.X}{R}:S.A::S.E.:\frac{tq,\frac{1}{2}BC}{R}$; it will be,

As the Radius	10.0000000
To the Sine of half the Sum of the Angles=67.26.58	9.9654564
So is the S. of the half Sum leſſened by DC=33.11.55¼	9.7384190
To a fourth Sine	9.7038754

As the fourth Sine	9.7038754
To the Sine of the half Sum leſſened by one of the adjacent Angles ſuppoſe the Angle C=9.3.18	9.1969563
So is the Sine of the half Sum leſſened by the other adjacent Angle B=25.11.44¾	9.6291170

To a Seventh Sine —— 9.1221979
To which add the Radius —— 10.0000000
Half that Sum —— 19.1221979
Will be —— 9.5610989

The Tangent of 20 deg. 00 min. 05 ſec. the half of the required Side BC equal to 40 deg. 00 min. 10 ſec.

Again,

11. Becauſe by the 10*th Axiom* of *Section* the 6*th*,

The $\frac{R}{S.\ A}\times\frac{R}{S.\ E}\times S.\ Z\times S.\ X = ctq,\ \frac{1}{2}\ BC.$

To the Arith. Comp. of the S. of ½ the Sum of the Angles leſſened by one of the adjacent Ang. C=9.3.18 } 0.8030437
Add the Arith. Comp. of the Sine of ½ the Sum of the Ang. leſſened by the other adja. Ang. B=25.11.44¾ } 0.3708830
Alſo the Sine of the half Sum of the three &c.=67.26.58 9.9654564
And the Sine of the half Sum leſſened by CD=33.11.55½ 9.7384190

Half the Sum of theſe four Sines —— 20.8778021

Will give —— 10.4389010

The Co-tangent of 20 deg. 00 min. 05 ſec. half the interjacent BC.

Again,

12. Becauſe by the 10*th Axiom* of *Section* the 6*th*,

The $\frac{S.\ A\times S.\ E}{R} : S.\ Z : : S.\ X : \frac{ctq,\ \frac{1}{2}\ BC}{R}$ it will be,

As the Radius —— 10.0000000
To the S. of ½ the Sum leſſened by one of the adjacent Angles ſuppoſe C=09.03.18 —— } 9.1969563
So is the Sine of the half Sum leſſened by the other adjacent Angle B=25.11.44¾ —— } 9.6291170
To a fourth Sine —— 8.8260733

As this fourth Sine —— 8.8260733
To the Sine of half the Sum=67.26.58 —— 9.9654564
So is the S. of the half Sum leſſened by DC=33.11.55¼ 9.7384490
To a ſeventh Sine —— 10.8778021

To which add the Radius ———————— 10.0000000

Half that Sum ———————— 20.8778021

Will be ———————— 10.4389010

The Co-ſine of 20 deg. 00 min. 05 ſec. the half of the Side BC, equal to 40 deg. 00 min. 10 ſec. as before ſound.

A great variety of other *Solutions* might be given to this and the former *Caſes*, as the *Reader* will eaſily ſee, if he conſults the *4th*, *5th*, and *6th*, preceding *Sections*.

In the Application of *Right-angled Spherical Triangles* to the *Solution* of the former *Caſes*, I have choſe to make Uſe of ſuch Proportions in which the Radius becomes the firſt Term, they being the readieſt for Practice, and require only Addition, and the ſame Method I ſhall make Uſe of hereafter in the Application of this and the former *Section* to *Aſtronomy*.

PART

PART V.

Containing the Application of the Projection of the SPHERE *and the Doctrine of* Spherical Triangles; *to the Solution of the moſt uſeful* Aſtronomical Problems.

SECT. I.

Aſtronomical Definitions.

THE *Axis* is that imaginary Line about which the *Heavens* ſeem to revolve, its two Extremities are call'd the Poles; the uppermoſt or viſible to us, the Northern Pole; the lower or inviſible to us, the Southern Pole.

2. The *Equinoctial* or *Equator*, is that Great Circle which is every way equally diſtant from the Poles, and conſequently divides the *Heavens* into two equal Parts, it is ſo called, becauſe when ever the Sun is in this Circle, the Days and Nights are of an equal Length to all the Inhabitants of the Earth.

3. *Meridians* or *Hour-Circles*, are thoſe imaginary Great Circles, which interſect each other in the Poles of the World, and cut the *Equinoctial* at Right-angles.

4. The *Ecliptick*, is a Great Circle interſecting the *Equator* in two oppoſite Points, and forming an Angle with it, equal to the Suns greateſt Declination; (which by the moſt Accurate Obſervations of the Ingenious Mr. *Flamſteed*, is found to be 23 deg. 29 min.) and is that Circle which the Sun is ſuppoſed to deſcribe by his Annual Motion round the Earth, (according to the Syſtem of *Ptolomy*.)

This Circle is uſually divided into 12 equal parts, call'd Signs (and conſequently each Sign muſt contain 30 Degrees) beginning from the interſection of the *Equator*, and number'd as follows.

♈	♉	♊	♋	♌	♍
Aries.	*Taurus.*	*Gemini.*	*Cancer.*	*Leo.*	*Virgo.*

♎	♏	♐	♑	♒	♓
Libra.	*Scorpio.*	*Sagittarius.*	*Capricornus.*	*Aquarius.*	*Piſces.*

The firſt ſix of theſe are called the Northern Signs, and poſſeſs that half of the Ecliptick which is to the Northward of the Equator; beginning at the firſt Point of *Aries*, and ending with the laſt Point of *Virgo*.

The latter ſix are called the *Southern Signs*, becauſe they poſſeſs the Southern half of the *Ecliptick*, beginning at the firſt Point of *Libra*, and ending with the laſt Point of *Piſces*.

5. *Circles of Longitude* in the Heavens, are thoſe Great Circles which are imagined to paſs thro', and conſequently interſect each other in the Poles of the *Ecliptick*; and cut the *Ecliptick* at Right-angles, as the *Meridians* do the *Equator*.

6. Of the *Meridians*, that which paſſes thro' the Interſection of the *Ecliptick* and *Equinoctial*, is called the *Equinoctial Colure*.

7. And that which cuts this at Right-angles, and paſſes thro' the Points of *Cancer* and *Capricorn* is called the *Solſtitial Colure*.

8. The *Horizon* is a Circle encompaſſing the Earth, and dividing it into Equal Parts; it is ſo called, as being the Terminator or Boundary, between the Illuminated Part of the Hemiſphere and the Luminated: And when ever the Sun, Moon, or Stars, arrive at this Circle, they are then ſaid to Riſe or Set. It is variable in different Latitudes, and has its Poles.

9. Its Uppermoſt or Viſible *Pole*, *viz.* the Point in the Heavens directly over our Heads, is called the *Zenith*.

10. The Lowermoſt or Inviſible, *viz.* the Point directly under our Feet, is called the *Nadir*.

11. *Vertical* or *Azimuth Circles*, are thoſe Great Circles, which interſect each other in the *Zenith* and *Nadir*, and cut the *Horizon* at Right-angles, as the *Meridians* do the *Equator*, and the Circles of Longitude the *Ecliptick*.

12. Of the *Vertical* or *Azimuth* Circles, that which paſſes thro' the Poles of the World, is called the *Meridian* of the Place for this reaſon, that when the Sun arrives at it, it is then *Mid-Day* or *High-Noon*; and whenever the Moon, Stars, or other Planets, are under it, they are then at the higheſt they can poſſibly be in that part of the Earth; and conſequently after that time, continually deſcend.

13. The *Prime Vertical*, or *Circle* of *Eaſt* and *Weſt*, is that *Vertical Circle* which cuts the *Meridian* at Right-angles.

14. Of the *Meridians* or *Hour-Circles*, (Explain'd in *Def.* 3.) that which cuts the *Meridian*, (deſcrib'd in *Def.* 12.) at Right-angles, is called the *Six a Clock Hour Circle*; becauſe when the Sun, Moon or Stars, are under this Circle, they then have been ſix Hours abſent from, or want ſix Hours of arriving at the *Meridian*.

15. All Small Circles parallel to the *Equator*, are called *Parallels of Declination*, when they are uſed with reſpect to the Sun, Moon or Stars; but *Parallels of Latitude*, when uſed with reſpect to the places on the Earth. Of theſe,

16. That which is diſtant from the *Equator*, by the Space of 23 deg. 29 min. towards the North, and which of conſequence muſt touch the *Ecliptick* in the firſt Point of *Cancer*, and which is the Parallel of the Suns Diurnal Courſe when he is in that Point of the *Ecliptic* is call'd the *Tropic of Cancer*: And

17. That which is diſtant from the *Equator* 23 deg. 29 min. towards the South Pole, and which therefore muſt touch the *Ecliptick* in the firſt Point of *Capricorn*, and become the Parallel of the Suns Diurnal Courſe, when he is converſant in that Point, is called the *Tropic of Capricorn*.

18. That Small Circle which is diſtant from the North Pole, by the Space of 23 deg. 29 min. is call'd the *Artic Circle*; and to any of thoſe Inhabitants whoſe *Zenith* falls within this Circle, the Sun is not ſeen when he is in the *Tropic of Capricorn* before deſcribed.

19. That Small Circle, which is as far diſtant from the Southern Pole, as the *Artic Circle* is from the Northern, is called the *Antartic Circle*. And to all thoſe Inhabitants (if any there be) whoſe *Zenith* falls between this Circle and the South Pole, the Sun never Exhibits himſelf when he is in the firſt Point of *Cancer*.

20. All Small Circles parallel to the *Ecliptic*, are called *Parallels of Latitude*, with regard to the Fixed Stars and Planets, becauſe they determine the diſtance of any of them from the *Ecliptic*.

21. All Small Circles parallel to the *Horizon*, are called *Almicanthers*, or *Parallels of Altitude*, becauſe they ſhew how far the Sun, Moon or Stars are aſcended above the *Horizon*.

22. *Longitude* of any Place upon the Earth, is an Arch of the *Equinoctial*, intercepted between the firſt *Meridian*, and the *Meridian* paſſing thro' the Place propoſed. Or, it is equal to the Angle formed by the firſt *Meridian*, and the *Meridian* paſſing thro' the Place given.

23. *The Latitude of any Place* is an Arch of the *Meridian*, paſſing thro' the Place, and intercepted between the Place and the *Equator*.

24. *The Right Aſcenſion of the Sun, Moon, or any Star*, is an Arch of the *Equinoctial*, intercepted between the firſt Point of *Aries*, and the Interſection of the *Meridian* paſſing thro' the Sun or Star; and is the ſame with the Angle formed by the *Equinoctial Colure*, and the *Meridian* paſſing thro' the Sun and Moon. It's ſo called, becauſe it is the Point of the *Equinoctial*, which riſeth with the Sun or any Star (in a right Sphere or ſuch a Sphere where the Poles lie in the *Horizon*,) and conſequently is the ſame Point that paſſes the *Meridian* with either of them.

25. *The Declination of the Sun, Moon, or any Star*, is an Arch of the *Meridian* paſſing thro' the Sun, Moon, or any Star; and intercepted between either of them and the *Equator*; and if the Sun, Moon, or Star, be to the Southward of the *Equator*, it is called *South Declination*: But if they lie to the Northward, or betwixt the *Equator* and the North Pole, it is called *North Declination*.

26. *The Place of the Sun*, is that Sign, Degree, Minute, *&c.* of the *Ecliptic*, that he poſſeſſes at any time; and his *Longitude* is the Arch of the *Ecliptic*, intercepted between this Place and the firſt Point of *Aries*.

27. *The Longitude of the Moon or any Star*, is an Arch of the *Ecliptic*, contained between the *Circle of Longitude*, paſſing thro' the Moon or Star, and the firſt Point of *Aries*. And that Degree of any Sign over which the Circle of Longitude paſſing thro' the Moon, or any Star runs, is called the *Place of the Moon or Star*.

28. *The Latitude of the Moon or any Star*, is equal to the Arch of the Circle of Longitude, paſſing over the Moon, or any Star, and intercepted between the Moon or Star, and the *Ecliptic*, and is called *South Latitude*, if the Moon or Star be to the Southward of the *Ecliptic*; but if to the Northward, it is called *North Latitude*.

29. The *Azimuth* is an Arch of the *Horizon*, contained between the *Meridian* and the *Vertical Circle*, paſſing thro' the Sun, Moon, or Star, at any time propoſed, when they are above the *Horizon*; and is the ſame with the Angle at the *Zenith*, made by the *Meridian* and the *Vertical Circle*.

30. The *Amplitude* is an Arch of the *Horizon*, contained between the Sun, Moon, or Star, and the Eaſt Point of the *Horizon* at their Riſing, or the Weſt Point at their Setting: Or it is equal to the Angle made by the *Prime Vertical*, and the Vertical Circle paſſing thro' the Sun, Moon, or Star, at their Riſing and Setting.

31. The *Altitude* of the Sun, Moon, or Star, is an Arch of the Vertical Circle, paſſing thro' the Sun, Moon, or Star, at any time, propoſed, and contained between the Sun, Moon, or Star, and the *Horizon*, and can never exceed 90 Degrees or a Quadrant.

32. *Zenith Diſtance*, is that Arch of the Vertical Circle, paſſing thro' the Sun, Moon, or Star, which is contained between either of them and the *Zenith*; and therefore muſt alway, together with the *Altitude*, make 90 Degrees or a Quadrant.

33. *Meridian Altitude*, is the Height of the Sun, Moon, or Star, when they are upon the *Meridian*; and conſequently is the greateſt Height to which they can arrive at that time.

34. *Oblique Aſcenſion* or *Deſcenſion*, is that Degree of the *Equator* which Riſes or Sets with the Sun, Moon, or any Star, in an *Oblique Sphere*; (or ſuch a *Sphere* where the Parallels of the Suns *Diurnal Courſe* cuts the *Horizon* at unequal Angles) which Poſition happens to all who live between the *Equator* and the Poles.

35. *Aſcenſional Difference*, is the Difference between the *Right* and *Oblique Aſcenſion* or *Deſcenſion*; or is that Arch of the *Equator*, which is contained between the Points of *Right* and *Oblique Aſcenſion* or *Deſcenſion*; and conſequently ſhews how long the Sun Riſes or Sets before or after the *Hour of Six*; or half the time that the Moon or any Star, continuing above or below our *Horizon*, exceed or want of Six Hours.

SECT. II.

Containing the Application of the two firſt Sections *of the Fourth Part, to the actual Projection of the* Sphere *upon the Planes of the principal* Great Circles.

D*Efinition.* To Project the Sphere *Stereographically* upon the Plane of any Great Circle, is ſo to deſcribe the Circles of the Sphere upon that Plane, as they would appear to the Eye placed in one of the Poles of the given Great Circle.

Prob. I.

Let it be required to draw the *Hour Circles*, or *Circles of Right Aſcenſion*, *Parallels of Declination*, *Tropics*, *Ecliptic*, &c. *Stereographically*, upon the Plane of the *Solſtitial Colure*; it being in this *Caſe* ſuppoſed to lie in the ſame Plane with the *Meridian*.

Let *APQS* (in *Plate* the *4th*) repreſent the Solſtitial Colure, upon the Plane of which the Sphere is to be projected, then will ♈ be its Center and the Pole of the Projection.

Becauſe the Eye in this *Caſe* is ſuppoſed in the firſt Point of Aries; or common Interſection of the Equator, Ecliptic, and Equinoctial Colure; theſe Circles will all be repreſented in the Projection, by Right-lines interſecting each other in the Center ♈, or Pole of the Projection, (by the 1ſt *Cor.* of the *Gen. Def.* of the 1ſt *Section.*) Wherefore,

Having drawn the Equator *A♈Q*, and at Right-angles to it the Axis, and in this *Caſe* the Equinoctial Colure *P♈S*; becauſe the Hour Circles or Circles of Right Aſcenſion interſect each other in the Poles *P* and *S*, their Centers will be found in the Equator *AQ*, at the diſtance of the Tangents of the ſeveral Angles that they form with the Meridian or Solſtitial Colure, from the Center ♈ or Pole of the Projection; ſo that if it were required to draw the Ten or Two a Clock Hour Circle, or Circle of Right Aſcenſion of 60 or 120 deg.

P, 10, 2 *S*

P, 10,2 *S* becauſe this Circle forms an Angle or 30 deg. with the Periphery of the Plane of Projection, we have no more to do but to ſet off the Tangent of 30 degrees to the Radius of the Projection, from ♈ to 2,10, then will 2,10 be the Center, and 10,*P* the Radius of the Circle *P*,10,2,*S* to be drawn; and after the ſame manner may any other Hour Circle or Circle of Right Aſcenſion be drawn.

Again, becauſe the Parallels of Declination, Tropics, Artic, and Antartic Circles are Small Circles, ſtanding at Right Angles to the Periphery of the Plane of Projection, their Centers will be found in the Axis or Equinoctial Colure *PS*, produced both ways from *P* and *S*, and if the Secants of their ſeveral diſtances from their neareſt Pole, be ſet off in the Axis, from ♈ the Center of the Projection on the ſame Side of the Equator as they lie, you will have their ſeveral Centers, and the Tangents of the ſame diſtance will be their Radij or Semi-diameter; thus ſuppoſe it were required to draw the Small Circle 50. *e*. 50. or Parallel of 50 deg. of Northern Declination, becauſe this Circle is diſtant from the Northern Pole *P* 40 deg. if the Secant of 40 deg. be ſet off in the Axis, from ♈ the Center of the Projection to *E*, you will have the Center of the Parallel to be drawn, whoſe Radius or Semi-diameter *E* 50, is the Tangent of the ſame diſtance from the Poſe *P*, or 40 deg.

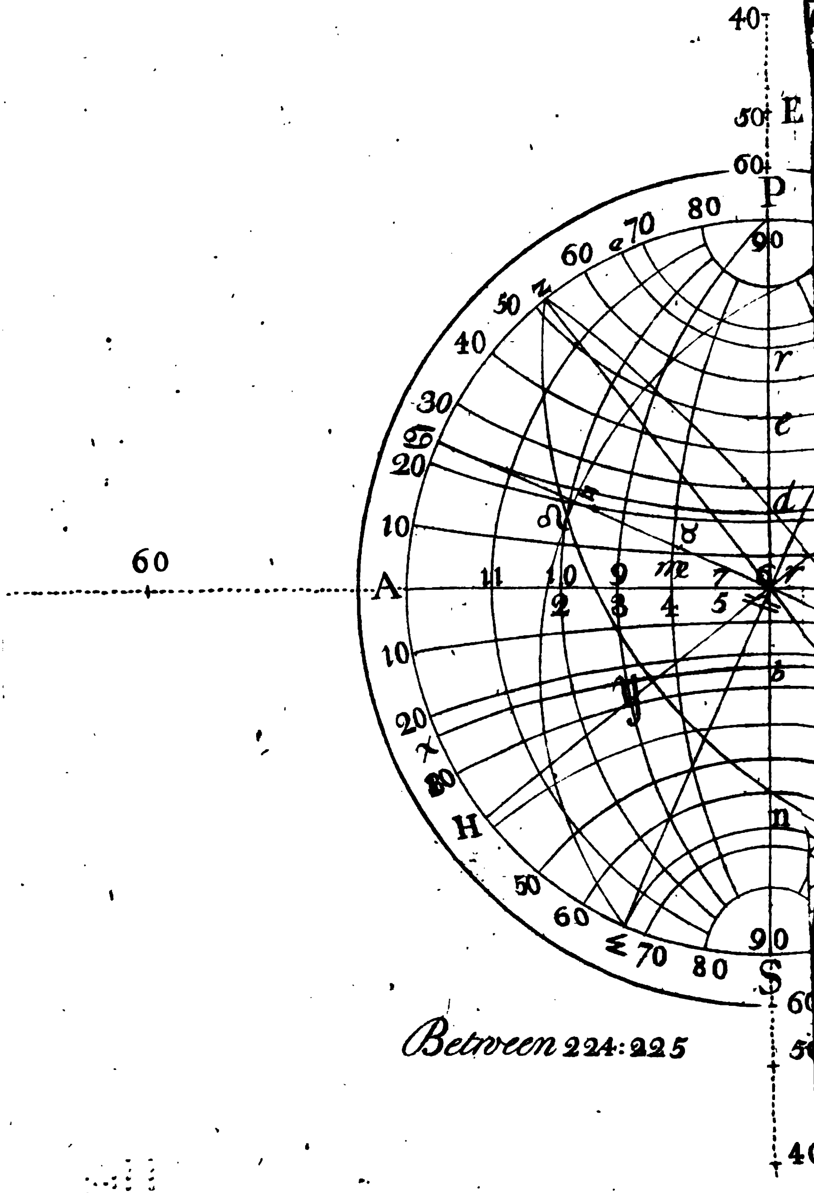
Again, Becauſe the Artic Circle is diſtant from the Pole 23 deg. 29 min. if the Secant of 23 deg. 29 min. be ſet off in the Axis, from ♈ towards *P*, it will give the Center; and the Tangent of the ſame diſtance will be the Semi-diameter of the Artic Circle to be drawn; in like manner the Secant of 66 deg. 30 min. ſet off from the Center ♈ will give the Center, and the Tangent of the ſame diſtance will be the Radius or Semi-diameter of the Tropic of *Cancer* ♋ *&c.*

After the ſame manner may the Tropic of *Capricorn* and Antartic Circle, or any other Parallel to the Equator be drawn.

The *Ecliptic* is a Great Circle, in this *Caſe* paſſing thro' the Eye, and forming an Angle with the Equator of 23 deg. 29 min. wherefore if the Chord of 23 deg. 29 min. be ſet off from *A* to ♋, or from *Q* to ♑, and the Right-line ♋♈♑ be drawn, it will be the Repreſentative of the Ecliptic in this Projection, whoſe Poles will be found in the Periphery of the Plane of the Projection at π & Σ, at a quadrantal Diſtance from its Interſection ♋ and ♑.

Circles of Cœleſtial Longitude, which interſect each other in π and Σ the Poles of the Ecliptic, will have their Centers in the Ecliptic it ſelf, and are projected after the ſame manner as the Circles of

Right



Between 224:225

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and *Leo*, *Aquarius* and *Sagittar*

After the ſame manner may
other Great Circle paſſing thro'
tor, *&c.*

There are other Methods fo
Circles, as the *Reader* will eaſil
firſt *Sections* of the former *Part*,
are the moſt ready for Practice,
ſerve hereafter.

Pr

Let it be required to draw
Circles of Right Aſcenſion, Pa
upon the Plane of the Horizon

In *Plate* the 5*th*, let *A M Q*
Plane of which the Circles of th
will *z* be its Center and the Po

Becauſe the Eye in this *Caſe*
Pole of the Horizon, the Vertic
ſtraight Lines in this Projection, i
or Pole of the Projection *z*, by
of *Section* the 1*ſt* of *Part* the 4*th*:

Draw the Diameter *AZQ*, an
cal or Circle of Eaſt and Weſt A
meter drawn at Right-angles to
of North and South Azimuth, a Angles that the ſe-
veral Azimuth Circles form with the Meridian, be laid off in the Periphery of the Plane of the Projection from *M* or *N*, towards *A* or *Q*, and thro' theſe points and the Center *z*, Right Lines be drawn,
they

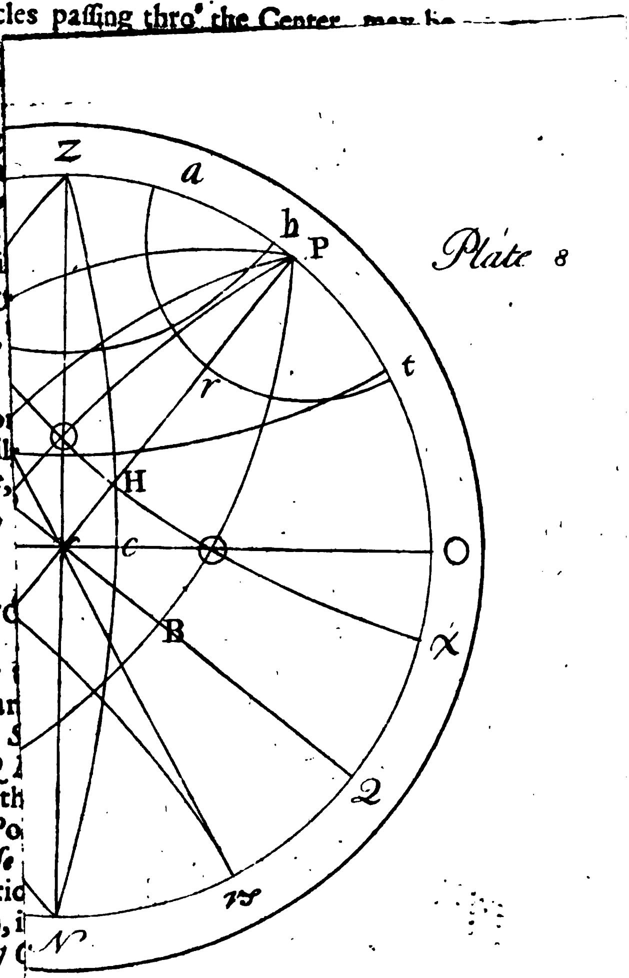

Plate 8

they will repreſent ſo many Azimuth Circles; thus, if it were required to draw a Vertical or Azimuth Circle that ſhall form an Angle of 30 degrees with the Meridian, ſet off 30 degrees in the Periphery of the Plane of the Projection from *M* to *b*, or from *N* to *a*, and draw the Diameter *b z a*, this will repreſent the Vertical or Azimuth Circle to be drawn; and by proceeding after the ſame manner, may any other Azimuth Circle be projected.

The Diſtance of the Pole from the Zenith of any Place, is equal to the Complement of the Latitude of that Place; wherefore if the Semi-tangent of the Complement of any Place (ſuppoſe *London*) equal to 38 deg. 28 min. be ſet off in the Meridian from *z* to *p*, it will give the Place of the North Pole *p* in this Projection.

The Equinoctial is a Great Circle, at 90 degrees or a Quadrantal Diſtance from the Pole, and conſequently is inclined to the Plane of the Horizon, at an Angle of 38 deg. 28 min. equal to the Complement of the Latitude of the Place; wherefore, if the half Tangent of 51 deg. 32 min. equal to the Latitude, be ſet off from *z* to *H*, it will give one Interſection of the Equator, and if the Tangent of 38 deg. 28 min. its Complement be ſet off from *z* to *g* on the contrary ſide of the Center, it will give the Center *g* of the Equator, about which Point *g* if the Circle *AHQ* be deſcribed at the diſtance of *g H*, it will be the Repreſentation of the Equator upon this Projection.

The Six a Clock Hour Circle is a Great Circle of the Sphere, which cuts the Meridian at Right-angles, and conſequently its Center will be found in the Diameter *MZN* produced towards *N*, and becauſe it paſſes thro' the Pole *P*, is inclined to the Plane of the Projection at an Angle of 51 deg. 32 min. equal to the Latitude of the Place; wherefore, if the Tangent of 51 deg. 32 min. be ſet off in the Meridian from *z* to *q* produced if need be, it will give the Center *q* of the Six a Clock Hour Circle, about which Point *q* if the Great Circle *APQ* be drawn at the diſtance of *q b*, it will be the Repreſentative of the Six a Clock Hour Circle in this Projection.

And becauſe the Planes of the ſeveral Hour Circles are inclined to the Plane of the Six a Clock Hour Circle, at Angles of 15 deg. 30 deg. 45 deg. *&c.* and conſequently muſt form the ſame Angles with it upon the Projection, if the Tangents of 15 deg. 30 deg. 45 deg. *&c.* to the Radius *p q* equal to its Semi-diameter be ſet off in the Line 45, 30, 45, (drawn at Right-angles to the Meridian *MN*, and paſſing thro' *q* the Center of the Six a Clock Hour-Circle) from *q* on each ſide towards 45, you will have the Centers of the ſeveral Hour Circles to be drawn; thus, if it were required to draw the Five a Clock Hour Circle 5 *p* 5, becauſe this Circle is inclined to the Plane of the Six a Clock Hour Circle at an Angle of 15 degrees, and conſequently muſt form the ſame Angle with it upon the Plane, if the Tangent of 15 degrees to the Radius *pq* equal to the Semidiameter of the Six a Clock Hour Circle, be ſet off in the Line *p*,45, from *p* to 15, it will give the Point 15, for the Center of the Hour Circle to be drawn, about which Point 15, if the Circle 5,*p*,5, be drawn at the diſtance of 15,*p*, it will be the Repreſentative of the Five a Clock Hour Circle upon this Plane; after the ſame manner by laying off the Tangents of 30 deg. 45 deg. *&c.* on each Side of the point *q* in the Line *q*, 45, you will have the Points 30.45, *&c.* for the Centers of the ſeveral Hour Circles to be drawn.

The Ecliptic is a Great Circle of the Sphere, inclined to the Plane of the Equator, at an Angle of 23 deg. 29 min. and conſequently (if the Southern half be Elevated above the Plane) muſt form an Angle of 14 deg. 59 min. with the Plane of the Horizon, equal to the Difference between the two Inclinations 38 deg. 28 min. and 23 deg. 29 min. but an Angle of 61 deg. 57 min. equal to the Sum of the Inclinations 38.28, and 23 deg. 29 min. if the Northern half of the Ecliptic be Elevated above the Plane of the Horizon; wherefore, if the Tangents of 14 deg. 59 min. and 61 deg. 57 min. be laid off in the Meridian, from *z* towards *m*, to *x* and *y*, you will have the Centers *x* and *y* of either Semicircle of the Ecliptic, about which points *x* and *y*, if the Semicircles *A*♈*Q* and *A*♎*Q* be drawn, you will have the Repreſentative of either half of the Ecliptic upon this Plane.

Circles of Longitude which interſect each other in the Poles of the Ecliptic, and cut the Ecliptic at Right-angles, are projected after the ſame manner as the Hour Circles or Circles of Right Aſcenſion are, which interſect each other in the Poles of the World, and cut the Equator at Right-angles: For *Example*, ſuppoſe it were required to draw the Circle of Longitude paſſing thro' the firſt point of *Taurus*, becauſe this Circle is inclined to the Plane of the Equinoctial Colure, at an Angle of 30 degrees: thro' the point *H* the Center of the Equinoctial Colure *AOQ*, draw the Right Line *H h k* perpendicular in this *Caſe* to the Solſtitial Colure *o p z N*, and make *H h* equal to the Tangent of 30 degrees, to the Radius *OH*, or draw the Line *O b* forming an Angle of 30 degrees at the point *o*, with the Solſtitial Colure *O z N*, where this interſects the Line *H b k* as in *b*, it will give the Center of the Circle of Longitude *O*♉ to be drawn.

After the ſame manner by laying off the Tangent of 30 degrees from *H* to *K* to the Radius *OH*, you will have the Center of the Circle of Longitude *O*♊, paſſing thro' the firſt Point of ♊, and the ſame Method is to be obſerved in drawing of any other Circle of Longitude whatſoever.

But if the Southern half of the Ecliptic *A* ♑ *Q* were to be divided, then the point *A* will become the firſt Point of ♎, the point *Q* the firſt Point of ♈, and *A* ♈ *Q* the Equinoctial Colure, and conſequently the Line *HK* in this *Caſe* muſt paſs thro' the Center of the Circle *A*♈*Q*, at the diſtance of the Tangent of 75 degrees from the Center *z* or Pole of the Projection, and then proceed as before.

Almicanthers or Parallels of Altitude, becauſe they are parallel to the Plane of the Projection, will have the Center and Pole of the Projection for their common Center and Pole, and their Radij or Semi-diameters will be equal to the Semi-tangents of their diſtances from the Pole it ſelf: For *Example*, ſuppoſe it were required to draw a Parallel of 40 degrees of Altitude, becauſe this Circle is diſtant from the Zenith 50 degrees equal to the Complement of 40 degrees, if the Small Circle 40.40,40.40. be deſcribed about the Center *z* at the diſtance of the Semi-tangent of 50 degrees, it will be the Repreſentative of the Parallel of 40 degrees of Altitude upon the Plane of the Projection, and the ſame Method muſt be obſerved in the drawing of any other Parallel of Altitude whatſoever.

The Tropics, Artic Circles, and Parallels of Declination, are projected by laying off the Semi-tangents of their greateſt and leaſt diſtances from the Zenith or Pole of the Projection, in the Meridian or Solſtitial Colure, on the ſame or contrary Sides of the Center, according as the *Caſe* directs: Suppoſe for *Example*, that it were required to project the Artic Circle, which is diſtant from the North Pole by the Space of 23 deg. 29 min. becauſe the diſtance between the Pole and the Zenith equal in our *Caſe* to 38 deg. 28 min. is greater than this diſtance, the Artic Circle will fall intirely on the ſame Side of the Zenith; wherefore if the Semi-tangent of the Sum of 38 deg. 28 min. and 23 deg. 29 min. equal to 61 deg. 57 min. be ſet off from *z* to *o*, you will have one Extremity of the Diameter of the Artic Circle, and if the Semi-tangent of the Difference between 38 deg. 28 min. and 23 deg. 29 min. equal to 14 deg. 59 min. be ſet off from *z* the ſame way to ♈, you will have the other Extremity of the Diameter *o r*, the middle point of which will be the Center; about which Center if the Small Circle *o r* be deſcribed, it will be the Repreſentative of the Artic Circle in this Projection.

Again, if the Tropic of Cancer were to be projected, becauſe the diſtance of the Tropic from the North or Upper Pole *66* deg. 31 min. is greater than the diſtance of the two Poles of the Horizon and Equator, in this *Caſe* 38 deg. 28 min. the Pole of the Projection *z* will lye within the Circle; wherefore, if the Semi-tangent of the Sum of *66* deg. 21 min. and 38 deg. 28 min. equal to 104 deg. 59 min. be ſet off from *z* to *l* it will give one Extremity of the Diameter; and if the Semi-tangent of the Difference 28 deg. 03 min. be ſet off from *z* the contrary way to ♋, it will give the other Extremity ♋ of the Diameter *l* ♋, the middle point of which Diameter will be the Center, about which if the Circle ♋ *l* be deſcribed, it will be the Repreſentative of the Tropic of Cancer in this Projection.

After the ſame manner may any other Parallel of Declination or Parallel to the Equator be drawn, if its diſtance from the Pole *p* be known or given.

Or,

Any Parallel of Declination, ſuppoſe the Tropic of Cancer, may more readily be deſcribed by the help of the *4th Cor.* of the *2d Prop.* of *Section* the *1ſt*, or the *3d Caſe* of *Prob.* the *6th*, of *Section* the *2d Part* the *4th*, after the following manner; thro' *p* the Pole of the Equator or Great Circle to which the Tropic is parallel, draw the Right Line *p* γ E at Right-angles to the Meridian *p z n*, and ſet off the diſtance of the Parallel from the Pole, equal to *66* deg. 31 min. in the Periphery of the Plane of the Projection from *M* to γ, and lay a Ruler over the Pole *H* of the Six a Clock Hour Circle and the point γ, to cut the Six a Clock Hour Circle in the point α, and thro' the point γ and the Center *q* of the Six a Clock Hour Circle, draw the Line *q* α γ to cut the Line *p* ω E in ω, then will *q* ω ſet off in the Meridian from *q* to *v* give the point *v* for the Center, and the diſtance *p* ω will be equal to the Semi-diameter of the Tropic to be drawn. Again,

If the Tropic of Capricorn *e* ♑ *f* were required to be drawn, becauſe this Circle is diſtant from the North or Uppermoſt Pole 113° 29 min.

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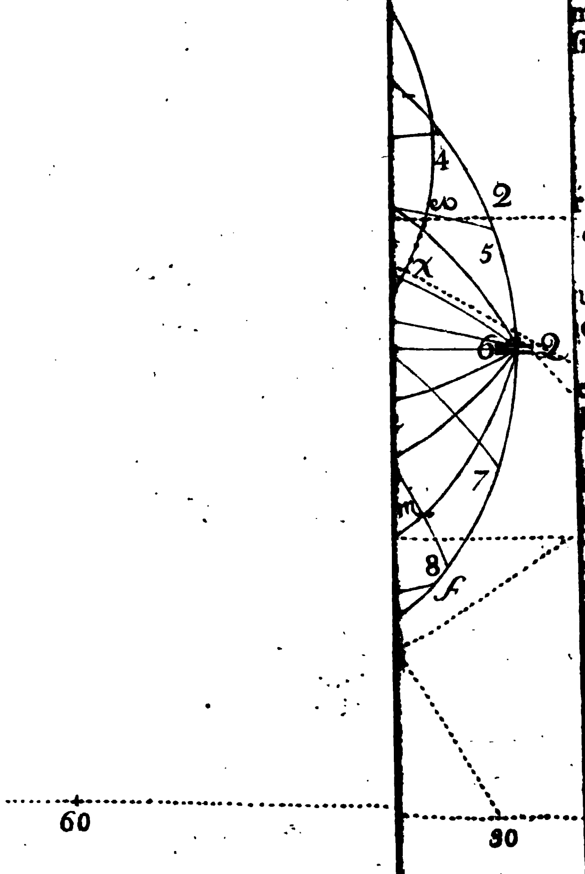

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and Lines be drawn from the ſeveral Diviſions thro' the Center, you will have a Number of Circles anſwering to the reſpective parts of an Hour.

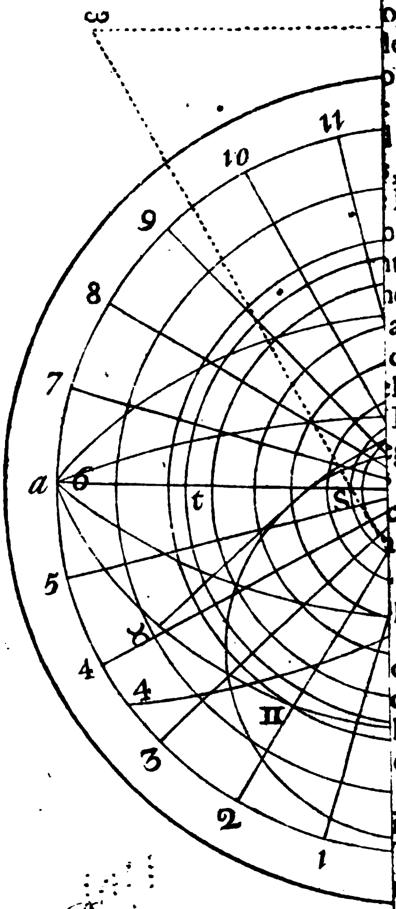
Again, becauſe the Parallels of Declination are all parallel to the Equator, and conſequently in this *Caſe* parallel to the Plane of the Projection, they will all (by the *1ſt Cor.* of the *1ſt Lemma* of the *1ſt Section*) become Circles in the Projection, and (by the *2d Cor.* of the ſame *Lemma*) will have the Center *p* or Pole of the Projection for their common Center and (by the *4th Cor.* of the ſame *Lemma*) their ſeveral Radij or Semi-diameters, will be equal to the Semi-tangents of their diſtances from the Pole of the Equator. Wherefore,

If about *p* as a Center, with the Semi-tangents of the ſeveral diſtances of the Parallels of Declination from the Pole, the ſeveral Small Circles 10.10, 20.20, 30.30, *&c.* be drawn, you will have the ſeveral Parallels of Declination deſcribed upon this Plane.

The Artic Circle is a Small Circle parallel to the Equator, at the diſtance of 23 deg. 29 min. from the North Pole, wherefore if about *p* as a Center, at the the diſtance of the Semi-tangent of 23° 29′ the Small Circle *s t r* be drawn, it will be the Repreſentative of the Artic Circle upon this Plane: In like manner the Small Circle *t ♋ x q* drawn parallel to the Equator *a n q m*, at the diſtance of the Semi-tangent of *66* deg. 30 min. the Complement of the Tropic's diſtance from the Equator, will be the Repreſentative of the Tropic of Cancer in this Projection; and becauſe the Southern Tropic is diſtant from the Northern Pole 113 deg. 29 min. if with the Semi-tangent of 113 deg. 29 min. a Small Circle be drawn about the Center *p*, it will be the Repreſentative of the Tropic of Capricorn, but this Circle falling without the Periphery of the Plane of the Projection, is not uſually drawn.

The Ecliptic is a Great Circle forming an Angle with the Equator of 23 deg. 29 min. equal to the Suns greateſt Declination; wherefore if the Tangent of 23 deg. 29 min. be ſet off in the Meridian from *p* to *f*, you will have the Center *ſ* of the Ecliptic *a ♋ q*, in this Projection.

Circles of Cœleſtial Longitude are projected in this *Caſe* after the ſame manner as the Hour Circles or Circles of Right Aſcenſion are in the former *Problem*, viz. by making the Semi-diameter or Radius *A b* of the Equinoctial Colure *a A q*, equal to the Tangent of 45 degrees, and laying off the Tangent of the Angle of Inclination of the Plane



Between 232:233

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paſſing thro' the 1*ſt* Point
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ls to the Ecliptic are drawn
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he ſame Latitude.

e Zenith z, (whoſe Center
Plane of the Projection by
itude of the given Place) is
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Angle, till it cut *y w* in *w* :
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30 degrees.

degrees to the Radius *z y*,
Angle of 30 degrees with
the

4

the Meridian zpy, then will w be the Center of the Azimuth Circle to be drawn, and wz the Radius, which Radius zw is likewiſe equal to the Secant of the ſame Angle to the Radius zy.

After the ſame manner may any other Azimuth Circle be drawn, or the Angle which any two Azimuth Circles make be determined.

Almicanthers or Parallels of Altitude, are drawn after the ſame manner in this *Caſe* as the Parallels of Declination were in the laſt *Problem*, by ſetting off the Semi-tangent of their greateſt and leaſt diſtances in the Meridian, from the Center or Pole of the Projection, (in this *Caſe* the Pole of the World) on the ſame or contrary Sides according as the *Caſe* requires, to obtain the two Extremities of their reſpective Diameters: Suppoſe for *Example*, it were required to draw a Parallel of 40 degrees of Altitude, becauſe this Circle is diſtant from the Zenith or Pole of the Horizon z 50 degrees, if the Semi-tangent of 88 deg. 28 min. the Sum of the diſtances of the two Poles, and the Small Circle's diſtance from the Zenith, be laid off in the Meridian from the Center to d, it will give one Extremity of the Diameter, and if the Semi-tangent of 11 deg. 32 min. the Difference of the two diſtances, be laid off from the Center p on the contrary Side of the Center, becauſe the Small Circle's diſtance from its Pole is greater than the diſtance of the Poles of the Equator and the Horizon, to c, it will give the other Extremity of the Parallel to be projected; the middle of which ♈ will be the Center, about which if the Small Circle cd be deſcribed, it will be the Repreſentative of a Parallel of 40 degrees of Altitude in this Projection; and the ſame Method muſt be obſerved in the drawing of any other Parallel of Altitude, but if ſeveral Parallels are to be drawn, they are beſt of all done by the help of the 3*d* *Caſe* of the 6*th* *Problem*, of the 2*d* *Section* of the 4*th* *Part*; an *Example* of which may be ſeen in the former *Problem* in the 229*th* *Page* in drawing of the two Tropics.

Prob. IV.

Let it be required to draw the Hour Circles, or Circles of Right Aſcenſion, Circles of Longitude, Parallels of Declination, Tropics, Ecliptic, &c. *Stereographically* upon the Plane of the Ecliptic.

Let ♈♋♎♑ (in *Plate* the 7*th*) repreſent the Ecliptic, upon the Plane of which the Sphere is to be projected, then will π be its Center, and the Pole of the Projection.

Becauſe the Eye in this *Caſe* is placed in one of the Poles of the Ecliptic, for *Example* (in the Southern Pole) the Circles of Cœleſtial Longitude will become ſtreight Lines in this Projection interſecting each other in the Center or Pole of the Projection *z*, by *Cor*. the *1ſt* of the *Gen. Def.* of *Sect*. the *1ſt*, of *Part* the *4th*. Wherefore,

Having drawn the Diameter ♈ π ♎ to repreſent the Equinoctial Colure, at Right-angles to it draw the Diameter ♋ π ♑, this will repreſent the Solſtitial Colure, and where theſe two Diameters interſect the Periphery of the Plane of the Projection, will give the four principal points of the Ecliptic ♈ ♋ ♎ ♑.

Again, becauſe the Circles of Longitude paſſing thro' the firſt points of ♉ and ♏, ♊ and ♐, *&c.* are inclined to the Plane of the Equinoctial Colure, at Angles of 30 deg. and 60 deg. if the Chords of 30 deg. and 60 deg. be laid off in the Periphery of the Plane of the Projection from ♈ and ♎ on each Side towards ♋ and ♑, and the Right-lines ♉ ♏, and ♊ ♐, *&c.* be drawn, they will be the Repreſentatives of the ſeveral Circles of Longitude paſſing thro' the ſeveral firſt points of ♉, ♊, *&c.* and where theſe interſect the Periphery or Ecliptic ♈ ♋ ♎ ♑, they will give the ſeveral firſt points ♉, ♊, *&c.*

After the ſame manner may any other Circle of Longitude forming any given Angle, with the Equinoctial Colure be drawn.

Circles of Cœleſtial Latitude, inaſmuch as they are parallel to the Ecliptic, and conſequently in this *Caſe* parallel to the Periphery of the Plane of the Projection, will have the Pole of the Projection π for their common Center and Pole, and their Semi-diameters will be equal to the Semi-tangent of the Complement of their reſpective diſtances from the Ecliptic: Suppoſe for *Example*, it were required to draw a Parallel of 10 degrees of Northern Latitude, becauſe this Circle is diſtant from the Pole of the Ecliptic π, by the ſpace of 80 degrees, if about the point π as a Center, with the Semi-tangent of 80 deg. the Small Circle *d*, *e*, *f*, *g*, be deſcribed, it will be the Repreſentative of the Parallel of 10 degrees of Latitude in this Projection; and proceeding after the ſame manner, may any other Parallel of Latitude be drawn.

The North Pole of the World is diſtant from the Pole of the Ecliptic by the ſpace of 23 deg. 29 min. wherefore if the Semi-tangent of 23 deg. 29 min. be ſet off in the Solſtitial Colure from *z* to *p*, the point *p* will be the Repreſentative of the North Pole. Again, Becauſe the Six a Clock Hour Circle is inclined to the Plane of the Projection at an Angle of *66* deg. 31 min. if the Tangent of *66* deg. 31 min. be laid off in the Solſtitial Colure from π the contrary way to the Pole, to *S*, the point *S* will be the Center of the Six a Clock Hour Circle, about which if the Circle *r p* ♎ be drawn, it will be the Repreſentative of the Six a Clock Hour Circle in this Projection.

And becauſe the Planes of the ſeveral Hour Circles are inclined to the Plane of this Circle, at Angles of 15 deg. 30 deg. *&c.* if the Tangent of 15 deg. 30 deg. *&c.* to the Radius *p S*, be laid off in the Line *S*,45, from *S* each way, to 15.30, *&c.* the points 15, 30, *&c.* will be the Centers of the ſeveral Hour Circles to be drawn, and if about the ſeveral points 15, 30, *&c.* Circles as *p* 7, *p* 8, *&c.* be drawn, at the diſtance of 15 *p*, 30 *p*, &c. they will be the Repreſentatives of the ſeveral Hour Circles upon this Projection.

The Equinoctial or Equator, is inclined to the Ecliptic, and conſequently to the Plane of the Projection, at an Angle of 23 deg. *29* min. equal to the Suns greateſt Declination: wherefore, if the Tangent of 23 deg. *29* min. be ſet off in the Solſtitial Colure from π towards *p* to *n*, this point *n* will be the Center of the Equator, about which point if the Great Circle ♈ *A* ♎ be drawn, it will be the Repreſentative of the Equator in this Projection.

The Artic Circle, Tropics, and Parallels of Declination are drawn after the ſame manner as in the 2*d Problem*, by ſetting off the Semi-tangents of their greateſt and leaſt diſtances from the Pole of the Ecliptic in the Solſtitial Colure, on the ſame or contrary Sides of the Center π, as the *Caſe* requires, in order to obtain the two Extremities of their Diameters: Suppoſe for *Example*, it were required to draw the Tropic of Cancer, becauſe this Circle is diſtant from the Pole of the World *66* deg. 31 min. if the Semi-tangent of *90* deg. equal to the Sum of *66* deg. 31 min. (the diſtance of the Tropic from the Pole) and 23 deg. *29* min. equal to the diſtance of the two Poles of the Ecliptic and Equator, be ſet off in the Solſtitial Colure from π to ♋, it will give one Interſection of the Diameter; and if the Semi-tangent of 43 deg. 02 min. (the Difference between) *66* deg. 31 min. and 23 deg. *29* min. be ſet off in the Solſtitial Colure from π the contrary way to *c*, it will give the other Extremity of the Diameter, the middle point of which will be the Center, about which ll be the Repreſen-

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jection, on the ſame or contrary Sides, according as the *Caſe* requires, to obtain their Diameters : thus, if it were required to draw a Parallel of 40 deg. of Altitude, becauſe this Circle is diſtant from the Pole of the Horizon or Zenith *z* 50 degrees, and that the Zenith is diſtant in this *Caſe* from the Pole of the Projection 14 deg, 59 min. if the Semi-tangent of 64 deg. 59 min. the Sum of the two given diſtances be ſet off from *π* the Pole of the Projection, in the Meridian towards *z* to *A*, it will give one Interſection of the Diameter of the Small Circle to be projected, and the Semi-tangent of 35 deg. 01 min. the difference of the two given diſtances be ſet off in the Meridian, from the Center of the Projection *π* on the contrary Side of the Center to *q*, will give the other Interſection ; wherefore, if about the middle point as a Center, the Small Circle *A q* be deſcribed to paſs thro' the points *A* and *q*, it will be the Repreſentative of a Parallel of 40 deg. in this Projection.

How any Part or Portion, of any projected Great or Small Circle may be meaſured, or how any Number of degrees may be laid off upon any Circle from a given point, in this or any of the former Projections, has been ſhewn at large in the *6th* and *7th Problem* of *Section* the *2d*, of *Part* the *4th*.

Any of the Projections thus prepared, will readily ſerve to repreſent all the *Phænomena* of the *Sphere*, and give *Solutions* to all *Problems* propoſed concerning it, and for this Purpoſe I ſhall make Uſe of the Repreſentation of the Circles of the *Sphere* upon the Plane of the Solſtitial Colure, it being in this *Caſe* ſuppoſed to lye under the Meridian as deſcribed in *Problem* the *1ſt*, not doubting but the *Reader* will eaſily be able to trace out the ſeveral Triangles in any of the other Repreſentations, and to avoid Confuſion, I ſhall deſcribe ſuch Circles only as are immediately neceſſary for the Buſineſs.

Wherefore, in *Plate* the *8th*, Let *Z P O Q N S H* repreſent the Meridian, or in this *Caſe* the Solſtitial Colure, *P* the North Pole, *S* the South Pole, *P S* the Axis or Six a Clock Hour-Circle, *AQ* the Equator, *a r t* the Artic Circle, and *a n t* the Antartic Circle, ♋ ☉ ☉ ♋, the Tropic of Cancer, or Parallel of the Suns diurnal Courſe when he is in the *1ſt* point of Cancer, ♑ *E* ♑ the Tropic of Capricorn, or Parallel of the Sun's diurnal Courſe when he is in the firſt point of Capricorn, and ♋ ♑ ♑ the Ecliptic.

Let Z repreſent the Zenith of any Place, ſuppoſe *London*, and *N* the Nadir, then will Z*N* be the Prime Vertical or Circle of Eaſt and Weſt Azimuth, and *HO* the Horizon, theſe things being premiſed, it is manifeſt,

1. That all thoſe People who have the Equator *A Q* paſs thro' their Zenith, in which *Caſe* the Axis *P* ♑ lyes in the Horizon, will have their Days and Nights of an equal length during the whole Year, for the Horizon *P S* cuts the Parallels ♋ ☉ ♋, ♑ *E* ♑, of the Sun's diurnal Courſe into two equal parts, whence the Semi-diurnal Arches or Parts above the Horizon, are equal to the Semi-diurnal Arches or Parts under the Horizon; and this is called a Right Poſition of the *Sphere*, becauſe the Parallel of the Sun's diurnal Courſe cuts the Horizon at Right-angles.

2. That all the Inhabitants (if any there be) who have either of the Poles *P* or *S* in their Zenith, (ſuppoſe the Northern Pole *P*) in which *Caſe* the Equator becomes their Horizon, will have but one Day and one Night during the whole Year, that is perpetual Day, (in this *Caſe*) during the Time the Sun is travelling from the firſt point of *Aries* to the firſt point of *Libra* thro' the Northern Signs, and perpetual Night while he is travelling from the firſt point of *Libra* to the firſt point of *Aries* thro' the Southern Signs, that is half the Year Day and half the Year Night; and the contrary to thoſe who have the Southern Pole in the Zenith, that is perpetual Day while the Sun is paſſing thro' the Southern Signs, and continual Night while he is travelling over the Northern Signs; for in either *Caſe* the Sun is removed at more than a Quadrantal diſtance from the Zenith for one half of the Year; and this Poſition of the Sphere is called a *Parallel* Poſition, becauſe the Parallels of the Sun's diurnal Courſe ♋ ☉ ♋, ♑ *E* ♑, are parallel to the Horizon *AQ*.

By Reaſon of the Excentricity of the Earths Orb, which is [illegible] parts nearly of the mean diſtance of the Earth from the Sun, the Earth is 186 Days nearly, in travelling thro' the Southern half of the Ecliptic, and but 179 Days in travelling thro' the Northern half, whence it comes to paſs that that part of the Earth which is under the North Pole, enjoys the preſence of the Sun longer by 7 Days, than that part of the Earth which is under the Southern Pole; and therefore ſtrictly ſpeaking, neither the Day nor the Night under either of the Poles can be ſaid to be half a Year long, but this Difference in Accounting for the General *Phænomena* is ſeldom taken Notice of.

And again, by Reaſon of the denſity of the Air near the Horizon, the Solar Rays are ſo refracted that the Sun appears above the Horizon before he is really Riſen, and is ſeen in the Evening after he is really Set, whence it comes to paſs that the Sun's whole Body may be ſeen by us and our Antipodes, at the ſame part of abſolute Time, which makes our Days really longer than they would otherwiſe be, and then is found by Calculation: And inaſmuch as this Refraction is greater the nearer you approach either Pole, occaſioned by the exceeding coldneſs of the Air, and it is probable that this may be ſo great near the Pole, as to increaſe the Length of the Day by Ten natural Rays, yet this Difference in the duration of the Days is not regarded as being accidental, and makes them generally ſaid to be of Half a Years continuance.

3. That thoſe People who dwell within the Artic Circle, have continual Night when the Sun is in the firſt point of *Capricorn*, for in this *Caſe* the Parallel of the Sun's diurnal Courſe is at a quadrantal diſtance from the Artic Circle, and thoſe who dwell within the Antartic Circle enjoy continual Day, becauſe no point within the Antartic Circle lyes at a greater diſtance than a Quadrant from the Tropic of *Capticorn*; and on the contrary, when the Sun is in the firſt point of *Cancer* the Inhabitants within the Artic Circle enjoy perpetual Day, but the Inhabitants within the Antartic Circle perpetual Night.

4. That by how much the nearer any Place within either Polar Circle is to the Pole, of the greater Number of Natural Days does their longeſt Days conſiſt, and on the contrary, the longer is their longeſt Night; and that their longeſt Day begins when the Declination of the Sun is equal to the Complement of the Latitude of the Place, and of the ſame Denomination, and that their longeſt Night begins when the Sun's Declination is equal to the Complement of the Latitude of the Place, but of a contrary Denomination, or on the contrary Side of the Equator. Thus for *Example*, in the Latitude of 80 degrees North, and which Place is removed 13 ½ within the Artic Circle, the longeſt Day begins on the *5th* of *April*, and continues to the *16th* of *Auguſt* following, and is equal to 133 Natural Days; and on the contrary, their longeſt Night begins on the *8th* of *October*, and ends on the *11th* of *February* following, and conſiſts of 126 Natural Days, ſhorter than the longeſt Day by 7 Days, occaſioned as before mentioned by the Excentricity of the Sun, and after the ſame manner may the length of the longeſt or ſhorteſt Day be determined in any given Place.

5. That all thoſe People who are ſcituated between the Equator and either of the Polar Circles, will have their Days and Nights longer or ſhorter, according as the Sun approaches nearer to, or receeds farther off from their Zenith, but their longeſt Day or longeſt Night will never exceed one Natural Day or 24 Hours, and the Poſition of the Sphere in this and the former *Caſe* is called an Oblique Poſition, becauſe the Equator and the Parallels of the Sun's diurnal Courſe cuts the Horizon at Oblique or unequal Angles.

6. To determine the Time of the Riſing and Setting of the Sun, the length of the Day and Night, his Amplitude, *&c.* in any of theſe Places ſet off the Latitude of the Place propoſed from *P* to *O*, and draw the Horizon *H O*, and at Right-angles to it the Prime Vertical Z*N*, and let the Sun be ſuppoſed to be in the firſt point of *Cancer* or moſt Northermoſt point of the Ecliptic, then will ♋ *K X* the Tropic of *Cancer*, bocome the Parallel of the Sun's diurnal Courſe in this *Caſe*, whence it is manifeſt, 1. That when the Sun arrives at *X* it is then Midnight to the Inhabitant at *Z*, that *OX* equal to the Difference (in this *Caſe*) between *O Q* the Complement of the Latitude of the Place, and the Sun's Declination *Q X* is equal to the Sun's greateſt Depreſſion, and that Z *X* equal to the Arch *O X* increaſed by a Quadrant *Z O*, is equal to the greateſt diſtance from the Vertex at that time.

As the Sun is carried on in the Parallel *X*♋ from *X* towards ♋, when he arrives at *O*, that is where the Parallel of the Sun's diurnal Courſe cuts the Horizon he there appears to Riſe; wherefore, if the Arch of the Equator *BQ* equal to the Arch *X O*, equal to the Semi-diurnal Arch, be converted into Time, it will give the Hour of the Sun's Riſing, but taken from a Semicircle will give the length of the Semi-diurnal Arch ☉ ♋; this therefore converted into Time will give the time of Sun Setting, for as the Number of Hours between Midnight and the Sun's Riſing gives the Hour of the Sun's Riſing, ſo the Number of Hours between Noon and Sun Setting, will give the time of the Setting.

As the degrees of the Equator anſwering to the point *B*, cut off by the Meridian *P* ☉ *B S* paſſing thro' the Sun, is called the Right-Aſcenſion of the Sun, becauſe it is the ſame point of the Equator that Riſes with the Sun in a Right Sphere, ſo the point of the Equator ♈ which aſcends with the Sun in this Sphere thus inclined, is called the Oblique Aſcenſion, and their Difference or the Arch ♈*B*, which is called the Aſcenſional Difference, is ever equal to the Exceſs or of the Semi-Solar Day above ſix Hours, or its Defect to the ſame; wherefore if ♈ *B* be converted into Time and ſubſtracted from ſix Hours, in this *Caſe*, it will give the time of the Suns Riſing, but added to ſix Hours will give the time of the Suns Setting.

Again, becauſe the Sun's Amplitude is equal to the Arch of the Horizon, intercepted between the Sun and the Eaſt point of the Horizon at his Riſing, or the Weſt point of the Horizon at his Setting, if the Arch ♈ ☉ be meaſured, it will give the Amplitude of the Sun in this *Caſe*; that is, how far he Riſes and Sets to the Northward of the Eaſt and Weſt points of the Horizon.

When the Sun arrives at *H* he is then ſix Hours diſtant from the Meridian, wherefore if the Vertical Circle *ZHN* be drawn thro' the Sun at this time, the Arch *HC* will give the Sun's Height, and the Arch of the Horizon *CO* will give his Azimuth from the viſible Pole *P*, which is Eaſterly if it be in the Morning, but Weſterly if it be in the Afternoon.

Again, if it be required to find when the Sun will appear due Eaſt or Weſt, and his Altitude at that time, thro' the Interſection *I* of the Parallel ♋ *X*, and the Prime Vertical Z ♈ *N*, let the Hour Circle or Meridian *PIS* be drawn, then will the Arch *I* ♈ be equal to the Sun's Height above the Horizon, and the Arch of the Equator ♈ *D* equal to the time after Six in the Morning when he will appear due Eaſt, or how long before Six in the Evening he will be ſeen in the Weſt, this therefore converted into Time, and added to, and ſubſtracted from Six Hours, will give the Time when the Sun will appear due Eaſt and Weſt.

Again, if the Arch of the Equator *DA* be converted into Time, it will ſhew how many Hours before Noon the Sun will appear due Eaſt, and the time in the Afternoon when he will appear due Weſt; for the Arches ♈ *D* and *DA* do together make a Quadrant or Six Hours.

Again, if the Hour of the Day and Azimuth of the Sun at any other time be required, his Altitude being firſt given, about the Zenith Z let the Parallel *qKb* be deſcribed, at the diſtance of the Complement of the Sun's Height at the time of Obſervation, where this interſects the Parallel of the Sun's diurnal Courſe as at *K*, it will give the Place of the Sun, thro' which if the Hour Circle *PKS* be drawn, the Arch of the Equator *RA* converted into time, will give the

Hour if it be in the Afternoon, but ſubſtracted from Twelve Hours will give the time, if the Obſervation was taken in the Morning.

Again, if thro' the ſame point *K*, a Vertical Circle as *ZKEN* be drawn, the Arch of the Horizon *EH* will give the Sun's Azimuth at that time, that is how much he appears to the Eaſtward of the Meridian if it be in the Morning, or how much to the Weſtward if it be in the Afternoon.

Laſtly, when the Sun arrives at ♋ it is then Mid-day or High Noon, and if the Declination *A*♋ in this *Caſe* be added to the Complement of the Latitude *AH*, it will give the Sun's greateſt Meridian Altitude ♋*H*, but ſubſtracted from the Arch *ZA* equal to the Latitude of the Place, will leave *Z*♋ the Meridional or leaſt Zenith Diſtance.

Let us now ſuppoſe the Sun to be in the firſt point of Capricorn, or moſt Southermoſt point of the Ecliptic, that is, as far diſtant from the Equator towards the South, as he was in the former *Caſe* to the Northward, then will ♑*Er* the Tropic of Capricorn, become the Parallel of his diurnal Courſe; whence it is obvious,

1. That *Or* the Sun's Depreſſion at Midnight in this *Caſe*, is equal to *H*♋ his Meridional Altitude in the former, for the Arches $OQ+Qr=HA+A\text{♋}$.

2. That *H*♑ his Meridional Altitude in this *Caſe*, is equal to *OX* his greateſt Depreſſion in the former, for the Arches $HA-A\text{♑}=OQ-QX$.

3. That his Aſcenſional Difference ♈*E* in this *Caſe*, is equal to the Aſcenſional Difference ♈☉ in the former, for the Triangles ♈*EF* and ♈☉*B* are equal: And becauſe the Oblique Aſcenſion in this *Caſe* is more than the Right Aſcenſion, but leſs in the former it follows,

4. That the time of the Sun Riſing in the preſent *Caſe*, is the ſame with the time of his Setting in the former *Caſe*, and conſequently the time of his Setting in the former, the ſame with the time of his Riſing in this.

5. That the Semi-diurnal Arch ♋☉ in the former *Caſe* equal to the Arch of the Equator *AB* is equal to the Semi-nocturnal Arch *rE* in this *Caſe*, equal to the Arch of the Equator *QF*, and conſequently the length of the Day in the former *Caſe*, is equal to the length of the Night in the preſent *Caſe*.

6. That the Semi-nocturnal Arch ☉*X* in the former *Caſe* equal to the Arch of the Equator *BQ*, is equal to the Semi-diurnal Arch

♑*E*

E ♈ in this *Caſe,* equal to the Arch of the Equator *AP*; whence it follows, that the length of the Night in the former *Caſe,* is the ſame with the length of the Day in the preſent.

7. That the Amplitude ♈ ☉ in the former *Caſe,* is the ſame with the Amplitude ♈ *E* in the preſent *Caſe,* but the contrary way, and conſequently the Sun Riſes and Sets as much to the Southward of the Eaſt and Weſt points in this *Caſe,* as he does to the Northward of the Eaſt and Weſt in the former; and conſequently,

8. The Amplitude is at all times the ſame way with the Declination, that is, if the Sun be to the Northward of the Equator, or have North Declination, he Riſes and Sets to the Northward of the Eaſt and Weſt points of the Horizon, but if he be to the Southward of the Equator and have South Declination, he Riſes and Sets to the Southward of the Eaſt and Weſt.

And theſe *Laws* obtain at all equal degrees of Declination, on contrary Sides of the Equator.

If the Place be in South Latitude, that is, if the South Pole be elevated above the Horizon, then the Semi-nocturnal Arch in the former *Caſe* becomes the Semi-diurnal Arch in this *Caſe,* and the Semi-nocturnal Arch in this *Caſe* becomes the Semi-diurnal Arch in the former; and conſequently the length of the Day in the former is equal to the length of the Night in the latter, and the time of the Sun's Riſing in the one, the ſame with the time of the Sun's Setting in the other; and this will happen at all equal degrees of Latitude and Declination, but of contrary Denominations.

Let us ſuppoſe the Sun in the Equator, then will *A* ♈ *Q* be its diurnal Path; whence it is manifeſt,

1. That the Midnight Depreſſion *OQ,* is equal to the Meridional Altitude *AH,* and each equal to the Complement of the Latitude *P Z.*

2. That the Semi-nocturnal Arch ♈ *Q* is equal to the Semi-diurnal Arch *A* ♈, for the Horizon *HO* divides the Equator *A Q* into two equal parts in the point ♈, and this Equality of Days and Nights happens to all the Inhabitants upon the *Earth,* except under the very Poles, at which time the Sun moves in the common Horizon of each, and the perpetual Day begins to the Inhabitants under the North Pole, and the continual Night to thoſe who dwell under the South Pole when the Sun enters into the firſt point of Aries, and the contrary when he enters into the firſt point of Libra, that is, the continual Day ends to the Northern Inhabitants, and the Sun ap-

pears to the Southern and conſequently their perpetual Day begins.

3. That the Aſcenſional Difference vaniſhes, for the Oblique Aſcenſion, Deſcenſion, and Right Aſcenſion are the ſame, and conſequently the Sun Riſes and Sets at the Hour of Six.

4. That there is no Amplitude, for the Sun Riſes in the Eaſt point of the Horizon, and Sets due Weſt.

5. That if about the point Z as a Pole, a Parallel be deſcribed at the diſtance of the Complement of the Altitude, the Arch of the Equator YA intercepted between this Interſection Y of the Parallel of Altitude, will ſhew the Hour from Noon.

6. That the Arch of the Horizon HE intercepted between the Vertical Circle $ZEYN$ and the Meridian ZHN will ſhew the Azimuth of the Sun at the ſame time.

After the ſame manner may the time of the Riſing, Setting, Culmination, *&c.* of any Star be determined, povided the Difference of the Right Aſcenſion or Arch of the Equator intercepted between the Meridian paſſing thro' the Sun and Star be known.

How the Doctrine of *Spherical Triangles* is applied to determine the length of any Part or Portion of any of the Great Circle in theſe Projections, and conſequently to determine the exact time of the Riſing and Setting of the Sun, his Amplitude, Azimuth, *&c.* ſhall be the Buſineſs of the next *Section*.

SECT. III.

Containing the Application of Spherical Trigonometry *to the Solution of the chief and primary Poblems of* Aſtronomy, *ariſing from the diurnal Motion of the* Sun, *or the Rotation of the* Earth *about her Axis.*

And which are of two Sorts.

1. GEneral, *viz.* ſuch as are common to all the Inhabitants upon the *Earth* at the ſame time, and ſuch are thoſe that require the Sun's Right Aſcenſion, and Declination, his Place being firſt given, *&c.*

Or,

2. Particular, that is variable in different Latitudes, and ſuch are thoſe that from the Latitude of the Place, and the Declination of the Sun require.

1. His Amplitude, Riſing, and Setting; that is, how many dedgrees he Riſes or Sets, from the true Eaſt or Weſt points of the Horizon.
2. His Aſcenſional Difference, or how long time he Riſes or Sets before or after the Hour of Six, and thence the exact time of his Riſing or Setting in any Place.
3. His Height at the Hour of Six.
4. His Azimuth, or upon what point of the Compaſs he appears at the ſame time.
5. His Height when he appears due Eaſt or Weſt.
6. The time when he appears due Eaſt or Weſt.
7. His Height at any time when he is in the Equator.
8. His Azimuth at any time when he is in the Equator.
9. His Height at any time, when he is in any other point of the Ecliptic.
10. His Azimuth at any time when he is in any point of the Ecliptic.

Of each of theſe in their Order.

And becauſe the Sun's Place in the Ecliptic is ſuppoſed to be firſt known or given in all *Problems* of the *Sphere*, let it be required in

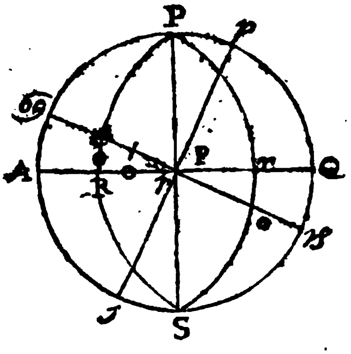
Prob. I.

From the Sun's Longitude and greateſt Declination (which according to the Correct Obſervations of the late Celebrated Mr. *Flamſteed*, is found to be 23 deg. 29 min.) to find his preſent Declination and Right Aſcenſion.

Example.

On the 7*th* of *May* 1720, at Noon, according to the *Flamſteedian Solar Tables*, the Sun was in *Taurus* 27 deg. 33 min. 57 ſec. let it be required from thence, to find his preſent Declination and Right Aſcenſion, his greateſt Declination or the Obliquity of the Ecliptic, being fixed as before at 23 deg. 29 min.

The Stereographic Solution.



1. Upon the Plane of the Solſtitial Colure.

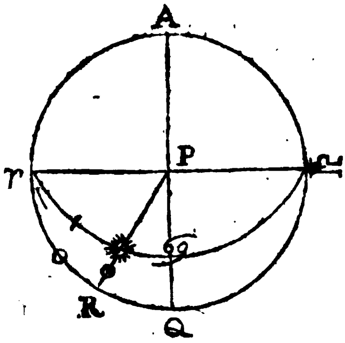
1, Thro' the Center ♈ of the Circle *P Q S A* repreſenting the Solſtitial Colure; draw the Equator *A* ♈ *Q*, and at Right-angles to it the Axis, or in this *Caſe* the Equinoctial Colure *P*♈*S*, then will *P* be the Northern Pole, and *S* the Southern.

2. Draw the Ecliptic ♋ ♈ ♑, forming an Angle of 23 deg. 29 min. with the Equator *AQ*, and croſs it at Right-angles with its Axis *P S*.

3. Set off the Semi-tangent of 57 deg. 33 min. 57 ſec. the Sun's Longitude from ♈ towards ♋ to ☉, then will ☉ repreſent the Place of the Sun in the Ecliptic.

4. Thro' ☉ and the Poles *P* and *S*, draw the Hour Circle *P*☉*R S*, then will ☉*R* be the Sun's preſent Declination, and ♈ *R* his Right Aſcenſion from ♈, each of which may be meaſured by *Caſe* the 2*d* and 3*d*, of *Problem* the 7*th*, of *Section* the 2*d*, of *Part* the 4*th*.

2. *The Stereographic Solution* upon the Plane of the Equator.

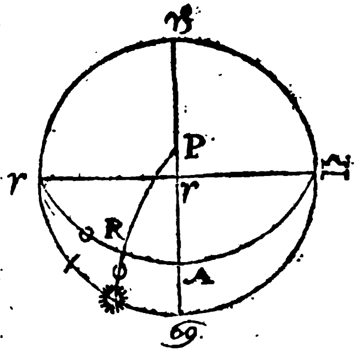


1. Thro' *P* the Pole of the Circle ♈ *A* ♎ *Q* repreſenting the Equator, draw the Equinoctial Colure ♈ *P* ♎, and at Right-angles to it the Solſtitial Colure *A P* ♋ *Q*.

2. Thro' the point ♈ draw the Ecliptic ♈ ♋ ♎, forming an Angle of 23 deg. 29 min. with the Plane of the Projection, (by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d* of *Part* the 4*th*.)

3. Set off the Sun's Longitude 57 deg. 33 min. 57 ſec. in the Ecliptic, from ♈ to ☉, and draw the Hour Circle *P* ☉ *R*, then will ☉ *R* be the Sun's preſent Declination, and ♈ *R* his Right Aſcenſion from the firſt point of *Aries*, each of which may be meaſured (by *Caſe* the 2*d* and 3*d* of *Problem* the 7*th*, of *Section* the 2*d of Part* the 4*th*.

3. *The Stereographic Solution* upon the Plane of the Ecliptic.



1. Thro' *p* the Pole of the Circle ♈ ♑ ♎ repreſenting the Ecliptic, draw the Circle of Cœleſtial Longitude ♈ *p* ♎, and at Right-angles to it the Solſtitial Colure ♑ *p* ♋.

2. Set off the Semi-tangent of 23 deg. 29 min. in the Solſtitial Colure from *p* to *P*, then will *P* be the place of the North Pole, about which as a Pole (or which is the ſame thing) thro' the point ♈ draw the Equator ♈ *A* ♎, at an Angle of 23 deg. 29 min. with the Ecliptic ♈ ♑ ♎ ♑.

3. Set off the Chord of 57 deg. 33 min. 57 ſec. in the Ecliptic from ♈ to ☉, and draw the Hour Circle *P R* ☉, by *Problem* the 1*ſt* of *Section* the 2*d* of *Part* the 4*th*, to cut the Equator in *R*, then will ♈ *R* be the Aſcenſional Difference, and *R* ☉ the preſent Declination, each of which may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d* of *Part* the 4*th*.

But to determine their Quantity by Calculation,

In the Spherical Triangle ♈ ☉ *R* (*See each of the three Projections*) Right-angled at *R*, are given ♈ ☉ the Sun's Longitude from the next Equinoctial point ♈, equal to 57 deg. 33 min. 57 ſec. and the Angle ☉ ♈ *R* the Sun's greateſt Declination, or the Obliquity of the Ecliptic, equal to 23 deg. 29 min. whence (by the 2*d Example* of *Caſe* the 5*th*, of *Right-angled Spherical Triangles*)

To find the preſent Declination.

It will be, As *R* . S. ☉ ♈ *R* : : S. ♈ ☉ : S. ☉ *R*; that is;

As the Radius ———————————— 10.0000000

To the S. of the Sun's Longitude from *Aries* 57.33.57 —9.9263467
So is the Sine of his greateſt Declination 23 29 ——9.6004090

To the Sine of the preſent Declination 19.39.10$\frac{3}{4}$ ——9.5267557

Which is North, becauſe the Sun is in the Northern Signs.

Let the Circle of Right Aſcenſion *P r o S* (*See the* 1*ſt Figure*) be drawn, to cut the Ecliptic in the point *o*, as far diſtant from the Equinoctial point ♈ in the Southern Semi-circle, as the point ☉ is diſtant from the Equinoctial point in the Northern Semicircle, then will the two Spherical Triangles ♈ ☉ *R* and ♈ *o r* Right-angled at *R* at *r* be equal in all reſpects, and conſequently the Side *r o* of the Triangle ♈ *o r*, equal to the Side ☉ *R* of the Triangle ♈ ☉ *R*, and becauſe the Pole or Center of the Circle *P Q S A*, is common to the two Equinoctial points *Aries* and *Libra*, it follows, that when the Sun is at the ſame diſtance from either of the Equinoctial points *Aries* or *Libra*, that then the Quantity of the Declination is the ſame, but of different Denominations, according as the Sun is converſant either in the Northern or the Southern Signs; ſo that in the preſent *Caſe* the Declination 19 deg. 39 min. 10 ſec. $\frac{3}{4}$, is common in the Northen Signs to

♉ 27° 33′ 57″, and ♌ 02° 26′ 03″.

And in the Southern Signs to

♏ 27° 33° 57″, and ♒ 02° 26′ 03″:

Each of which points are diſtant 57 deg. 33 min. 57 ſec. from the next Equinoctial point.

2. To find the Right Aſcenſion, it will be by the 2*d Example* of the 4*th Caſe* of *Right-angled Spherical Triangles*,

As *R* : cs, ⊙ *R* : : t, ♈ ⊙ : : t, ♈ *R* ; that is,

As the Radius —————————— 10.0000000

To the Co-ſine of the Sun's greateſt Declin. 23.29 — 9.9624527

So is the T. of the Sun's Longitude from ♈ 57.33.57 — 10.1969144

To the Tang. of the Right Aſcenſion 55.17.03¾ — 10.1593671

And becauſe in the two *Right-angled-Spherical Triangles* ♈ ⊙ *R* and ♈ *o r* (*See the 1ſt Figure*) the Side ♈ *R* of the Triangle ♈ ⊙ *R*, is equal to the Side ♈ *r* of the Triangle ♈ *o r*, it is manifeſt, that the ſame Quantities of Right Aſcenſion numbred from the neareſt Equinoctial points, correſpond with the ſame Quantities of Longitude numbred from the ſame Equinoctial Points. Wherefore, to find the Right-Aſcenſion from the firſt point of *Aries* obſerve,

1. If the Sun be in the firſt Quadrant of the Ecliptic, that is, between the firſt point of *Aries* and the firſt point of *Cancer*, then the Right Aſcenſion found is the Right Aſcenſion numbred from the firſt point of *Aries*; thus in the preſent *Example*, becauſe the Sun is ſuppoſed to be in the firſt Quadrant of the Ecliptic, that is in *Taurus* 27 deg. 33 min. 57 ſec. the Arch of the Equator ♈ *R* equal to 55 deg. 17 min. 03 ſec. ¾ found by Calculation, is the Right Aſcenſion numbred from the firſt point of *Aries*.

2. If the Sun be in the ſecond Quadrant of the Ecliptic, that is, between the firſt point of *Cancer* and the firſt point of *Libra*, then the Arch of the Equator given by Calculation, ſubſtracted from 180 deg. (the Right Aſcenſion of the firſt point of *Libra*) will give the Right-Aſcenſion numbred from the firſt point of *Aries*; for *Example*, Suppoſe the Sun at ⊙ in the ſecond Quadrant of the Ecliptic, to be in *Leo* 2 deg. 26 min. 03 ſec. which is as far diſtant from the Equinoctial point *Libra*, as in the former *Caſe* the Sun was diſtant from the Equinoctial point *Aries*; then in this *Caſe*, if the Arch of the Equator ♎ *R* equal to ♈ *R*, equal to 55 deg. 17 min. 03 ſec. ¾ be taken from 180 deg. the Right Aſcenſion of the firſt point of *Libra*, the Remainder 124 deg. 42 min. 56 ſec. ¼, will be the Right Aſcenſion of the Sun numbred from the firſt point of *Aries*.

3. If the Sun be in the third Quadrant of the Ecliptic, that is, between the firſt point of *Libra* and the firſt point of *Capricorn*, then the Arch of the Equator found by Calculation, added to the Right

Aſcenſion of the firſt point of *Libra* equal to 180 degrees, will give the Right Aſcenſion numbred from the firſt point of *Aries*, as ſuppoſe the Sun at *o* in 27 deg. 33 min. 57 ſec. of the Sign of *Scorpio*, which is in the third Quadrant of the Ecliptic, and as far diſtant from the Equinoctial point *Libra*, as in the firſt *Caſe* the Sun was diſtant from the Equinoctial point *Aries*; wherefore in this *Caſe*, if the Arch of the Equator ♎ *r* equal to ♈ *R*, equal to 55 deg. 17 min. 03 ſec. ¼ be added to 180 degrees, the Right Aſcenſion of the firſt point of *Libra*, the Sum 235 deg. 17 min. 03 ſec. ¼, will be the Sun's Right Aſcenſion numbred from the firſt point of *Aries*.

4. If the Sun be in the fourth and laſt Quadrant of the Ecliptic, that is between the firſt point of *Capricorn* and the firſt point of *Aries*, then the Arch of the Equator found by Calculation ſubſtracted from 360 deg. will leave the Right Aſcenſion numbred from the firſt point of *Aries*; as ſuppoſe the Sun to be at *o*, in 2 deg. 26 min. 03 ſec. of *Aquarius*, which is in the fourth Quadrant of the Ecliptic, and as far diſtant from the Equinoctial point *Aries* as he was in the firſt *Caſe*, but on contrary Sides, wherefore if the Arch ♈ *r* equal to ♈ *R* equal to 55 deg. 17 min. 03 ſec. ¼ be taken from 360 deg. the Right Aſcenſion of the laſt point of *Piſces* or of the firſt point of *Aries*, the Remainder 305 deg. 42 min. 56 ſec. ¾, will be the Right Aſcenſion of the Sun numbred from the firſt point of *Aries*.

After the ſame manner were the following Tables of the Sun's Right Aſcenſion and Declination made, from the Sun's Place delivered in the firſt Table, and how the Sun's Place it ſelf may be Calculated, ſhall be ſhewn in its due Place.

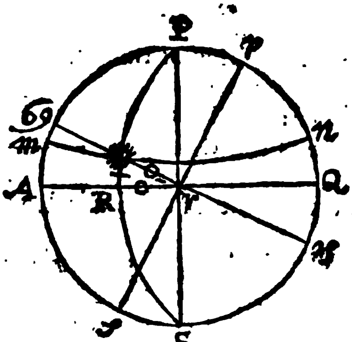
Prob. I.

Caſe 2.

Given the Sun's greateſt Declination or Obliquity of the Ecliptic, and his preſent Declination, to find his Longitude and Right Aſcenſion.

Example.

Suppoſe the Sun's greateſt Declination (as before fixed) 23 deg. 29 min. and his preſent Declination 19 deg. 39 min. 10 ſec. ¾ North and increaſing, and let it be required to find his Longitude, and Right Aſcenſion from the firſt point of *Aries*.



The Stereographic Solution upon the *Plane* of the *Solſtitial Colure.*

1. Having drawn the Solſtitial Colure *PQSA*, the Equator *AQ*, and at Right-angles to it the Equinoctial Colure *P* ♈ *S*, ſet off the Chord of 23 deg. 29 min. the greateſt Declination from *A* to ♋ or from *Q* to ♑, and draw the Ecliptic ♋ ♈ ♑, and at Right-angles to it its Axis *p s*.

2. About the point *P* as a Pole, becauſe the Declination is North, deſcribe the Parallel *m* ☉ *n* at the diſtance of 70 deg. 20 min. 49 ſec. ¼, equal to the Complement of the Sun's preſent Declination 19 deg. 39 min. 10 ſec. ¾.

3. Thro' the Interſection ☉ of the Parallel *m* ☉ *n*, with the Ecliptic ♋ ☉ ♑, draw the Circle of Right Aſcenſion *P* ☉ *R S* and the thing is done; and the Arch of the Ecliptic ♈ ☉ will be the Sun's Longitude, and ♈ *R* his Right Aſcenſion, to find each of which by Calculation, in the Spherical Triangle ♈ ☉ *R* Right-angled at *R*, are given, ☉ *R* the Sun's preſent Declination, and the Angle ☉ ♈ *R* his greateſt Declination, whence to find firſt the preſent Right Aſcenſion ♈ *R*, it will be by *Caſe* the *9th* of *Right-angled Spheric Triangles*,

As t, ∠ ☉ ♈ *R* : t, ☉ *R* : : *R* : S. *R* ♈ ; that is,

As the Tangent of the greateſt Declination 23.29 —— 9.6379563

To the Tangent of the preſent Declination 19.39.10¾ —— 9.5528217
So is the Radius —————————— 10.0000000

To the Sine of the Right Aſcenſion 55.17.03¼ —— 9.9148654

Now becauſe the Declination is North and increaſing, the Sun is in the firſt Quadrant of the Ecliptic, and conſequently the Arch laſt found 55 deg. 17 min. 03 ſec ¼, is his Right Aſcenſion from *Aries*

2. To find the Longitude ♈ ☉, it will be by the *10th Caſe* of *Right-angled Spherical Triangles*,

As S. ☉ ♈ *R* : S. ☉ *R* :: *R* : S. ☉ ♈; that is,

As the Sine of the Sun's greateſt Declination 23.29 —— 9.6004090

To the Sine of the preſent Declination 19.39.10¾ —— 9.5267557

So is the Radius —— 10.0000000

To the Sine of the Sun's Longitude 57.33.57 —— 9.9263467

Which Arch of 57 deg. 33 min. 57 ſec. becauſe the Sun is in the firſt Quadrant of the Ecliptic, muſt be counted from the firſt point of *Aries*, and conſequently the Sun is in ♉ 27 deg. 33 min. 57 ſec.

If the Declination had been North and decreaſing, then the Sun had been in the ſecond Quadrant of the Ecliptic, and conſequently the Arch of the Equator 55 deg. 17 min. 03 ſec. ¼ laſt found, taken from 180 degrees, the Right Aſcenſion anſwering to the firſt point of *Libra*, will leave 124 deg. 42 min. 56 ſec. ¾, the Right Aſcenſion from the firſt point of *Aries*; and the Arch of the Ecliptic 57 deg. 33 min. 57 ſec. numbred back from the firſt point of *Libra*, will give the Sun's Place in *Leo* 2 deg. 26 min. 03 ſec..

If the Declination had been South and increaſing, that is, if the Sun had been in the third Quadrant of the Ecliptic, then the Arch of the Equator 55 deg. 17 min. 03 ſec. ¼ added to 180 degrees, will give 235 deg. 17 min. 03 ſec. ¼, for the Right Aſcenſion from *Aries*, and the Arch of the Ecliptic 57 deg. 33 min. 57 ſec. counted forwards from the firſt point of *Libra*, will give 27 deg. 33 min. 57 ſec. in the Sign of *Scorpio*, for the Sun's Place.

If the Declination had been South decreaſing, that is, if the Sun had been in the fourth or laſt Quadrant of the Ecliptic, then the Arch of the Equator 55 deg. 17 min. 03 ſec. ¼ taken from 360 deg. will leave 304 deg. 42 min. 56 ſec. ¾, for the Sun's Right Aſcenſion from *Aries*, and the Arch of the Ecliptic 57 deg. 33 min. 57 ſec. reckoned backward from the laſt point of *Piſces*, will give 2 deg. 26 min. 03 ſec. in the Sign of *Aquarius*, for the Sun's Place.

This Problem is of Uſe in finding the Sun's true Place in the Ecliptic, and thence the Error of the Solar Tables, if any there be; ſince from the Sun's Meridional Altitude obtained by a good Quadrant, the Latitude of the Place of Obſervation being firſt known, the Declination of the Sun at that time is eaſily had, as ſhall be ſhewn hereafter.

Prob.

Prob. I.

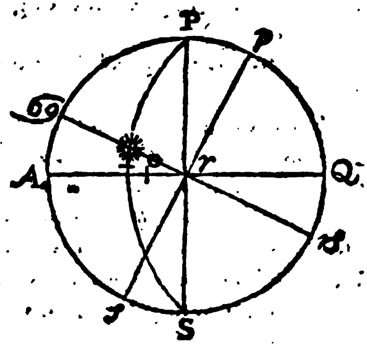
Caſe 3.

Given the greateſt Declination or Obliquity of the Ecliptic, and the Right Aſcenſion, to find the preſent Declination, Longitude, and Angle formed by the Ecliptic and Meridian, or Circle of Right Aſcenſion paſſing thro' the Sun, *&c.*

Example

The greateſt Declination or Obliquity of the Ecliptic, being 23 deg. 29 min. and the Right Aſcenſion 55 deg. 17 min. 03 ſec. ¼, I demand the Declination, Longitude, and Angle formed by the Ecliptic and Meridian.

The Stereographic Solution.



1: Having drawn the Solſtitial Colure *P Q S A*, the Equator *AQ*, and at Right-angles to it the Equinoctial Colure *A* ♈ *Q*, ſet off the greateſt Obliquity of the Ecliptic, from *A* to ♋ or from *Q* to ♑, and draw the Ecliptic ♋ ♑, and at Right Angles to it its Axis *p s*.

2. Becauſe the Right Aſcenſion is leſs than a Quadrant, ſet off the Semi-tangent of 55 deg. 17 min. 03 ¼, in the Equator from ♈ to *R*, and draw the Meridian or Circle of Right Aſcenſion *P* ☉ *R S* and the thing is done; and the Longitude ♈ ☉ may be meaſured by *Caſe* the 2*d* of *Prob.* the 7*th*, of *Sect.* the 3*d*; the Declination ☉ *R* by *Caſe* the 3*d* of the ſame *Problem*, and the Angle of the Ecliptic and Meridian ♈ ☉ *R* by *Caſe* the 3*d* of *Problem* the 10*th* of the ſame *Section*; but to determine their juſt Quantities by Calculation, in the Spherical Triangle ♈ ☉ *R*, Right-angled at *R* are given, the Angle ☉ ♈ *R* the greateſt Obliquity of the Ecliptic, the Side ♈ *R* the Right Aſcenſion, whence to find

1. The Declination ☉ *R*, it will be by the 2*d* *Example* of the 8*th* *Caſe* of *Right Angled Spherical Triangles*;

As *R* : S. ♈ *R* : : t, ⊙ ♈ *R* : t, ⊙ *R* ; that is,

As the Radius ———————————— 10.0000000

To the Sine of the Sun's Right Aſcenſion 55.17.03¼ —— 9.9148651
So is the Tangent of the greateſt Obliquity 23.29 —— 9.6379563

To the Tangent of the preſent Declination 19.39.10¾ — 9.5528214

Which is North, becauſe the Right Aſcenſion is leſs than 180 degrees.

But if the Right Aſcenſion had been more than 180 degrees, the Declination had been South.

2. To find the correſpondent Longitude, it will be by the *2d Example* of the *6th Caſe* of *Right-angled Spherical Triangles*,

As *R* : c s, < ⊙ ♈ *R* : : c t, ♈ *R* : c t, ⊙ ♈ ; that is,

As the Radius ———————————— 10.0000000

To the Co-ſine of the greateſt Declination 23.29 —— 9.9624527
So is the Co-tang. of the Right Aſcenſion 55.17.03¼ —— 9.8406339

To the Co-tang. of the correſponding Long. 57.33.57 9.8030866

Now becauſe the Right Aſcenſion is leſs than a Quadrant or 90 Degrees, the point in the Ecliptic correſponding to the given Degree of Right Aſcenſion is ♉ 27 deg. 33 min. 57 ſec.

And accordingly the competent Longitude found, reckoned backward or forward from either Equinoctial point, as the point of Right Aſcenſion happens to be in either of the four Quadrants of the Equator, will give the correſpondent point in the Ecliptic anſwering to it.

Thus if the Right Aſcenſion had been 124 deg. 42 min. 56 ſec. ¾, then the correſpondent point in the Ecliptic had been ♌ 02 deg. 26 min. 03 ſec. if the Right Aſcenſion had been 235 deg. 17 min. 3 ſec. ¼, then the correſpondent point had been ♏ 27 deg. 33 min. 57 ſec. if the Right Aſcenſion had been 304 deg. 42 min. 56 ſec. ¾, then the correſpondent point had been ♒ 2 deg. 26 min. 3 ſec. each of which points being equally diſtant from the next Equinoctial point.

3. To find the Angle ♈ ⊙ *R*, formed by the Ecliptic and the proper Meridian, it will be by the *2d Example* of the *7th Caſe* of *Right-angled Spherical Triangles*,

As *R* : c s, ♈ *R* : : S. ☉♈ *R* : c s, ♈ ☉ *R*; that is,

As the Radius ———————————— 10.0000000

To the Co-ſine of the Right Aſcenſion 55.17.03¼ ——— 9.7554981
So is the S. of the greater Obliquity 23.29 ————— 9.6004090

To the Co-ſine of the Angle of the Ecliptic and Meridian 76.52.59¼ ———————— 9.3559071

This Problem is of Uſe in finding out the Nonageſimal Degree or higheſt point of the Ecliptic, its Altitude above the Horizon at any given time, whence the true Altitude of the Sun, Moon, or Planet, may be inveſtigated, and thence their Parallaxes in Altitude, Right Aſcenſion, Declination, Longitude and Latitude, may be determined, which is neceſſary to be known, in the Calculation of Solar Eclipſes, and the Moon's Tranſits over the fixed Stars, of great Uſe in pronouncing the difference of Longitude between theſe Places whence they can be obſerved, as ſhall be made appear in the latter part of this Treatiſe.

There may be as many Variations of this *Problem* as there are *Caſes* in *Right-angled Spherical Triangles*, but as well in this as in the following *Problems*, I ſhall give an actual *Solution* to ſuch only as I ſhall judge more immediately uſeful, and leave the reſt to Exerciſe the Young Beginner.

Prob. I.

Caſe 4.

Given the Sun's Right Aſcenſion 55 deg. 17 min. 03 ſec. ¼, and the preſent Declination 19 deg. 39 min. 10 ſec. ¾, to find the greateſt Declination, Longitude, and the Angle formed by the Ecliptic and the proper Meridian.

Anſwer.

The Sun's Longitude is 57 deg. 33 min. 57 ſec. his greateſt Declination 23 deg. 29 min. and Angle formed by the Ecliptic and proper Meridian 76 deg. 52 min. 59 ſec. ¼.

Caſe 5.

Given the Sun's Longitude 57 deg. 33 min. 57 ſec. and his preſent Declination 19 deg. 39 min. 10 ſec. ¼, required his greateſt Declination, Right Aſcenſion, and the Angle formed by the Ecliptic and proper Meridian.

Anſwer.

The greateſt Declination is 23 deg. 29 min. the Right Aſcenſion 55 deg. 17 min. 03 ſec. ¾, and the Angle formed by the Ecliptic and the proper Meridian 76 deg. 52 min. 59 ſec. ½.

Caſe 6.

Given the Sun's Longitude 57 deg. 33 min. 57 and his Right Aſcenſion 55 deg. 17 min. 03 ſec. ¾, to find his greateſt and preſent Declination, and Angle formed by the Ecliptic and the proper Meridian.

Anſwer.

The greateſt Declination is 23 deg. 29 min. the preſent Declination 19 deg. 39 min. 10 ſec. ¼, and the Angle formed by the Ecliptic and the proper Meridian is 76 deg. 52 min. 59 ſec. ½.

Caſe 7.

Given the greateſt Declination 23 deg. 29 min. and the Angle formed by the Ecliptic and the proper Meridian, to find the Longitude, Right Aſcenſion, and preſent Declination.

Anſwer.

The Longitude is 57 deg. 33 min. 57 ſec. the preſent Declination is 19 deg. 39 min. 10 ſec. ¼, and the Right Aſcenſion 55 deg. 17 min. 03 ſec. ¾.

Caſe 8.

Given the Sun's Longitude 57 deg. 33 min. 57 ſec. and the Angle formed by the Ecliptic and the Meridian 76 deg. 52 min. 59 ſec. ¼, to find the Sun's Right Aſcenſion, greateſt and preſent Declination.

Anſwer.

The Sun's Right Aſcenſion is 55 deg. 17 min. 03 ſec. ¼, his greateſt Declination 23 deg. 29 min. and his preſent Declination 19 deg. 39 min. 10 ſec. ¼.

Caſe 9. *Doubtfull.*

Given the Sun's Right Aſcenſion 55 deg. 17 min. 03 ſec. ¼; and the Angle formed by the Ecliptic and the Meridian 76 deg. 52 min. 59 ſec. ¼, required his Longitude, greateſt and preſent Declination.

Anſwer.

His Longitude is 57 deg. 33 min. 57 ſec. his greateſt Declination 23 deg. 29 min. and his preſent Declination 19 deg. 39 min. 10 ſec. ¼.

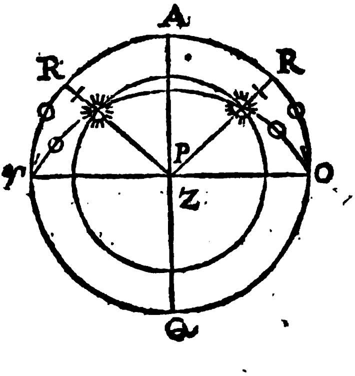
Caſe 10.

Given the Sun's preſent Declination 19 deg. 39 min. 10 ſec. ¼, and the Angle formed by the Ecliptic and the Meridian 76 deg. 52 min. 59 ſec. ¼, to find his greateſt Declination, Longitude, and Right Aſcenſion.

Anſwer.

The greateſt Declination is 23 deg. 29 min. the Longitude is 57 deg. 33 min. 57 ſec. and the Right Aſcenſion 55 deg. 17 min. 03 ſec. ¼.

The Obliquity of the Ecliptic, or the Inclination of the Axis of the Earth to the Plane of the Ecliptic, is the Principal Poſtulatum upon which all the Calculations and Concluſions relating to the Sun, Moon, Planets and Fixed Stars intirely depend, and a Knowledge of the juſt Quantity it contains is of that Importance in all *Aſtronomical Matters*, that the chief *Aſtronomers* in all times have made it one of their firſt Endeavours, to determine it as exactly as poſſible; *Ptolomy*, who lived 140 Years after *Chriſt*, fixed it at 23 deg. 51 min. 20 ſec. the ſame as *Ariſtarchus*, *Eratoſthenes*, and *Hipparchus* had done; each of which lived before *Chriſt*, and the firſt of them 280

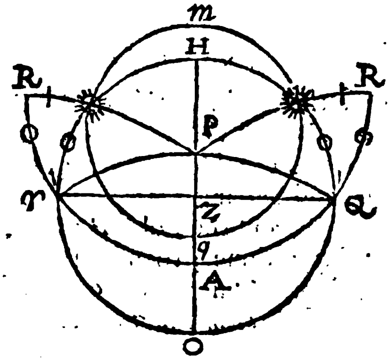


2. About the Point *z* as a Pole, deſcribe the Great Circle ♈ ☉ ☉ *O*, by ſetting off the Tangent of 38 deg. 28 min. from *P* towards Z for the Center, this will repreſent the Horizon.

3. About the point *P* deſcribe the Parallel of Declination, or Parallel of the Sun's diurnal Courſe ☉ *m* ☉, at the diſtance of the Semi-tangens of 70 deg. 29 min. 49 ſec. $\frac{1}{4}$, the Complement of the Declination in this *Caſe*, becauſe the Sun is to the Northward of the Equator.

4. Thro' the Interſection ☉ of this Parallel, with the Horizon before drawn, draw the Hour Circle *P* ☉ *R* and the thing is done; then will ♈ *R* be the Sun's Aſcenſional Difference, and ♈ ☉ the Amplitude, each of which may be meaſured by the *1ſt* and *3d Caſes* of *Problem* the *7th*, of *Section* the *2d*.

3*d*, *The Stereographic Solution of the ſame Caſe upon the Plane of the Horizon.*

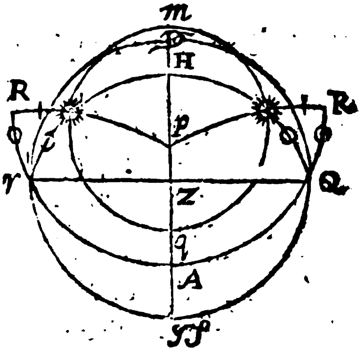


1. Having drawn the Horizon ♈ *HQO*, the Prime Vertical ♈ *Z Q*, and at Right-angles to it the Meridian *H P A Z* ☉, ſet off the Semitangent of 38 deg. 28 min. the Complement of the Latitude from *Z* to *P*, and draw the Six a Clock Hour Circle ♈ *P Q*, by ſetting off the Tangent of 51 deg. 32 min. the Latitude of the Place from Z to *C* for the Center.

2. About the Pole point *P*, draw the Equator ♈ *A Q*; (by *Caſe* the *3d*, of *Problem* the *2d*, of *Section* the *2d*) and the Parallel of the Sun's diurnal Courſe *m* ☉ *q*, at the diſtance of 70 deg. 29 min. 40 ſec. $\frac{1}{4}$, the Complement of the Sun's Declination because the Sun is on the Northern Side of the Equator, (by *Caſe* the *3d*, of *Problem* the *4th*, of *Section* the *2d*.

3. Thro' the Point of Interſection ☉ of this Parallel with the Horizon, draw the Hour Circle *P* ☉ *R*, by *Problem* the 1*ſt* of *Section* the 2*d*; then may ♈ *R* the Sun's Amplitude, and ♈ ☉ the Aſcenſional Difference, be meaſured by the 1*ſt* and 3*d* *Caſes* of *Problem* the 7*th*, of *Section* the 2*d*.

4*th*, *The Stereographic Solution of the ſame Caſe upon the Plane of the Ecliptic.*



1. Having drawn the Ecliptic ♈ ♋ ♎ ♑, the Meridian ♋ *p* π *A* ♑, and at Right-angles to it ♈ π ♎, ſet off the Semi-tangent of 23 deg. 29 min. the diſtance between the Pole of the World and the Ecliptic from π to *p*, then will *p* repreſent the Northern Pole.

2. About the Pole *p*, draw the Equator ♈ *A* *Q*. (by *Caſe* the 3*d*, of *Problem* the 2*d*, of *Section* the 2*d*) and the Parallel *m* ☉ *q* of the Sun's Diurnal Courſe, at the diſtance of 70 deg. 29 min. 40 ſec. ¼. the Complement of the Sun's Declination, becauſe the Sun is to the Northward of the Equator, by *Caſe* the 3*d* of *Problem* the 4*th*, of *Section* the 2*d*.

3. Set off the Semi-tangent of 14 deg. 59 min. the diſtance between the Pole of the Ecliptic and the Zenith, from π to *Z*, and about the point Z as a Pole, deſcribe the Great Circle ♈ ☉ *H* ☉ *Q*, by *Caſe* the 3*d* of *Problem* the 2*d*, of *Section* the 2*d*, this will repreſent the Horizon.

4. Thro' the Interſection ☉ of the Parallel before drawn, and the Horizon, draw the Hour Circle *P* ☉ *R* and the thing is done; and ♈ *R* the Sun's Amplitude, and ♈ ☉ his Aſcenſional Difference, may be meaſured by the 1*ſt* and 3*d* *Caſes* of *Problem* the 7*th*, of *Section* the 2*d*.

But to determine their juſt Quantity by Calculation,

In the Spherical Triangle ♈ ☉ *R* Right-angled at *R*, (*See each of the four Constructions*) are given, ☉ *R* the Sun's preſent Declination, and the Angle ☉ ♈ *R* equal to the Arch *Q O*; the Complement of the Latitude of the Place; whence

Sun touches the Horizon at a quadrantal diſtance from the Eaſt or Weſt points of the Horizon, that is in the North point of the Horizon if the Latitude of the Place and Declination of the Sun are both North, but in the Southern point of the Horizon if the Latitude and Declination are both South, after which it becomes infinite, and the Sun never reaches the Horizon.

2. To find the Aſcenſional Difference ♈ *R*, it will be by the *9th Caſe* of *Right-angled Spherical Triangles*,

As the t, $<\odot$ ♈ *R* : *R* : : t, $\odot$ *R* : S. ♈ *R*; that is,

As the Co-tangent of the Latitude 51.32———9.9000865

To the Radius——— 10.0000000

So is the Tangent of the Declination 19.39.10 ¾———9.5528217

To the Sine of the Aſcenſional Difference 26 42 43———9.6527352

Or Difference between the Right Aſcenſion and the Oblique Aſcenſion, or Deſcenſion.

This in the preſent *Caſe* where the Latitude and Declination are both the ſame way, that is both North, ſubſtracted from the Right Aſcenſion 55 deg. 17 min. 03 ſec. ¼, determined in the firſt *Problem*, will leave 28 deg. 34 min. 20 ſec. ¼, for the Oblique Aſcenſion for the point of the Equator ♈, (*See the firſt Figure*) that Riſes with the Sun is in Antecedence of the point *R* that paſſes the Meridian with it.

And the Sun's Aſcenſional Difference 26 deg. 42 min. 43 ſec. added to the Right Aſcenſion 55 deg. 17 min. 03 ſec. ¼, will give 81 deg. 59 min. 46 ſec. ¼, for the Oblique Deſcenſion for the point ♈, that deſcends with the Sun in the ſame *Figure*, is in conſequence of the point ♈, that paſſes the Meridian with him.

Again, in *Figure* the *5th*, where the Declination is Southerly, that is where the Latitude and Declination are contrary, the ſame Aſcenſional Difference 26 deg. 42 min. 43 ſec. added to the correſpondent Right Aſcenſion 235 deg. 17 min. 03 ſec. ¼, will give 261 deg. 59 min. 46 ſec. ½ for the Oblique Aſcenſion, but ſubſtracted from the Right Aſcenſion 235 deg. 17 min. 03 ſec. ¼, will leave 208 deg. 34 min. 20 ſec. ¼ for the Oblique Deſcenſion or point of the Equator that Sets with him, whence the Right Aſcenſion of the *Mid-Heaven*, or point of the Equator then Culminating is eaſily had, for as in the former *Caſe*, the point of the Equator that aſcends with the Sun, was in Antecedence of the Point of the Equator that Culminated with him, and therefore the Difference muſt be ſubſtracted; ſo here the point of the Equator that aſcends with him being in conſequence, the Difference muſt be added to obtain the aſcending point; on the contrary, as the point that deſcended with the Sun in the former *Caſe* was in conſequence of the point that paſſed the Meridian with him, and therefore the Difference muſt be added; ſo here the point that deſcended with him being in Antecedence of the point that Culminated with him, the Difference muſt be ſubſtracted, to obtain the diſtance of the deſcending point from the *1ſt* point of *Aries*; and this *Law* muſt be obſerved in all the variety of *Caſes* that may be.

This Aſcenſional Difference ♈ *R* and ♈ *R* (*See the 1ſt and 5th Figures*) is ever equal to the Exceſs or Defect of the Semi-Solar Day above or under ſix Hours; and conſequently being added to or ſubſtracted from ſix Hours, according as the *Caſe* requires, will give the Semi-length of the Day and Night, and thence the true time of the Riſing or Setting of the Sun, at any Time or in any Place, thus in *Fig.* the *1ſt*, where the Latitude of the Place and Declination are both North, the Aſcenſional Difference ♈ *R* added to the quadrantal Arch of the Equator ♈ *A*, gives the Arch of the Equator *AR*, correſponding to the Sun's Semi-diurnal Arch *m* ☉; or the ſame Arch of Aſcenſional Difference ♈ *R* taken from the quadrantal Arch ♈ *Q*, leaves the Equatoral Arch *R Q* correſponding to the Semi-nocturnal Arch ☉ *n*; wherefore in the preſent *Caſe* if to 6 h. 00 m. 00 ſec. be added the Aſcenſional Difference 26 deg. 42 min. 43 ſec. which being reduced into Time, by allowing 15 Degrees to an Hour, *&c.* makes 1 h. 46 m. 51 ſec. the Sum 7 h. 46 m. 51 ſec. will be the Semi-length of the Day or the time of the Sun's Setting, and the ſame Aſcenſional Difference 1 h. 46 m. 51 ſec. ſubſtracted from 6 h. 00 m. 00 ſec. will leave 4 h. 13 m. 9 ſec. for the Semi-length of the Night or time of the Sun's Riſing.

On the contrary, in *Fig.* the *5th*, where the Declination of the Sun and Latitude of the Place are contrary, that is one North and the other South, the Arch of Aſcenſional Difference ♈ *R* taken from the quadrantal Arch ♈ *A*, leaves *R A* for the Equatoral Arch anſwering to the Semi-Solar Arch ☉*m*, but added to ♈ *Q* gives *RQ* anſwering to the Semi-nocturnal Arch ☉*n*: wherefore in this *Caſe*, if the Aſcenſional Difference equal to 1 h. 46 m. 51 ſ. be taken from

6 h 00 m. 00 ſ. the Remainder 4 h. 13 m. 09 ſ. gives half the length of the Day or time of the Sun's Setting, and the ſame Arch of 1 h. 46 m. 51 ſ. added to 6 h. 00 m. 00 ſ. gives 7 h. 46 m. 51 ſ. for the Semi-length of the Night or time of the Sun's Riſing.

Or,

Becauſe the Sun is 6 h. 00 m. 00 ſ. in travelling from *q* to *x* or from *x* to *m*, and that in whatſoever place of the Ecliptic he is, is always ſix Hours diſtant from the Meridian when he arrives at the Circle *P* ♈ *S*, and for which Reaſon it is called the Six a Clock Hour Circle, it is evident that the Aſcenſional Difference added to or ſubſtracted from 6 h. 00 m. 00 ſ. gives the time of the Sun's Riſing or Setting: Thus in *Fig.* the 1*ſt*, where the Latitude and Declination are both North, becauſe the Sun Riſes when he arrives at ☉, that is before he arrives at the Six a Clock Hour Circle *P* ♑, If

	h.	m.	ſ.
From 90 degrees	6	00	00
Be taken the Aſcenſional Difference	1	46	51
There will remain the time of the Sun's Riſing	4	13	09

And inaſmuch as the Sun in his return back from the Meridian, arrives at the Six a Clock Hour Circle *P* ♈ *S*, before he gets to the Horizon in the point ♁, it is evident that

	h.	m.	ſ.
If to	6	00	00
Be added the Aſcenſional Difference	1	46	51
The Sum will give the time of the Sun's Setting	7	46	51

On the contrary, in *Fig.* the 5*th* where the Latitude and Declination are contrary, that is, one North and the other South, becauſe the Sun arrives at the Six a Clock Hour Circle *P* ♈ *S* before he aſcends the Horizon.

	h.	m.	ſ.
If to	6	00	00
Be added the Aſcenſional Difference	1	46	51
The Sum makes the time of the Sun's Riſing	7	46	51

And becauſe the Sun in his return back, arrives at the Horizon ⊙ before he arrives at the Six a Clock Hour Circle *P⊙S*, it is manifeſt, that,

	h.	m.	ſ.
If from	6	00	00
Be taken the Aſcenſional Difference	1	46	51
There will remain the time of his Setting	4	13	09

Whence it follows univerſally, that if the Latitude of the Place and Declination of the Sun are both North or both South, the Aſcenſional Difference ſubſtracted from 6 h. 00 m. 00 ſ. will give the time of the Sun's Riſing, and added to 6 h. 00 m. 00 ſ. will give the time of his Setting.

On the contrary, if the Latitude of the Place and Declination of the Sun are of a different Denomination, that is, if one be North and the other South, then the Aſcenſional Difference added to Six Hours, will give the time of the Sun's Riſing, and ſubſtracted from 6 h. 00 m. 00 ſ. will give the time of the Setting.

And becauſe the Hour of the Sun's Setting ſhews the Quantity of time elapſed between Noon and the End of the Day, and which is therefore equal to half the length of the Day, if the Hour of Sun Setting be doubled, it gives the length of the Day or Number of Hours the Sun is viſible above the Hoizon.

Thus in *Caſe* the *1ſt*, where the Hour of the Sun's Setting was 7 h. 46 m. 51 ſ. if this Sum be doubled or multiplyed by 2, the Product 15 h. 33 m. 42 ſ. will give the length of the Day.

And again in the *2d Figure*, where the time of the Sun's Setting was 4 h. 13 m. 09 ſ. this doubled will give 8 h. 26 m. 18 ſ. for the length of the Night.

And again, becauſe the Hour of Sun Riſing ſhews the Quantity of time elapſed, between Midnight and the time of the ſaid Sun Riſing, and is therefore equal to half the length of the Night, if the time of the Sun Riſing be doubled it will give the length of the Night.

Thus in the *1ſt Caſe* if the time of the Sun Riſing 4 h. 13 m. 09 ſ. be doubled, the Sum 8 h. 13 m. 18 ſ. will give the length of the Night, and in the *2d Example* the time of Sun Riſing 7h. 46m. 51ſ. being added to it ſelf, produces 15 h. 33 m. 42 ſ. for the length of the Night, and the ſame *Law* muſt be obſerved in all *Caſes*.

And inaſmuch as in the ſame place, the Sine of the Aſcenſional Difference is to the Tangent of the Declination in a conſtant Ratio, that is, as the Co-tangent of the Latitude is to the Radius, it follows that the greater is the Declination the greater is the Aſcenſional Difference, and the greater is the Inequality between the length of the Days and Nights; and on the contrary, the leſſer is the Declination, the leſſer is the Aſcenſional Difference, and the leſſer is the Inequality between the length of the Days and Nights, and conſequently when the Declination ceaſes the Aſcenſional Difference vaniſhes, and the ſame point of the Equator that Riſes with the Sun, Culminates and Deſcends with him, whence the Oblique Aſcenſion, Right Aſcenſion and Oblique Deſcenſion are the ſame, the Sun Riſes at ſix in the Morning and Sets at ſix in the Evening, and the length of the Day is equal to the length of the Night, to all the Inhabitants upon the Earth.

Again, the Declination remaining the ſame, the only variable parts of the Triangle being the Latitude of the Place, and the Co-tangents of the reſpective diſtances of the Places from the Equator, being reciprocally proportional to the Sines of the reſpective Aſcenſional Differences, and always in a conſtant Ratio, it follows, that the nearer the Places are to the Equator the leſs is the Aſcenſional Difference, or Inequality between the length of the Day and Night, and conſequently under the Equator the Aſcenſional Difference ceaſes, and the Sun conſtantly Riſes at ſix and Sets at ſix, and the length of the Day is equal to the length of the Night throughout the whole Year.

And on the contrary, the greater the Latitude, or the nearer the Places are ſcituated to the Pole, the greater is the Aſcenſional Difference, and the greater the Inequality between the length of the Days and Nights, till when, the Latitude of the place becomes equal to the Complement of the Sun's Declination, and the ſame way the Aſcenſional Difference is the greateſt, and the Sun never Sets but continues above the Horizon for the ſpace of one natural Day or 24 Hours, but if the Declination and Latitude are contrary the Sun never appears, and the length of the Night becomes equal to the length of the Day in the oppoſite part, after which the Aſcenſional Difference becomes infinite; and the time of the Sun's continuance above or belove the Horizon, according to the different times of the Year, conſiſts of more than 24 Hours, as ſhall be ſhewn in the Sequel of this *Problem*.

And becauſe in the ſame Latitude to the ſame Declination, there will always be the ſame Aſcenſional Difference, and conſequently the ſame Inequality between the length of the Day and the length of the Night, and inaſmuch as the Declination depends upon the diſtance of the Sun from the Equinoctial points, and is the ſame at equal diſtances, it follows,

1. That when the Sun is at the ſame diſtance from either of the Equinoctial points, and on the ſame Side of the Equator, that the time of his Riſing and Setting and length of the Day and Night, is the ſame: For *Example*, when the Sun is in ♉ 27 deg. 33 min. 57 ſec. which happens upon the *7th* of *May*, or ♌ 2 deg. 26 min. 03 ſec. which happens on the *14th* of *July*, he Riſes at 4 h. 13 m. 09 ſ. and Sets at 7 h. 46 m. 51 ſ. the length of the Day at each time is 15 h. 33 m. 42. ſ. and the length of the Night is 8h 26m. 18 ſ. for the Semi-diurnal Arch *m* ☉, and the Semi-nocturnal Arch ☉ *q* (*See the 1ſt Figure*) in each *Caſe* is the ſame; in like manner when the Sun is in 27 deg. 33 min. 57 ſec. of *Scorpio*, which happens on the 8*th* of *November*, or in ♒ 2deg. 26min. 03ſec. which happens upon the 12*th* of *January*, at each of which times the Sun is at the ſame diſtance from, and on the ſame Side of the Equator, he Riſes at 7 h. 46 m. 51 ſ. and Sets at 4 h. 13 m. 09 ſ. the length of each Day is 8 h. 26 m. 18 ſ. and the length of the Night 15h. 33m. 42 ſ. for the Semi-diurnal and Semi-nocturnal Arches (*See the 5th Figure*) *m L* and *L n* are the ſame.

2. That when the Sun is at the ſame diſtance from either of the Equinoctial points, but on contrary Sides of the Equator, that the time of the Riſing and length of the Night in one *Caſe*, is the ſame with the time of the Setting and length of the Day in the other *Caſe* and the contrary,

Thus for *Example*, on the *7th* of *May*, when the Sun is in ♉ 27 deg. 33 min. 57 ſec. or on the *14th* of *July* when he is in *Leo* 02 deg. 26 min. 03 ſec. the time of his Riſing 4 h. 13 m. 09 ſ. and the length of the Night 8 h. 26 m. 18 ſ. is the time of his Setting and length of the Day, on the 8*th* of *November* when he is in *Scorpio* 27 deg. 33 min. 57 ſec. or on the 12*th* of *January*, when he is in *Aquarius* 02 deg. 26 min. 03 ſec. at which two latter times he is as far diſtant from either of the Equinoctial points in the Southern Semi-circle of the Ecliptic, as he is at the two former times in the Northern Semi-circle, and the ſame will happen at all oppoſite points of the Ecliptic, or equal Diſtances from either Equinoctial point numbred in contrary Semi-circles, as is very evident to the Intelligent *Reader* if he compares the *1ſt* and *5th Figures* together, whence it will plainly appear, that the Aſcenſional Difference ♈ *R* in one, is equal to the Aſcenſional Difference ♈ *R* in the other, and conſequently the Semi-nocturnal Arch ☉ *q*, or time of the Riſing in the *1ſt Figure*, is the ſame, or equal to the Semi-diurnal Arch ☉ *x*, or time of Riſing in the *5th Figure*, and the Difference between the Semi-diurnal and Semi-nocturnal Arches in each *Caſe*, being equal in Quantity to the double of the Aſcenſional Difference.

The Oblique Aſcenſion and Deſcenſion, the time of the Sun's Riſing and Setting, and the length of the Day and Night may be inveſtigated directly, without the help of the Aſcenſional Difference.

For in the Spherical Triangle ☉ *P O* (*See the 1ſt Figure*) Right-angled at *O*, are given, ☉ *P* equal to the Complement of the Sun's Declination, or his diſtance from the Pole, and *O P* the Latitude, or height of the Pole above the Horizon, whence to find the Angle at *P* equal to the Arch of the Equator *Q R*, equal to the Semi-nocturnal Arch or half the length of the Night, and conſequently the time of Sun Riſing, it will be by the *12th Caſe of Right-angled Spherical Triangles*,

As the *R* : t, *OP* : : c t, *P*☉ : c s, ☉*PO*; that is,

As the Radius	10.0000000
To the Tangent of the Latitude 51.32	10.0999135
So is the Tangent of the Declination 19.39.10¾	9.5528217
To the Co-ſine (in this *Caſe*) of the Semi-nocturnal Arch *R Q*, equal to 63.17.17	9.6527352

Which reduced into time gives 4 h. 13 m. 09 ſ. for the time of the Sun's Riſing, and ſubſtracted from 12 h. gives 7 h. 46 m. 51 ſ. for the time of Setting.

Again, the ſame Semi-nocturnal Arch 4 h. 13 m. 09 ſ. being doubled, gives 8 h. 26 m. 18 ſ. for the length of the Night, and this taken from 24 h. gives 15 h. 33 m. 42 ſ. for the length of the Day.

Again, the Complement of this Arch 26 h. 42 m. 43 ſ. ſubſtracted from and added to the Sun's Right Aſcenſion 55 deg. 17 min. 03 ſec. ¼, will give for the Oblique Aſcenſion 28 deg. 34 min. 20 ſec. ¼, and for the Oblique Deſcenſion 81 deg. 59 min. 46 ſec. as before determined.

Or,

The ſame Arch of 63 deg. 17 min. 17 ſec. added to, and ſubſtracted from the Sun's Right Aſcenſion 55 deg. 17 min. 03 ¼, will give 118 deg. 34 min. 20 ſec. ¼, and 351 deg. 59 min. 46 ſec. ¾ for the Right Aſcenſion or Degree under the lower part of the Meridian, at the time of the Sun's Riſing and Setting, whence the aſcending and deſcending Points are eaſily had.

But if the Latitude of the Place and Declination of the Sun are contrary, that is, if one be North and the other South, then the fourth Arch before found gives the Semi-diurnal Arch, or half the length of the Day or time of the Sun's Setting.

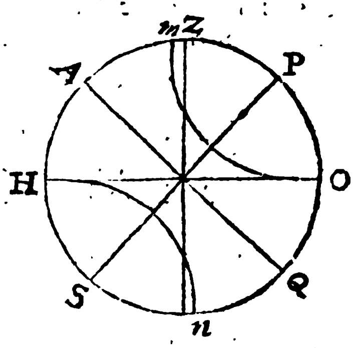
Thus in *Figure* the *5th*, where the Declination is the ſame in Quantity as in the *1ſt Figure* but South, and the Latitude North, the Angle *HS* ☉ equal to the Angle ☉ *PO* of the Triangle ☉ *PO* in the *1ſt Figure*, equal to the Arch *AR*, or Semi-diurnal Arch, gives 4 *h.* 13 m. 09 ſ. for the time of the Sun's Setting, and taken from 12 Hours, gives 7 h. 46 m. 51 ſ. for the time of the Sun's Riſing.

And the ſame Arch 4 h. 13 m. 09 ſec. doubled, gives 8h. 26m. 18 ſec. for the length of the Day, and this taken from 24 Hours, gives 15 deg. 33 min. 42 ſec. for the length of the Night.

In like manner, the ſame Arch of 63 deg. 17 min. 17 ſec. taken from the Sun's Right Aſcenſion 235 deg. 17 min. 03 ſec. ¼, will leave 171 deg. 59 min. 46 ſec. ¾ for the Right Aſcenſion of the Mid-heaven, at the time of the Sun's Riſing; and added to the Right Aſcenſion 235 deg. 17 min. 03 ſec. ¼, will give 298 deg. 34 min. 20 ſec. ¼, for the Right Aſcenſion of the Mid-heaven, at the time of the Setting, whence the Oblique Aſcenſion and Deſcenſion is eaſily had; and the ſame *Law* muſt be obſerved where the Latitude is South, and the Declination either the ſame or contrary; all which is abundantly manifeſt to any who will diligently conſider and compare the two *Conſtructions* together.

It has been aſſerted in *Page* the *268th* that when the Declination of the Sun becomes equal to the Complement of the Latitude of the Place, and of the ſame Denomination, that then the Sun never deſcends below the Horizon, but touches in the oppoſite part of the Meridian, and on the contrary, that when the Declination is of a different Denomination, that then the Sun never aſcends above the Horizon, but only touches it when upon the Meridian.

Let it therefore be required to find the Beginning, Duration and End of the longeſt Day and the longeſt Night at the North *Cape* in the Latitude of 71 deg. 25 min. North.



In the adjacent *Figure*, having ſet off the Latitude of the North *Cape* 71 deg. 25 min. from *O* to *P*, and drawn the Axis *P S*, and at Right-angles to it the Equator *AQ*, it is manifeſt, that when the Declination of the Sun or his Diſtance from the Equator towards the North Pole *P*, becomes equal to *OQ*, that is in the preſent *Caſe* equal to 18 deg. 35 min. (the Complement of the Latitude, that then the Parallel *m O* which is called the the Circle of perpetual Apparition, becomes the Parallel of the Semi-diurnal Courſe, and that in his Revolution round the Heaven's he never deſcends below, but only touches the Horizon in the point *O*, when he is upon the Northern part of the Meridian, and after that he continues viſible above the Horizon, till having paſſed thro' all the varietys of Northern Declinations, from 18 deg. 35 min. North, to the greateſt 23 deg. 29 min. and from thence back again to 18 deg. 35 min. North, the ſame Parallel *m O* becomes the Parallel of his diurnal Courſe.

On the contrary, when his Declination or Diſtance from the Equator towards the South becomes equal to *AH* or *QN* in the preſent *Caſe* 18 deg. 35 min. the Parallel *H n*, or Circle of perpetual Occultation, becomes the Parallel of his Nocturnal Courſe, at which time he never aſcends above the Horizon but only touches it in the point *H*, when he is upon the Southern part of the Meridian, and after that he continues inviſible below the Horizon, till having paſſed thro' all the varietys of the Southern Declination from 18 deg. 35 min. South, to the greateſt 23 deg. 29 min. and from thence back again to 18 deg. 35 min. the ſame Parallel *H n* becomes again the Parallel of his Nocturnal Courſe.

And to determine the exact time of the Begining, End and Duration of the longeſt Day and Night by Calculation, it will be by the 12th *Caſe* of *Problem* the 1ſt, of this *Section*.

As the Sine of the greateſt Declination 23.29 ——— 9.6004090

To the Co-ſine of the given Latitude 71.25 ——— 9.5603597
So is the Radius ——— 10.0000000

To the Sine of the correſponding Arch of Long. 53.6.20 9.9029507
counted from the next Equinoctial point

This when the Sun's Declination is North and increaſing, gives the Sun's Place in *Taurus* 23 deg. 06 min. 20 ſec. and which happens upon the 3*d* of *May*, when the longeſt Day begins

But when the Declination is North and decreaſing, it ſhews the Sun to be in *Leo* 6 deg. 53 min. 40 ſec. and this happens upon the 19*th* of *July*, when the longeſt Day ends; and conſequently the longeſt Day conſiſts of 77 Natural Days.

When the Sun's Declination is South and increaſing, it gives his Place in *Scorpio* 23 deg. 06 min. 20 ſec. and which happens upon the 4*th* of *November*, when the longeſt Night begins.

And laſtly, when the Sun's Declination is South and Decreaſing, it gives his Place in *Aquarius* 6 deg. 53 m. 40 ſec. and this happens on the 16*th* of *January*, when the longeſt Night ends; and conſequently the length of the longeſt Night conſiſts of 73 Natural Days, ſhorter than the longeſt Day by four Natural Days.

On the contrary, in the Latitude of 71 deg. 25 min. South, the longeſt Day begins on the 4*th* of *November*, and ends on the 16*th* of *January* following, and contains 73 Natural Days, and the longeſt Night begins on the 3*d* of *May*, and ends on the 19*th* of *July*, and is equal in Quantity to 77 Natural Days, longer than the longeſt Day by four Natural Days; and the ſame *Law* muſt be obſerved in all other *Caſes* of this kind.

By the help of the Table of the Sun's Declination, the beginning and end of the longeſt and ſhorteſt Day and Night, in any Place may be found by Inſpection, by finding out the given Declination in the given Tables, and the top of the Column will ſhew the Month, and in the firſt Column upon the left Hand you have the required Day, when either the longeſt or ſhorteſt Day and Night begins or ends.

And inaſmuch as the Sine of the correſpondent Arch of Longitude is reciprocally proportional to the Sine of the Latitude, it follows, that the greater the Latitude, the leſs will be the correſpondent Arch of Longitude, till at laſt the Latitude being at the greateſt, the correſpondent Arch of Longitude vaniſhes, and the civil Day is equal to the length of the Year, and this happens under either Pole.

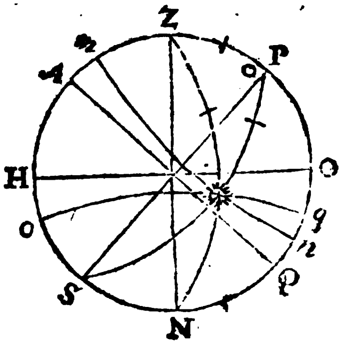
In the Year 1695, at a Town called *Pello*, in the Latitude of 65 deg. 53 min. ten Miles to the Northward of *Forneo* in the Weſtern *Bothnia*, on the 14*th* of *June*, at twelve Hours *P. M.* when the Center of the was Sun depreſſed 40 min. below the Horizon, he was ſeen by the means of the Refraction at the Altitude of two Diameters; and the *Dutch* in *Nova Zembla* for the ſame Cauſe have ſeen him when he ought to have been depreſſed 4 deg. below the Horizon, and if at the *North Cape*, where our Ships that go to *Archangel* conſtantly paſs and repaſs, we allow but two Degrees for the Refraction, he will be ſeen conſtantly above the Horizon, from the 26*th* of *April* to the 27*th* of *July* following, during the ſpace of 92 Natural Days, that is longer by 15 Natural Days than he would do were it not for this Refraction, and for the ſame Reaſon the longeſt Night will not commence till the 13*th* of *November*, and will end on the 7*th* of *January* following, and will be equal in Quantity to 55 Natural Days, only, ſhorter than it would otherwiſe be were it not for the Refraction, by 18 Natural Days; whence it comes to paſs that the apparent longeſt Day, or time that the Sun is viſible above the Horizon, is longer than the apparent longeſt Night or time that he is abſent, is equal to 37 Natural Days, and this Refraction muſt be taken Notice of in all *Calculations* whatſoever, to obtain the length of the apparent time of the Sun's Abſence or Preſence, and the ſame will hold good in the *Fixed Stars* or *Planets*.

This I judged proper to take Notice of, to obviate any Objection that might appear againſt the former *Calculations* to thoſe who are ignorant of the Cauſe, and it would be of no ſmall Service to the *Science*, if our *Mariners* as they Sail to and fro in thoſe Parts, would Note in their *Journals*, the exact time when the Sun begins to be ſeen above the Horizon during the whole 24 Hours, and how long he continues ſo to do; for as the exact time may eaſily be obtained by *Calculation*, this compar'd with the apparent, will readily ſhew the true Quantity of the Refraction.

It is confirmed by daily Experience, that the *Atmoſphere* continues illuminated for ſome time after the Sun is Set, and begins to be illuminated before he Riſes, and by *Aſtronomers* it is found to laſt in the Evening till the Sun is deſcended 18 degrees below the Horizon, when the ſmalleſt Fixed Stars that can be ſeen with the naked Eye, and ſuch are thoſe of the *6th* Magnitude, become viſible, and that it begins in the Morning when the Sun is within 18 degrees of aſcending the Horizon, and when theſe Stars begin to vaniſh, this commencement of the Illumination of the *Atmoſphere*, when the Sun begins to ſcatter his Light thro' the Heaven's or firſt dawning of the Day, is commonly called the break of Day or begining of Morning Twilight, as in the Evening when this Illumination ceaſes, and the *Atmoſphere* is wholly deſtitute of the Solar Rays, it is called the end of Evening Twilight, or beginning of Total Darkneſs.

Let it therefore be required on the *7th* of *May*, when the Sun's Northern Declination is 19 deg. 39 min. 10 ſec. ¾, to find the begining of Twilight in the Morning, and the end of the Twilight in the Evening at *London* in the Latitude of 51 deg. 32 min. North.

The Stereographical Solution.



Having drawn the Meridian Z *O N H*, the Horizon *HO*, the Prime Vertical Z*N*, the Axis *PS*, the Equator *AQ*, and the Parallel of Nothern Declination *m* ⊙ *n*, as in the firſt *Figure*, about the Zenith Z draw the Small Circle *O* ⊙ *q*, at the diſtance of 108 deg. or which is the ſame thing, a Parallel of Depreſſion of 18 degrees, where this interſects the Parallel of Declination before drawn, as in ⊙, gives the Place of theSun at the beginning of Twilight in the Morning, or at the end of Twilight in the Evening.

2. Thro' the point ⊙ and the Zenith and Nadir, draw the Vertical Circle Z ⊙ *N*, alſo thro' the Poles *P* and *S*, and the ſame point ⊙, draw the Hour Circle *P* ⊙ *S* and the thing is done; and the Angle ⊙ *P Q* or Arch of the Equator, is equal to the time of the beginning of Twilight in the Morning, and the Angle *A P* ⊙, or Arch of the Equator, *A x*, will give the end of the Twilight in the Evening, each of which may be meaſuredby *Caſe* the *2d* of *Problem* the *10th*, of *Section* the *2d*.

And to find it by *Calculation*, by the *12th Caſe* of *Oblique-angled Spherical Triangles.*

To *P*☉ equal to the Complement of the Declination (in this *Caſe*) equal to 70 deg. 20 min. 50 ſec. add *ZP*, equal to the Complement of the Latitude of 38 deg. 28 min. and *Z*☉ equal to the Depreſſion of the Sun plus 90 degrees, equal to 108 degrees, and from half the Sum equal to 108 deg. 24 min. 25 ſec. ſubſtract *Z*☉ the diſtance of the Sun from the Zenith, equal to 108 degrees, and keep the Remainder 24 min. 25 ſec. then,

To the Arith. Comp. of the Co-ſine of the Latit.=51.32 0.2061683
Add the Arith. Comp. of the Co-ſine of the Decl. 19.39.10 0.0260658
Alſo the Sine of the half Sum=108.24.25 ——— 9.9771919
And the Sine of the Remainder=00.24.25 ——— 7.8513207

Half the Sum of theſe four Sines ——— 9.0303734

Will be the Sine of 6 deg. 09 min. 23 ſec. which being doubled gives 12 deg. 18 min. 46 ſec. for the Angle ☉*PQ* or Arch of the Equator *XQ*, and being reduced into time, gives 0 h. 49 m. 15 ſ. for the begining of Twilight in the Morning, and taken from twelve Hours gives the Angle *AP*☉ or Arch of the Equator *A x*, equal to 11 h. 10 m. 45 ſ. for the end of wilight in the Evening.

It has been already determined, that when the Sun has 19 deg. 39 min. 10 ſec. ¾ of North Declination, that then he Riſes at 04 h. 13 min. 09 ſec. when the Morning Twilight ends; wherefore to find the duration of Twilight in the Morning,

From the end of Morning Twilight ——— 4 13 09
Take the begining of Morning Twilight ——— 0 49 15

Remains the duration of Twilight in the Morning ——— 3 25 54

And becauſe when the Sun Sets the Evening Twi-light begins,

h. m. ſ.
From the end of the Evening Twilight ——— 11 10 45
Take the time of the Sun's Setting ——— 7 46 51

Remains the duration of Twilight in the Evening ——— 3 23 54

And becauſe the Sun has the ſame Northern Declination on the 14*th* of *July* that he has on the 7*th* of *May*, it follows, that on each of thoſe Days, the Day breaks, or firſt begins to appear at 4 h.

15 m. in the Morning, and the Sun Riſes at 4 h. 13 m. 09 ſ. and Sets at 7 h. 46 m. 51 ſ. and the Twilight ends at 11 h. 10 m. 45 ſ. at Night, when the duration of Twilight is 3 h. 23 m. 54 ſ. When the Declination of the Sun becomes equal to the Difference between the Complement of the Latitude and the Depreſſion, and both the ſame way, the Solar Rays never quit the Hemiſphere; therefore, from the Complement of the Latitude of *London* equal to 38 deg. 28 min. take the Depreſſion 18 deg. 00 min. the Remainder 20 deg. 28 min. is equal to the Declination when the total Darkneſs ceaſes, and this anſwering to the 11*th* of *May* and the 11*th* of *July*, according to *Caſe* the 2d of *Problem* the 1*ſt*, of this *Section*, it follows, that in *London* from the 11*th* of *May* to the 11*th* of *July*, there is no total Darkneſs, and that the Twilight continues from the time of the Setting of the Sun, till the time of his Riſing again.

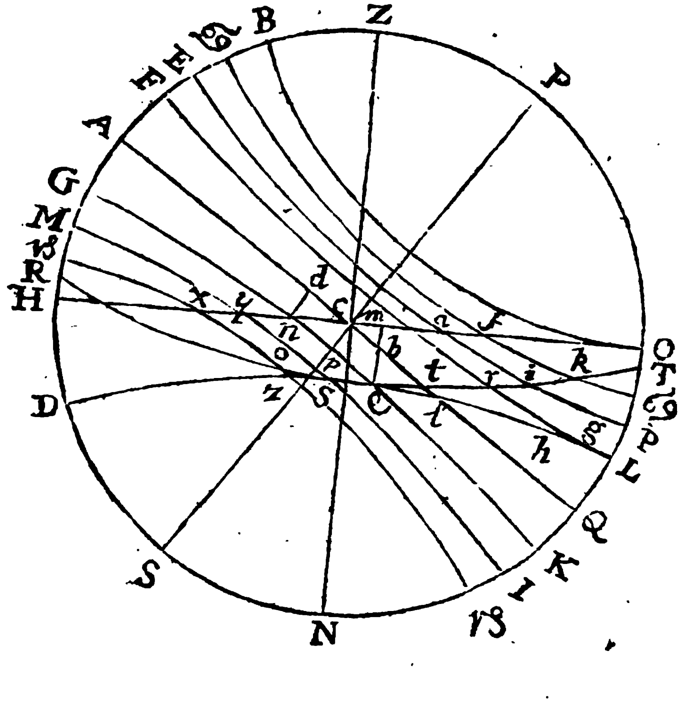
After the ſame manner of Computation it will be found that on the 12*th* of *January* and the 8*th* of *November*, when the Sun has the ſame Quantity of South Declination as in the former *Example*, he had North Declination, that the begining of Twilight in the Morning or firſt dawn of the Day, will happen at 5 h. 42 m. 05 ſ. and that the end of Twilight in the Evening or beginning of total Darkneſs, will be at 6 h. 17 m. 55 ſ. after it has continued for the ſpace of 02 h. 04 m. 16 ſ. ſhorter than the duration in the former *Caſe*, when the Sun had the ſame Quantity of North Declination, by the ſpace of 01 h. 19 m. 08 ſ. the Reaſon of which will be very evident from what follows.

In the adjacent Figure, let *ZONH* repreſent the Meridian, *HO* the Horizon, *AQ* the Equator, ♋ ♋; *Ep*, *FL*, *GK*, *HI*, and ♑ ♑, ſeveral Parallels of Declination and *Dep* the Parallel of Depreſſion or Circles bounding the Twilight, at which Circle, whenſoever the Sun by his apparent diurnal Motion arrives, the Morning Twilight begins or the Evening Twilight ends, in whatſoever Parallel of Declination the Sun is in.

In this Figure the ſeveral Arches *xz*, *ys*, *ne*, &c. of the ſeveral Parallels of Declination, intercepted between the Horizon *HO* and the Parallel of Depreſſion *Dep*, will be the ſeveral Arches of Twilight, and as each of theſe Arches bears a greater or leſſer Proportion to the Circles or Parallels of Declination, of which each Arch is a part, ſo the duration of Twilight will be longer or ſhorter, when the Sun is in theſe reſpective Parallels.

Let the Great Circle *R e F* be drawn, to touch the Parallel of Depreſſion as in *e*, and the Circle of perpetual Apparition *BO*, and becauſe the Horizon *HO* touches the ſame Circle of perpetual Apparition in the point *O*, the Angles *d c n*, *m c b*, and *b e t*, formed by the Interſection of the Equator and theſe two Circles, *viz.* the Great Circle *R e T* and the Horizon *H c O* will be equal, and the ſeveral Arches of Declination *f K*, *a i*, *m r*, &c. intercepted between the Horizon *HO* and the Great Circle *R e T* will be ſimilar, whence it follows,

1. That while the Sun is on the ſame ſide of the Equator with the viſible Pole, the Twilights do conſtantly grow longer according as he approaches nearer to the ſame Pole, for the Twilight Arch *b m* is longer in being deſcribed than *b c*; the Twilight Arch *g a* is longer than *b m*, and the Twilight Arch *p a* is longer than *b m*, till at laſt when the Sun is got to ſuch a diſtance from the Equator, that the Twilight Arch *f* [illegible] interſects the Meridian in ſome point between *p* and *o*, it will never meet the Parallel of Depreſſion or bounding Circle *R e F*, but continue during the whole Night.



And inaſmuch as the ſeveral Arches *f k*, *a i*, *m r*, *c t*, &c. are ſimilar, it follows, that when the Sun deſcribes the Parallel of Declination *n e*, that cuts the Parallel of Depreſſion or bounding Circle *D e p* in the ſame point *e*, when the Great Circle *R e T* touches it, that then duration of Twilight will be the ſhorteſt, for when the Sun deſcribes the Parallel of Declination *k s*, or the Southern ſide of the Parallel *n e*, the Twilight Arch *s y* is longer than *y p*, equal to *n e*, and when the Sun deſcribes the Parallel of Declination ♑ ♑, the Arch of Twilight *z y* greater than *x o*, equal to *y p*, equal to *n e*, is greater than *y s*.

Hence we are taught how to account for that great Difference that ariſes between the increaſe and decreaſe of the Twilight, and the increaſe and decreaſe of the Days and Nights: For tho' in all Northern Latitudes the Days conſtantly decreaſe as the Nights increaſe, during the time the Sun is moving from the *Summer Solſtice* till he arrives at the *Winter Solſtice*, and that the Twilights and Days are at the longeſt when the Sun is in the firſt point of *Cancer*, and both begin to decreaſe together, yet the duration of Twilights do not continually decreaſe till the Sun gets into the firſt point of *Capricorn*; but there is a certain point between them, to which when the Sun arrives, the Twilights are then at the ſhorteſt; after which they begin to increaſe again in length, tho' the Days go on to decreaſe, and continue ſo to do the farther the Sun goes to the Southward; and on the contrary, tho' the Days begin to increaſe after the Sun is paſt the firſt point of *Capricorn*, and continue ſo to do till he arrives at the firſt point of *Cancer*, yet the Twilights begin to ſhorten, and continue ſo to do till he comes to a certain point, as far diſtant from the Solſtitial point in the aſcending Semi-circle of the Ecliptic, as in the former *Caſe* he was diſtant in the deſcending Semi-circle, when the duration again becomes ſhorteſt; after which they begin to increaſe again together, and to determine the diſtance of the Parallel of Declination from the Equator when this ſhall happen.

Thro' the point *e*, where the Arch of ſhorteſt Twilight cuts the Parallel of Depreſſion *D e p*, draw the Vertical Circle *Z b e*, to cut the Equator in the point *b*, and becauſe the Great Circle *R e T*, which touches the Circle of perpetual Apparition, touches likewiſe the Parallel of Depreſſion in the ſame point *e*, the Angles *d m b* and *b e t* of the Triangles *d m b* and *b e t* will be both Right-angles, and becauſe the Horizon touches the ſame Circle of Apparition, the Angles *m c b*, and *b t e* formed by the Equator, and the Horizon and the Great Circle *R e T*, will be alſo equal; but the Angles *c b m* and *e b t* are equal being oppoſite, wherefore the Triangles *c b m* and *b e t* are Equiangular, and conſequently Equilateral, and conſequently the Arch *m b* equal to *b e*, is equal to half the Arch *m e*, equal to half the Depreſſion, equal to 9 Degrees; and the Arch *e t* will be equal to *c m*, and to find its length by Calculation, in the Spherical Triangle *b c m* Right-angled at *m* are given, the Arch *b m* equal to half the Depreſſion, equal to 9 Degrees, and the Angle *m c b* equal to the Arch *OQ*, equal to the Complement of *PO* the Elevation of the Pole, whence to find the Arch *c m*, it will be by the *9th Caſe of Right-angled Spherical Triangles*,

As t, <*b c m* : *R* : : t, *b m* : S. *c m*; that is,

As the Tangent Complement of the Latitude of the Place,
To the Radius;
So is the Tangent of the half of the Depreſſion,
To the Sine of a fourth Arch.

Again,

Thro' the point *n*, where the Parallel of ſhorteſt Twilight cuts the Horizon, draw the Arch of the Meridian *h d*, then will the Arch *n d* be equal to the Declination of the Sun, or the diſtance of the Parallel of Declination from the Equator, when the ſhorteſt Twilight happens, and becauſe the Arch *n e* of ſhorteſt Twilight is parallel to the Equator, the Arches *n c* and *e t* will be equal, and conſequently will be equal to *c m*, the fourth Arch before found; wherefore in the Spherical Triangle *n c d* Right-angled at *d*, are given, the Angle *n c d* equal to *m c b*, equal to the Arch *OQ* the Complement of the Latitude, and the Arch or Side *n c* equal to *c m*, the fourth Arch before found, whence to find the Arch of Declination *n d*, it will be by the *5th Caſe of Right-angled Spherical Triangles*,

As *R* S. <*n c d* : : S. *n c* : S. *n d*; that is,

As the Radius,
To the Sine of the Complement of the Latitude of the Place;
So is the Sine of the fourth Arch before found,
To the Sine of the Arch of Declination ſought.

Again,

Becauſe the t, <*c* : *R* : : t, *b m* : S. *c m*; and that the Tangent of an Arch is to the Radius, as the Sine of the ſame Arch is to its Coſine.

The S. $<c$: c s, $<c$ :: t, mb : S. cm.

Wherefore, the c s, $<c$: S. $<c$ :: S. cm : t, mb, and by the latter Proportion the R : S. $<c$ :: S. cn = S. cm : S. nd.

Wherefore, becauſe the two middle Terms in each Set of Proportionals are the ſame, it will be

As R : c s, $<c$ :: t, mb : S. nd; that is univerſally,

As the Radius,

To the Sine of the Latitude of the Place;

So is the Tangent of half the Depreſſion,

To the Sine of the Declination of the Sun when the ſhorteſt Twilight happens.

Wherefore if it be required to find when the ſhorteſt Twilights happen at *London* in the Latitude of 51 deg. 32 min. North, it will be

As the Radius	10.0000000
To the Sine of the Latitude of 51.32	9.8937452
So is the Tangent of the half Depreſſion 9.00	9.1997125
To the Sine of the Declination South 7.07⅓	9.0934577

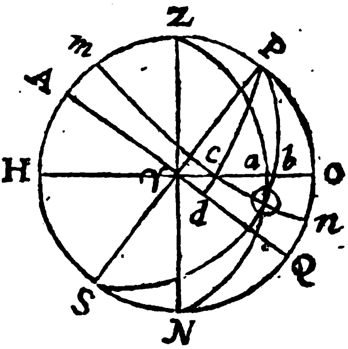
And this happening between the 14*th* and 15*th* of *February*, and the laſt Day of *September* and the firſt of *October*, it follows, that upon theſe Days at *London* the duration of Twilight will be the ſhorteſt, when it will continue for the ſpace of 1 h. 56 m. 32 ſ. only.

And becauſe the Sine of the Declination when the ſhorteſt Twilight happens, is to the Tangent of the half Depreſſion in a conſtant Ratio; that is, as the Sine of the Latitude of the Place to the Radius, it follows, that when the Latitude vaniſhes the Declination of the Arch of ſhorteſt Twilight ceaſes, and the Sun then moves in the Equator; and this happens to all thoſe People who dwell under the Line, when the Twilights at all times and even when they are at the greateſt, and which happens at the times of the *Solſtices*, are quickly over, becauſe the Sun conſtantly deſcends almoſt perpendicularly; and on the contrary, the greater the Obliquity of the Sphere, that is, the greater the Latitude of the Place the more obliquely does the Sun deſcend, and the longer is the duration of Twilight, till when the Latitude of the Place becomes equal to 48 deg. 31 min. the Twilight upon the longeſt Day continues from Sun Set to Sun Riſe, that is during the whole Night; and this, as the Latitude of the Place increaſes, increaſes likewiſe in Number of Days before and after the longeſt Day, till at laſt when the Latitude becomes equal to 90 Deg. that is, in a Parallel Sphere the Twilights laſt for ſeveral Months; ſo that by the help of this and the Refraction the Inhabitants (if any there be) have the Benefit of the Sun's Light for almoſt all the Year.

This Illumination of the Heav'ns after the Sun is Set and before he Riſes, proceeds from the Reſiſtance that the Solar Rays meet with in paſſing thro' the *Atmoſphere*, by which Means they are ſtrongly reflected back upon us, and we made to receive the Benefit of his Light for ſome time before he Riſes and after he Sets; and as this middle State between Day and Night is of a longer or ſhorter duration, according to the different Heights of the *Atmoſphere*, it follows, that in *Winter* time when the Air by Cold is condenſed, that the *Atmoſphere* is low, and that the Twilights are ſooner over; and on the contrary, in the *Summer* time when the Air is rarified by the Heat of the Sun, the *Atmoſphere* is higher, and conſequently the Twilights laſt longer; ſo that in an Evening after a warm Day they are of a longer duration than they are in a Morning, and after a Series of hot Days than after a Series of cold Weather; and this is one Reaſon that with us ever after the *Summer Solſtice*, when the Weather has been fine, that a Man may ſee to Read almoſt at Midnight.

And here we ſee one great Uſe and Benefit of the *Atmoſphere* that encompaſſeth the Earth, for was it not for this, the Sun immediately before his Setting and after his Riſing, would ſhine as bright as at Noon, and the Moment that he is deſcended below the Horizon at his Setting, it would be as dark as at Midnight, as it would continue to be till the Moment the Sun aſcends in the Morning, which ſuddain Change from the greateſt Light to the greateſt Darkneſs, would certainly be very inconvenient to Mankind: And beſides we have this farther Benefit from the *Atmoſphere*, that even while the Sun ſhines it illuminates the whole Face of the Heavens, and makes it appear bright; and were it not for this, that part of the Heavens only would appear to ſhine in which the Sun is placed, and a Spectator if he ſhould turn his Back upon the Sun, would immediately perceive it as Dark as Night, and the ſmalleſt Stars would be ſeen to appear as they do now in a clear Night, ſince there would be no Subſtance to Reflect the Solar Rays to our Eyes, and even at Night our Artificial Lights would be but of little Service to us.

After the ſame manner as we find the Time of the Beginning and End of Morning and Evening Twilight, we may find the true Time of the apparent Riſing and Setting of the Sun, and the Length of the apparent Day and Night; for the Horizontal Refraction at *London*, being 33 min. 00 ſec. when the Sun's Center really touches the ſenſible Horizon, it appears by means of the Refraction at the Altitude of 33 min. 00 ſec. and conſequently when his Center appears to Set it is really then depreſſed 33 min. 00 ſec. below the Horizon, whence it comes to paſs, as has been before hinted, that the Time of the true Setting of the Sun happens before the Time of the apparent Setting; and conſequently that the Length of the apparent Day is longer than the Length of the Aſtronomical Day; and on the contrary, that the apparent Riſing preceeds the true Riſing, and cauſes the Length of the apparent Night to be leſs than the Length of the Aſtronomical. For tho' the Sun apparently riſes when he comes to the point ☉, at which Time the Arch *a* ☉ of the Vertical Circle *Z* ☉ *a*, is 33 min. 00 ſec. yet he does not really aſcend the Horizon, till he arrives at the point *c*, and on the contrary, in returning from *m* back again, tho' he really Sets when he arrives at *e*, yet he does not apparently Set till he comes to ☉, and to determine the Arch of the Equator *d r*, or Difference of Time between the real and apparent Times of the Riſing and Setting of the Sun, in the Oblique-angled Spherical Triangle *Z P* ☉ are given *Z P* the Complement of the Latitude of the Place 38 deg. 28 min. 00 ſec. *P* ☉ the diſtance of the Sun from the North Pole, 70 deg. 20 min. 49 ſec. ½, and Z ☉ the diſtance of the Sun from the Vertex, at the Time of the Beginning and End of the apparent Day 90 deg. 33 min. 00 ſec. whence by the help of the 11*th Caſe* of *Oblique-angled Spherical Triangles*, and according to the former Method of Calculation, the Angle ☉ *P Q*, the Time of the apparent Riſing of the Sun, will be found to be 62 deg. 14 min. 36 ſec. equal in Time to 4 h. 08 m. 58 ſ. ½, whence the length of the apparent Night will be 8 h. 17 min. 57 ſec. and the Angle *Z P* ☉ the Time of the apparent Setting,



will be 117 deg. 45 min. 24 ſec. equal in Time to 7 h. 51 m. 01 ſ. ½, whence the Length of the apparent Day will be 15 h. 42 m. 03 ſ. and by comparing theſe with the true Times of the Riſing, Setting, *&c.* already determined; we ſhall find, that upon the Account of the Refraction, the Arch of the Equator *d r*, or Difference of Time between the true and apparent Riſing and Setting is 1 deg. 2 min. 39 ſec. equal in Time to 4 min. 10 ſec. ½, and conſequently the Sun apparently Riſes ſooner than the true Time by 4 min. 10 ſec. ½, and Sets apparently ſo much later, whence the apparent Day is longer in this *Caſe* than the Real or Aſtronomical Day, by 8 min. 21 ſec. and conſequently the apparent Night is ſo much ſhorter than the true Night.

As this Refraction occaſions an Error in the Time of the Sun's Riſing and Setting, ſo it likewiſe viciates the Amplitude, for tho' the Sun really Riſes and Sets at the Diſtance ♈ *e*, from the Eaſt or Weſt points of the Horizon, yet he apparently Riſes and Sets at the Diſtance of ♈ *a* from the ſame points; and to find the Arch of Difference *a c*, in the Oblique-angled Spherical Triangle Z *P* ☉, from the ſame Data, and by the help of the 11*th Caſe* of *Oblique-angled Spherical Triangles*, the Angle *P* Z ☉ or Arch *a* ☉, will be found to be 56 deg. 27 min. 06 ſec. whence the apparent Amplitude ♈ *a* will be 33 deg. 32 min. 54 ſec. greater than the true Amplitude before found 32 deg. 43 min. 43 ſec. by 49 min. 11 ſec. and the ſame *Law* is to obſerved in finding the apparent Amplitude, time of the Riſing and Setting, *&c.* of any of the Fixed Stars and Planets, regard being had to the different horizontal Parallaxes, in different Places and at different Times.

And inaſmuch as in the ſmall Triangle ☉ *a b*, the Arch ☉ *b* the Hypothenuſe of the Triangle ☉ *a b*, or Difference of the Declination of the Sun at the Time given, and at that Time when his real Semidiurnal Arch is equal to the Semi-apparent Arch, at this Time is nearly equal to the Arch ☉ *a* of the ſame Triangle ☉ *a b*, equal to the the horizontal Parallax or Depreſſion of the Sun at the Time of the apparent Riſing and Setting of the Sun; if to the preſent Declination 19 deg. 39 min. 10 ſec. ¾, be added the horizontal Parallax 33 min. 00 ſec. the Sum 20 deg. 12 min. 16 ſec. North, will be the Declination of the Sun nearly, when the Time of his real Riſing and Setting, and the length of the real Day and Night, will be nearly the ſame with the Time of the apparent Riſing and Setting, *&c.*

in the preſent *Caſe* as the *Reader* will ſee, if he will be at the Pains to make the Calculatian; wherefore, to find the Time nearly of the apparent Riſing and Setting, *&c.* of the Sun and his Amplitude, to the Declination of the Sun (if the Declination of the Sun and Latitude of the Place are both North, or both South) add the horizontal Refraction; but if the Latitude of the Place and Declination of the Sun are contrary, that is, one North and the other South, from the Declination of the Sun ſubſtract the horizontal Refraction, and the Sum or Difference according as the *Caſe* is, will be the apparent Declination of the Sun at the Time of his apparent Riſing and Setting: whence and by the help of the Latitude of the Place, may the apparent Amplitude and apparent time of the Riſing, Setting, *&c.* be Calculated, after the ſame manner as the true Time of the Riſing and Setting is in the former *Example.*

And inaſmuch as the Refraction obtains at all Times and in all Places, it ought to be allowed for in all Computations, and more eſpecially in ſuch Places as lie pretty far from the Equator, and at ſuch Times as the Declination is large.

In the former Inveſtigation, the Declination of the Sun has been ſuppoſed to be the ſame at the time of the Riſing and Setting, as it was at Noon; this tho' it may ſerve well enough for ordinary and common Uſes, yet in Calculations, where greater degrees of Exactneſs are required, ſuch as finding the Hour of the Day and Night, from the obſerved Altitudes of the Sun and Fixed Stars, the prediction of Solar and Lunar Eclipſes, and the Appulſes or Occultation of the Moon to the Fixed Stars, *&c.* it is neceſſary to know the true Declination of the Sun at the time when ſuch Appearance is found to happen: For ſince about the *Equinoxes*, the Sun's Declination alters about one Minute in an Hour, every one who is but meanly skill'd in theſe Things, muſt readily allow, that if the Altiude of the Sun was taken at Four in the Afternoon or Eight in the Morning, in order to pronounce the true Time of the Day, that an Error of Four Minutes in the Declination, which will neceſſarily happen if the Meridional or Tabular Declination be made Uſe of, will cauſe a conſiderable Error in the Hour deduced by Calculation; and that if the Altitude of a Star were taken about Midnight, to determine the Time of the Night, that an Error of two Minutes in the Sun's Right Aſcenſion, will cauſe an Error of two Minutes in the Time; and therefore *Aſtronomers* in all Computations,

ons, Calculate the Place of the Sun, and thence his Declination and Right Aſcenſion as near to the Time as poſſible, or if good Tables be ready Calculated, the ſame may be had by making proportionable Allowances, as ſhall be ſhewn at the End of the Tables of the Sun's Place, *&c.*

At 04 h. 13 m. 09 ſ. the time of the Sun's Riſing, by the former Calculation, the Sun is in 27 deg. 15 min. 10 ſec. of *Taurus*, at which time his Declination will be found by Calculation to be 19 deg. 34 min. 54 ſec. whence his Aſcenſional Difference will be found to be 26 deg. 35 min. 55 ſec. equal in Time to 1 h. 46 m. 23 ſ. $\frac{2}{3}$, and conſequently the true time of the true Riſing of the Center of the Sun at 4 h. 13 m. 36 ſ. $\frac{1}{3}$, later than by the former Computation 27 ſec. $\frac{1}{3}$.

Again at 7 h. 16 m. 51 ſ. the time of the Sun's Setting, the Sun was in 27 deg. 52 min. 34 ſec. of *Taurus*, hence his Declination will be found to be 19 deg. 43 min, 22 ſec. his Aſcenſional Difference 26 deg. 49 min. 33 ſec. equal in Time to 1 h. 47 min. 18 ſec. $\frac{1}{3}$; and conſequently the true time of the Setting will be at 7 h. 47 m. 18 ſ. $\frac{1}{3}$, later than by the former Calculation 27 ſec. $\frac{1}{3}$, whence the Length of the Day will be 15 h. 33 m. 41 ſ. $\frac{2}{3}$; and the nearer the Time happens to the *Equinoxes* the greater is the Error, and the nearer the *Solſtice* the leſs will be the Error.

It has been ſhewn that by Reaſon of the horizontal Refraction of the *Atmoſphere*, that the Sun at this time appears 4 m. 10 ſ. $\frac{1}{2}$ ſooner, and continues viſible 4 m. 10 ſ. $\frac{1}{2}$ longer in the Evening than he would otherwiſe do were it not for this Refraction, and conſequently in this *Caſe* the Center of the Sun apparently Riſes at 4 h. 09 m. 26 ſ. and Sets apparently at 7 h. 51 m. 28 ſ. $\frac{7}{10}$, after it has continued viſible above the Horizon 15 h. 42 m. 02 ſ. $\frac{7}{10}$.

The Sun's mean horizontal Semidiameter is 16 m. 6 ſ. for a few Seconds in this *Caſe* amount to nothing, and conſequently when the Sun's Center is really depreſſed 16 m. 06 ſ. below the Horizon, or by Reaſon of the Refraction when it is depreſſed 49 m. 06 ſ. his upper Limb begins to appear in the Morning, wherefore in the Spherical Triangle Z P ☉ (*See the Figure in Page 205th*) are given, ZP equal to 38 deg. 28 min. 00 ſec. P ☉ equal in this *Caſe* to 70 deg. 25 min. 06 ſec. and Z ☉ equal to 90 deg. 49 min. 06 ſec. whence the Angle ☉PO or true time when the upper Limb aſcends the Horizon, or the Sun firſt begins to appear, is 61 deg. 49 min. 15 ſec. equal in Time to 4 h. 07 m. 17 ſ.

In like manner, when the Sun's Center is depreſſed 49 m. 06 ſ. below the weſtern Edge of the Horizon, the upper Limb of the Sun deſcends, and the Sun totally diſappears, ſo that in the ſame Triangle Z *P* ⊙ we have given, Z *P* equal to 38 deg. 28 min. 00 ſec. Z ⊙ equal to 90 deg. 49 min. 06 ſec. as before, and *P* ⊙ (in this *Caſe*) 70 deg. 16 min. 38 ſec. whence the Angle *ZP*⊙ or the true time when the upper Limb apparently ſets, or the Sun begins totally to diſappear will be 118 deg. 24 min. 28 ſec. equal in Time to 7 h. 53 m. 37 ſ. $\frac{6}{7}$ after it has continued viſible above the Horizon for the ſpace of 15 h. 46 m. 20 ſ. $\frac{6}{7}$ longer than by the common Computation 12 m. 38 ſ. $\frac{6}{7}$.

After the ſame manner it may be found when the Sun's lower Limb Riſes and Sets, and conſequently how long the Sun's whole Body continues viſible above the Horizon.

And by adding to or ſubſtracting from the Sun's preſent Declination, the Sum or Difference of the horizontal Refraction and the Sun's Mean Semi-diameter, according as the Declination is North or South; after the ſame manner taught in *Page* the 284, 285, may the Declination of the Sun be found, whence the true time of his central Riſing and Setting, will give the apparent time of the Riſing or Setting of his upper or lower Limb.

An exact *Solution* of this *Problem* is of greater Uſe than appears at firſt ſight, and 'tis for this Reaſon that I have been ſo very particular and exact in giving the true time of the apparent Riſing and Setting of the upper or lower Limb, for as an exact Knowledge of the true time of the Day is not only of great Uſe in all Aſtronomical Obſervations, but is the principal Thing wanting at preſent to render the ſeveral Methods hitherto propoſed by *Aſtronomers*, for finding the Longitude at Sea practicable; and as an Obſervation of this Appearance, requires no Aſtronomical Inſtrument, but only a careful and diligent looking out and watching the Time when the Sun firſt begins to appear and to diſappear, and which any Body is capable of doing; and as the ſame may be done by every Fixed Star, there can ſcarce happen any great ſpace of Time, but the careful and diligent *Mariner* may have an Opportunity of comparing his *Watch* with the true time, and diſcovering the Error, if any there be, and how much it is.

And becauſe in the Inveſtigation of the time of the Riſing and Setting of the Sun or Stars, there is nothing requiſite but the Latitude of the Place and the Declination of the Sun; and inaſmuch as the Declination of the Sun may be determined to the greateſt degree of Exactneſs imaginable, and that an Error of Five Minutes in the Latitude will ſcarce make an Error of Twenty Seconds in the Time, there is no Way more ſure and certain than this, as I know of, that can be put in Practice at Sea.

And even upon the Shore when Perſons are not furniſhed with proper Inſtruments, this Method of diſcovering the true time of the Day in proper Places and at proper Times, may be very uſeful and ſerviceable to them if due Care be taken.

To ground the *Learner* well in the *Projection of the Sphere* and this very uſeful part of *Science,* I have given the *Stereographic Solution* of this *Problem,* and the 1*ſt Caſe* of the 1*ſt Problem* upon the Planes of the ſeveral Great Circles, whence the Inteligent *Reader* will readily perceive why *Aſtronomers* have choſen the Plane of the *Meridian,* for the *Solution* of the uſual *Problems* ariſing from the diurnal Motion of the Sun, it being much more natural and eaſy for the Purpoſe; and therefore in the *Solution* of the following *Problems,* I ſhall confine my Self to the Plane of the *Meridian* only, not doubting but the skilful *Reader* will eaſily ſupply the reſt himſelf.

This *Caſe* is doubtful, as is manifeſt from the *9th Caſe of Right-angled Spherical Triangles,* and from the *Conſtruction* it ſelf upon the ſeveral Planes, but inaſmuch as the Sides and Angles of the great Triangle, are only the Supplements of the Sides and Angles of the leſſer Triangle, the ſame Anſwers are produced by counting from the oppoſite points of the Equator and Horizon; thus (in *Fig.* the 1*ſt*) if ♈ repreſents the Eaſt point of the Horizon, as ♈ ☉ 32.43.42 ¾, repreſents the Amplitude in the Morning, reckoned from the Eaſt towards the North, ſo its Supplement 147.16.11 ¼, or the Hypothenuſe ♈ ☉ of the great Triangle reckoned from the ſame Eaſt point towards the North, gives N. 57.16.11 ¾ Weſterly, for the Co-Amplitude at the Setting; in like manner as the Baſe or Side ♈*R* 26.42.43 equal to 1 h. 46 m 51 ſ. ſhews how long the Sun Riſes before the Hour of Six in the Morning, ſo its Supplement of the great Baſe or Side ♈*R*=153° 17′ 17″ equal to 10h. 13m: 09 ſ. reckoned backward from the Six a Clock Hour Circle, will give 4 h. 13 m. 09 ſ. for the Setting of the Sun before Midnight, or 7 h. 46 m. 51 ſ. after Mid-day or Noon, as before found; and the ſame will happen when the Declination of the Sun and Latitude of the Place are of a contrary Denomination.

Prob.

Prob. II.

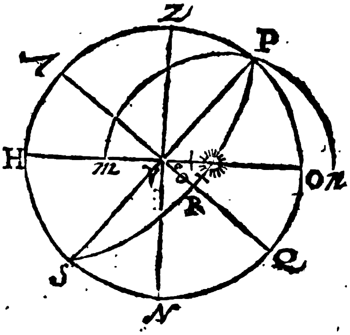
Caſe 2.

Given the Sun's Amplitude and preſent Declination, to find the Latitude of the Place, Aſcenſional Difference, time of the Sun's Riſing, Setting, &c.

Example

The Sun having 19 deg. 39 min. 10 ſec. ¾ of North Declination, and the Amplitude being found to be 32 deg. 43 min. 42 ſec. ¾ Northerly, in the Morning. I demand the Latitude of the Place of Obſervatton, time of the Sun's Riſing, &c.

The Stereographic Solution upon the Plane of the Meridian.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical Z ♈ *N*, ſet off the Semi-tangent of the Amplitude 32 deg 43 min. 42 ſec. ¾ in the Horizon from the Eaſt point ♈ to ⊙, in the Northen Semircircle *ZON*, this will give the Place of the Sun.

2. About the Point ⊙ as a Pole, deſcribe the Small Circle *m P n* at the diſtance of 70 deg. 20 min 39 ſec. ¼, the Complement of the Sun's Declination or his diſtance from the Pole (by *Caſe* the 3*d* of *Problem* the 4*th*, of *Section* the 2*d*) where this interſects the Meridian as in *P*, it will give the Place of the North Pole.

3. Thro' the Point *P* draw the Axis *P* ♈ *S*, and at Right-angles to it the Equator *A* ♈ *Q*.

4. Thro' the Poles *P* and *S* and the Point ⊙, draw the Hour-Circle *P* ⊙ *S*, and the thing is done; and ♈ *R* the Aſcenſional Difference, may be meaſured (by *Caſe* the 2*d* of *Problem* the 7*th*, of *Section* the 2*d*,) and the Angle ⊙ ♈ *R* or the Arch *O Q*, or its Comple-

plement *OP* the Latitude of the Place, may be meaſured by the *Line of Chords*.

And to determine their Quantities by Calculation,

In the Spherical Triangle ♈ *R* ☉ Right-angled at *R*, are given, ♈ ☉ the Amplitude, and ☉ *R* the preſent Declination, whence to find firſt the Angle ☉ ♈ *R* the Complement of the Latitude, it will be by the 13*th Caſe of Right-angled Spherical Triangles*,

As the S. ♈ ☉ : S. ☉ *R* : : *R* : S. < ☉ ♈ *R*; that is,

As the Sine of the Amplitude 32.43.42¾ ———— 9.7329240

To the Sine of the Declination 19.39.10¾ ———— 9.5267557
So is the Radius ———— 10.0000000

To the Co-ſine of the Latitude 51.32 ———— 9.7938317

Which is North, if the Sun Riſes before, or Sets after the Hour of Six.

2. To find the Aſcenſional Difference ♈ *R*, it will be (by *Caſe* the 14*th*) of *Right-angled Spherical Triangles*.

As the c s, ☉ *R* : c s, ♈ ☉ : : *R* : c s, ♈ *R*; that is,

As the Co-ſine of the Declination 19.39.10¾ ———— 9.9739342

To the Co-ſine of the Amplitude 32.43.42¾ ———— 9.9249204
So is the Radius ———— 10.0000000

To the Co-ſine of the Aſcenſional Differ. 26.42.43 —9.9509862

Which Aſcenſional Difference converted into Time, and ſubſtracted from ſix Hours, according to the Direction given in the former *Caſe*, will give 4 h. 13 m. 09 ſ. for the time of the Sun's Riſing or Hour of the Day, if the Latitude and Declination are both the ſame way, but, *&c.*

The Variation of the *Compaſs* being known, this *Problem* is of Uſe in finding the Latitude of the Place and the Hour of the Day, at the time of the Obſervation.

Caſe 3.

Given the Latitude of the Place and Amplitude of the Sun, required the preſent Declination, Aſcenſional Difference, time of Riſing, and Setting, *&c.*

Example

In the Latitude of 51 deg. 32 min. North, the Amplitude at Setting being found to be 32 deg. 43 min. 42 ſec. ¾ Weſt: I demand the preſent Declination, Aſcenſional Difference, time of Setting, *&c.*

Anſwer.

The preſent Declination is 19 deg. 39 min. 10 ſec. ¾, the Aſcenſional Difference 26 deg. 42 min. 43 ſec. the Time of Setting 7 h. 46 m. 51 ſ.

Caſe 4.

Given the Latitude of the Place, and the time of the Sun's Riſing or Setting, to find the Amplitude and preſent Declination.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun was obſerved to Riſe at 4 h. 13 m. 09 ſ. I demand his Amplitude and preſent Declination.

Anſwer.

The Amplitude is Eaſt 32 deg. 43 min. 42 ſec. ¾ North, and the preſent Declination 19 deg. 39 min. 10 ſec. ¾ North.

Caſe 5.

Given the time of the Sun's Riſing or Setting, and preſent Declination, to find the Latitude of the Place, Amplitude, *&c.*

Example.

The Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination, is found by Obſervation to Riſe at 4 h. 13 m. 39 ſ. I demand the Latitude of the Place, and how far he Riſes to the Northward of the Eaſt point of the Horizon.

Anſwer.

The Latitude of the Place is 51 deg. 32 min. North and the Amplitude 32 deg. 43 min. 42 ſec. ¾ Eaſt Northward.

Caſe 6.

Given the Amplitude and the time of the Sun's Riſing or Setting, to find the Latitude of the Place and preſent Declination.

Example.

I demand the Latitude of the Place and Declination of the Sun; when he is found to Riſe at 4 deg. 13 min. 09 ſec. and 32 deg. 43. min. 42 ſec. ¾ to the Northward of the Eaſt point of the Horizon.

Anſwer.

The Latitude is 51 deg. 32 min. North, and the Declination is 19 deg. 39 min. 10 ſec. ¾ North.

Caſe 7.

Given the Latitude of Place and Angle formed by the Meridian, paſſing thro' the Sun and Horizon, to find the preſent Declination, Amplitude, and time of the Sun's Riſing, Setting, *&c.*

Example.

In the Latitude of 51 deg. 32 min. North, the Angle formed by the Horizon and proper Meridian being 56 deg. 14 min. 35 ſec. I demand the preſent Declination, Amplitude, time of Riſing, *&c.*

Anſwer.

The preſent Declination is 19 deg. 39 min. 10 ſec. ¾, the Amplitude 32 deg. 43 min. 42 ſec. ¾, and the time of the Sun's Riſing 4 h. 13 m. 09 ſ. if the Declination be Northerly, but if Southerly, at 7 h. 46 m. 51 ſ. for the Data in the Queſtion is not ſufficient to limit the *Problem* in that *Caſe.*

Caſe 8.

Given the preſent Declination, and Angle formed by the Horizon and proper Meridian, to find the Amplitude, Latitude of the Place, and time of the Sun's Riſing and Setting.

Example.

The Declination of the Sun being 19 deg. 39 min. 10 ſec. ¾ North, and the Angle formed by the Horizon and proper Meridian being 56 deg. 14 min. 35 ſec. I demand the Latitude, Amplitude, time of Riſing, *&c.*

Anſwer.

The Amplitude is 32 deg. 43 min. 42 ſec. ¾, the Latitude 51 deg. 32 min. and the time of the Sun's Riſing 4 deg. 13 m. 09 ſ. if the Latitude be North, but 7 h. 46 m. 51 ſ. if the Place be in South Latitude, for the Data in the *Problem* is not ſufficient to determine it.

Caſe 9.

Given the Amplitude and Angle which the Horizon forms with the proper Meridian, to find the Latitude of the Place, the Declination of the Sun, and time of his Riſing and Setting.

Example.

I demand the Latitude of the Place, preſent Declination, and time of the Sun's Riſing, his Amplitude being 32 deg. 43 min. 42 ſec. ¾ North, and the Angle which the Horizon forms with the proper Meridian 56 deg. 14 min. 35 ſec.

Anſwer.

The Declination is 19 deg. 39 min. 10 ſec. ¾, the Latitude is 51 deg. 32 min. the time of the Sun's Riſing 4 h. 13 m. 09 ſ. if the Place of Obſervation be in North Latitude, but 7 h. 46 m. 51 ſ. if the Place be in Southern Latitude, for the Queſtion is not in that ſufficiently determined.

Caſe 10.

Given the time of the Sun's Riſing and Setting, and the Angle formed by the Horizon and the proper Meridian, to find the Latitude of the Place, preſent Declination, and Amplitude.

Example.

I demand the Latitude of the Place, Declination of the Sun, and his Amplitude, he being found to riſe at 4 h. 13 m. 09 ſ. and the Angle formed by the Horizon and proper Meridian being 56 deg. 14 min. 35 ſec.

Anſwer.

The Amplitude is 32 deg. 43 min. 42 ſec. ¾, the Declination 19 deg. 39 min. 10 ſec. ¾, and the Latitude 51 deg. 32 min. and are all the ſame Way; that is, all North or all South, becauſe the Sun Riſes before Six, but there is not ſufficient Data in the *Problem* to determine which.

This *Caſe* is Ambiguous, as appears by the *2d Example* of the *9th Caſe* of *Right-angled Spherical Triangles*, but as the Sides and Angles required in the great Triangle are only the Supplements of the Sides and Angles of the ſmall Triangle, they give the ſame *Anſwers*, being counted from the oppoſite points of the Equator and Horizon.

Prob. III.

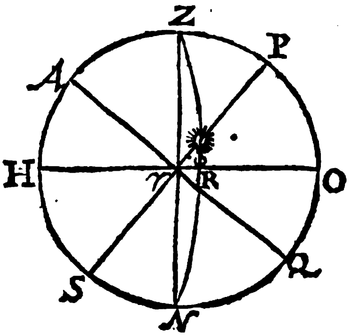
Caſe 1.

The Latitude of the Place, and the Declination of the Sun being given, to find his Height and Azimuth at the Hour of Six.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun having 19 deg. 39 min. 10 ſec. $\frac{1}{4}$ North Declination. I demand his Heighth and Azimuth at the Hour of the Six.

The Stereographic Solution upon the Plane of the Meridian.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical Z ♈ *N*.

2. Set off the Latitude 51 deg. 32 min. from *O* to *P*, and draw the Six a Clock Hour Circle *P* ♈ *S*, and at Right-angles to it the Equator *AQ*.

3. Set off the Half-tangent of the Sun's Declination 19 deg. 39 min. 10 ſec. $\frac{1}{4}$, in the Six a Clock Hour Circle from ♈ to ☉, then will ☉ repreſent the Place of the Sun, thro' which and the Zenith and Nadir, if the Vertical Circle Z ☉ *R N* be drawn the thing is done; and *R O* the Sun's Azimuth from the Meridian may be meaſured by *Caſe* the *2d* of *Problem* the *7th*, of *Section* the *2d*, and ☉ *R* the Altitude may be meaſured by *Caſe* the *3d* of the ſame *Problem*,

And to determine the Lengths by Calculation,

In the Spherical Triangle ♈ ☉ *R*, Right-angled at *R* are given, the Hypothenuſe ♈ ☉ the Sun's Declination, and the Angle ☉ ♈ *R* equal to the Arch *O P*, equal to the Latitude, whence to find

1. The Altitude at the Hour of Six, it will be by the *2d Example* of the *5th Caſe* of *Right-angled Spherical Triangles*,

As *R* : S. ☉ ♈ *R* : : S. ♈ ☉ : S. ☉ *R*; that is,

As the Radius	10.0000000
To the Sine of the Latitude of 51.32	9.8937452
So is the Sine of the Declination 19.39.10 ¾	9.5267557
To the S. of the Sun's Height at the Hour of Six 15.16.4	9.4205009

Which is the ſame both Morning and Afternoon.

2. To find the Azimuth, it will be by the *2d Example* of the *4th Caſe* of *Right-angled Spherical Triangles*.

As the *R* : c s, ☉ ♈ *R* : : t, ☉ ♈ : t, ♈ *R*; that is,

As the Radius	10.0000000
To the Co-ſine of the Latitude 51.32	9.7938317
So is the Tangent of the Declination 19 39.10 ¾	9.5528217
To the Co-tangent of the Azimuth from the Meridian 77 deg. 28 min. 29 ſec ¾,	9.3466534

And which muſt always be counted from the viſible Pole, that is, if the Place be in North Latitude it muſt be reckoned from the North, but if the Place be in South Latitude it muſt be counted from the South; and conſequently in the preſent *Caſe* if it be in the Morning, the Azimuth is North 77 deg. 28 min. 29 ſec. ¾; Eaſt, but if it be in the Afternoon it is North 77 deg. 28 min. 29 ſec. ¾ Weſt, or in the *Nautical Phraſe* the Sun bears Eaſt by North nearly at Six in the Morning, but Weſt by North nearly at Six in the Evening.

Hence the Hour being given, the Variation of the *Compaſs* is eaſily found, or the Variation of the *Compaſs* being known the Hour is eaſily had, or the Altitude being carefully taken, the Hour and Variation of the *Compaſs* is diſcovered.

And becauſe in the ſame Place the Latitude remaining the ſame, the Altitude and Azimuth depend upon the Sun's Declination, and are ever proportional to it, it follows, that the greater the Declination the greater the Altitude, and the leſs the Azimuth; and on the contrary, the leſſer the Declination the leſſer the Altitude, and the greater the Azimuth; till at laſt the Declination vaniſhing the Altitude vaniſhes, and the Azimuth becomes equal to a Quadrant or 90 Degrees, and conſequently at that time the Sun will appear due Eaſt in the Horizon at Six in the Morning, and due Weſt in the Horizon at Six in the Evening, at which time he Riſes and Sets at the Hour of Six.

For in the Triangle ♈ ⊙ *R* when the Side ♈ ⊙ becomes equal to nothing, the points ♈, ⊙ and *R*, coincide, and the Triangle ♈⊙*R* vaniſhes.

And again, becauſe the Declination remaining the ſame, the Sines of the Altitude are directly proportional to the Sines of the Latitude of the Places, it follows, that the greater the Latitude the greater the Altitude at the Hour of Six, till at laſt when the Latitude becomes equal to 90 Degrees or a Quadrant, the Height at Six becomes equal to the Declination, and this happens to all thoſe who have that Pole in their Zenith, as is the ſame Way with the Declination that is to the Northern Inhabitants when the Sun has North Declination, but to the Southern Inhabitants when the Sun has South Declination.

For when the point *P* returns to and coincides with the point Z, that is, when the Axis coincides with the Prime Vertical, the Side ♈*R* of the Triangle ♈*R*⊙ vaniſhes, and ⊙*R* becomes equal to ♈ ⊙.

On the contrary, as the Latitude decreaſes the Height and Azimuth at Six decreaſes, till at laſt when the Latitude vaniſhes, the Altitude at the Hour of Six vaniſhes; and the Azimuth becomes equal to the Complement of the Amplitude, equal to the Declination, and the Sun appears in the Horizon, and this happens to all thoſe Inhabitants who dwell under the Equator.

For when the Pole *P* coincides with the point *C*, that is, when the Axis becomes the Horizon, the Side ♈ ⊙ of the Triangle ♈ ⊙ *R* vaniſhes, and ♈ *R* becomes equal to ♈ ⊙.

And conſequently when the Declination is of a contrary Denomination to the Latitude of the Place, the Sun does not appear above the Horizon till after the Hour of Six, and the fourth Number found in this *Caſe* ſhews his Depreſſion at that time.

Prob. III.

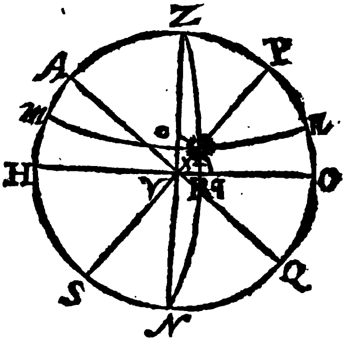
Caſe 2.

Given the Sun's Declination and his Altitude at the hour of Six, to find the Azimuth and Latitude of the Place of Obſervation.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ¾ of North Declination, his Altitude at the hour of Six is found to be 15 deg. 16 min. 04 ſec. I demand his Azimuth and the Latitude of the Place of Obſervation

The Stereographic Solution upon the Plane of the Meridian.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *Z* ϒ *N*, about the Center ϒ with the Semi-tangent of the Sun's Declination 19 deg. 39 min. 10 ſec. ¾ deſcribe the Parallel *O* ☉ *q*.

2. At the diſtance of 74 deg. 43 min: 56 ſec. the diſtance of the Sun from the Zenith, or Complement of the Altitude, deſcribe the Parallel or Small Circle *m* ☉ *n*, (by *Caſe* the 2*d* of *Prob.* the 4*th*, of *Sect.* the 2*d*) where this interſects the former Parallel in ☉, it will give the Place of the Sun; thro' which point and the Center ϒ, draw the Six a Clock Hour Circle *P* ϒ *S*, and at Right-angles to it the Equator *A* ϒ *Q*.

3. Thro' the Point ☉ and the Zenith and Nadir, draw the Vertical Circle Z ☉ *N* and the thing is done; and *R O* the Sun's Azimuth may be meaſured (by *Caſe* the 2*d* of *Prob.* the 7*th*, of *Sect.* the 2*d* of *Part* the 4*th*,) and the Angle ☉ ϒ *R* or Arch *P O*, equal to the Latitude of the Place of Obſervation, may be meaſured by the *Line of Chords.*

Logarithmically, or by Calculation.

In the Spherical Triangle ☉ ϒ *R* Right-angled at *R* are given, ☉ ϒ the Sun's Declination, and ☉ *R*, his Altitude, whence to find ϒ *R* the Complement of the Azimuth from the Meridian, it will be (by the 14*th Caſe* of *Right-angled Spherical Triangles.*

As c s, ♈ *R* : *R* : : c s, ⊙ ♈ : c s, ♈ *R*, that is,

As the Co-ſine of the Altitude 15.16.04 ———————— 9.9843948

To the Radius ———————————————— 10.0000000

So is the Co-ſine of the Declination 19.39.10¾ ———— 9.9739342

To the S. of the Azimuth from the Meridian 77.28.29¾ 9.9895394

Which becauſe the Declination is North, muſt be counted from the North towards the Eaſt if it be in the Morning, but towards the Weſt if it be in the Afternoon.

2. To find the Latitude or Angle ⊙ ♈ *R*, it will be by the 13*th Caſe of Right-angled Spherical Triangles*,

As the S. ♈ ⊙ : *R* : : S. ⊙ *R* : S. ⊙ ♈ *R*; that is,

As the Sine of the Declination 19.39.10¾ ———— 9.5267557

To the Radius ———————————————— 10.0000000

So is the Sine of the Height 51.16.04 ———————— 9.4205009

To the Sine of the Latitude 51.32 ———————— 9.8937452

Which is Northerly, becauſe the Declination is Northerly.

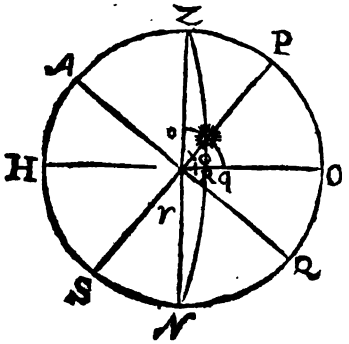
By the help of this *Problem* the Latitude of the Place may be obtained from the height of the Sun at the hour of Six, which may ſometimes be of Uſe to the Induſtrious *Navigator*.

Prob. III.

Caſe 3.

Given the Azimuth at the hour of Six, and the Declination, to find the Altitude, and the Latitude of the Place of Obſervation.

Example. The Sun having 19.39.10¾ N. Declination, his Azimuth at ſix in the Morning was found to be N. 77.28.29¾ E. I demand, *&c.*



The Stereographic Solution upon the Plane of the Meridian.

1. Having drawn the Meridian *Z O N H*, the Horizon *HO*, and at Right-angles to it the Prime Vertical Z♈ *N*, with the Semi-tangent of the Sun's Declination 19 deg. 39 min. 10 ſec. ¾, deſcribe the Parallel *O* ⊙ *q* about the Center ♈ as a Pole.

2. Set off the Semi-tangent of the Complement of the Azimuth 12 deg. 31 min. 30 ſec. ¼, from ♈ in the Horizon to *R*, or with the Secant of the Azimuth 77 deg. 28 min. 29 ſec. ¾, ſetting one foot of the Compaſſes in Z or *N*, the other Foot reſting in the Horizon *H* ♈ *O* produced, deſcribe the Vertical Circle *Z R N*, where this interſects the Parallel *o* ☉ *q* before drawn, it will give the Place of the Sun, thro' which and the Center ♈, if the Hour Circle *P* ☉ ♈ *S* be drawn the thing done; and ☉ *R* the Sun's Altitude may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*, and the Angle ☉ ♈ *R* or the Arch *O P* the Latitude may be found by the *Line of Chords.*

Logarithmically, or by Calculation.

In the Spherical Triangle ☉ ♈ *R* Right-angled at *R*, are given, ☉ ♈ the Sun's Declination, ♈ *R* the Complement of the Sun's Azimuth, whence to find firſt ☉ *R* his Altitude, it will be by the 2*d* *Example* of the 14*th Caſe* of *Right-angled Spherical Triangles*,

As the c s, ♈ *R* : *R* : : c s, ♈ ☉ : c s, ☉ *R*; that is,

As the Sine of the Azimuth 77.28.29 ¾ ———— 9.9895394

To the Radius ———— 10.0000000

So is the Co-ſine of the Declination 19.39.10¾ ———— 9.9739342

To the Co-ſine of the Altitude 15.16.04 ———— 9.9843948

2. To find the Angle ☉ ♈ *R* the Latitude, it will be by the 2*d* *Example* of the 12*th Caſe* of *Right-angled Spherical Triangles*,

As *R* : t, ♈ *R* : : ct, ♈ ☉ : c s, < ☉ ♈ *R*; that is,

As the Radius ———— 10.0000000

To the Co-tangent of the Azimuth 77.28.29¾ ———— 9.3466534

So is the Co-tangent of the Declination 19.39.10 ¾ ———— 10.4471783

To the Co-ſine of the Latitude 51.32 ———— 9.7938317

Which is North, becauſe the Declination is Northerly.

The Variation of the *Compaſs* being known, the Latitude of the Place of Obſervation may be found by the help of this *Problem.*

Caſe 4.

Given the Altitude of the Sun, and his Azimuth at the hour of Six, to find the Declination, and Latitude of the Place of Obſervation.

Example.

At Six in the Evening, the Altitude of the Sun was found to be 15 deg. 16 min. 04 ſec. and his Azimuth 77 deg. 28 min. 29 ſec. ¾ North Weſtward. I demand the Declination of the Sun, and the Latitude of the Place of Obſervation ?

Anſwer.

The Sun's Declination is 19.39.10¾ N. and the Latitude 51.32 N.

Caſe 5. *Doubtful.*

Given the Latitude of the Place and the Sun's height at the hour of Six, to find the Declination and Azimuth.

Example

In the Latitude of 51.32 North, the Sun's height at Six in the Morning was found to be 15.06.04. I demand his Declination and Azimuth.

Anſwer.

His Declination is 19 deg. 39 mn. 10 ſec. ¾, and Azimuth North 77 deg. 28 min. 29 ſec. ¾ Eaſt.

Caſe 6.

Given the Latitude of the Place, and Azimuth of the Sun at the hour of Six, to find the Altitude and Declination.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun's Azimuth at Six in the Morning was found to be North 77 deg. 28 min. 29 ſec. ¾ Eaſt. I demand the Altitude and preſent Declination?

Anſwer.

The Altitude is 15 deg. 16 min. 04 ſec. and Declination 19 deg. 39 min. 10 ſec. ¾ North.

Caſe 7.

Given the Latitude of the Place and Angle formed by the Vertical and Six a Clock hour Circle, to find the Sun's Height and Azimuth at the hour of Six, and his preſent Declination.

Ex-

Example.

In the Latitude of 51 deg. 32 min. North, the Sun being upon the Six a Clock hour Circle in the Morning, the Angle formed by the Vertical and Six a Clock hour Circle is found to be 40 deg 09 min. 07 ſec. $\frac{1}{4}$. I demand the Sun's Declination, his Height and Azimuth at that time.

Anſwer.

The Sun's Declination 19 deg. 39 min. 10 ſec. $\frac{3}{4}$ North, his Altitude 15 deg. 16 min. 04 ſec. and his Azimuth North 77 deg. 28 min. 29 ſec. $\frac{3}{4}$ Eaſt.

Caſe 8.

Given the Sun's Declination, and the Angle formed by the Vertical Circle paſſing thro' the Sun and the Six a Clock hour Circle, to find his Altitude, Azimuth and Latitude of the Place, he being ſuppoſed to be on the Six a Clock hour Circle.

Example.

The Sun being upon the Six a Clock hour Circle, there is given, the Declination equal to 19 deg. 39 min: 10 ſec. $\frac{3}{4}$, and the Angle formed by the Vertical and Six a Clock hour Circle equal to 40 deg. 09 min. 07 ſec. $\frac{1}{4}$. I demand the Sun's height and Azimuth, and Latitude of the Place?

Anſwer.

The Sun's Height is 15 deg. 16 min. 04 ſec. his Azimuth is North 77 deg. 28 min. 29 ſec. $\frac{3}{4}$ Eaſt, (if in the Morning) the Latitude 51 deg. 32 min. North.

Caſe 9.

There is given the Sun's Height at the hour of Six, and the Angle formed by the Vertical and Six a Clock hour Circle, to find his Azimuth, Declination, and the Latitude of the Place.

Example.

The Sun being upon the Six a Clock hour Circle, there is given his Height equal to 15 deg. 16 min. 04 ſec. and the Angle formed by the Vertical and Six a Clock hour Circle equal to 40 deg. 09 min. 07 ſec. $\frac{1}{4}$. I demand the Declination, Azimuth and Latitude of the Place?

Anſwer.

The Declination is 19 deg. 39 min. 10 ſec. ¾, Azimuth 77 deg. 28 min. 29 ſec. ¾, and Latitude 51 deg. 32 min. but whether the Declination and Latitude be North or South, or whether the Azimuth muſt be counted from the North or South, is not to be determined by the Data in the Queſtion.

Caſe 10. *Doubtful.*

There is given the Sun's Azimuth at the hour of Six, and the Angle formed by the Vertical and Six a Clock hour Circle, to find his Declination, Altitude and Latitude of the Place.

Example.

The Sun being upon the Six a Clock hour Circle, there is given, his Azimuth North 77 deg. 28 min. 29 ſec. ¾ Eaſt, and the Angle formed by the Vertical and Six a Clock hour Circle equal to 40 deg. 09 min. 07 ſec. ¼, to find the Altitude, Declination, and Latitude of the Place.

Anſwer.

The Sun's Altitude is 15 deg. 16 min. 04 ſec. his Declination 19 deg. 39 min. 10 ſec. ¾, and the Latitude of the Place 51 deg. 32 min. North,

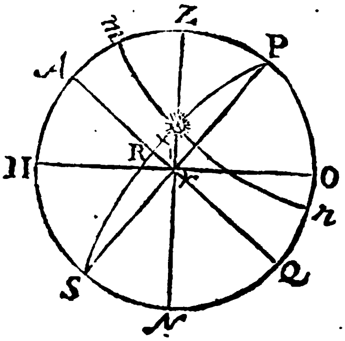
Prob. IV.

Caſe 1.

The Latitude of the Place, and the Declination of the Sun being given, to find his Height and the time when he will be upon the Prime Vertical.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun's Declination being 19 deg. 39 min. 10 ſec. ¾ N. I demand his Height and the time when he will appear due Eaſt and Weſt.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical, or Circle of Eaſt and Weſt *Z♈N*, ſet off the Latitude of the Place 51 deg. 32 min. from *O* to *P*, and draw the Axis *PS*, and at Right-angles to it the Equator *AQ*.

2. About the Pole *P* at the diſtance of 70 deg. 20 min. 49 ſec ¼, the Complement of the Sun's Decli-

nation, deſcribe the Parallel of Declination or Small Circle *m* ⊙ *n*, by *Caſe* the *2d* of *Prob.* the *4th*, of *Sect.* the *2d*, where this interſects the Prime Vertical Z ♈ *N* in ⊙, it gives the Place of the Sun when he appears due Eaſt or Weſt.

3. Thro' the Point ⊙ and the Poles *P* and *S*, draw the Meridian or hour Circle *P* ⊙ *S*, and the thing is done, and ♈ ⊙ the Sun's Altitude, and ♈ *R* the time before and after the hour of Six, may be meaſured by *Caſe* the *2d* of *Prob.* the *7th*, of *Sect.* the *2d*.

Logarithmically, or by Calculation.

In the Right-angled Spherical Triangle ♈ ⊙ *R* Right-angled at *R*, are given, ⊙ *R* the Sun's preſent Declination, and the Angle ⊙ ♈ *R* equal to the Latitude, whence to find firſt ♈ *R* his Altitude upon the Prime Vertical, it will be by *Caſe* the *10th* of *Right-angled Spherical Triangles*,

As S. < ⊙ ♈ *R* : *R* : : S. ⊙ *R* : S. ⊙ ♈ ; that is,

As the Sine of the Latitude [illegible].32 ———————— 9.8937452

To the Radius ———————————————— 10.0000000

So is the Sine of the Sun's Declination 19.39.10 ¾ ——— 9.5267557

To the S. of the height upon the Prime Vert. 25.26.19¾ — 9.6330105

Which is the ſame whether the Sun appears due Eaſt or Weſt.

2. To find the time when he will appear due Eaſt or Weſt, it will be by the *9th Caſe* of *Right-angled Spherical Triangles*,

As the t, < ⊙ ♈ *R* : *R* : : t, ⊙ *R* : S. ♈ *R* ; that is,

As the Tangent of the Latitude 51.32 ———— 10.0999135

To the Radius ———————————————— 10.0000000

So is the Tangent of the Declination 19.39.10 ¾ —— 9.5528217

To the Sine of the Arch ♈ *R* equal to 16.28.29 ———— 9.4529082

the time before or after the hour of Six, or to the Co-ſine of the Arch *AR* the time from Noon, equal to 73 deg. 31 min. 01 ſec. this converted into Time, will give 4 h. 54 m. 04 ſ. for the time when the Sun will appear due Weſt in the Afternoon, but ſubſtracted from 12 h. 00 m. 00 ſ. will give 7 h. 05 m. 56 ſ. for the time when he will appear due Eaſt in the Morning.

Or,

The Arch ♈ *R* equal to 16 deg. 28 min. 59 ſec. equal to 1h. 05m. 56 ſ. ſubſtracted from ſix hours, will give 4 h, 54 m. 04 ſ. for the time when the Sun will appear due Weſt, but added to ſix hours will give 7 h. 5 m. 50 ſ. for the time when the Sun will appear due Eaſt, as before determined; at each of which times he will be ſeen 25 deg. 20 min. 20 ſec. above the Horizon.

This *Caſe* is doubtful, as appears by the *9th Caſe of Right-angled Spherical Triangles*; but inaſmuch as the Sides of the Great Triangle are the Supplements of the Sides of the leſſer Triangle, if they are counted the contrary way they will produce the ſame Anſwers, for as the Arch ♈ *R* equal to 1 h. 05 min. 50 ſec. counted from ſix in the Morning, gives 7h. 05 min. 50 ſec. for the time when the Sun will appear due Eaſt, its Supplement 10 h. 54 min. 10 ſec. or the Arch ♈ *A R* the Side of the great Triangle counted from the ſame hour will give 4 h. 54 m. 10 ſ. for the time when the Sun will appear due Weſt, and as the Arch ♈ ☉ 25 deg. 26 min. 19 ſec. ¾, counted from the Horizon, gives the height of the Sun in the Morning, or when he appears due Weſt, ſo its Supplement 153 deg. 33 min. 40 ſec. ¼, or the Side of the great Triangle counted from the ſame point in the ſame Vertical Circle, will give 63 deg. 33 min. 40 ſec. ¼ for his Zenith Diſtance, or 25 deg. 26 min. 19 ſec. ¾ for his Altitude, when he appears due Weſt in the Afternoon, as before determined.

By the help of this *Problem* likewiſe, if the hour of the Day be known, the Variation of the *Compaſs* may eaſily be found, or the Variation of the *Compaſs* being known the hour is eaſily had, or the Altitude being carefully taken at that time, the Hour of the Day and the Variation of the *Compaſs* is readily diſcovered, as is very evident to the Inteligent *Reader*.

This and the former *Problem* are uſeful in finding the Declination of Walls, *&c.*

And becauſe in the ſame place the Latitude remaining the ſame, the Sines of the Altitudes are ever as the Sines of the Declinations, and the Sines of the Times directly as the Tangents of the ſame Declination, it follows, that the greater the Declination the greater the Altitude, and the greater the Diſtance of Time from the hour of Six, and on the contrary, the leſſer the Declination the leſſer the Altitude, and the nearer does the time approach to the hour of Six; till at laſt, when the Declination ceaſes the Altitude vaniſhes, and the Sun appears in the Horizon upon the Prime Vertical, at the hour of Six in the Morning and Evening.

For in the Triangle ♈ ⊙ *R* where the Side ♈ ⊙ vaniſhes, the points ⊙, *R*, and ♈ coincide.

And becauſe, the Declination remaining the ſame, the Sines of the Altitudes are reciprocally proportional to the Sines of the ſeveral Latitudes, and the Sines of the times from Six reciprocally, as the Tangents of the reſpective Latitudes, it follows, that the greater the Latitude the leſs will be the Altitude, and the leſs the time from the hour of Six, till when, the Latitude becomes equal to a Qudrant, the Altitude becomes equal to the Declination, and the time becomes equal to ſix Hours; and this happens at all times when the Sun is viſible to thoſe who have either Pole in their Zenith.

For when the Point *P* returns into Z, that is when the Arch *PZ* vaniſhes, the point *R* returns into ♈, and ⊙ ♈ becomes equal to ⊙ *R*, and the Triangle ♈ ⊙ *R* vaniſhes.

On the contrary, the leſſer the Latitude the greater will be the Altitude, and the nearer does the time approach to Noon, till when the Latitude becomes equal to the Declination, and the ſame Way the Sun appears at a quadrantal Diſtance from the Horizon, in the Prime Vertical upon the Meridian, and conſequently in the Zenith of the Place where he is due Eaſt or Weſt, and this happens at certain times to all thoſe who inhabit between the *Tropics*, that is, to the Northern Inhabitants when the Sun has North Declination, and to the Southern Inhabitants when he has South Declination.

For in the Triangles ⊙ ♈ *R*, when ⊙ *R* becomes equal to *A* Z, the points ⊙ and Z, *R* and *A* coincide, and ♈ ⊙ and ♈ *R* become equal to ♈ Z and ♈ *A*, that is equal to Quadrants.

But when the Latitude of the Place becomes leſs than the Declination, the Sun never arrives upon the Prime Vertical, but conſtantly keeps on the ſame Side of it, that is either to the Northward or Southward of it, according as the Place is ſcituated either to the Northward or Southward of the Equator: For when the Arch *PO* equal to Z*A* is leſs than *Am*, the point *m*, and conſequently the whole Parallel *m*⊙*n* falls on the ſame Side, or in this *Caſe* to the Northward of the Prime Vertical Z ♈ *N*; and conſequently when the Latitude of the Place and the Declination are contrary, the Sun never appears till he is paſt the Prime Vertical, and the Number found in this *Caſe* ſhews the Depreſſion.

Prob. IV.

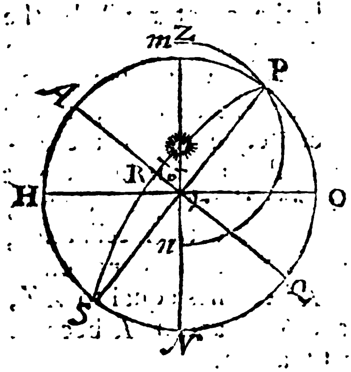
Caſe 2.

The Sun being upon the Prime Vertical, there is given his Altitude and preſent Declination, to find the Latitude of the Place and hour of the Day.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ¼ North Declination, and 25 deg. 26 min. 19 ſec. ¾ of Altitude upon the Prime Vertical. I demand the Latitude of the Place of Obſervation and hour of the Day.

The Stereograbic Solution upon the Plain of the Meridian.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical Z*N*; ſet off the Semi-tangent of 25 deg. 26 min. 19 ſec. ¾, the Sun's Altitude from ♈ to ☉, this will repreſent the Place of the Sun.

2. About the Point ☉ as a Pole, at the diſtance of 70 deg. 20 min. 49 ſec. ¾ the Complement of the Sun's Declination, deſcribe the Small Circle *m P n*, where this interſects the Meridian, it will give the Place of the North Pole.

3. Thro' the Pole and the Center ♈ draw the Six a Clock Hour Circle P ♈ *S*, and at Right-angles to it the Equator *A Q*.

4. Thro' the Poles *P* and *S* and the point ☉, draw the Meridian *P* ☉ *S*, and the thing is done; and *RA* the Hour from Noon may be meaſured by *Caſe* the 2*d* of *Problem* the 7*th* of *Section* the 2*d*, and the Angle ☉ ♈ *R* equal to the Arch *A* Z the Latitude, may be meaſured by the *Line of Chords*.

Logarithmically, or by Calculation.

In the Spherical Triangle ☉ ♈ *R*, Right-angled at *R* are given ☉ ♈ the Sun's Altitude, and ☉*R* his preſent Declination, whence to find firſt the Angle ☉ ♈ *R* equal to the Latitude, it will be by the 13*th Caſe* of *Right-angled Spherical Triangles*, As the S. ☉ ♈ : S. ☉ *R* : : *R* : S. ☉ ♈ *R* ; that is,

As the Sine of the Altitude 25.26.19¾ ———— 9.6330105

To the Sine of the Declination 19.39.10¾ ———— 9.5267557
So is the Radius ———— 10.0000000

To the Sine of the Latitude 51.32 ———— 9.8937452

Which is North becauſe the Declination is North.

2. To find the Time, it will be by the 14*th Caſe of Right-angled Spherical Triangles*,

As cs, ☉ *R* : *R* : : cs, ☉ ♈ : cs, *R* ♈ ; that is,

As the Co-ſine of the Declination 19.39.10¾ ———— 9.9739342

To the Radius ———— 10.0000000
So is the Co-ſine of the Altitude 25.26.19¾ ———— 9.9557691

To the Sine of the hour from Noon 73.31.31 ———— 9.9817749

Equal to 4h. 54m. 04ſ. when the Sun will appear due Weſt in the Afternoon, and this taken from 12 Hours will leave 7h. 5m. 56ſ. for the time when the Sun will appear due Eaſt in the Morning.

The Variation of the *Compaſs* being known, the time when he appears due Eaſt or Weſt may be had, and from his Height obſerved at that time, the Latitude of the Place may be found by the help of this *Problem.*

Prob. IV.

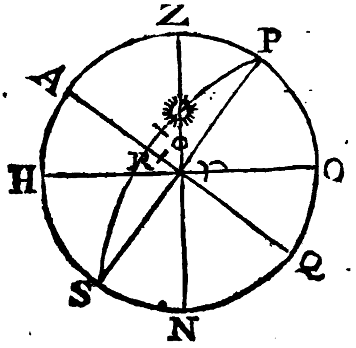
Caſe 3.

The Sun being upon the Prime Vertical there is given, the Declination and the time of the Day, to find the Altitude and Latitude of the Place.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ¾ of North Declination, was obſerved to be upon the Prime Vertical at 4 h. 54 m. 04ſ. in the Afternoon. I demand his Altitude and the Latitude of the Place of Obſervation.

The Stereographic Solution upon the Plane of the Meridian.



1. Having drawn the Meridian Z O *N H*, the Six a Clock Hour Circle *P* ♈ *S*, and at Right-angles to it the Equator *A Q*, ſet off the Semi-tangent of the Complement of the hour from Noon 16 deg. 28 min. 59 ſec. from ♈ to *R*, and draw the Meridian *P R S.*

2. Set off the given Declination 19 deg. 39 min. 10 ſec. ¾ from *R* to ☉, by *Caſe* the 3*d* of *Problem* the 6*th* of *Section* the 2*d*, then will ☉ repreſent the Place of the Sun.

3. Thro' the Point ☉ and the Center ♈ draw the Prime Vertical Z ☉ ♈ *N* and the thing is done; and ♈ ☉ the Sun's Altitude may be meaſured (by *Caſe* the 2*d* of *Prob.* the 7*th*, of *Sect.* the 2*d*) and the Angle Z ♈ *A* or the Arch Z *A* the Latitude, may be meaſured by the *Line of Chords.*

Logarithmically, or by Calculation.

In the Spherical Triangle ☉ ♈ *R* Right-angled at *R* are given, ☉ *R* the preſent Declination, and ♈ *R* the hour from Six, whence to find ☉ ♈, the Sun's Altitude, it will be (by the 1*ſt Caſe* of *Right-angled Spherical Triangles.*

As *R* : c s, ☉ *R* : : c s, ♈ *R* : c s, ☉ ♈; that is,

As the Radius ———————— 10·0000000

To the Co-ſine of the Declination 19·39·10¾ ——— 9·9739342
So is the Sine of the hour from Noon 73·31.01 ——— 9·9817749

To the Co-ſine of the Altitude 25·26.19¾ ——— 9·9557091

2. To find the Latitude of the Place of Obſervation, it will be by *Caſe* the 2*d* of *Right-angled Spherical Triangles,*

As the S. ♈ *R* : *R* : : t, ☉ *R* : t, < ☉ ♈ *R*; that is,

As the Co-ſine of the hour from Noon 73·31.01 ——— 9.4529082
To the Radius ———————— 10.0000000
So is the Tangent of the Declination 19·39.10¾ ——— 9.5522187
To the Tangent of the Latitude 51.32 ——— 10·0999135

Which is North becauſe the Sun's Declination is Northerly.

The time when the Sun is upon the Prime Vertical being known, he Latitude of the Place of Obſervation, and the Variation of the *Compaſs* may be obtained by the help of this *Problem.*

Caſe 4.

Given the Latitude of the Place and Altitude of the Sun upon the Prime Vertical, to find the Declination of the Sun and Hour of the Day.

Example.

In the Latitude of 51.32 North, the Sun's Height upon the Prime Vertical was found to be 25 deg. 26 min. 19 ſec. ¾. I demand the preſent Declination and hour of the Day.

Anſwer.

The preſent Declination 19 deg. 39 min. 10 ſec. ¾, and the Hour 4 h. 54 m. 4 ſ. in the Afternoon if the Sun was in the Weſtern Semicircle, but 7 h. 5 m. 56 in the Morning, if the Sun was in the Eaſtern Semicircle.

Caſe 5.

Given the Altitude of the Sun and hour of the Day, when the Sun appears upon the Prime Vertical, to find the Declination, and Latitude of the Place of Obſervation.

Example

The Sun being upon the Weſtern Quadrant of the Prime Vertical at 4 h. 54 m. 04 ſ. his Altitude was found to be 25 deg. 26 min. 19 ſec. ¾ I demand his Declination, and the Latitude of the Place of Obſervation ?

Anſwer.

The Sun's Declination is 19 deg. 39 min. 10 ſec. ¾, and the Latitude of the Place is 51 deg. 32 min. and both the ſame way, but whether North or South is not ſufficiently determined by this Data.

Caſe 6.

Given the Latitude of the Place, and hour of the Day when the Sun appears upon the Prime Vertical, to find the Declination and Altitude above the Horizon.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun was obſerved to be upon the Prime Vertical at 7 h. 5 m. 56 ſ. in the Morning. I demand his Declination and Altitude at that time.

Anſwer.

His Altitude is 25 deg. 26 min. 19 ſec. ¾, and his Declination 19 deg. 39 min. 10 ſec. ¾ North.

Caſe 7.

The Sun being upon the Prime Vertical there is given the Latitude of the Place, and the Angle which the Meridian paſſing thro' the Sun forms with the Prime Vertical, to find the Sun's Declination his Altitude, and hour of the Day.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun being in the Weſtern Semicircle, the Angle that the Meridian paſſing thro' the Sun formed with the Prime Vertical being 41 deg. 20 min. 27 ſec. I demand his Altitude, Declination, and hour of the Day.

Anſwer.

The Altitude is 25 deg. 26 min. 19 ſec. ¾, the Declination 19 deg. 39 min. 10 ſec. ¾ North, and the hour 4 h. 54 m. 4 ſ. in the Afternoon.

Caſe 8.

The Sun being upon the Prime Vertical there is given, his Declination, and the Angle that the Meridian paſſing thro' the Sun forms with the Prime Vertical, to find the Latitude of the Place, the Altitude of the Sun, and hour of the Day.

Example.

The Sun being upon the Eaſtern Quadrant of the Prime Vertical, and having 19 deg. 39 min. 10 ſec. ¾ North Declination, the Angle that the Meridian paſſing thro' the Sun formed with the Prime Vertical, was found to be 41 deg. 20 min. 27 ſec. I demand the Altitude, hour of the Day, and Latitude of the Place.

Anſwer.

The Sun's Altitude is 25 deg. 26 min. 19 ſec. ¾, the Latitude 51 deg. 32 min. North, and the hour 07 h. 05 m. 56 ſec. in the Morning.

Caſe 9.

The Sun being upon the Prime Vertical there is given, his Altitude, and the Angle which the Meridian paſſing thro' the Sun forms with the Prime Vertical, to find the Latitude of the Place, Declination of the Sun, and Hour of the Day.

Example.

The Sun being upon the Prime Vertical, his Altitude in the Morning was found to be 25 deg. 26 min. 19 ſec. ¾: and the Angle which the Meridian paſſing thro' the Sun, formed with the Prime Vertical was 41 deg. 20 min. 27 ſec. I demand the Latitude of the Place, Declination of the Sun, and hour of the Day.

Anſwer.

The Hour is 7 h. 5 m. 56 ſ. the Declination 19 deg. 39 min. 10 ſec. ¾, and the Latitude 51 deg. 32 min. the ſame way with the Declination, but whether they be North or South is not ſufficiently determined by the Data in the *Problem.*

Caſe 10. *Doubtful.*

The Sun being upon the Prime Vertical there is given, the hour of the Day, and Angle which the Meridian paſſing thro' the Sun, forms with the Prime Vertical, to find the preſent Declination, Altitude, and Latitude of the Place.

Example.

At 4 h. 54 m. 04 ſ. in the Afternoon, the Sun being then upon the Prime Vertical, the Angle which the Meridian paſſing thro' the Sun formed with the Prime Vertical being 41 deg. 20 min. 27 ſec. I demand the Sun's Altitude, his Declination, and the Latitude of the Place of Obſervation.

Anſwer.

The Sun's Altitude is 25 deg. 26 min. 19 ſec. ¾, the Declination 19 deg. 39 min. 10 ſec. ¾, and the Latitude of the Place of Obſervation 51 deg. 32 min. the ſame way with the Declination, but whether North or South is not ſufficiently determined by the Data in the *Queſtion*.

Prob. V.

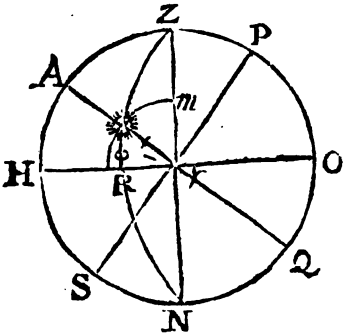
Caſe 1.

The Sun being upon the Equator, the Latitude of the Place is given and the hour of the Day, to find the Sun's Height, and Azimuth at that time.

Example.

In the Latitude of 51 deg. 32 min. North. I demand the Sun's Height and Azimuth at half an Hour paſt Three in the Afternoon, or at half an Hour paſt Eight in the Morning, each of which times are Three Hours and a half diſtant from Noon.

The Stereographic Solution upon the the Plane of the Meridian.



1. Having drawn the Meridian *Z O N H*, the Horizon *H O*, and at Right-angles to it the Prime Vertical *ZN*, ſet off the Latitude of the Place 51 deg. 32min. from *O* to *P*, and draw the Six a Clock Hour Circle *P ♈ S*, and at Right-angles to it the Equator *A Q*.

2. Set off the Semi-tang. of 37 deg. 30 min. the Complement of the hour from Noon reduced into Degrees, &c. in the Equator from ♈. to ☉, then will ☉ repreſent the Place of the Sun thro' which point, and the Zenith and Nadir, if the Azimuth or Vertical Circle Z ☉ *R N* be drawn, the thing is done; and *H R* the Azimuth from the Meridian, may be meaſured by *Caſe* the 2*d* of *Problem* the 7*th*, of *Section* the 2*d*, and ☉ *R* the Altitude may be meaſured by *Caſe* the 3*d* of the ſame *Problem*.

Logarithmically, or by Calculation.

In the Spherical Triangle ☉ ♈ *R* Right-angled at *R*, are given, ☉ ♈ the hour from Six, and the Angle ☉ ♈ *R* equal to the Arch *A H*, the Complement of the Latitude of the Place, whence to find ☉ *R* the Sun's Altitude at that time, it will be by the 2*d Example* of the 5*th Caſe of Right-angled Spherical Triangles*, As *R* : S. ☉ ♈ : : S. ☉ ♈ *R* : S. ☉ *R*; that is,

As the Radius ——————————— 10.0000000

To the Co-ſine of the hour from Noon 52.30 ——— 9.7844471
So is the Co-ſine of the Latitude 51.32 ——— 9.7938317

To the Sine of the height at that Hour 22.15.08¼ ——— 9.5782788

Whether it be Forenoon or Afternoon.

2. To find the Azimuth, it will be by the *2d Example* of the *4th Caſe* of *Right-angled Spherical Triangles*,

As *R* : c s, ☉ ♈ *R* : : t, ☉ ♈ : t, ♈ *R*; that is,

As the Radius ——————————— 10.0000000

To the Sine of the Latitude 51.32 ——————— 9.8937452
So is the Co-tangent of the hour from Noon 52.30 —— 9.8849865

To the Co-tang. of the Azim. from the Merid. 59.00.10 9.7787257

Which muſt be counted from the inviſible Pole; ſo that it if the time given be in the Forenoon, it is South 59 deg. 00 min. 10 ſec. Eaſt, but if the time given be in the Afternoon, it is South 59 deg. 00 min. 10 ſec. Weſt, or according to the *Nautical Phraſe*, the Sun at half an Hour paſt Eight in the Morning appears S. E. by E. ¼ E. nearly, and in the Afternoon at half an Hour paſt Three he appears S. W. by W. ¼ Weſterly nearly.

And becauſe in the ſame Place the Latitude remaining the ſame, the Sines of the Altitude are directly proportional to the Co-ſines of the hour from Noon, it follows, that the nearer the time is to Noon the greater is the Altitude, till when the Sun arrives upon the Meridian, the Altitude becomes equal to the Complement of the Latitude, and the ſame will happen to all the Inhabitants upon the Earth.

For when the point ☉ arrives at *A*, ♈ ☉ will become equal to ♈ *A* or a Quadrant, and ☉ *R* will become equal to *A H*.

And on the contrary, the greater is the diſtance of the time from Noon, the leſſer is the Altitude, and the greater is the Azimuth, till when the Sun is got at a Quadrantal, or at the diſtance of Six Hours from the Meridian, the Altitude vaniſhes, and the Azimuth becomes equal to 90 Degrees or a Quadrant, and the Sun appears at Riſing and Setting in the Eaſt or Weſt points of the Horizon; and this happens at this time to all the Inhabitants upon the Earth, except juſt under either of the Poles: For when the point ☉ arrives at ♈, the points ☉, *R*, and ♈, coincides, and conſequently the Arch ☉ *R* vaniſhes, and the Arch *H R* becomes equal to *H* ♈.

Again, the Hour remaining the ſame, becauſe the Sine of the Height is directly proportional to the Co-ſine of the Latitude, and the Co-tangent of the Azimuth to the Sine of the Latitude, it follows, that the greater is the Latitude, the leſſer will be the Altitude and the Azimuth from the Meridian at the ſame time; and conſequently when the Latitude is equal to a Quadrant or 90 Degrees the Altitude vaniſhes, and the Sun appears in the Horizon at the Azimuthal diſtance of the Hour converted into Degrees from the Meridian, and this happens particularly to thoſe who dwell under either of the Poles.

For when the Pole Point *P* coincides with the Zenith Point *Z* the Equator *A Q* coincides with the Horizon *H O*, at which time the Arch ☉ *R* vaniſhes, and *H R* becomes equal to *A* ☉.

And on the contrary it follows, that the leſſer is the Latitude, the greater will be the Altitude and Azimuth from the Meridian at the ſame time, till when the Latitude ceaſes, the Altitude becomes equal to the Complement of the Hour from Noon converted into Degrees, and the Azimuth becomes conſtantly equal to a Quadrant or 90 Degrees, and conſequently the Prime Vertical becomes the Path of the Sun's diurnal Courſe; and this happens particularly at this time, to all thoſe who dwell under the Equator.

For when the Pole Point *P* coincides with the Northermoſt Point of the Horizon *O*, the Axis or Six a Clock Hour Circle coincides with the Horizon, and the Equator *A Q* coincides with the Prime Vertical Z *N*, at which time the point *R* returns into ♈, the Arch ♈ *R* vaniſhes; and *H R* becomes equal to *H* ♈, and ♈ ☉ becomes equal to *H m*.

Prob. V.

Caſe 2.

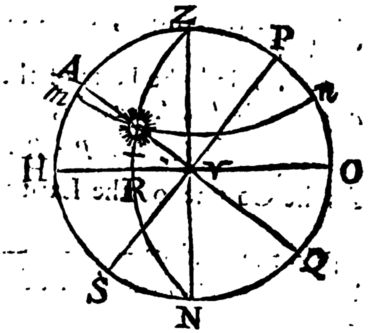
The Sun being in the Equator there is given, the Latitude of the Place and the Altitude of the Sun, to find the Azimuth of the Sun and hour of the Day.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun being in the Equator, his Altitude is found by Obſervation to be 32 deg. 15 min. 08 ſec. ¼. I demand the Azimuth and hour of the Day.

The Stereographic Solution upon the Plane of the Meridian.

1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *ZN*, ſet off the Latitude of the Place from *O* to *P*, and draw the Six a Clock hour Circle *PS*, and at Right-angles to it the Equator *AQ*.



2. About the Zenith as a Pole, deſcribe the Small Circle *m* ☉ *n*, at the diſtance of 67 deg. 44 min. 52 ſec. the Complement of the given Altitude, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*, where this interſects the Equator as in the point ☉, it will give the Place of the Sun.

3. Thro' the points Z, ☉, and *N*, let the Vertical Circle Z ☉ *N* be drawn, and the thing is done; and *A* ☉ the Hour from Noon and *HR* the Azimuth from the Meridian, may be meaſured by *Caſe* the 2*d* of *Problem* the 7*th*, of *Section* the 2*d*.

Logarithmically, or by Calculation.

In the Spherical Triangle ♈ ☉ *R* Right-angled at *R*, are given, the Angle ☉ ♈ *R* equal to the Arch *A H*, equal to the Complement of the Latitude, and ☉ *R* the Sun's Altitude, whence to find firſt ♈ *R* the Complement of the Azimuth from the Meridian, it will be by the *9th Caſe* of *Right-angled Spherical Triangles*, As the t, ⊙ ♈ *R* : *R* : : t. ⊙ *R* : S. *R* ♈ ; that is,

As the Co-tangent of the Latitude 51.32 —————— 9.9000865

To the Radius —————————————— 10.0000000
So is the Tangent of the Altitude 22.15.08¼ ———— 9.6118905

To the Co-ſine of the Azimuth from the Meridian 59.00.10 ———————— } 9.7118040

This if the Altitude was obſerved when the Sun was aſcending or in the Morning, is South 59 deg. 00 min. 10 ſec. Eaſt, or S. E. by E ¼ E. but if the Altitude was taken when the Sun was deſcending or in the Afternoon, it is South 59 deg. 00 min. 10 ſec. or S. W. by W. ¼ W.

2. To find the hour of the Day, it will by *Caſe* the 10*th of Right-angled Spherical Triangles,*

As S. ⊙ ♈ *R* : *R* : : S. ⊙ *R* : S. ⊙ ♈ ; that is,

As the Co-ſine of the Latitude 51.32 —————— 9.7938317

To the Radius —————————————— 10.0000000
So is the Sine of the Altitude 22.15.08¼ ———————— 9.5782788

To the Co-ſine of the Hour from Noon 52.30 ——— 9.7844471

Which being reduced into Time, will give 3 h. 30 m. for the hour of the Day in the Afternoon, if the Sun was in the Weſtern Semi-circle, but ſubſtracted from 12 hours will give 8 h. 30 m. in the Morning, if the Sun was in the Eaſtern Semi-circle.

This *Caſe* is doubtful, as appears by the 9*th Caſe of Right-angled Spherical Triangles*, and the *Conſtruction*; and as ♈ ⊙ gives the Hour reckoned from Six, ſuppoſe in the Morning, its Supplement or the Side of the great Triangle gives the hour in the Afternoon, counted from the ſame hour of Six; and the Supplement of *R* ♈ or Side of the ſame Triangle gives the Azimuth in the Afternoon, counted from the Eaſt point of the Horizon.

Prob.

Prob. V.

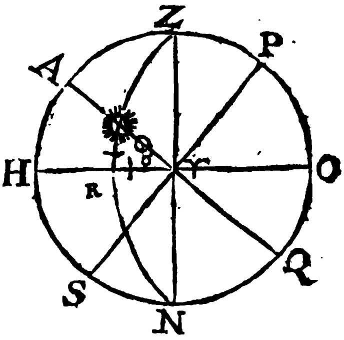
Caſe 3.

The Sun being in the Equator there is given, his Altitude and Azimuth, to find the Hour of the Day and Latitude of the Place.

Example.

The Sun being in the Equator his Altitude was found by Obſervation to be 22 deg. 15 min. 08 ſec. ¼, and at the ſame time his Azimuth was determined to be South 59 deg. 00 min. 10 ſec. Eaſt. I demand the Hour of the Day and Latitude of the Place of Obſervation.

The Stereographic Solution upon the Plane of the Meridian.



1. Having drawn the Meridian *Z O N H*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *Z N*, ſet off the Semi-tangent of 30 deg. 59 min. 50 ſec. the Complement of the Azimuth from the Meridian, from ♈ to *R*, and draw the Vertical Circle Z ☉ *R N*.

2. Set off the Altitude of the Sun 22 deg. 15 min. 08 ſec. ¼, in the Vertical Circle laſt drawn, from *R* to ☉, by *Caſe* the 3*d* of *Problem* the 6*th*, of *Section* the 2*d*, this will give the Place of the Sun.

3. Thro' ☉ and the Center ♈, draw the Equator *A* ♈ *Q*, and at Right-angles to it the Six a Clock hour Circle *P* ♈ *S*, and the thing is done; and *A* ☉ the hour from Noon may be meaſured by *Caſe* the 2*d* of *Problem* the 7*th*, of *Section* the 2*d*, and the Angle ☉ ♈ *R* equal to the Arch *AH*, equal to the Complement of the Latitude, may be meaſured by the *Line of Chords*.

Logarithmically, or by Calculation.

In the Spherical Triangle ☉ ♈ *R* Right-angled at *R*, are given, ♈ *R* the Complement of the Azimuth from the Meridian, and ☉ *R* the Sun's Altitude, whence to find the Angle ☉ ♈ *R* the Complement of the Latitude, it will be by *Caſe* the 2*d* of *Right-angled Spherical Triangles*,

As S, ♈ *R* : *R* : : t, ☉ *R* : t, ☉ ♈ *R*; that is,

As the Co ſine of the Azim. from the Merid. 59.00.10 —— 9.7118040

To the Radius —————————— 10 0000000
So is the Tangent of the Altitude 22.15.08 ¼ ———— 9.6118905

To the Co-tangent of the Latitude 51.32 ———— 9.9000865

Which is Northerly, becauſe the Azimuth counted from the Southern part of the Horizon is leſs than a Quadrant.

2. To find the Hour of the Day, it will be by *Caſe* the 1*ſt* of *Right-angled Spherical Triangles*,

As the *R* : c s, ♈ *R* : : c s, ☉ *R* ; c s, ☉ ♈ ; that is,

As the Radius —————————— 10.0000000

To the Sine of the Azim. from the Meridian 59 00.10 —— 9.9330782
So is the Co-ſine of the Altitude 22.15.08¼ ———— 9.9663884

To the Sine of the hour from Noon 52.30 ———— 9.8994666

This being reduced into Time, and ſubſtracted from twelve hours (becauſe the Azimuth is Eaſterly) or becauſe the Sun is in the Eaſtern Semi-circle, will give 8 h. 30 m. for the time of the Day in the Forenoon.

Prob. V.

Caſe 4.

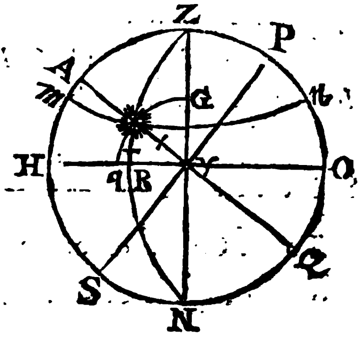
The Sun being in the Equator there is given, his Altitude and the hour of the Day, to find the Azimuth of the Sun and the Latitude of the Place.

Example.

The Sun being in the Equator at 30 minutes paſt Eight in the Morning, the Sun's Altitude was found by Obſervation to be 22 deg. 15 min. 08 ſec. ¼. I demand the Azimuth and Latitude of the Place.

The Stereographic Solution upon the Plane of the Meridian.

1. Having drawn the Meridian *Z O N H*, the Horizon *H O*, and at Right-angles to it the Prime Vertical Z *N*, about the Center ♈, at the diſtance of 37 deg. 30 min. the Semi-tangent of the hour from Six, deſcribe the Small Circle *G* ⊙ *q*.



2. At the diſtance of 67 deg. 44 min. 52 ſec. the Complement of the Sun's Altitude, about the Zenith as a Pole, deſcribe the Small Circle *m* ⊙ *n*, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*, where this interſects the Small Circle before drawn as at ⊙, it will give the Place of the Sun.

3. Thro' the point ⊙ and the Center ♈, draw the Equator *A*♈*Q*, and the Vertical Circle Z ⊙ *N* thro' the ſame point ⊙ and the points Z and *N*, and the thing is done; and *H R* the Azimuth from the Meridian, may be meaſured by *Caſe* the 2*d* of *Problem* the 7*th*, of *Section* the 2*d*; and the Angle ⊙ ♈ *R* equal to the Arch *A H*, equal to the Complement of the Latitude by the *Line of Chords*.

Logarithmically, or by Calculation.

In the Spherical Triangle ⊙ ♈ *R* Right-angled at *R*, are given, ⊙ ♈ the Complement of the hour from Noon, and ⊙ *R* the Altitude of the Sun, whence to find ♈ *R* the Complement of the Azimuth from the Meridian, it will be by *Caſe* the 14*th* of *Right-angled Spherical Triangles*,

As c s, ⊙ *R* : *R* : : c s, ♈ ⊙ : c s, *R*. ♈; that is,

As the Co-ſine of the Altitude 22.15.08 ¼ ———————— 9.9663881

To the Radius ———————————————— 10.0000000

So is the Sine of the hour from Noon 52.30 ———— 9.8994667

To the Sine of the Azim. from the Merid. 59.00,10 — 9.9330786

To be counted Eaſterly, becauſe the Obſervation was made in the Forenoon, but whether from the North or South is not ſufficiently determined in the *Queſtion.*

2. To find the Latitude, it will be by the 13*th Caſe of Right-angled Spherical Triangles,*

As S. ♈ ☉ : S. ☉ *R* : : *R* : S. ☉ ♈ *R*; that is,

As the Co-ſine of the hour from Noon 52.30 ———— 9.7844471

To the Sine of the Altitude 22.15.08¼ ———— 9.5782789
So is the Radius ———— 10.0000000

To the Co-ſine of the Latitude 51.32 ———— 9.7938318

But whether it be North or South doth not appear from the Data.

Caſe 5.

The Sun being in the Equator there is given, the hour of the Day and Azimuth of the Sun, to find his Altitude, and the Latitude of the Place of Obſervation.

Example.

The Sun being in the Equator at half an hour paſt Three in the Afternoon, the Azimuth of the Sun was found by Obſervation to be South 59 deg. 00 min. 10 ſec. Weſt. I demand the Latitude of the Place and height of the Sun.

Anſwer.

The Sun's Altitude is 22 deg. 15 min. 08 ſec. ¼, and the Latitude of the Place of Obſervation 51 deg. 32 min. North.

Caſe 6.

The Sun being in the Equator there is given, the Latitude of the Place and Azimuth of the Sun, to find the Altitude and hour of the Day.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun's Azimuth being found by Obſervation to be South 59 deg. 00 min. 10 ſec. Eaſt. I demand the hour of the Day and Altitude of the Sun, he being ſuppoſed to be in the Equator.

Anſwer.

The Sun's Altitude is 22 deg. 15 min. 08 ſec. ½, and the Hour half an Hour paſt Eight in the Morning.

Caſe 7.

The Sun being in the Equator there is given, the Latitude of the Place, and Angle formed by the Vertical Circle, paſſing thro' the Sun and the Equator, to find the Altitude and Azimuth of the Sun, and hour of the Day.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun being in the Equator there is given, the Angle that the Vertical Circle paſſing thro' the Sun forms with the Equator, equal to 57 deg. 46 min. 35 ſec. to find the hour of the Day, Altitude and Azimuth of the Sun.

Anſwer.

The Altitude is 22 deg. 15 min. 08 ſec. ½, Azimuth South 59 deg. 00 min. 10 ſec. Eaſt or Weſt, and the Hour 8 h. 30 m. in the Morning, or 3 h. 30 m. in the Afternoon, according as the Sun is in the Eaſtern or Weſtern Semicircle.

Caſe 8.

The Sun being in the Equator there is given, the hour of the Day and Angle formed by the Vertical Circle paſſing thro' the Sun and the Equator, to find the Altitude and Azimuth of the Sun, and the Latitude of the Place.

Example.

The Sun being in the Equator at half an Hour paſt Three in the Afternoon, the Angle formed by the Equator and the Vertical Circle paſſing thro' the Sun was found to be 57 deg. 16 min. 35 ſec. I demand the Altitude and Azimuth of the Sun, and the Latitude of the Place.

Anſwer.

The Sun's Altitude is 22 deg. 15 min. 08 ſec. ½, the Azimuth 59 deg. 00 min. 10 ſec. Weſt, and Latitude 51 deg. 32 min. but whether the Latitude be North or South, and whether the Azimuth

muth muſt be counted from the North or South, is not ſufficiently determined in the *Queſtion.*

Caſe 9.

The Sun being in the Equator there is given, his Altitude, and the Angle formed by the Equator and the Vertical Circle paſſing thro' the Sun, to find the Latitude of the Place, Azimuth of the Sun, and hour of the Day.

Example

The Sun being in the Equator, the Altitude of the Sun in the Eaſtern Semicircle was found to be 22 deg. 15 min. 08 ſec. ¼, when the Angle formed by the Equator and the Vertical Circle paſſing thro' the Sun was 57 deg. 46 min. 35 ſec. I demand the Latitude of the Place, Azimuth of the Sun, and hour of the Day.

Anſwer.

The hour of the Day is 30 minutes paſt Eight in the Morning, the Azimuth is 59 deg. 00 min. 10 ſec. Eaſterly, and the Latitude 51 deg. 32 min. but whether the Latitude be North, or whether the Azimuth muſt be counted from the North or South points, of the Horizon, is not ſufficiently determined.

Caſe 10. *Doubtful.*

The Sun being in the Equator there is given his Azimuth, and the Angle formed by the Equator, and the Vertical Circle paſſing thro' the Sun, to find the Latitude of the Place, Altitude of the Sun, and the hour of the Day.

Example.

The Sun being in the Equator, his Azimuth in the Morning was found by Obſervation to be South 59 deg. 00 min. 10 ſec. Eaſt, at the ſame time that the Angle formed by the Equator and the Vertical Circle paſſing thro' the Sun is 57 deg. 46 min. 35 ſec. I demand the Latitude of the Place, Altitude of the Sun, and hour of the Day.

Anſwer.

The Latitude of the Place is 51 deg. 32 min. North, the Altitude of the Sun is 22 deg. 15 min. 08 ſec. ¼, and the hour of the Day is half an hour paſt Eight in the Morning.

Prob.

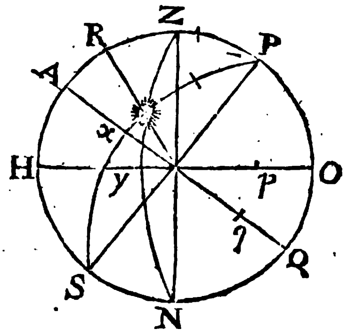
Prob. VI.

The Sun being in any point of the Ecliptic, his Declination is given, together with the hour of the Day and Latitude of the Place, to find his Altitude and Azimuth at that time.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun having 19 deg. 39 min. 10 ſec. $\frac{3}{4}$ North Declination, his Altitude and Azimuth is required at half an hour paſt Eight in the Morning, or at half an hour paſt Three in the Afternoon, at each of which times he is equally diſtant from the Meridian.

The Stereographic Solution upon the Plane of the Meridian.



1. Having drawn the Meridian Z *O N H*, the Horizon *H O*, and at Right-angles to it the Prime Vertical Z *N*, ſet off the Latitude of the Place from *O* to *P*, and draw the Six a Clock hour Circle *P S*, and at Right-angles to it the Equator *A Q*.

2. Thro' the Pole *P* draw the Hour Circle *P* ⊙ *S*, forming an Angle of 52 deg. 30 min. with the Meridian, equal to the time from Noon reduced into Degrees, &c. by *Caſe* the *3d* of *Problem* the *9th*, of *Section* the *2d*.

3. Set off the Declination of the Sun (in this *Caſe* to the Northward of the Equator, becauſe the Declination is North) in the hour Circle *P x* from *x* to ⊙ (by *Caſe* the *3d* of *Problem* the *6th*, of *Section* the *2d*) this will give the Place of the Sun.

4. Thro' the Zenith and Nadir and the point ⊙, draw the Vertical or Azimuth Circle Z ⊙ *N*, and the thing is done; and the Angle *PZ*⊙ or Arch of the Horizon *y O*, the Azimuth from the viſible Pole *P*, may be meaſured by *Caſe* the *2d* of *Problem* the *10th*, or *Caſe* the *2d* of *Problem* the *7th*, of *Section* the *2d*, and the Arch *Z* ⊙ the

Sun's Zenith Diſtance, or the Complement of his Altitude may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of the ſame *Section*.

To find the Requiſites by Calculation.

In the Oblique-angled Spherical Triangle *Z P* ☉ are given, *Z P* equal to the Complement of the Latitude, *P* ☉ in this *Caſe* equal to the Complement of the Sun's Declination, or his diſtance from the North Pole, and the Angle *Z P* ☉ the hour from Noon, whence to find firſt the Angle *P Z* ☉ the Azimuth from the Meridian, it will be by the 10*th Caſe* of *Oblique-angled Spherical Triangles*, having firſt let fall the Perpendicular ☉ *R*.

As the *R* : cs, *Z P* ☉ : : t, ☉ *P* : t, *P R* ; that is,

As the Radius ———————————————— 10.0000000

To the Co-ſine of the hour from Noon 52.30 ———— 9.7844471

So is the Co-tangent of the Declination 19.39.10¾ ——— 10.4471783

To the Tangent of a fourth Arch *P R* 59.36.08 — 10.2316254

If from the fourth Arch *P R* equal to 59 deg. 36 min. 08 ſec. be taken the Complement of the Latitude *Z P* equal to 38 deg. 28 min. there will remain Z *R* equal to 21 deg. 08 min. 08 ſec. which we will call a fifth Arch, for *P R*—Z *P*=Z *P* (*See the Figure*) wherefore, by the ſame *9th Caſe* of *Oblique*, &c. it will be,

As S. *Z R* : S. *P R* : : t, <*P* : t, <*Z* ; that is,

As the Sine of the 5*th* Arch *Z R* 21.08.08 ———— 9.5569965

To the Sine of the 4*th* Arch *P R* 59.36.08 ———— 9.9357759

So is the Tangent of the hour from Noon 52.30 ——— 10.1150195

To the Tang. of the Azim. from the Merid. 72.12.54 10.4937989

Which in the preſent *Caſe* becauſe the Latitude is North, and the Angle *P Z* ☉ Obtuſe, muſt be counted from the South point of the Horizon ; wherefore if the time given was in the Forenoon, the Azimuth is South 72 deg. 12 min. 54 ſec. Eaſt, or E. S. E. ¼ E. nearly, but if the time given be in the Afternoon, the Azimuth is South 72 deg. 12 min, 54 ſec. Weſt, or W. S. W. ¼ W. nearly.

2. To find the Altitude, it will be by the 10*th Caſe of Oblique-angled Spherical Triangles*,

As the cs, *P R* : cs *Z R* : : cs, *P* ☉ : cs, *Z* ☉ ; that is,

As the Co-ſine of the 4*th* Arch *P R* 59.36.08 ———— 9.7041508

To the Co-ſine of the 5*th* Arch *Z R* 21.08.08 ———— 9.9697559
So is the Sine of the Declination 19.39.10 ¾ ———— 9.5267557

To the Sine of the Altitude 38.18.46 ½ ———— 9.7923608

Which is the ſame both Forenoon and Afternoon.

Or,

The Azimuth being firſt obtained, the Altitude from thence may be found, by the help of the 3*d Axiom* of *Spherical Triangles* in *Page* the , or the Altitude being firſt Inveſtigated, the Azimuth may from thence be Calculated by the help of the ſame *Axiom*.

Prob. VI.

Caſe 2.

Given the Latitude of the Place, and the Declination of the Sun together with his Altitude, to find the hour of the Day and Azimuth.

Example.

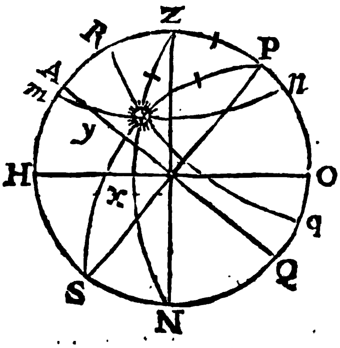
In the Latitude of 51 deg. 32 min. North, the Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination, his Altitude was found by Obſervation to be 38 deg. 18 min. 46 ſec. ½. I demand the hour of the Day and Azimuth of the Sun.

The Stereographic Solution upon the Plane of the Meridian.

1. Having drawn the Meridian *Z O N H*, the Horizon *H O*, and at Right-angles to it the Prime Vertical *Z N*, ſet off the Latitude of the Place from *O* to *P*, and draw the Six a Clock hour Circle *P S*, and at Right-angles to it the Equator *A Q*.

2. About the Pole *P*, at the diſtance of 70 deg. 20 min. 49 ſec. ¼, the Complement of the Sun's Declination becauſe the Declination is North, draw the Small Circle *R* ☉ *q*, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*.

3. At



3. At the diſtance of 51 deg. 41 min. 13 ſec. $\frac{3}{4}$, the Complement of the Sun's Altitude, about the Zenith Z, deſcribe the Small Circle *m* ☉ *n*, by the ſame *Caſe*, where this Circle interſects the Small Circle *R* ☉ *q* as in the point ☉; it will give the Place of the Sun at that time.

4. Thro the point ☉ and the Zenith and Nadir, draw the Vertical Circle Z ☉ *N*, alſo thro' the ſame point ☉ and the two Poles, draw the hour Circle *P* ☉ *S*, and the thing is done; and the Angle ☉ Z *P* or Arch of the Horizon *x O*, the Azimuth in this *Caſe* from the North, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, or *Caſe* the 2*d* of *Problem* the 7*th*, of *Section* the 2*d*, and the Angle Z *P* ☉ or Arch of the Equator *y Q* the hour from Noon, may be meaſured by the ſame *Caſes*.

Among the great variety of *Logarithmical Solutions* that may be given to this moſt uſeful *Problem* (as the *Reader* may ſee if he Conſult the 11*th Caſe* of *Oblique-angled Spherical Triangles* in *Page* the 109*th* or the *6th*, *7th*, and *8th Spherical Axioms*, contained between the 133*d* and 151*ſt Pages*) I ſhall make Uſe of two or three that are the moſt ready and eaſy for that Purpoſe.

1. If the Latitude of the Place and Declination of the Sun are both North or both South, ſubſtract the Declination from 90 Degrees or a Quadrant, but if the Latitude and Declination are contrary, that is if one be North and the other South, add the Declination to 90 Degrees or a Quadrant, and call the Sum or Difference the Diſtance of the Sun from the Pole, then having let fall the Perpendicular ☉ *R* from the point ☉ upon the Meridian *Z O N H*, it will be by the *6th Spherical Axiom*.

$$\text{As } t, \tfrac{1}{2} ZP : t, \tfrac{1}{2}\overline{\odot P + \odot Z} : : t, \tfrac{1}{2}\overline{\odot P - \odot Z} : t, \tfrac{1}{4} PR - \tfrac{1}{2} ZP.$$

As the Tang. of ½ the Comp. of the Latitude 19.14.00—9.5426877

To the T. of ½ the Sum of the Sun's diſt. from the Pole and the Comp. of the Latitude 61.01 01 ⅛ } 10.2565541

So is the T. of ½ the Diff. between the diſt. of the Sun from the Pole and the Comp. of Lat. 9 19.47⅞ } 9.2156196

To the Tangent of a *4th* Arch 40.22.08——————9.9294860

Which fourth Arch 40 deg. 22 min. 08 ſec. added to half the Complement of the Latitude 19 deg. 14 min. 00 ſec. will give 59 deg. 36 min. 08 ſec. for a fifth Arch *P R*, and this fifth Arch *PR* leſſened by the Complement of the Latitude 38 deg. 28 min. 00 ſec. will give 21 deg. 08 min. 08 ſec. equal to Z *R*, for a ſixth Arch, whence to find the Azimuth *P* Z ⊙, it will be by the *12th Caſe of Right-angled Spherical Triangles*,

$$\text{As } R : ct, Z\odot : : t, ZR : cs, \odot ZR; \text{ that is,}$$

As the Radius ————————————10.0000000

To the Tangent of the Altitude 38.18.46½——————9.8976923

So is the Tangent of the ſixth Arch Z *R* 21.08.08———9.5872404

To the Co-ſine of the Azimuth 72.12.54——————9.4849327

Which becauſe the Latitude is North, is South 72 deg. 12 min. 54 ſec. Eaſt, or E. S. E. half E. nearly, if the Altitude of the Sun was taken in the Morning, but if the Sun's Altitude was obſerved in the Afternoon, it is South 72 deg. 12 min. 54 ſec. Weſt, or W. S. W. half W. nearly.

Again, to find the hour of the Day, it will be by the *12th Caſe of Right-angled Spherical Triangles*,

$$\text{As } R : ct, P\odot : : t, PR : cs, ZP\odot; \text{ that is,}$$

As the Radius ————————————10.0000000

To the Tangent of the Sun's Declination 19 39.10¾———9.5528217

So is the Tangent of the *5th* Arch *P R* 59.36.08 ——10.2316254

To the Co-ſine of the hour from Noon 52.30——————9.7844471

Which Converted into Time will give 3 h. 30 m. for the Hour in the Afternoon, if the Sun's Altitude was taken in the Weſtern Semicircle, but ſubſtracted from 12 hours, will give 8h. 30m. for the Hour in the Morning, if the Altitude of the Sun was taken in the Eaſtern Semicircle.

Or,

To the Complement of the Latitude of the Place 38 deg. 28 min. equal to Z *P*, add the Complement of the Altitude of the Sun 51 deg. 41 min. 13 ſec. ½ equal to Z ⊙, and the diſtance of the Sun from the Pole 70 deg. 20 min. 49 ſec. ¼ equal to *P* ⊙, into one Sum, and from half that Sum 80 deg. 15 min. 01 ſec. ⅜, ſubſtract the Sun's diſtance from the Pole equal to 70 deg. 20 min. 49 ſec ¼, and keep the Difference; then ſay,

As the Radius	10 0000000
To the Co-ſine of the Latitude 51.32	9.7938317
So is the Co-ſine of the Altitude 38.18.46 ½	9.8946686
To a fourth Sine	9.6885003
As the fourth Sine	9.6885003
To the Sine of the half Sum 80.15.01 ⅜	9.9936845
So is the Sine of the Difference 9.54.12 ⅛	9.2354940
To a Seventh Sine	19.5406782
To which add the Radius	10.0000000
And the Co-ſine of half that Sum	19.5406782
59.53.32¾, being doubled will	9.7703391

Give 107 deg. 47 min. 05 ſec. ½ for the Angle *P* Z ⊙, or Arch of the Horizon ✕ *O*, the Azimuth to be counted from the North point of the Horizon, becauſe the Latitude is North, and therefore being taken from 180 deg. 00 min. 00 ſec. will leave 72 deg. 12 min. 54 ſec. ½ for the Azimuth, to be counted from the South as before determined.

Or,

To the Arithmetical Complement of the Co-ſine of the Latitude 51.32	0.2061683
Add the Arithmetical Complement of the Co-ſine of the Altitude 38.18.46½	0.1053314
Alſo the Co-ſine of the half Sum 80.15.01¼	9.9936845
And the Co-ſine of the Difference 9.54.12¼	9.2354940
Half the Sum of theſe four Sines	19.5406782
Will give the Co-ſine of 53.53.32¾	9.7703391

The half of the Azimuth, which being doubled, will give 107 deg. 47 min. 05 ſec: ½ for the whole Azimuth, to be counted from the North, as before found.

Or,

From half the Sum of the Complement of the Latitude, Complement of the Altitude, and diſtance of the Sun from the Pole equal to 80 deg. 15 min. 01 ſec. ½, ſubſtract firſt, the Complement of the Latitude 38 deg. 28 min. and call the Remainder (41 deg. 47 min. 01 ſec. ¾) the firſt Difference; then ſecondly, from the ſame half Sum Subſtract the Complement of the Altitude 51 deg. 41 min. 13 ſec. ½, and call the Remainder (28 deg. 33 min. 48 ſec. ¼) the ſecond Difference; then ſay,

As the Radius	10.0000000
To the Co-ſine of the Latitude 51.32	9.7938317
So is the Co-ſine of the Altitude 38.18.46½	9.8946686
To a fourth Sine	9.6885003
As the fourth Sine	9.6885003
To the Sine of the firſt Difference 41.47.01¾	9.8236842
So is the Sine of the ſecond Difference 28.33.47⅞	9.6795455
To a Seventh Sine	9.8147294
To which add the Radius	10.0000000
The Sine of half that Sum	19.8147294
Will be 53.53.33	9.9073647

Which being doubled, will give 107 deg. 47 min. 06 ſec. the Azimuth from the North, and this taken from 180 deg. will leave 72 deg. 12 min. 54 ſec. for the Azimuth from the South, as before determined.

Or,

To the Arith. Comp. of the Co-ſine of the Lat. 51.32	0.2061685
Add the Arith. Comp. of the Co-ſine of the Alt. 38.18.46½	0.1053314
Alſo the Sine of the *1ſt* Difference 41.47.01 ¼	9.8236842
And the Sine of the *2d.* difference 28.33.48¼.	9.6795463
Half the Sum of theſe four Sines	19.8147302
Will be the Sine of 53.53.33¾	9.9073651

Which being doubled, will give 107 deg. 47 min. 06 ſec. for the Azimuth from the North, and this being ſubſtracted from 180 deg. 01 min. 00 ſec. will give the Azimuth from the South, the ſame as in the former *Example.*

The Azimuth being thus obtained, the hour of the Day may be had by the *3d Axiom of Spherical Triangles.* For,

As S. *P* ⊙ : S. ∠*PZ* ⊙ : : S. *Z* ⊙ : S. *ZP* ⊙ ; that is,

As the Co-ſine of the Declination 19.39.10¾	9.9739342
To the Sine of the Azimuth 107.47.06	9.9787325
So is the Co-ſine of the Altitude 38.18.46½	9.8946686
To the Sine of the hour from Noon 52.30.00	9.8974669

Which converted into Time, gives 3h. 30m. 00ſ. if the Altitude was taken in the Afternoon, but ſubſtracted from 12 Hours gives the time of the Day if the Altitude was obſerved in the Morning.

Again, by the help of the *7th* and *8th Axioms of Spherical Triangles,* may the Hour of the Day be inveſtigated directly, without finding the Azimuth, after the Manner following,

From the half Sum of the Complement of the Latitude, the Complement of the Altitude, and the diſtance of the Sun from the viſible Pole, equal to 80 deg. 15 min. 01 ſec. ¾, ſubſtract the Complement of the Sun's Altitude equal to 51 deg. 41 min. 13 ſec. ½, and note the Difference 28 deg. 33 min. 47 ſec. ⅞, and it will be, As the Radius —————— 10.0000000

To the Co-ſine of the Latitude 51.32 —————— 9.7938317
So is the Co-ſine of the Declination 19.39.10¾ —————— 9.9739342

To a fourth Sine —————— 9.7677659

As the fourth Sine —————— 9.7677659

To the Sine of half the Sum 80.15.01½ —————— 9.9936845
So is the Sine of the Difference 28.33.47⅞ —————— 9.6795455

To a Seventh Sine —————— 9.9054641
To which add the Radius —————— 10.0000000

And the Co-ſine of half the Sum of theſe four Sines — 19.9054641

Will be the Co-ſine of 26.14.59 —————— 9.9527320

Which being doubled, will give 52 deg. 29 min. 58 ſec. for the Hour from Noon, and this reduced into Time, will give as in the former *Operation*, either 3 h. 30 m. 00 ſ. or 8 h. 30 m. 00 ſ. according as the Sun was in the Eaſtern or Weſtern Semicircle.

Or,

To the Arith. Comp. of the Co-ſine of the Lat. 51.32 — 0.2061683
Add the Arith. Comp. of the Co-ſine of the Decl. 19.39.10¾ 0.0260658
Alſo the ſine of the half Sum 80.15.10½ —————— 9.9936845
And the Sine of the Remainder 28.33.47⅞ —————— 9.6795455

Half the Sum of theſe four Sines —————— 19.9054641

Will give the Co-ſine of 26.14.59 —————— 9.9527320

Which being doubled, will give 52 deg. 29 min. 58 ſec. equal in Time to 3 h. 30 m. 00 ſ. as in the former Calculation.

The Hour being thus obtained, the Azimuth from thence may be Calculated, by the help of the 3d *Axiom of Spherical Triangles*. For, As the S. Z ⊙ ; S. <Z *P* ⊙ : : S. *P* ⊙ : S. <⊙ Z *P* ; that is,

As the Co-ſine of the Altitude 38.18.46$\frac{1}{2}$ ———— 9.8946686

To the Sine of the Hour from Noon 52.30.00 ———— 9.8994667
So is the Co-ſine of the Declination 19.39.10$\frac{3}{4}$ ———— 9.9739342

To the Sine of the Azimuth 107.47.06 ———— 9.9787323

The ſame as was deduced from the former Calculation.

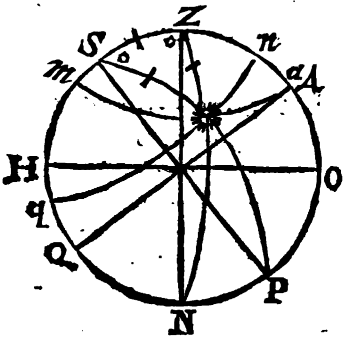
Prob. VI.

Caſe 2.

Example 2.

Given the Latitude of the Place 51 deg. 32 min. South, the Declination of the Sun 19 deg. 39 min. 10 ſec. $\frac{3}{4}$ South, and his Altitude 38 deg. 18 min. 46 ſec. $\frac{1}{2}$, to find the Azimuth and the Hour of the Day.

The Stereographic Solution upon the Plane of the Meridian.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical Z *N*, ſet off the Latitude of the Place, (in this *Caſe* South) from *H* to *S*, and draw the Six a Clock Hour Circle *SP*, and at Right-angles to it the Equator *AQ*.

2. About the Pole *S*, at the diſtance of 70 deg. 20 min. 49 ſec. $\frac{1}{4}$, the Complement of the Sun's Declination, draw the Small Circle *n* ⊙ *q*, alſo about the point Z at the diſtance of 51 deg. 41 min. 13 ſec. $\frac{2}{4}$ the Complement of the Sun's Altitude, draw the Small Circle *A* ⊙ *m*, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*, where theſe two Circles interſect each other, it will give the Place of the Sun ⊙.

3. Thro' the point ☉ and the Poles *S* and *P*, draw the Hour Circle *S*☉*P*, alſo thro' the ſame point ☉ and the Zenith and Nadir, draw the Azimuth Circle Z ☉ *N*, then may the Angle *P* Z ☉ the Azimuth, and the Angle ☉ *P* Z the Hour, be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*.

And becauſe the Numbers in this *Example* are the ſame with the Numbers in the former *Example*, the Hour from Noon (by the ſame Method of *Calculation*) will be found to be the ſame, that is 3h. 30m. 00ſ. if the Altitude was taken in the Afternoon, but 8 h. 30 m. 01ſ. if the Altitude were obſerved in the Morning; alſo the Azimuth will be the ſame in Quantity in this *Example* as it was in the former, but muſt be counted from the oppoſite point of the Horizon, becauſe the oppoſite or Southern Pole in this *Caſe* is elevated above the Horizon, and conſequently the Azimuth in our preſent *Caſe* is North 72 deg. 12 min. 54 ſec. Eaſt, or E. N. E. half E. nearly, if the Sun be in the Eaſtern Semicircle, but North 72 deg. 12 min. 54 ſec. Weſt, or W. N. W. half W. nearly, if the Sun was obſerved in the Weſtern-Semicircle.

Prob. VI.

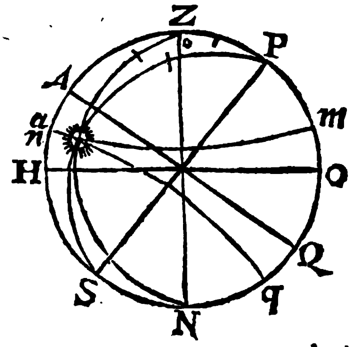
Caſe 2.

Example 3.

Given the Latitude of the Place 51 deg. 32 min. 00 ſec. North, the Sun's Declination 19 deg. 39 min. 10 ſec. ¼ South, and his Altitude 15 deg. 30 min. 20 ſec. I demand the Hour of the Day and Azimuth of the Sun.

The Stereographic Solution upon the Plane of the Meridian.

1. Having drawn the Meridian Z *O N H*, the Horizon *HO*, and at Right-angles to it the Prime Vertical Z *N*, ſet off the Latitude of the Place 51° 32', from *O* to *P*, and draw the Axis *PS*, and at Right-angles to it the Equator *A Q*.



2. About the Pole *S* (becauſe the Declination is South) at the diſtance of 70 deg. 20 min. 49 ſec. ¾, the Complement of the Sun's Declina-

tion, deſcribe the Small Circle *a* ⊙ *q*, alſo at the diſtance of 74 deg. 29 min. 40 ſec. the Complement of the Sun's Altitude, about the Zenith as a Pole, deſcribe the Small Circle *n* ⊙ *m*, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*, where theſe two interſect each other at the point ⊙, it will give the Place of the Sun.

3. Thro' the point ⊙ and the Poles *P* and *S*, draw the Hour Circle *P* ⊙ *S*, alſo thro' the ſame point ⊙ and the Zenith and Nadir, draw the Azimuth Circle Z ⊙ *N*, then may the Azimuth ⊙ *Z P* and the Hour from Noon *Z P* ⊙, be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*.

Logarithmically, or by Calculation.

Becauſe the Latitude is North and the Declination South, to 90 Degrees or a Quadrant add the Declination 19 deg. 39 min. 10 ſec. ¾, the Sum 109 deg. 39 min. 10 ſec. ¾, will give *P* ⊙ the diſtance of the Sun from the North Pole. Then,

To the Complement of the Latitude 38 deg. 28 min. 00 ſec. add the Complement of the Altitude 74 deg. 29 min. 40 ſec. and the diſtance of the Sun from the North or viſible Pole 109 deg. 39 min. 10 ſec. ¾ into one Sum, from half that Sum 111 deg. 18 min. 25 ſec. ⅜, ſubſtract the diſtance of the Sun from the Pole 109 deg. 39 min. 10 ſec. ¾, and keep the Difference and it will be,

As the Radius	10.0000000
To the Co-ſine of the Latitude 51.32.00	9.7938317
So is the Co-ſine of the Altitude 15.30.20	9.9838988
To a fourth Sine	9.7777305
As this fourth Sine	9.7777305
To the Sine of the half Sum 111.18.25⅜	9.9692512
So is the Sine of the Difference 1.39.14⅝	8.4603647
To a ſeventh Sine	8.6518854
To which add the Radius	10.0000000
And half that Sum	18.6518854
Will be the Co-ſine of 77.46.08	9.3259427

Which being doubled, will give 155 deg. 32 min. 36 ſec. for the Azimuth from the viſible Pole (in this *Caſe* North) this therefore taken from 180 deg. 00 min. 00 ſec. will give 24 deg. 27 min. 24 ſec. for the Azimuth, to be counted from the oppoſite point, and conſequently if the Obſervation be made in the Eaſtern Semicircle, the Azimuth is South 24 deg. 27 min. 24 ſec. Eaſt, or S. S. E. ¼ E. nearly, but if the Altitude be obſerved in the Weſtern Semicircle, the Azimuth will be South 24 deg. 27 min. 24 ſec. Weſt, or S. S. W. a quarter W. nearly,

Or,

To the Arith. Comp. of the Co-ſine of the Lat. 51.32.00 0.2061683
Add the Arith. Comp. of the Co-ſine of the Alt. 15.30.20 0.0161012
Alſo the Sine of the half Sum 111.18.25¼ ——— ——— — 9.9692513
And the Sine of the Difference 1.39.14¾ ——— ——— 8.4603647

Half the Sum of theſe four Sines ——— ——— ——— — ——18.6518854

Will be the Co-ſine of 77.46.18 ——— ——— ———9.3259427

Which being doubled, will give for the Azimuth 155 deg. 32 min. 36 ſec. as before found; whence to find the Hour of the Day, it will be by the 3*d Axiom*,

As the S. *P*⊙ : S. Z⊙ : : S. <Z : S. <*P*; that is,
As the Co-ſine of the Declination 19.39.10¾ ——— —9.9739342

To the Co ſine of the Altitude 15.30.20 ——— ———9.9838988
So is the Co-ſine of the Azimuth 155.32.36 ——— 9.6170055

To the Sine of the Hour from Noon 25.03.46⅝ ——— —9.6269701

Which reduced into Time, will give 1h. 40m. 15ſ. for the Hour if the Altitude be taken in the Afternoon, but ſubſtracted from 12 Hours, will give 10 h. 19 min. 45 ſec. for the Hour in the Forenoon, if the Altitude were taken in the Morning.

Or,

From half the Sum of the Complement of the Latitude, Complement of the Altitude, and diſtance of the Sun from the Pole equal to 111 deg. 18 min. 25 ſec. ¼, take the Complement of the Altitude equal to 74 deg. 29 min. 40 ſec. and keep the Difference 36 deg. 48 min. 45 ſec. ¾; then,

To the Arith. Comp. of the Co-ſine of the Latit. 51.32—0.2061683
Add the Arith. Comp. of the Co-ſine of the Decl. 19.39.10$\frac{3}{4}$ 0.0260658
Alſo the Sine of the half Sum 111.18.25 $\frac{3}{8}$————9.9692512
And the Sine of the Difference 36.48.45$\frac{3}{8}$ ————9.7775727

Half the Sum of theſe four Sines———— 19.9790580

Will give the Co-ſine of 12.31:52$\frac{3}{4}$ ———— 9.9895290

Which being doubled, will give 25 deg. 03 min. 45 ſec. $\frac{1}{2}$, and this reduced into Time, will give 1 h. 40 m. 15 ſ. the ſame with the former Calculation.

The Hour being thus obtained, the Azimuth may be inveſtigated by the help of the 3*d Axiom*.

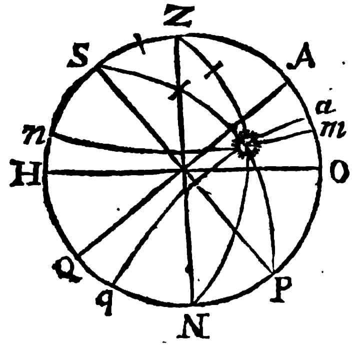
Prob. VI.

Caſe 2.

Example 4.

Given the Latitude of the Place 51 deg. 32 min South, the Declination of the Sun 19 deg. 39 min. 10 ſec. $\frac{3}{4}$ North, and the Altitude 15 deg. 30 min. 20 ſec. to find the Hour of the Day, and Azimuth.

The Stereographic Solution upon the Plane of the Meridian.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *ZN*, the Six a Clock Hour Circle *SP*, and the Equator *AQ* as in the 2*d Example* of this *Caſe*, about the point Z at the diſtance of 74 deg. 29 min. 40 ſec. the Complement of the Altitude, deſcribe the Small Circle *n* ☉ *m*

2. About the Pole *P* (becauſe the Declination is North) deſcribe the Small Circle *a* ☉ *q*, at the diſtance of 70

70 deg. 20 min. 50 ſec. the Complement of the Declination, where this cuts the Small Circle *n* ⊙ *m* as in the point ⊙, it will give the Place of the Sun; and if thro' this point and the Zenith and Nadir, and the two Poles *P* & *S*, be drawn an Azimuth Circle as *Z* ⊙ *N*, and an Hour Circle as *S* ⊙ *P*, the thing is done; and the Azimuth *S Z* ⊙ and the Hour *Z S* ⊙ may be meaſured by *Caſe* the *2d* of *Problem* the 10*th*, of *Section* the 2*d*.

And inaſmuch as the Numbers in this *Example* are the ſame with the Numbers in the former *Example*, the Hour from Noon will be found 1 h. 40 m. 15 ſ. if the Altitude be taken in the Afternoon, but 10 h. 19 m. 45 ſ. if the Altitude be taken in the Morning; alſo the Quantity of the Azimuth will be the ſame in this *Example* as in the former, but muſt be counted from the oppoſite point of the Horizon; and conſequently if the Sun was in the Eaſtern Semi-circle the Azimuth will be North 24 deg. 27 min. 24 ſec. Eaſt, or N. N. E. a quarter E. nearly, but if the Altitude was taken in the Weſtern Semicircle, the Azimuth will be North 24 deg. 27 min. 24 ſec. Weſt, or N. N. W. a quarter W. nearly.

I have been the more particular in the Solution of this Caſe, *it being the principal* Problem *made Uſe of by* Aſtronomers, *to find the true Time of the Day, in order to Correct their Clocks, and by the* Navigators, *to find the true Azimuth, in order to find the* Variation *of the* Compaſs.

Prob. VI.

Caſe 2.

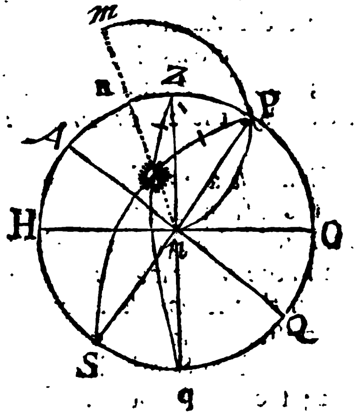
Given the Sun's Altitude and Declination, together with his Azimuth, to find the Hour of the Day and Latitude of the Place of Obſervation.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ½ North Declination, his Altitude was found to be 38 deg. 18 min. 46 ſec. ½, and at the ſame time his Azimuth South 72 deg. 12 min. 54 ſec. Eaſt. I demand the Hour of the Day and Latitude of the Place of Obſervation.

The Stereographic Solution.

1. Having drawn the Meridian *Z O N H*, the Horizon *H O*, and at Right-angles to it the Prime Vertical *Z N*, thro' the point *Z*



draw the Vertical Circle Z ⊙ *N*, forming an Angle with the Meridian of 72 deg. 12 min. 54 ſec. equal to the Azimuth, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*, and ſet off the Complement of the Sun's Altitude 51 deg. 41 min. 13 ſec. $\frac{1}{2}$ in it, from Z downward to ⊙, by *Caſe* the 3*d* of *Problem* the 6*th*, of the ſame *Section*, then will ⊙ repreſent the Place of the Sun.

2. About the point ⊙ as a Pole, at the diſtance of 70 deg. 20 min. 49 ſec. $\frac{1}{4}$, the Complement of the Sun's Declination from the Pole, deſcribe the Small Circle *m P n* by *Caſe* the 3*d* of *Problem* the 4*th*, of *Section* the 2*d*; where this Circle interſects the Meridian as in *P*, it gives the Place of the North Pole.

3. Thro' the point *P* draw the Six a Clock Hour Circle *P S*, and at Right-angles to it the Equator *A Q*, and thro' the point ⊙ and the points *P* and *S* and Z and *N*, draw the Hour Circle *P* ⊙ *S*, and the Azimuth Circle Z ⊙ *N*, and the thing is done; and the Hour from Noon, or the Angle Z *P* ⊙ may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*, and the Arch *P O* the Latitude of the Place may be meaſured by the *Line of Chords*.

Logarithmically, or by Calculation.

Having let fall the Perpendicular ⊙ *R* from ⊙ upon the Meridian, it will be by *Caſe* the 3*d* of *Oblique-angled Spherical Triangles*. Firſt to find the Arch *Z R*.

As *R* : c s, ⊙ *Z P* : : t, Z ⊙ : t, *Z R*; that is,

As the Radius ———————————— 10 0000000

To the Co-ſine of the Azimuth 72.12.54 ———— 9.4849345
So is the Co-tangent of the Altitude 38.18.46$\frac{1}{2}$ ———— 10.1023077

To the Tangent of a fourth Arch 21.08.08 ———— 9.5872422

And again, As c s, Z ☉ : c s, P ☉ : : c s, Z R : c s, P R; that is,
As the Sine of the Altitude 38.18.46½ ——— ——— 9.7923608

To the Sine of the Declination 19.39.10¾ ——— ——— 9.5267557
So is the Co-ſine of the fourth Arch 21.08.08 ——— 9.9697559

To the Co-ſine of a fifth Arch 59 36.08 ——— ——— 9.7041508

But P R—R Z=Z P. Wherefore,

	°	′	″
If from the fifth Arch ——— ——— ———	59	36	08
Be taken the fourth Arch ——— ——— ———	21	08	08
There will Remain the Complement of the Latitude—	38	28	00
Which taken from ——— ——— ———	90	00	00
Leaves the Latitude of the Place of Obſervation ———	51	32	00

Again, to find the Hour of the Day, it will be by *Caſe* the *4th of Oblique-angled Spherical Triangles,*

As the S. P ☉ : S. Z ☉ : : S. ∠Z ☉ P : S. ∠Z P ☉; that is,
As the Co-ſine of the Declination 19.39.10¾ ——— 9.9739342

To the Co-ſine of the Altitude 38.18.46¼ ——— ——— 9.8946686
So is the Sine of the Azimuth 72.12.54 ——— ——— 9.9787325

To the Sine of the Hour from Noon 52.30.00 ——— ——— 9.8994669

Equal in Time to 3 h. 30 m. 00 ſ. whence the Hour of the Day may be had by the former *Rules*.

Or, the Azimuth being firſt found, the Hour of the Day might have been found by the help of the *3d Axiom*, or the Hour being firſt calculated, the Azimuth might have been determined by the help of the ſame *Axiom*, as in the former *Caſe*.

When the Altitude of the Sun becomes equal to the Declination, then the Azimuth becomes equal to the Hour, for when Z ☉ is equal to P ☉, then the Angle P becomes equal to the Angle Z.

But, when the Altitude of the Sun is leſs than the Declination, (as will happen every Day except when the Sun is in the Equator, and in every Place except juſt under either Pole) the *Caſe* will be Ambiguous: For, the Arch Z ☉ the Sun's Zenith Diſtance, will be greater than *P* ☉ his Diſtance from the Pole.

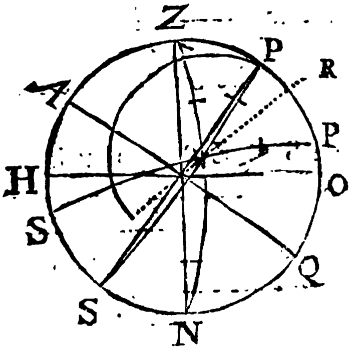
Prob. VI.

Caſe 3.

Example 2.

Let the Sun's Azimuth be ſuppoſed North 74 deg. 03 min. 11 ſec. ¼ Weſt, his Altitude 12 deg. 30 min. 00 ſec. and his Declination 19 deg. 39 min. 10 ſec. ¾ North, and let it be required to find the Hour of the Day and Latitude of the Place.

The Stereographic Solution.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, the Prime Vertical Z*N*, the Azimuth Circle Z ☉ *N* forming an Angle with the Meridian of 74 deg. 03 min. 11 ſec. ¼, and made Z ☉ equal to 77 deg. 30 min. 00 ſec. about the Point ☉ deſcribe a Small Circle at the diſtance of 70 deg. 20 min. 49 ſec. ¾, this will cut the Meridian in the points *P* and *P*, and conſequently the Arches *PO* and *PO* will be the Latitudes of the two Places, and the Angles Z*P*☉, Z*P*☉, the Hour at each of theſe Places; wherefore, having let fall the Perpendicular *PR*, it will be

As *R* : c s, ☉ Z *P* : : t, ☉ Z : t, Z *R*; that is,

As the Radius ———————— 10.0000000

To the Co-ſine of the Azimuth 74.03.11¼ ———— 9.4389312
So is the Co-tangent of the Altitude 12.30.00 ———— 10.6542448

To the Tangent of a fourth Arch 51.05.58½ ———— 10.0931760

And again, As $cs, Z\odot : cs, P\odot :: cs, ZR : cs, PR$; that is,

As the Sine of the Altitude 12,30.00 —————— 9.3353368

To the Sine of the Declination 19.39.10¾ —————— 9.5267557

So is the Co-ſine of the fourth Arch 51.05.58½—— —— 9 7979380

To the Sine of a fifth Arch 12.37.58½ —————— 9.9893569

But $ZR \pm PR = ZP$, or $R\odot \pm PR = PO$; that is,

	°	′	″
If to the Complement of the 4th Arch *Z R* equal to—	38	54	01½
Be added the fifth Arch *P R* equal to ——————	12	37	58½
The Sum will give one Latitude equal to ——————	51	32	00

Again,

	°	′	″
If from the Comp. of the 4th Arch *Z R* equal to —	38	54	01½
Be taken the fifth Arch *P R* equal to ——————	12	37	58½
The Remainder will be the other Latitude equal to—	26	16	03

And to find the Hour at each of thoſe Places it will be,

As $S.P\odot : S.Z\odot :: S.\angle Z : S.\angle P$; that is,

As the Co-ſine of the Declination 19.39.10¾—— —— 9.9739342

To the Co-ſine of the Sun's Altitude 12.30.00———— 9.9895815

So is the Sine of the Azimuth from the Merid. 74.03.11¼ 9.9829568

To the Sine of the Hour from Noon 85.24.31 ———— 9.9986041

Which reduced into Time, gives 5 h. 41 m. 38 ſ. for the Hour in the Afternoon, in the Latitude of 26 deg. 16 min. 03 ſec. North, but 6 h. 18 m. 22 ſ. for the Hour in the Afternoon, in the Latitude of 51 deg. 32 min. becauſe the Azimuth is Weſterly; but if the Azimuth had been Eaſterly in the Latitude of 26 deg. 16 min. 03 ſec. the Hour in the Forenoon had been 6 h. 18 m. 22 ſ. and in the

Latitude of 51 deg. 32 min. North, the Hour had been 5 deg. 41 min. 38 ſec.

Prob. VI.

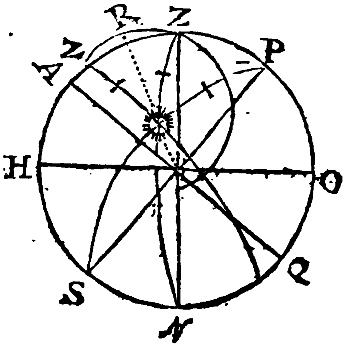
Caſe 4.

Given the Altitude of the Sun and his preſent Declination, together with the Hour of the Day, to find the Latitude of the Place and Azimuth from the Meridian.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ¾ of North Declination, at 3 h. 30 m. 00 ſ. in the Afternoon, his Altitude was found to be 38 deg. 18 min. 46 ſec. ½. I demand the Latitude of the Place, and Azimuth of the Sun.

The Stereographic Solution



1. Having drawn the Meridian Z *O N H*, the Six a Clock Hour Circle *P S*, and the Hour Circle *P* ⊙ *S*, forming an Angle of 52 deg. 30 min. 00 ſec. with the Meridian Z*O*·*NH*, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*.

2. Set off the Complement of the Sun's Declination 70 deg. 20 min. 49 ſec. ¼ from *P* to ⊙, and about the point ⊙ as a Pole, at the diſtance of 51 deg. 41 min. 13 ſec. ¼ the Complement of the Sun's Altitude, deſcribe the Small Circle Z Z, by *Caſe* the 3*d* of *Problem* the 4*th*, of *Section* the 2*d*, to cut the Meridian in the points Z, Z.

3. Thro' the points Z, Z, and the Sun ⊙, draw the Vertical Circles Z ⊙, Z ⊙, and the thing is done; and Z *P*, Z *P* the Complement of the Latitude, may be meaſured by the *Line of Chords*, and the Angle Z the Azimuth, by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*.

This *Caſe* is doubtful, as appears by the *3d Caſe of Oblique-angled Spherical Triangles*, and by the *Conſtruction*; and to determine the Requiſites by Calculation, having let fall the Perpendicular ⊙ *R*, from ⊙ upon the Meridian, it will be by the *3d Caſe of Oblique-angled Spherical Triangles*,

As *R* : cs, *Z P* ⊙ : : t, ⊙ *P* : t, *P R*; that is,

As the Radius ———————— 10.0000000

To the Co-ſine of the Hour from Noon 52.30.00 —— 9.7844471
So is the Co-tangent of the Declination 19.39.10$\frac{3}{4}$ — 10.4471783

To the Tangent of a fourth Arch *P R* 59.36.08 —— 10.2316254

And again, As c s, ⊙ *P* : c s, *Z* ⊙ : : c s, *P R* : c s, *Z R*; that is,
As the Sine of the Declination 19.39.10$\frac{3}{4}$ ———— 9.5267557

To the Sine of the Altitude 38.18.46$\frac{1}{2}$ ———— 9.7923608
So is the Co-ſine of the fourth Arch *P R* 59.36.08 —— 9.7041508

To the Co-ſine of the fifth Arch *Z P* 21.08.08 —— 9.9697559

But $PR \pm ZR = ZP$; wherefore, if

To the fourth Arch *P R* equal to 59 deg. 36 min. 08 ſec. be added the fifth Arch *Z P* equal to 21 deg. 08 min. 08 ſec. the Sum 80 deg. 44 min. 16 ſec. taken from 90 deg. will leave 9 deg. 15 min. 44 ſec. for the leſſer Latitude. And,

If from the fourth Arch *P R* 59 deg. 36 min. 08 ſec. be taken the fifth Arch *ZR* equal to 21 deg. 08 min. 08 ſec. the Remainder 38 deg. 28 min. 00 ſec. taken from 90 deg. will leave 51 deg. 32 min. 00 ſec. for the greater Latitude.

But if in the former *Caſe* the Sum ſhould exceed 90 degrees or a Quadrant, the Exceſs will give the Latitude the contrary way; that is if the Declination be North, then the Latitude would be South.

Or,

The Comp. of $PR \pm ZR$ = Latit. that is, $30.23.52 \pm 21.08.08 =$

$$\left\{\begin{array}{l}51.32.00\\9.15.44\end{array}\right\}\begin{array}{l}\text{Greater}\\\text{Leſſer}\end{array}\left.\vphantom{\begin{array}{l}1\\1\end{array}}\right\}\text{Latitude.}$$

Again, for the Azimuth from the Meridian, it will be by the *1ſt Caſe* of *Oblique-angled Spherical Triangles,*

As the S. Z ☉ : S. *P* ☉ : : S.<ZP☉ : S.<*P*Z☉ ; that is,

As the Co-ſine of the Altitude 38.18.46½ ———— 9.98946686

To the Co-ſine of the Declination 19.39.10¾ ———— 9.97393421
So is the Sine of the Hour from Noon 53.30.00 —— 9.8994667

To the Sine of the Azimuth from the Merid. 72.12.54—9.9787323

Wherefore,

If the Latitude be 51 deg. 32 min. 00 ſec. the Azimuth in the Afternoon is South 72 deg. 12 min. 54 ſec. Weſt, or W. S. W. half W. nearly, but in the Latitude of 9 deg. 15 min. 44 ſec. North, the Azimuth is North 72 deg. 12 min. 54 ſec. Weſt, or W. N. W. half W. nearly, as is manifeſt by the *Figure*; and the contrary, if the Time given had been in the Forenoon.

Again, when the Sun's Altitude in this *Caſe* becomes leſs than the Declination, the *Caſe* will be limited to one *Anſwer* only, for then Z ☉ will be greater than *P* ☉, and the Length of the Arch *P Z* will be limited, as will appear by the following *Example.*

Prob. VI.

Caſe 4.

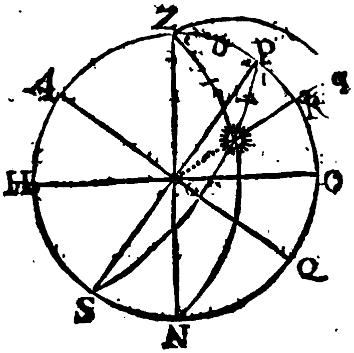
Example 2.

Suppoſe the Sun having 19 deg. 39 min. 10 ſec. ¾ of North Declination, his Altitude at 6 h. 18 m. 22 ſ. in the Afternoon, was found to be 12 deg, 30 min. 00 ſec. I demand the Latitude of the Place and Azimuth of the Sun.

The Stereographic Solution.

1. Haveing drawn the Meridian *Z O N H,* the Six a Clock Hour Circle *P S*, the Equator *A Q*, the Hour Crircle *P* ☉ *S* forming an Angle of 94 deg. 35 min. 30 ſec. with the Meridian *P Z H N*, and ſet off the Complement of the Declination 70 deg. 20 min. 39 ſec.¼ from *P* to ☉, to obtain the Place of the Sun.

2. About



2. About the point ⊙ as a Pole, at the diſtance of 77 deg. 30 min. 00 ſec. the Complement of the Sun's Altitude, deſcribe the Small Circle *q z*, where this cuts the Meridian in Z, it will give the Zenith; thro' which and the Nadir *N*, draw the Vertical Circle Z ⊙ *N*, and the thing is done; and the Angle Z the Azimuth, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*, and *O P* the Latitude by the *Line of Chords*.

That this *Caſe* is limited is manifeſt by the *Conſtruction*, for the Small Circle *q*Z cuts the Meridian but on one Side of the Pole, and that in the point Z; wherefore to find the Latitude by Calculation, having let fall the Perpendicular ⊙ *R*, it will be by the 3*d Caſe* of *Oblique-angled Spherical Triangles*,

As *R* : c s, Z *P* ⊙ : : t, *P* ⊙ : t, *P R*; that is,

As the Radius —————————— 10.0000000

To the Co-ſine of the Hour *94.35.39* ——————— 8.9033820
So is the Co-tangent of the Declination *19.39.10¾*—10.4471783

To the Tangent of a fourth Arch 12.38.05————— 9.3505603

And again, As c s, ⊙ *P* : c s, Z ⊙ : : c s, *P R* : c s, Z *R*; that is,
As the Sine of the Declination *19.39.10¾* ———— 9.5267557

To the Sine of the Altitude 12.30.00———— 9.3353368
So is the Co-ſine of the fourth Arch 12.38.05———— 9.9893538

To the Co-ſine of a fifth Arch 51.06.00————— 9.7979349

Now if from the fifth Arch Z*R* equal to 51 deg. 06 min. 00 ſec. be taken away the fourth Arch *P R*, equal to 12 deg. 38 min. 05

ſec. the Remainder taken from 90 Degrees or a Quadrant, will leave 51 deg. 32 min. 05 ſ. for the Latitude North.

And to find the Azimuth, it will be by *Caſe* the 1*ſt* of *Oblique-angled Spherical Triangles,*

As S. Z ☉ : S. *P* ☉ : : S. <*P* : S. <Z ; that is,

As the Co-ſine of the Altitude 12.30.00 ——— ——— 9.9895815

To the Co-ſine of the Declination 19.39.10¾ ————— 9.9739342
So is the Sine of the Hour 94.35.30 ——— — ——— 9.9986039

To the Sine of the Azimuth 74.03.11 ——— ——— 9.9829566

Which becauſe the Latitude is North and the Time given in the Afternoon is North 74 deg. 03 min. 11 ſec. Weſt, or W: N. W. ½ W nearly.

Prob. VI.

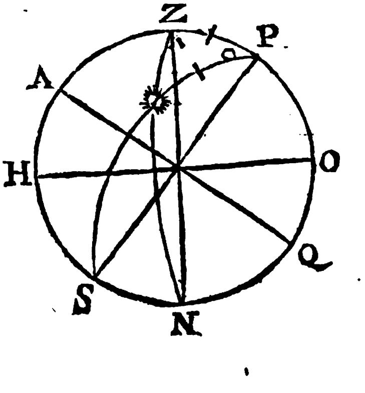
Caſe 5.

Given the Latitude of the Place the Declination of the Sun, and the Azimuth, to find the Hour of the Day and Altitude.

Example.

In the Latitude of 51 deg. 32 min. 00 ſec. North, the Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination. I demand the Latitude of the Place and hour of the Day, the Azimuth of the Sun being South 72 deg. 12 min. 54 ſec. Weſt.

The Stereographic Solution.



1. Having drawn the Meridian Z *O NH*, the Horizon *HO*, and at Right-angles to it the Equator *A Q*, ſet off the Latitude of the Place 51 deg. 32 min. 00 ſec. from *O* to *P*, and draw the Six a Clock Hour Circle *P S*, and at Right-angles to it the Equator *AQ*.

2. Thro' the Zenith *Z* draw the Vertical Circle Z ☉ *N*, forming an Angle of 72 deg. 12 min. 54

ſec. with the Meridian, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*.

3. At the diſtance of 70 deg. 20 min. 49 ſec. $\frac{3}{4}$, the Complement of the Sun's Declination, draw the Small Circle *n* ⊙ *q*, about the Pole *P*, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*, where this cuts the Vertical Circle in the point ⊙, it will give the Place of the Sun.

4. Thro' the point ⊙ and the points *Z* and *N*, draw the Vertical Circle Z ⊙ *N*, alſo thro' the ſame point ⊙ and the Poles *P* and *S* draw the Hour Circle *P* ⊙ *S*, and the thing is done; and Z ⊙ the Sun's Zenith Diſtance may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*, and the Hour from Noon Z *P* ⊙, by *Caſe* the 2*d* of *Problem* the 10*th*, of the ſame *Section*.

To find the Requiſites by Calculation.

In the Oblique-angled Spherical Triangle Z *P* ⊙ are given, Z *P* equal to the Complement of the Latitude, *P* ⊙ equal to the Complement of the Sun's Declination, and the Angle ⊙ Z *P* the Azimuth, whence to find the Arch Z ⊙ the Complement of the Altitude, it will be by the 3*d* *Caſe* of *Oblique-angled Spherical Triangles*, having firſt let fall the Perpendicular *P R*.

As *R* : c s, < *P* Z *R* : : t, *P* Z : t, Z *R*; that is,
As the Radius ———————— 10.0000000

To the Co-ſine of the Azimuth 72.14.54 ———— 9.4849345
So is the Co-tangent of the Latitude 51.32.00 ——— 9.9000865

To the Tangent of a fourth Arch Z *R* 13.38.25$\frac{1}{2}$ ——— 9.3850210

And again, As c s, Z*P*: c s, ⊙*P* : : c s, Z*R* : c s, ⊙*R*; that is,
As the Sine of the Latitude 51.32.00 ———————— 9.8937452

To the Sine of the Declination 19.39.10$\frac{1}{4}$ ———— 9.5267557
So is the Co-ſine of the fourth Arch 13.38.25$\frac{1}{2}$ ——— 9.9875746

To the Co-ſine of a fifth Arch 65.19.39 ———— 9.6205851

Now if from the fifth Arch 65 deg. 19 min. 39 ſec. be taken the fourth Arch 13 deg. 38 min. 25 ſec. $\frac{1}{2}$, the Remainder will be Z⊙, equal to 51 deg. 41 min. 13 ſec. ½, this therefore taken from 90 Degrees or a Quadrant, will give 38 deg. 18 min. 46 ſec. ½, for the Altitude.

Or,

The fourth Arch 13 deg. 38 min. 25 ſec. ½, added to the Sine of the fifth Arch 24 deg. 40 min. 21 ſec. will give 38 deg. 18 min. 46 ſec. ½, for the Altitude required.

For the Hour it will be by the firſt *Caſe of Oblique-angled Spherical Triangles*,

As the S. $P\odot$: S. $Z\odot$: : S. $<Z$: S. $<P$; that is,

As the Co ſine of the Declination 19.39.10¾	*9.9739342*
To the Co-ſine of the Altitude 38.18.46 ½	*9.8946686*
So is the Sine of the Azimuth 72.12.54	*9.9787325*
To the Sine of the Time 52.30.00	*9.8994669*

This being reduced gives 3 h. 30 m. 00 ſ. for the Hour of the Day in the Afternoon, becauſe the Azimuth was Weſterly.

But if it happen that the Latitude of the Place is leſs than the Sun's Declination and both the ſame way, the *Caſe* will be doubtful and admit of two *Anſwers*, as the following *Example* will make appear.

Prob. VI.

Caſe 5.

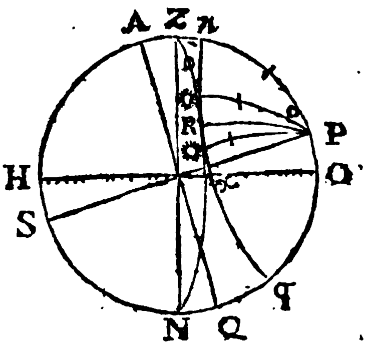
Example 2.

Let it be required to find the Hour of the Day and Altitude of the Sun when he will appear upon the Weſt North Weſt or Eaſt North Eaſt Azimuth Circle at *Barbadoes*, in the Latitude of 13 deg. 30 min. North, at the Time of the *Summer Solſtice*, or when the Sun has 23 deg. 29 min. of Northern Declination, which happens there upon the 10*th* of *June*.

The Stereographic Solution.

1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *ZN*, ſet off the Latitude of the Place 13 deg. 30 min. from *O* to *P*, and draw the Six a Clock Hour Circle *PS*, and at Right-angles to it the Equator *AQ*.

2. Thro'



2. Thro' the Zenith and Nadir draw the Vertical Circle Z ◉ ☉ *N*, forming an Angle of 67 deg. 30 min. 00 ſec. equal to the Azimuth, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*.

3. About the Pole *P*, at the diſtance of the Complement of the Sun's Declination 66 deg. 31 min. 00 ſec. draw the Small Circle *x* ◉ *q*, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*, where this interſects the Vertical Circle before drawn in the points ☉, ◉, it gives the two different places in the Heavens where the Sun is, when he appears under the ſame Azimuth, and *x* ◉, *x* ☉, will be the different Altitudes of the Sun at thoſe times, each of which may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*.

4. Thro' the points ☉, ◉, and *P* and *S*, draw the Hour Circles *P* ◉ *S*, *P* ☉ *S*, and the Angles Z *P* ☉, Z *P* ◉, will give the times from Noon, each of which may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*.

Logarithmically, or by Calculation.

In the Oblique-angled Spherical Triangle *Z P* ☉ are given, *Z P* equal to Complement of the Latitude, the Angle Z the Azimuth, and P◉ the Sun's diſtance from the North Pole, whence to find Z☉ the Complement of the Altitude, it will be by *Caſe* the 3*d* of *Oblique-angled Spherical Triangles*, having firſt let fall the Perpendicular *PR*.

As *R* : cs, <Z : : t, Z *P* : t, Z *R*; that is,

As the Radius —————————— 1000000000

To the Co-ſine of the Azimuth 67.30.00 —————— 9.5828397
So is the Co-tangent of the Latitude 13.30.00 ——— 10.6196463

To the Tangent of a fourth Arch 57.53.51½ ————— 10.2024860

And again, As c s, ZP : c s, $P\odot$ :: c s, ZR : c s, $\odot R$; that is,
As the Sine of the Latitude 13.30.00 ———— 9.3681853

To the Sine of the Declination 23.29 ———— 9 6004090
So is the Co-ſine of the fourth Arch ZR 57.53.51$\frac{1}{2}$ — 9.7254489

To the Co-ſine of the fifth Arch $\odot R$ 24.53.14$\frac{3}{4}$ ———— 9.9576726

But $ZR \pm \odot R = Z\odot$, that is, if to the fourth Arch 57 deg. 53 min. 51 ſec. $\frac{1}{2}$, be added the fifth Arch 24 deg. 53 min. 14 ſec. $\frac{3}{4}$, the Sum 82 deg. 47 min. 06 ſec. $\frac{1}{4}$, taken from 90 Degrees, will leave 7 deg. 12 min. 53 ſec. $\frac{3}{4}$ for the leaſt Altitude $x\odot$. But,

If from the fourth Arch 57 deg. 53 min. 51 ſec. $\frac{1}{2}$, be taken the fifth Arch 24 deg. 53 min. 14 ſec. $\frac{3}{4}$, the Remainder 33 deg. 00 min. 36 ſec. $\frac{3}{4}$, taken from 90 Degrees, will leave 56 deg. 59 min. 23 ſec. $\frac{1}{4}$ for the greater Altitude; whence to find the Time from Noon will be

As the Co-ſine of the Declination 23.29.00 ———— 9.9624527

To the Sine of the Azimuth 67.30.00 ———— 9.9656153
So is the Co-ſine of the leaſt Altitude 7.12.53$\frac{3}{4}$ —— 9.9965476

To the Sine of the correſpondent Time 87.54.26 ———— 9.9997102
anſwering to 5 h. 51 m. 37 ſ. $\frac{3}{4}$

Again,

As the Co-ſine of the Sun's Declination 23.29.00 —— 9.9624527

To the Sine of the Azimuth 67.30.00 ———— 9.9656153
So is the Co-ſine of the greateſt Altitude 56.59.23$\frac{1}{4}$ —— 9.7362193

To the Sine of the correſpondent Time 33.16.55 ———— 9.7393819
anſwering to 2 h. 13 m. 07 ſ. $\frac{3}{4}$

Whence it is manifeſt, that at *Barbadoes* on the 10*th* of *June*, when the Sun has his greateſt Declination, he appears upon the ſame point of the *Compaſs* twice in the Forenoon, and twice in the Afternoon; that is, he appears in the E. N. E. Azimuth at 6 h. 08 m. 22 ſ. $\frac{1}{4}$ in the Morning, and again at 9 h. 46 m. 52 ſ. $\frac{1}{4}$ the ſame

Morning, at the firſt of which Times he has 7 deg. 12 min. 53 ſec. ¾ of Altitude, and at the ſecond time 56 deg. 59 min. 23 ſec. ¾.

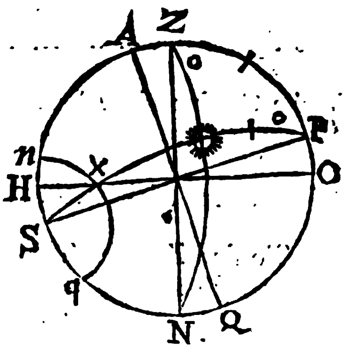
And again,

He appears in the W. N. W. Azimuth at 2 h. 13 m. 07 ſ. ¾ in the Afternoon, and again at 5 h. 51 m. 37 ſ. ¾ the ſame Afternoon, at the firſt of which times he has 56 deg. 59 min. 23 ſec. ¼ of Altitude, and at the ſecond only 7 h. 12 m. 53 ſ. ¾.

Again, Suppoſe it were required to find at the ſame Place and on the ſame Day, what ſhall be the Sun's greateſt Azimuth from the Meridian, at what Time it ſhall happen, and at what Altitude?

The Stereographic Solution.

1. Having drawn the Meridian *ZONH*, the Horizon *HO*, the Prime Vertical Z *N*, the Six a Clock Hour Circle *PS*, and the Equator *AQ*, about the South Pole *S* draw a Small Circle *n x q*, at the diſtance of the Sun's Declination (in this *Caſe* 23 deg. 29 min.) to interſect the Horizon in *x*.



2. About the point *x* as a Pole, deſcribe the Vertical Circle Z ☉ *N* by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*, and thro' the points *P*, *x*, and *S*, draw the Hour Circle *P* ☉ *S*, by the firſt *Problem* of the ſame *Caſe*, where theſe two Circles interſect each other as in the point ☉, it will give the Place of the Sun; then will Z☉ be the Complement of the Altitude, (and which may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*) the Angle *P* Z ☉ the Azimuth, and the Angle Z *P* ☉ the Hour from Noon, each of which may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of the ſame *Section*, but to determine them by Calculation.

In the Spherical Triangle *Z P* ☉ Right-angled at ☉, are given, *Z P* equal to the Complement of the Latitude, *P* ☉ equal to the Complement of the Declination, whence to find firſt, the Hour from Noon, it will be by *Caſe* the 12*th of Right-angled Spherical Triangles*, As ct, *P* ☉ : c.t, Z *P* : : *R* : c s, <*P*; that is,

As the Tangent of the Declination 23.29 ———— 9.6379563

To the Tangent of the Latitude 13.30 ———— 9.3803537
So is the Radius ———— 10.0000000

To the Co-ſine of the Hour from Noon 56.27.20 ———— 9.7423974

Which converted into Time will give 3 h. 45 m. 49 ſ. ⅓.

2. To find the Angle Z or the Azimuth, it will be by *Caſe* the 13*th of Right-angled Spherical Triangles*,

As S. Z *P* : S. Z ☉ : : *R* : S. <Z ; that is,

As the Co-ſine of the Latitude 13.30.00 ———— 9.9878315

To the Co-ſine of the Declination 23.31.00 ———— 9.9624527
So is the Radius ———— 10.0000000

To the Sine of the Azimuth 70.36.09½ ———— 9.9746212

3. To find Z ☉ the Complement of the Altitude, it will be by *Caſe* the 14*th of Right-angled Spherical Triangles*,

As c s, P ☉ : c s, Z *P* : : *R* : c s, Z ☉ ; that is,

As the Sine of the Declination 23 29.00 ———— 9.6004090

To the Sine of the Latitude 13.30.00 ———— 9.3680853
So is the Radius ———— 10.0000000

To the Sine of the Altitude 35.51.43½ ———— 9.7677763

Whence it follows, that the Sun's greateſt Azimuth happens at 8 h. 14 m. 10 ſ. ⅔ in the Morning, when he appears North 70 deg. 36 min. 09 ſec. half Eaſt, or Eaſt North Eaſt a quarter Eaſt nearly, or at 3 h. 45 m. 49 ſ. ⅓ in the Afternoon, when he appears North 70 d. 36 m. 09 ſ. ⅓ half Weſt, or Weſt North Weſt half Weſt nearly, at each of which Times he has 35 deg. 51 min. 43 ſec. ½ of Altitude; and which is therefore the propereſt Time for obſerving the Azimuth of the Sun, becauſe the Variation is then the leaſt.

The Sun's Amplitude by the *1ſt Caſe* of the *2d Aſtronomical Problem*, *Page* the *262d*, will be found to be 24 deg. 11 min. 33 ſec. and conſequently his diſtance from the North point of the Horizon will be 65 deg. 48 min. 27 ſec. whence by the help of the preceding *Calculation*, he will be found to have the ſame Azimuth at 10h. 03 m. 36 ſ. ½ in the Morning, or at 1 h. 56 m. 23 ſ. ½ in the Afternoon, when his Altitude will be found to be 50 deg. 43 min. 36 ſec. ſo that between the time of the Sun's Riſing at 5 h. 36 m. 00 ſ. and 8 h. 14 m. 10 ſ. when he has the greateſt Azimuth, and which is equal to 2 h. 38 m. 10 ſ. he alters his Azimuth but 4 deg. 47 min. 42 ſec. that is not quite half a point, during which time he aſcends 35 deg. 51 min. 43 ſec. above the Horizon; and again, between 8 h. 14 m. 10 ſ. and 10 h. 03 m. 36 ſ. where his Azimuth becomes equal to the Complement of the Amplitude, and which is equal to 1 h. 49 m. 26 ſ. during which time he gains 14 deg. 51 min. 53 ſ. of Altitude, he alters his Azimuth again, but 4 h. 47 m. 42 ſ. and the ſame in the Afternoon between the hours 1 h. 56 m. 27 ſ. and 3 h. 45 m. 49 ſ. and again, between 3 h. 45 m. 49 ſ. and the time of the Sun's ſetting, ſo that between the time of the Sun's Riſing at 5 h. 36 m. 00 ſ. and 10 h. 03 m. 36 ſ. in the Forenoon, and between 1 h. 56 m. 24 ſ. in the Afternoon, and the time of the Setting Sun which is at 6 h. 24 m. 00 ſ. and which is equal to 4 h. 27 m. 36 ſ. during which time the Sun aſcends and deſcends 50 deg. 43 min. 36 ſec. he alters his Azimuth but 4 ¾, whence it is that the Variation in theſe Places may be obſerved with the greateſt Exactneſs, ſince a conſiderable Error in Time or Altitude, will make little or no Error in the Azimuth; and 'tis for this Reaſon that our *Mariners* ought to omit no Opportunity at ſuch Times and in ſuch Places, to make all Obſervations they can poſſible. And,

Inaſmuch as the ſame *Caſe* will happen in all Places ſcituated between the *Tropics* at one Time or other, and no Notice being taken of it in the Books now in Uſe, I have been the more particular in Exemplifying it, becauſe many good Obſervations have been loſt, and many good Opportunities neglected, for want of Perſons being better informed.

Prob. VI.

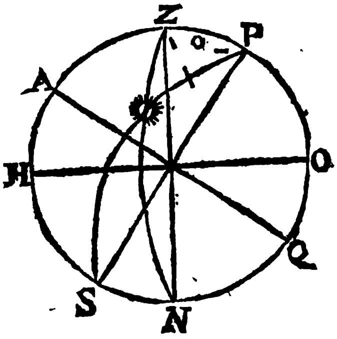
Caſe 6.

Given the Azimuth of the Sun, his Declination, together with the Hour, to find the Altitude of the Sun, and Latitude of the Place.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination, his Azimuth at 8 h 30 m. 00 ſ. in the Morning was found to be South 72 deg. 12 min. 54 ſec. Eaſt: I demand his Altitude, and the Latitude of the Place of Obſervation.

The Stereographic Solution.



1. Having drawn the Meridian Z *O N H*, the Six a Clock Hour Circle *P S*, and at Right-angles to it the Equator *A Q*, thro' the point *P* draw the Hour Circle *P* ⊙ *S* forming an Angle of 52 deg. 30 min. 00 ſec. with the Meridian equal to the Hour from Noon, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*, and ſet off 70 deg. 20 min. 39 ſec. ¼, the Complement of the Sun's Declination from *P* to ⊙, by *Caſe* the 3*d* of *Problem* the 6*th*, of the ſame *Section*, then will ⊙ be the Place of the Sun.

2. Thro' the point ⊙ draw the Vertical Circle *Z* ⊙ *N*, forming an Angle with the Meridian of 72 deg. 12 min. 54 ſec. equal to the given Azimuth, by *Caſe* the 4*th* of *Problem* the 9*th*, of *Section* the 2*d*, and the thing is done; and Z ⊙ the Sun's Zenith Diſtance may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*, and *O P* the Latitude of the Place may be meaſured by the *Line of Chords*.

Logarithmically, or by Calculation.

In the Oblique-angled Spherical Triangle *Z P* ⊙ are given, *P* ⊙ the Complement of the Sun's Declination, *Z P* ⊙ the Hour from

Noon, and the Angle P Z ⊙ the Azimuth from the Meridian, whence to find, firſt, the Complement of the Latitude Z *P*, it will be by the *6th Caſe of Oblique-angled Spherical Triangles*, having firſt let fall the Perpendicular *O R*.

As *R* cs, Z *P* ⊙ : : t, ⊙ *P* : t, *P R* ; that is,
As the Radius ——————————————10.0000000

To the Co-ſine of the Hour from Noon 52.30.00 ———9.7844471
So is the Co-tangent of the Declination 19.39.10¾ —10.4471783

To the Tangent of *P R* a fourth Arch 59.36.08 ————10.2316254

And again, As t, <Z : t, <*P* : : S. *P R* : S. Z *R*; that is,
As the Tangent of the Azimuth 72.12.54 ————10.4937989

To the Tangent of the Hour 52.30.00 ——————10.1150195
So is the Sine of the fourth Arch *P R* 59.36.08 ————9.9357759

To the Sine of a fifth Arch *Z R* 21.08.08 ——————9.5569965

But *P R*—Z *R*=Z *P* ; that is, if from the fourth Arch 59 deg. 36 min. 08 ſec. be taken the fifth Arch Z *R* equal to 21 deg. 08 min. 08 ſec. there will remain Z *P* equal to the Complement of the Latitude 38 deg. 28 min. 01 ſec. this therefore taken from 90 deg. or a Quadrant, will give *P O* the Latitude, equal to 51 deg. 32 min.

Or,

If to 30 deg. 23 min. 52 ſec. the Complement of the fourth Arch, be added the fifth Arch 21 deg. 08 min. 08 ſec. the Sum 51 deg. 32 min. 00 ſec. will give the Latitude of the Place of Obſervation.

Again, to find the Sun's Altitude, it will be by the ſame *Caſe* of *Oblique-angled Spherical Triangles*,

As c s, *P R* : c s, Z *R* : : c s, P ⊙ : c s, Z ⊙ ; that is,
As the Co-ſine of the fifth Arch *PR* 59.36:08 ————9.7041508

To the Co-ſine of the fourth Arch Z *R* 21.08.08 ———9.9697557
So is the Sine of the Declination 19.39.10¾ ————9.5267559

To the Sine of the Altitude 38.18.46 ——— ————9.7923608

Or,

Or the Altitude might have been found directly from the Data in the *Queſtion*, without letting fall a Perpendicular, by the 1ſt *Caſe of Oblique-angled Spherical Triangles*,

When the Hour from Noon is greater than the Azimuth from the Meridian, the *Caſe* becomes doubtful; as the following *Example* will ſhew.

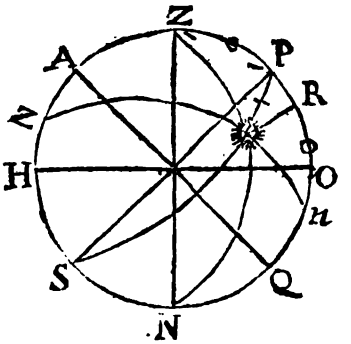
Prob. VI.

Caſe 6.

Example 2.

The Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination, at 6 h. 18 m. 22 ſ. in the Afternoon, his Azimuth was found by Obſervation to be North 74 deg. 03 min. 11 ſec. Weſt. I demand the Latitude of the Place and Altitude of the Sun.

The Stereographic Solution.



1. Having drawn the Meridian Z O *N H*, the Six a Clock Hour Circle *P S*, and at Right-angles to it the Equator *A Q*, thro' the Pole *P* draw the Hour Circle *P* ⊙ *S*, forming an Angle with the Meridian equal to the given Time, equal to 94 deg. 35 min. 29 ſec. by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*, and lay off the Complement of the Sun's Declination equal to 70 deg. 20 min. 49 ſec. ¼ from *P* to ⊙, by *Caſe* the 3*d* of *Problem* the 6*th*, of the ſame *Section*, this will repreſent the Place of the Sun.

2. Thro' the point ⊙ draw the Vertical Circle Z ⊙ *N*, z ⊙ *n*, (by *Caſe* the 4*th* of *Problem* the 9*th*, of *Section* the 2*d*.) forming an Angle *P* Z ⊙, *P* z ⊙, with the Meridian, equal to the given Azimuth 74 deg. 03 min. 11 ſec. and the thing is done; and the Zenith diſtance Z ⊙, z ⊙, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*, and the Complements of the Latitudes Z *P*, z *P*, may be meaſured by the *Line of Chords*.

That this *Caſe* is doubtful, is manifeſt by the *Conſtruction*, and by the 2*d* and 6*th Caſes of Oblique-angled Spherical Triangles*; and therefore to find the different Latitudes Z *O*, z *O*, in the Spherical Triangle Z *P* ☉ are given, the Angle Z the Azimuth, and the An- *P* the Hour, and *P* the diſtance of the Sun from the Pole, whence by the 2*d* and 6*th Caſes of Oblique-angled Spherical Triangles*, having firſt let fall the Perpendicular ☉ *R* it will be,

As *R* : cs, < *P* : : t, *P* ☉ : t, *P R*; that is,

As the Radius ———————————— 10 0000000

To the Co-ſine of the Hour from Noon 94.35.29 ——— 8.9033558
So is the Co-tang. of the Sun's Declination 19.39.10$\frac{3}{4}$ 10.4471783

To the Tang. of a fourth Arch *P R* 12.38.02 ———— 9.3505341

And again, As t, < Z : t, < *P* : : S. *P R* : S. Z *R*; that is,
As the Tangent of the Azimuth 74.03.11 ——— 10.5440236

To the Tangent of the Hour 94.35.29 ———— 11.0952482
So is the Sine of the fourth Arch *P R* 12.38.02 ——— 9.3398894

To the Sine of the fifth Arch Z *R* 51.05.59$\frac{1}{2}$ ——— 9.8911140

But Z *P*—*P R*=Z *P*; that is, if from the fifth Arch 51 deg. 05 min. 59 ſec. $\frac{1}{2}$, be taken the fourth Arch *P R* 12 deg 38 min. 02 ſec. the Remainder Z *P* equal to 38 deg. 27 min. 57 ſec. $\frac{1}{2}$, taken from 90 Degrees or a Quadrant will leave *P O*, equal to 51 deg. 32 min. 02 ſec. $\frac{1}{2}$ for one Latitude, which is North.

Again, becauſe the Supplement of Z *R*—*P R* is equal to z *P*, if from the Supplement of Z *R* 128 deg. 54 min. 00 ſec. $\frac{1}{2}$, be taken *P R*, equal to 12 deg. 38 min. 02 ſec. the Remainder 116 deg. 15 min. 58 ſec. $\frac{1}{2}$, it will give z *P*, from which taking away a Quadrant or 90 Degrees, it will leave 26 deg. 15 min. 58 ſec. $\frac{1}{2}$, for the other Latitude which is South, for when Z *P* is more than a Quadrant, the contrary Pole is elevated above the Horizon.

Again, to find the Sun's Altitude, it will be by the firſt *Caſe* of *Oblique-angled Spherical Triangles*, As S.<Z : S.<*P* : : S. *P* ⊙ : S. Z ⊙ ; that is,

As the Sine of the Azimuth 74.03.11 ——————— 9.9829568

To the Sine of the Hour 94.35.29 ——————— 9.9986041
So is the Co-ſine of the Declination 19.39.10¾ ——— 9.9739342

To the Sine of Z ⊙ equal to 77.30.00 ——————— 9.9895815

This taken from a Quadrant will leave 12 deg. 30 min. 00 ſec. for the Altitude of the Sun in the Latitude of 51 deg. 32 min. 02 ſec. ½ North, but taken from a Semicircle, will leave 102 deg. 30 min. 00 ſec. for the diſtance of the Sun from the Zenith *z*, of a Place in the Latitude of 26 deg. 15 min. 58 ſec. ½ South, that is the Altitude of the Sun in the Latitude of 26 deg. 15 min. 58 ſec. ½ South.

Prob. VI.

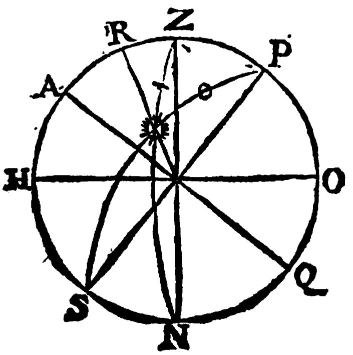
Caſe 7.

Given the Altitude of the Sun, his Azimuth, and the time of the Day, to find the Latitude of the Place and Declination of the Sun

Example.

At half an Hour paſt Three in the Afternoon, the Sun's Altitude was obſerved to be 38 deg. 18 min. 46 ſec. ½, when his Azimuth was South 72 deg. 12 min. 54 ſec. Weſt I demand the Latitude of the Place of Obſervation, and Declination of the Sun.

The Stereographic Solution.



1. Having drawn the Meridian Z *O N H*, the Horizon *H O*, and at Right-angles to it the Prime Vertical Z *N*, draw the Vertical Circle Z ⊙ *N*, forming an Angle *A Z* ⊙, of 72 deg. 12 min. 54 ſec. with the Meridian, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*, and lay off the 51 deg. 41 min. 13 ſec. ½, the Complement of the Sun's Alti-

tude from Z to ⊙, by *Caſe* the *3d* of *Problem* the *6th* of the ſame *Section*, then will ⊙ repreſent the Place of the Sun.

2. Thro' the point ⊙ draw the Hour Circles ⊙ *P*, ⊙ *p*, by *Caſe* the *4th of Problem* the *9th*, of *Section* the *2d*, forming an Angle of 52 deg. 30 min. 00 ſec. equal to the Hour reduced into Time, with the Meridian *Z P* ⊙, and the thing is done; and ⊙ *P*, ⊙ *p*, the reſpective diſtances of the Sun from the Pole, may be meaſured by *Caſe* the *3d* of *Problem* the *7th*, of *Section* the *2d*, and the Latitudes *O P*, *o p*, may be meaſured by the *Line of Chords*.

That this *Caſe* is Ambiguous is plain from the *Conſtruction*, and the *2d Example* of the *6th Caſe of Oblique-angled Spherical Triangles*, and thence to find the Latitudes *O P*, *o p*, by *Calculation*, in the Spherical Triangles *Z P* ⊙, *Z p* ⊙, are given Z ⊙ the Sun's Zenith diſtance, the Angle Z the Azimuth, and the Angle *P* the Hour from Noon, whence to find *P O*, *p O*, the Latitudes, it will be by *Caſe* the *6th of Oblique-angled Spherical Triangles*, having firſt let fall the Perpendicular ⊙ *R*.

As the *R* : c s, <Z : : t, Z ⊙ : t, Z *R*; that is,

As the Radius ———————— 10.0000000

To the Co-ſine of the Azimuth 72.12.54 ——————— 9.4849345
So is the Co-ſine of the Altitude 38.18.46 ½ ——————— 10.1023077

To the Tangent of a fourth Arch 21.08.08 ——————— 9.5872422

And again, As t, <*P* : t, <*Z* : : S. Z *R* : S. *PR*; that is,
As the Tangent of the Hour 52.30.00 ——————— 10.1150195

To the Tangent of the Azimuth 72.12.54 ——————— 10.4937989
So is the Sine of the fourth Arch 21.08.08 ——————— 9.5569965

To the Sine of a fifth Arch 59.36.08 ——————— 19.9357779

But *P R*—Z *R*=Z *P*; that is, as the fiſth Arch *P R* equal to 59 deg. 36 min. 08 ſec. leſſened by the fourth Arch *Z P*, equal to 21 deg. 08 min. 08 ſec. will give *Z P*, equal to 38 deg. 28 min. 00 ſec. and this taken from 90 Degrees or a Quadrant, will leave 51 deg. 32 min. 00 ſec. equal to *P O* the Latitude North.

Again, Becauſe the Supplement of $R\,P = R\,p - Z\,R = Z\,p$, if from 120 deg. 23 min. 52 ſec. the Supplement of the fifth Arch, be taken the fourth Arch $z\,R$ equal to 21 deg. 08 min. 08 ſec. the Remainder 99 deg. 15 min. 44 ſec. is equal to $Z\,p$, from whence taking away 90 Degrees equal to $Z\,O$, there will remain $O\,p$, equal to 9 deg. 15 min. 44 ſec. for the other Latitude which is South, for becauſe $Z\,p$ is greater than a Quadrant, the oppoſite Pole to P is elevated above the Horizon.

And again, to find the Sun's-diſtance from the Pole, it will be by the *1ſt Caſe of Oblique-angled Spherical Triangles,*

As S. $\angle P$: S. $\angle Z$: : S. Z ☉ : S. P ☉ ; that is,

As the Sine of the Hour from Noon 52.30.00 ——— 9.8994667

To the Sine of the Azim. from the Merid. 72.12.54 —— 9.9787325

So is the Co-ſine of the Altit. of the Sun 38.18.46½ — 9.8946686

To the Sine of P ☉ 70.20.49¼ ——————— 9.9739344

This taken from 90 Degrees, will leave 19 deg. 39 min. 10 ſec. ¾ for the Sun's Declination North, in the Latitude of 51 deg. 32 min. 00 ſec. North, but taken from 180 Degrees, will leave 109 deg. 39 min. 10 ſec. ¾ for p ☉ the diſtance of the Sun from the Pole, in the Latitude of 9 deg. 15 min. 44 ſec. South, whence his Declination is given again 19 deg. 39 min. 10 ſec. ¾ North.

But when the Hour from Noon is greater than the Azimuth from the Meridian, the *Caſe* is then limited and confin'd to one *Anſwer*, as the following *Example* will make appear.

Prob. VI.

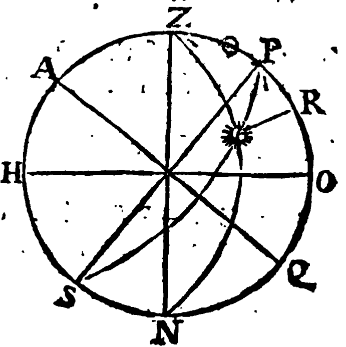
Caſe 7.

Example 2.

At 6 h. 18 m. 22 ſ. in the Afternoon, the Sun's Altitude was found by Obſervation to be 12 deg. 30 min. 00 ſec. when his Azimuth was North 74 deg. 03 min. 11 ſec. Weſt. I demand the Latitude of the Place, and Declination of the Sun.

The

The Stereographic Solution.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, the Prime Vertical Z *N*, and the Vertical Circle Z ☉ *N*, forming an Angle of 74 deg. 03 min. 11 ſec. with the Northern part of the Meridian Z*ON*, by *Caſe* the *3d* of *Problem* the *9th*, of *Section* the *2d*, and laid off the Sun's Zenith diſtance from Z to ☉, in the ſame Circle, by *Caſe* the *3d* of *Problem* the *6th* of the ſame *Section*, thro' the point ☉ the place of the Sun, draw the Hour Circle *P* ☉ *S*, forming an Angle Z *P* ☉, of 94 deg. 35 min. 29 ſec. equal to the time from Noon, with the Meridian *AZP*, and the thing is done; and *P* ☉ the Sun's diſtance from the Pole may be meaſured by *Caſe* the *3d* of *Problem* the *7th* of *Section* the *2d*, and *P O* the Latitude of the Place may be meaſured by the *Line of Chords*.

That this *Caſe* is limited is plain, from the *1ſt Example* of the *6th Caſe* of *Oblique-angled Spherical Triangles*, and from the *Conſtruction*, and therefore to find the Latitude *O P* by Calculation, in the Spherical Triangle Z*P* ☉ are given, Z ☉ the Sun's diſtance from the Pole, the Angle Z the Azimuth, and the Angle *P* the Hour, whence to find firſt *O P* the Latitude, it will be, having firſt let fall the Perpendicular ☉ *R*, by the *6th Caſe* of *Oblique*, &c.

As *R* : cs, < Z : : t, Z ☉ : t, Z*R*; that is,

As the Radius —————————— 10.0000000

To the Co-ſine of the Azimuth 74.03.11¼ —————— 9.4389312
So is the Co-tangent of the Altitude 12.30.00 —— 10.6542448

To the Tangent of a fourth Arch 51.05.58½ —————— 10.0931760

And again, as c s, Z ☉ : c s, *P* ☉ : : c s, Z R : c s, *P R* ; that is,

As the Sine of the Altitude 12.30.00 ——— 9.3353368

To the Sine of the Declination 19.39.10¾ ——— 9.5267557

So is the Co-ſine of the fourth Arch 51.05.58½ ——— 9.7979380

To the Co-ſine of the fifth Arch 12.37.58½ ——— 9.9893569

But $ZR - PR = ZP$, that is, the fourth Arch 51 deg. 05 min. 58 ſec. ½, leſſened by the fifth Arch 12.37.58½, will leave 38 deg. 28 min. 00 ſec. for Z *P*, which taken from 90 Degrees, will leave 51 deg. 32 min. 00 ſec. for *P O* the Latitude North.

And for the Declination, it will be by the 1*ſt Caſe of Oblique*, &c.

As S. < *P* : S. < Z : : S. Z ☉ : S. *P* ☉ ; that is,

As the Sine of the Hour 94.35.29 ——— 9.9986041

To the Sine of the Azimuth 74.03.11¼ ——— 9.9829568

So is the Co-ſine of the Sun's Altitude 12.30.00 ——— 9.9895815

To the Co-ſine of the Declination 19.39.10¾ ——— 9.9739342

Prob. VI.

Caſe 8.

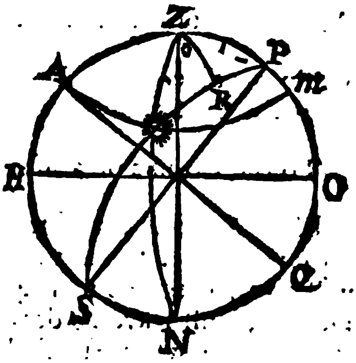
Given the Latitude of the Place, the Altitude of the Sun, and the Hour of the Day, to find the Sun's Azimuth, and his Declination.

Example

In the Latitude of 51 deg. 32 min. 00 ſec. North, at 30 minutes after Three in Afternoon, the Sun's Altitude was found to be 38 deg. 28 min. 46 ſec. ½. I demand the Sun's Azimuth and Declination.

The

The Stereographic Solution.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *ZN*, ſet off the Latitude 51 deg. 32 min. 00 ſec. from *O* to *P*, and draw the Six a Clock Hour Circle *PS*, and at Right-angles to it the Equator *AQ*.

2. Thro' the Point *P*, draw the Hour Circle *P* ☉ *S*, forming an Angle *ZP*☉ of 52 deg. 30 min. 00 ſec. equal to the Time from Noon, with the Meridian *Z O N H*, by *Caſe* the 3*d* of *Problem* the 9*th* of *Section* the 2*d*.

3. About the point Z as a Pole, at the diſtance of 51 deg. 41 min. 13 ſec. ½ the Complement of the Sun's Altitude, draw the Parallel *A* ☉ *m*, by *Caſe* the 2*d* of *Problem* the 4*th* of the ſame *Section*, where this cuts the Hour Circle before drawn in ☉, it will give the Place of the Sun.

4 Thro' the point ☉ and the Zenith and Nadir, draw the Vertical Circle Z ☉ *N*, and the thing is done; and *P* ☉ the Complement of the Sun's Declination, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*, and the Angle Z the Azimuth, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of the ſame *Section*.

Logarithmically, or by Calculation.

In the Oblique-angled, *&c.* Z *P* ☉ are given, *Z P* the Complement of the Latitude, Z ☉ the Complement of the Altitude, and the Angle *P* the Hour from Noon, whence to find 1ſt *P* ☉, it will be (having firſt let fall the Perpendicular Z *R*) by the 1*ſt Example* of the 3*d Caſe of Oblique-angled*, &c.

As *R* : c s, ∠*P* : : t, *Z P* : t, *P R*; that is,

As the Radius —————— 10.0000000

To the Co-ſine of the Hour 53.30.00 —————— 9.7844471

So is the Co-tangent of the Latitude 51.32.00 —————— 9.9000865

To the Tangent of a fourth Arch *P R* 25.48.39 —————— 9.6845336

And

And again, As c s, Z *P* : c s, Z ☉ : : c s, *P R* : c s, ☉ *R* ; that is,

As the Sine of the Latitude 51.32.00 ——————— 9.8937452

To the Sine of the Altitude 38.18.46½ ——— ——— 9.7923608
So is the Co-ſine of the fourth Arch 25.48.39 ——— ——— 9.9543566

To the Co-ſine of a fifth Arch 44.32.10¾ ——— ——— 9.8529722

But *P R* + ☉ *R* equal to *P* ☉, that is, the fourth Arch 25 deg. 48 min, 39 ſec. added to the fifth Arch 44 deg. 32 min. 10 ſec. ¾, will give 70 deg. 20 min. 49 ſec. ¾ for *P* ☉, which taken from 90 Degrees, will leave 19 deg. 39 min. 10 ſec. ¼ for the Sun's Declination North; whence to find the Azimuth or Angle at *P*, it will be by the 1ſt *Caſe* of *Oblique*, &c.

As S. Z ☉ : S. *P* ☉ : : S. <*P* : S. <Z ; that is,

As the Co-ſine of the Altitude 38.18.46½ ——— ——— 9.8946686

To the Co-ſine of the Declination 19.39.10¼ ——— ——— 9.9739342
So is the Sine of the Hour 52.30.00 ——— ——— 9.8994667

To the Sine of the Azimuth 72.12.54 ——— ——— 9.9787323

Which is South Weſterly.

Or the Azimuth might have been found directly, by the *4th Caſe* of *Oblique-angled Spherical Triangles*, and thence the Declination, by *Caſe* the 1ſt.

But when the Altitude of the Sun is greater than the Latitude of the Place the *Caſe* is not limited, and admits of two *Anſwers*, as the following *Example* will make appear.

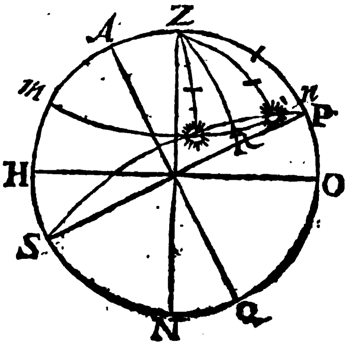
Prob. VI.

Caſe 8.

Example 2.

In the Latitude of 13 deg. 30 min. 00 ſec. North, the Sun having 60 deg. 30 min. 00 ſec. of Altitude at Two of the Clock in the Afternoon. I demand the Azimuth of the Sun, and his Declination

The Stereographic Solution.



1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *ZN*, ſet off the Latitude of the Place 13 deg. 30 min. 00 ſec from *O* to *P* and draw the Six a Clock Hour Circle *PS*, and at Right-angles to it the Equator *AQ*.

2. Thro' the Pole *P* draw the Hour Circle *P* ⊙ *S*, forming an Angle of 30 deg. 00 min. 00 ſec. (equal to the Hour from Noon) with the Meridian, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*.

3. About the Zenith Z draw the Small Circle *m* ⊙ ◉ *n*, at the diſtance of 29 deg. 30 min. 00 ſec. the Complement of the Sun's Altitude, by *Caſe* the 2*d* of *Problem* the 4*th* of the ſame *Section*.

4. Thro' the Interſections ⊙ and ◉, of this Small Circle, with the Hour Circle before drawn, draw the Vertical Circles Z◉*N*, Z⊙*N*, and the thing is done; and *P* ◉, *P* ⊙, the Complements of the Sun's Declination, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th* of *Section* the 2*d*, and the Angles *PZ* ⊙, *PZ* ◉, the Azimuths, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of the ſame *Section*.

That this *Caſe* is doubtful, and admits of two *Anſwers* is manifeſt; from the 3*d* and 4*th Caſes of Oblique-angled Spherical Triangles*, and from the *Conſtruction*; and therefore to find firſt the Declination, it will be by the 3*d Caſe of Oblique*, &c. having firſt let fall the Perpendicular ⊙*R*.

As the *R* : c s, <*P* : : t, *P* Z : t, *PR*; that is,

As the Radius ——————— 10.0000000

To the Co-ſine of the Hour from Noon 30.00.00 ——— 9.9375306
So is the Co-tangent of the Latitude 13.30.00 ——— 10.6196463

To the Tangent of a fourth Arch 74.30.20 ——— 10.5571769

Again, cs, *Z P* : cs, *Z* ☉ : : cs, *P R* : cs, ☉ *R*; that is

As the Sine of the Latitude 13.30.00 —— 9.3681853

To the Sine of the Altitude 60:30.00 —— 9.9396968
So is the Co-ſine of the fourth Arch *P R* 74.00.20 —— 9.4267469

To the Co-ſine of a fifth Arch ☉ *R* 5.07.30 —— 9.9982584

Now *P R*+☉ *R*=*P* ☉, that is, if to *PR* the fourth Arch 74 deg. 30 min. 20 ſec. be added ☉ *R* the fifth Arch 5 deg. 07 min. 40 ſec. the Sum 79 deg. 38 min. 00 ſec. taken from 90 Degrees, will leave 10 deg. 22 min. 00 ſec. for one Declination North.

Again, ~~If from~~ *P R* the fourth Arch 74 deg. 30 min. 20 ſec. be taken ☉ *R* the fifth Arch, the Remainder *P* ☉ equal to 67 deg. 22 min. 40 ſec. taken from 90 Degrees, will leave 20 deg. 37 min. 10 ſec. for the other Declination North.

Whence to find the Correſpondent Azimuth, it will be

As S. Z ☉ : S. *P* ☉ : : S. ∠*P* : S. ∠ Z; that is,

As the Co-ſine of the Altitude 60.30.00 —— 9.6923388

To the Co-ſine of the Declination 10.22.00 —— 9.9928922
So is the Sine of the Hour 30.00.00 —— 9.6989700

To the Sine of the Azimuth 87.12.21¼ —— 9.9994834

Anſwering to 10 deg. 22 min. 00 ſec. North Declination.

And again,

As the Co-ſine of the Altitude 60.30.00 —— 9.6923388

To the Co-ſine of the Declination 20.37.20 —— 9.9712402
So is the Sine of the Hour 30.00.00 —— 9.6989700

To the Sine of the Azimuth 71.51.52½ —— 9.9778714

Anſwering to 20 deg. 37 min. 00 ſec. North Declination.

And becauſe the firſt of theſe Declinations correſponds pretty near to the *6th* of *April* and the 15*th* of *Auguſt*, and the latter to the 12*th* of *May* and the 10*th* of *July*, it follows, that at *Barbadoes* upon each of theſe times the Sun has the ſame Altitude at the ſame Hour, that is at two Hours from Noon the Sun has 60 deg. 30 min. of Altitude, but on the two firſt Days, *viz.* the *6th* of *April* and the 15*th* of *Auguſt*, his Azimuth in the Afternoon is North 87 deg. 12 min. 21 ſec. Weſt, or Weſt a quarter North nearly, and in the Morning North 87 deg. 12 min. 21 ſec. Eaſt, or Eaſt a quarter North nearly; but on the two latter Days, *viz.* the 12*th* of *May* and the 16*th* of *July*, his Azimuth in the Afternoon is North 71 deg. 51 min. 52 ſec. ½ Weſt, or Weſt North Weſt a quarter Weſt nearly, but in the Morning it is North 71 deg. 51 min. 52 ſec. Eaſt, or Eaſt North Eaſt a quarter Eaſt nearly.

Prob. VI.

Caſe 9.

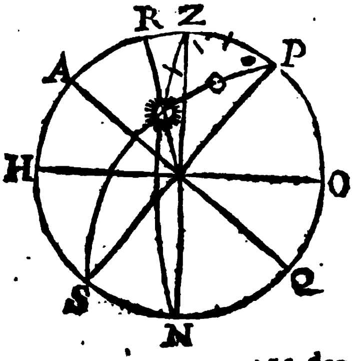
Given the Latitude of the Place, the Altitude of the Sun, and his Azimuth, to find the Declination and the Hour of the Day.

Example.

In the Latitude of 51 deg. 32 min. 00 ſec. North, the Sun having 38 deg. 18 min. 46 ſec. ½, his Azimuth at that time was found to be South 72 deg. 12 min. 54 ſec. Eaſt. I demand the Declination of the Sun, and the Hour of the Day.

The Stereographic Solution.

1. Having drawn the Meridian *ZONH* the Horizon *HO*, and at Right-angles to it the Prime Vertical *ZN*, ſet off 51 deg. 32 min. the Latitude from *O* to *P*, and draw the Six a Clock Hour Circle *P S*, and at Right-angles to it the Equator *A Q*.



2. Thro' the Zenith Z draw the Vertical Circle Z ⊙ *N*, forming an Angle of 72 deg. 12 min. 54 ſec. with the Meridian, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d*, and lay off

51 deg. 41 min. 13 ſec. ½, the Complement of the Sun's Altitude, in it from Z to ☉, by *Caſe* the 3*d* of *Problem* the 6*th* of the ſame *Section*, then will ☉ repreſent the Place of the Sun.

3. Thro' the Poles *P* and *S*, and the point ☉, draw the Hour Circle *P* ☉ *S*, and the thing is done; and the Angle *P* the Hour from Noon, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th* of *Section* the 2*d*, and *P*☉ the Complement of the Sun's Declination, may be meaſured by *Caſe* 3*d* of *Problem* the 7*th*, of the ſame *Section*.

Logarithmically, or by Calculation

In the Oblique-angled Spherical Triangle *P Z* ☉ are given, *Z P* the Complement of the Latitude, *P* ☉ the Complement of the Altitude, and the Angle Z the Azimuth, whence to find the Angle *P* the Hour from Noon, having firſt let fall the Perpendicular *R* ☉ it will be, by the 2*d Example* of the 9*th Caſe* of *Oblique*, &c.

As the *R*: c s, <Z : : t, Z ☉: t, *Z R*; that is,

As the Radius —————————— 10 0000000

To the Co-ſine of the Azimuth 72.12.54 ———— 9.4849345
So is the Co-tangent of the Altitude 38.18.46½ ——— 10.1023077

To the Tangent of a fourth Arch 21.08.08 ———— 9.5872422

But *Z R*+Z *P*=*P R*, that is, if to the fourth Arch 21 deg. 08 min. 08 ſec. be added the Complement of the Latitude 38 deg. 28 min. 00 ſec. the Sum 51 deg. 36 min. 08 ſec. will give a fifth Arch *P R*; wherefore,

As the S. *P R* : S. Z *R* : : t, <Z : t, <*P*; that is,

As the Sine of the fifth Arch 59.36.08 ———— 9.9357759

To the Sine of the fourth Arch 21.08.08 ———— 9.5569965
So is the Tangent of the Azimuth 72.12.54 ———— 10.4937989

To the Tangent of the Hour 52.30.00 ———— 10.1150195

Which reduced into Time and taken from twelve Hours, will give 8 h. 30 m. 00 ſ. for the Hour in the Morning, becauſe the Azimuth was Eaſterly.

Again, by the 2*d Example* of the 10*th Caſe* of *Oblique*, &c. to find the Declination, it will be

As the c s, *Z R* : c s, *P R* : : c s, Z ☉ : c s, *P* ☉; that is,

As the Co-ſine of the fourth Arch 21.08.08 ——————9.9697559

To the Co-ſine of the fifth Arch 59.36.08—————— 9.7041508
So is the Sine of the Altitude 38.18.46½——————9.7923608

To the Sine of the Declination 19.39.10¾—————— 9.5267557

Or the Hour being firſt obtained, the Declination might have been found by the firſt *Caſe of Oblique*, &c. or the Declination being determined, the Hour might have been inveſtigated by the ſame *Caſe*.

Prob. VI.

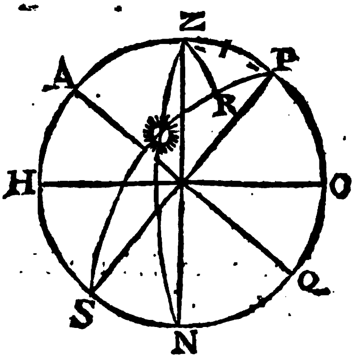
Caſe 10.

Given the Latitude of the Place, the Hour of the Day and Azimuth of the Sun, to find the Sun's Altitude, Declination, and the Angle formed by the Vertical and Hour Circle paſſing thro' the Sun.

Example.

In the Latitude of 51 deg. 32 min. 00 ſec. at 3 h. 30 m. 00 ſ. in the Afternoon, the Sun's Azimuth was found to be South 72 deg. 12 min. 54 ſec. Weſt. I demand his Altitude, his Declination, and the Angle formed by the Vertical and Hour Circle paſſing thro' the Sun.

The Stereographic Solution.



1. Having drawn the Meridian *Z O N H*, the Horizon *H O*, and at Right-angles to it the Prime Vertical *Z N*, ſet off the Latitude 51 deg. 32 min. 00 ſec. from *O* to *P*, and draw the Six a Clock Hour Circle *P S*, and at Right-angles to it the Equator *A Q*.

2. Thro' the Pole *P* draw the Hour Circle *P* ☉ *S*, forming an Angle with the Meridian of 52 deg. 30 min. 00 ſec. by *Caſe* the 3*d* of *Pro-*

blem the *9th* of *Section* the *2d*; alſo thro' the point Z draw the Vertical Circle Z ☉ *N*, forming an Angle with the Meridian of 72 deg. 12 min. 54 ſec. by the ſame, and the thing is done; and the Complement of the Altitude Z ☉, may be meaſured by *Caſe* the *3d* of *Problem* the *7th* of *Section* the *2d*, and *P* ☉ the Complement of the Declination, may be meaſured by the ſame *Caſe*.

Logarithmically, or by Calculation.

In the Oblique-angled Spherical Triangle Z *P* ☉ are given, Z *P* the Complement of the Latitude, the Angle Z the Azimuth, and the Angle *P* the Hour from Noon, whence to find Z ☉ the Complement of the Altitude, it will be (having firſt let fall the Perpendicular Z *R*) by *Caſe* the *8th* of *Oblique*, &c.

As the *R* : c s, Z *P* : : t, <*P* : c t, *P* Z *R*; that is,

As the Radius ———————————— 10.0000000

To the Sine of the Latitude 51.32.00 ———————— 9.8937452
So is the Tangent of the Hour 52.32.00 ———————— 10.1150195

To the Co-tang. of the <*PZR* or a *4th* Arch 44.25.18¾ 10.0087647

But the Angle *P* Z ☉—*P* Z *R*=☉ Z *R*, that is, if from the Azimuth counted from the North 107 deg. 47 min. 06 ſec. be taken the fourth Arch or Angle *P* Z *R* 44 deg. 25 min. 18 ſec. ¾, there will remain a fifth Arch or the Angle ☉ Z *R* 63 deg. 21 min. 47 ſec. ¼; wherefore,

As c s, ☉ Z *R* : c s, *P* Z *R* : : t, *P* Z : t, ☉ Z; that is,

As the Co-ſine of the *5th* Arch or <☉Z*R* 63.21.47¼ — 9.6516021

To the Co-ſine of the *4th* Arch or <*PZR* 44.25.18¾ — 9.8538233
So is the Co-tangent of the Latitude 51.32.00 ———— 9.9000865

To the Co-tangent of the Altitude 38.18.46½ ———— 10.1023077

Whence to find the Declination, it will be by the firſt *Caſe* of *Oblique*, &c.

As the S. <*P* : S. <Z : : S. Z ⊙ : S. *P* ⊙ ; that is,

As the Sine of the Hour 52.30.00 ——————— 9.8994667

To the Sine of the Azimuth 72.12.54 ——————— 9.9787325
So is the Co-ſine of the Altitude 38.18.46½ ——————— 9.8946686

To the Co-ſine of the Declination 19.39.10¾ ——————— 9.9739344

And to find the Angle Z ⊙ *P*, formed by the Azimuth and Hour Circle paſſing thro' the Sun, uſually called the Angle of Poſition, it will be by the *1ſt Example* of the *7th Caſe* of *Oblique*, &c.

As the S. <*P* Z *R* : S. <⊙ Z *R* : : c s, <*P* : c s, < ⊙ ; that is,
As the Co-ſine of the <*PZR* or *4th* Arch 44.25.18¾ — 9.8450583

To the Sine of the <⊙Z*R* or *5th* Arch 63.21.47¼ ——— 9.9512724
So is the Co-ſine of the Hour from Noon 52.30 ——— 9.7844471

To the Co-ſine of the Angle of Poſition Z⊙*P* 38.58.27 9.8906612

Or the Altitude being found, the Angle of Poſition might have been found by *Caſe* the *1ſt of Oblique-angled Spherical Triangles.*

Prob. VI.

Caſe 11.

Given the Latitude of the Place, the Hour of the Day, and the Angle formed by the Azimuth and Vertical Circle paſſing thro' the Sun or Angle of Poſition, to find the Altitude of the Sun, his Azimuth and preſent Declination.

Example.

In the Latitude of 51 deg. 32 min. 00 ſec. North, at 3 h. 30 m. in the Afternoon the Angle of Poſition was found to be 38 deg. 58 min. 27 ſec. I demand the Altitude of the Sun, his Azimuth, and the preſent Declination.

Anſwer.

The Altitude is 38 deg. 18 min. 46 ſec. ½, the Declination 19 deg. 39 min. 10 ſec. ¾ Weſt, and the Azimuth South 72 deg. 12 min. 54 ſec. Weſt.

This *Caſe* conſidered as a Spherical Triangle is Ambiguous, by the 2*d*, 5*th*, and 6*th Caſes of Oblique-angled Spherical-Triangles*, inaſmuch as the Angle of Poſition is leſs than the Hour Angle at the Pole, but when conſidered as an Aſtronomical Solar *Problem* it is limited to one *Anſwer* only, becauſe to the leaſt Azimuth from the Meridian which is South 18 deg. 56 min. 28 ſec. ½ Weſt, the Zenith diſtance will be 128 deg. 18 min. 46 ſec. ½, that is, the Sun will be depreſſed 38 deg. 18 min. 46 ſec. ½ below the Horizon, and his diſtance from the North Pole will be 161 deg. 16 min. 28 ſec. $\frac{5}{7}$, that is, the Declination will be 71 deg. 16 min. 28 ſec. $\frac{5}{7}$ South, which is impoſſible.

Prob. VI.

Caſe 12.

Given the Sun's Declination, the Hour of the Day, and the Angle of Poſition, to find the Sun's Altitude, his Azimuth, and Latitude of the Place.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination, at 3 h. 30 m. in the Afternoon, the Angle of Poſition was found to be 38 deg. 58 min. 27 ſec. I demand the Altitude of the Sun, his Azimuth, and the Latitude of the Place of Obſervation.

Anſwer.

The Altitude of the Sun is 38 deg. 18 min. 46 ſec. ½, the Azimuth South 72 deg. 12 min. 54 Weſt, and the Latitude of the Place of Obſervation 51 deg. 32 min. 00 ſec. North.

Prob. VI.

Caſe 13.

Given the Altitude of the Sun, the Hour of the Day, and the Angle of Poſition, to find the Sun's Declination, his Azimuth, and the Latitude of the Place of Obſervation.

Ex-

Example.

The Sun having 38 deg. 18 min. 46 ſec. ½ of Altitude, at 30 minutes paſt Eight in the Forenoon, the Angle of Poſition was found to be 38 deg. 58 min. 27 ſec. I demand the Sun's Azimuth, his Declination, and the Latitude of the Place of Obſervation.

Anſwer.

The Sun's Azimuth is South 72 deg. 12 min. 54 ſec. Eaſt, his Declination is 19 deg. 39 min. 10 ſec. ¾, and the Latitude 51 deg. 32 min. 00 ſec. but whether North or South is not ſufficiently limited by the Data in the *Queſtion.*

Prob. VI.

Caſe 14.

Given the Hour of the Day, the Azimuth of the Sun, and the Angle of Poſition, to find the Sun's Altitude, his Declination, and the Latitude of the Place of Obſervation.

Example.

At 30 minutes paſt Three in the Afternoon, the Sun's Azimuth was found to be South 72 deg. 12 min. 54 ſec. Weſt, at the ſame time that the Angle of Poſition was 38 deg. 58 min. 27 ſec. I demand the Sun's Altiude, his Declination, and the Latitude of the Place.

Anſwer.

The Sun's Altitude is 38 deg. 18 min. 46 ſec. ½, his Declination 19 deg. 39 min. 10 ſec. ¾ North, and the Latitude of the Place 51 deg. 32 min. 00 ſec. Weſt

Prob. VI.

Caſe 15.

Given the Latitude of the Place, the Declination of the Sun, and the Angle of Poſition; to find the Sun's Altitude, Azimuth, and Hour of the Day.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination, the Angle of Poſition is found to be 38 deg. 58 min. 27 ſec. I demand the Altitude of the Sun, his Azimuth, and the Hour of the Day.

Anſwer.

This *Caſe* is Ambiguous; and the Altitude of the Sun may be 38 deg. 18 min. 46 ſec. ½, when his Azimuth will be South 72 deg. 12 min. 54 ſec. Weſt, and the Hour 3 h. 30 m. if the Sun be in the Weſtern Semicircle, but if the Sun be in the Eaſtern Semicircle the Azimuth will be South 72 deg. 12 min. 54 ſec. Eaſt, and the Hour 8 h. 30 m. in the Morning.

Again, the Sun's Altitude may be 11 deg. 01 min. 55 ſec. ½, when his Azimuth will be 72 deg. 12 min. 54 ſec. Eaſt or Weſt, according as the Sun is in the Eaſtern or Weſtern Semicircle, and the Hour will be either 9 h. 01 min. 46 ſec. ½ in the Morning, or 2 h. 58 m. 13 ſ. ½ in the Afternoon.

Prob. VI.

Caſe 16.

Given the Latitude of the Place, the Altitude of the Sun, and the Angle of Poſition, to find the Sun's Azimuth, his Declination, and the Hour of the Day:

Example.

In the Latitude of 51 deg. 32 min. the Sun having 38 deg. 18 min. 46 ſec. ½ of Altitude in the Morning, the Angle of Poſition at the ſame time being 38 deg. 58 min. 27 ſec. I demand the Azimuth of the Sun, his Declination, and the Hour of the Day.

Anſwer.

The Sun's Declination is 19 deg. 39 min. 10 ſec. ¾ North, his Azimuth South 72 deg. 12 min. 54 ſec. Eaſt, and the Time 8 h. 30 ſ, in the Forenoon.

Altho' this *Caſe* is doubtful conſidered as a Spherical Triangle, yet it is limited as a Solar *Problem*, becauſe the other *Anſwer* gives the Sun's Declination to be 71 deg. 16 min. 28 ſec. ¼ North, which is impoſſible.

Prob.

Prob. VI.

Caſe 17.

Given the Sun's Azimuth, his Altitude, and the Angle of Poſition, to find the Sun's Declination, the Latitude of the Place, and the Hour of the Day.

Example.

The Sun having 38 deg. 18 min. 46 ſec. $\frac{1}{2}$ of Altitude in the Morning, his Azimuth was found to be South 72 deg: 12 min. 54 ſec. Eaſt, and at the ſame time the Angle of Poſition was 38 deg. 58 min. 27 ſec. I demand the Declination of the Sun, the Latitude of the Place, and Hour of the Day.

Anſwer.

The Declination of the Sun is 19 deg. 39 min. 10 ſec. $\frac{3}{4}$ North, the Latitude of the Place is 51 deg. 32 min. and the Hour of the Day is 8 h. 30 m. in the Forenoon.

Prob. VI.

Caſe 18.

Given the Sun's Declination, his Azimuth, and the Angle of Poſition, to find the Altitude of the Sun, the Latitude of the Place, and Hour of the Day.

Example.

The Sun having 19 deg. 39 min. 10 ſec. $\frac{3}{4}$ North Declination, his Azimuth was found to be South 72 deg. 12 min. 54 ſec. Weſt, and at the ſame time the Angle of Poſition was 38 deg. 58 min. 27 ſec. I demand the Latitude of the Place, Altitude of the Sun, and Hour of the Day.

Anſwer.

The Latitude is 51 deg. 32 min. North, the Altitude is 38 deg. 18 min. 46 ſec. $\frac{1}{2}$, and the Hour 30 minutes paſt Three in the Afternoon.

Prob.

Prob. VI.

Caſe 19.

Given the Sun's Declination, his Altitude, and the Angle of Poſition, to find the Sun's Azimuth, the Latitude of the Place and Hour of the Day.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination, his Altitude in the Weſtern Semicircle was found to be 38 deg. 18 min. 46 ſec. ½, and at the ſame time the Angle of Poſition was 38 deg. 58 min. 27 ſec. I demand the Latitude of the Place, the Azimuth of the Sun, and the Hour of the Day.

Anſwer.

The Latitude of the Place is 51 deg. 32 min. North, the Azimuth 72 deg. 12 min. 54 ſec. Weſt, and the time 3 hours 30 min. in the Afternoon.

Prob. VI.

Caſe 20.

Given the Latitude of the Place, the Azimuth of the Sun, and the Angle of Poſition, to find the Sun's Altitude, his Declination, and the Hour of the Day.

Example.

In the Latitude of 51 deg. 32 min. North, the Sun's Azimuth was found to be South 72 deg. 12 min. 54 ſec. Weſt, and at the ſame time the Angle of Poſition was 38 deg. 58 min. 27 ſec. I demand the Altitude of the Sun, his Declination, and the Hour of the Day.

This *Caſe* tho' doubtful as a *Caſe* in *Spherical Triangles*, yet it is limited as a *Solar Problem*, and the true *Anſwer* is as follows,

The Sun's Altitude is 38 deg. 18 min. 46 ſec. ½, his Declination 19 deg. 39 min. 10 ſec. ¾ North, and the Hour 30 minutes paſt Three in the Afternoon.

From what has been ſaid it is very manifeſt, that in every *Caſe* where any two of the Data are oppoſite one to the other, that the *Caſe* is limited or not limited, according as the Quantitys of the Parts when compared amongſt themſelves are greater or leſſer the one than the other, and that the ſame *Problem* that is doubtful, when conſidered as a Spherical Triangle, is ſome times limited when it becomes a *Solar Problem*, upon account of the Limitation of the Sun's diſtance from the Pole, and his Altitude or Depreſſion above or below the Horizon; and that the ſame *Problem* which is limited with regard to the Sun, is unlimited when applied to the fixed Stars, as will appear hereafter; and theſe are Things that the *Reader* ought to be very well acquainted with.

Prob. VII.

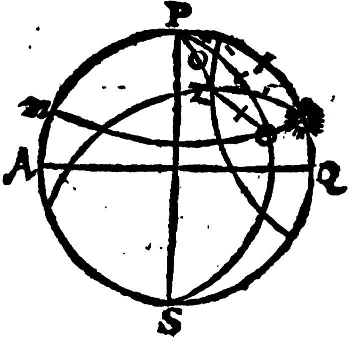
Two unequal Altitudes of the Sun being given, together with the Declination of the Sun, and the Difference of time between the Obſervations, to find the Azimuth of the Sun, the Hour of the Day and Latitude of the Place of Obſervation.

Example.

The Sun having 19 deg. 39 min. 10 ſec. ½ North Declination, his Altitude in the Forenoon was found to be 38 deg. 18 min. 46 ſec. ½, and at the End of an Hour and a half the ſame Morning, the Altitude was found to be 50 deg. 24 min. 49 ſec. ½. I demand the Sun's Azimuth, the Hour of the Day at each Obſervation, and the Latitude of the Place.

The Stereographic Solution.

1. Having drawn the Meridian *P Q S A*, the Equator *A Q*, and at Right-angles to it the Six a Clock Hour Circle *P S*, about the Pole *P*, at the diſtance of 70 deg. 20 min. 49 ſec. ½ the Complement of the Sun's Declination, deſcribe the Small Circle ☉ *O m*, by *Caſe* the 2d of *Problem* the 4th, of *Section* the 2d.



2. Thro' the Pole *P*, draw the Hour Circle *P* ☉ *S* forming an Angle

of 22 deg. 30 min. equal to the Difference of Times between the Obſervations, by *Caſe* the 3*d* of *Problem* the 9*th* of *Section* the 2*d*, then will ☉ and *O* repreſent the two different Places of the Sun, at the different Times of Obſervation.

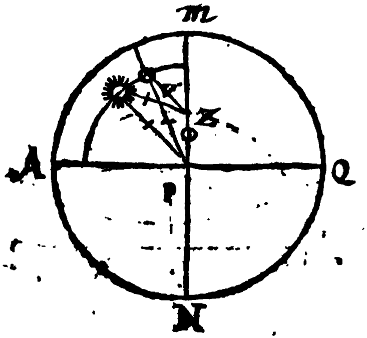
3 About the point ☉ at the diſtance of 51 deg. 41 min. 13 ſec. ½, the Complement of the leaſt Altitude, deſcribe the Small Circle ☉ Z, by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d*.

Alſo about the point ☉ deſcribe the Small Circle ☉ Z, at the diſtance of 39 deg. 35 min. 10 ſec. ¼, the Complement of the greateſt Altitude, by *Caſe* the 3*d* of the ſame *Problem*, to interſect the former Circle in Z.

4. Thro' the points Z and *P*, and Z and ☉, and Z and *O*, and *O* and ☉, draw the great Circle Z *P*, Z ☉, Z *O*, and *O* ☉, by *Problem* the 1*ſt* of *Section* the 2*d*, and the thing is done; and the Angle Z *P O*, and Z *P* ☉, the Times from Noon when each Obſervation was made, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d*, the Angles *P* Z *O*, *P* Z ☉, the correſpondent Azimuths from the Meridian may be meaſured by *Caſe* the 3*d* of the ſame *Problem*, and the Arch *P* Z the Complement of the Latitude, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th* of the ſame *Sect.*

As the Projection of the *Sphere* upon each particular Plane, has its peculiar Advantage, ſo this *Problem* is very eaſy and naturally repreſented upon the Plane of the *Equator*, after the following manner.

The Stereographic Solution.



1. Having drawn the Equator *m Q N A*, with the Semi-tangent of 70 deg. 20 min. 49 ſec. ½, the Complement of the Sun's Declination, about the point *p* as a Center and Pole, draw the Small Circle ☉ *O*, and make the Angle ☉ *p O*, equal to 22 deg. 30 min. equal to the difference of Times between the Obſervations, then will the points *O* and ☉ repreſent the Place of the Sun at the two different Times of Obſervation.

2. At the diſtance of 51 deg. 41 min. 13 ſec. the Complement of the Sun's leaſt Altitude, about the point ⊙ as a Pole, draw the Small Circle ⊙ Z, by *Caſe* the 3*d* of *Problem* the 4*th* of *Section* the 2*d*, and about the point *O* at the diſtance of 39 deg. 35 min. 10 ſec. ¼, the Complement of the greateſt Altitude, deſcribe the Small Circle *O* Z by the ſame, and thro' the Interſection Z and the Pole *p*, draw the Meridian *m* Z *p N*, and the thing is done; and the Angles *p* Z *O*, and *p* Z ⊙, the Azimuths from the Meridian, may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th* of *Section* the 2*d*, and the Angles Z *p O* and Z *p* ⊙ the times from Noon, may be meaſured by the *Line of Chords*, and the Arch *p* Z the Complement of the Latitude may be meaſured by the *Line of Semi-tangents.*

Logarithmically, or by Calculation.

Having let fall the Perpendicular *p R*, in the Spherical Triangle *p* ⊙ *R* Right-angled at *R* are given, *p* ⊙ equal to the Complement of the Sun's Declination, the Angle ⊙ *p R* equal to half the Difference of the times; whence,

1*ſt* To find the Angle *p* ⊙ *R*, it will be by the 3*d Caſe* of *Right-angled Spherical Triangles*,

As *R* : c s, ⊙ *p* : : t, ⊙ *p R* : c t, *p* ⊙ *R*; that is,

As the Radius —————— 10.0000000

To the Sine of the Declination 19.39.10¾ —————— 9.5267557
So is the Tangent of ½ Difference of the Times 11.15 9.2986618

To the Co-tang. of < *p* ⊙ *R*, or a 4*th* Arch 86.10.21 — 8.8254175

2*d*, To find the Arch ⊙ *R*, it will be by the 5*th Caſe of Right-angled Spherical Triangles*,

As *R* : S. *p* ⊙ : : S. < *p* ⊙ *R* : S. ⊙ *R*; that is,

As the Radius —————— 10.0000000

To the Co-ſine of the Declination 19.39.10¾ —————— 9.9739342
So is the Sine of the half Diff. of the Time 11.15 — 9.2902357

To the Sine of a fifth Arch *R* ⊙ 10.35.12¼ —————— 9.2641699

Which being doubled becauſe the Triangles *O p R* and ⊙ *p R* are equal, will give 21 deg. 10 min. 25 ſec. ¼, a ſixth Arch, for the length of the Arch *O* ⊙, or diſtance between the Sun at the two different Times of Obſervation. Wherefore,

3. In the Oblique-angled Spherical Triangle, *O* Z ⊙ are given, Z *O* equal to 39 deg. 35 min. 10 ſec. ¼, the Complement of the greateſt Altitude, Z ⊙ equal to 51 deg. 41 min. 13 ſec. ½, the Complement of the leaſt Altitude, and the Arch ⊙ *O* equal to 21 deg. 10 min. 25 ſec. ¼, whence to find the Angle Z ⊙ *O* (by the 11*th Caſe of Oblique*, &c.) having added the three Arches Z*O*, Z ⊙, and *O* ⊙ together, and from half of their Sum 56 deg. 13 min. 24 ſec. ½, having taken away the Arch Z.*O* equal to 39 deg. 35 min. 10 ſec. ¼, to find the Difference 16 deg. 38 min. 14 ſec. ¼.)

To the Arithm. Complement of the Co-ſine of the leaſt Altitude 38.18.46½	0.1053314
Add the Arithm. Complement of the Co-ſine of the ſixth Arch *O* ⊙ 21.10.25¼	0.4422567
Alſo the Sine of the half Sum 56.13.24½	9.9197119
And the Sine of the Difference 16.38.14¼	9.4568396
Half the Sum of theſe four Sines will give the Co-ſine 23.57.57⅛	9.9241356
Which being doubled will give 47.11.54¾	9.9620698

for a ſeventh Arch or the Angle Z ⊙ *O*.

But the Angle *p* ⊙ *O*—<Z ⊙ *O*=Z ⊙ *p*; that is, if from the 4*th* Arch *p* ⊙ *O* equal to 86 deg. 10 min. 21 ſec. ⅝, be taken away the ſeventh Arch or Angle Z ⊙ *O*, equal to 47 deg. 11 min. 54 ſec. ¾, there will remain 38 deg. 58 min. 26 ſec. ⅞ for an eighth Arch, or the Angle of Poſition Z ⊙ *p*, wherefore in the Triangle Z *p* ⊙ are given, Z ⊙ the Complement of the leaſt Altitude 51 deg. 41 min. 13 ſec. ½, the Arch *p* ⊙ the Complement of the Sun's Declination, equal to 70 deg. 20 min. 49 ſec. ¼, and the Angle of Poſition *p* ⊙ Z equal to 38 deg. 58 min. 26 ſec. ⅞; whence

4*th*, To find the 9*th* Arch ⊙ *r*, (a Perpendicular having been firſt let fall from Z upon *p* ⊙) it will be by the 9*th Caſe of Oblique Spheric*, or the 4*th Caſe* of *Right-angled Spheric*.

As *R* : cs, Z ☉ *p* : : t, Z ☉ : t, ☉ *R* ; that is,
As the Radius ———————————— 10.0000000

To the Co-ſine of the Angle *Z* ☉ *p* 38.58.26^{7} ———— 9.8906614
So is the Co-tangent of the leaſt Altitude 38.18.46$\frac{1}{2}$ —— 10.1023077

To the Tangent of a *9th* Arch ☉ *R* 44.32 10$\frac{1}{7}$ ———— 9.9929691

But *p* ☉ — *R* ☉ = *p R* ; that is, if from the Complement of the Sun's Declination 70 deg. 29 min. 49 ſec. $\frac{1}{4}$, be taken *R* ☉ equal to 44 deg. 32 min. 10 ſec. $\frac{1}{7}$, there will remain 25 deg. 48 min. 38 ſec. $\frac{7}{7}$, for the length of *p r* a *10th* Arch, whence to find the Complement of the Latitude *Z p*, it will be by the 10*th Caſe of Oblique Spheric.*

As cs, ☉ *R* : cs, *P r* : : cs, Z ☉ : cs, Z *P* ; that is,
As the Co-ſine of the ninth Arch 44.32.10$\frac{1}{7}$ ———— 9.8529722

To the Co-ſine of the tenth Arch 25.48.38$\frac{7}{7}$ ———— 9.9543568
So is the Sine of the leaſt Altitude 38.18.46$\frac{1}{2}$ ———— 9.7923608

To the Sine of the Latitude 51.32 ———————— 9.8937454

Again, to find the time of the firſt Obſervation, it will be by the *9th Caſe of Oblique Spheric.*

As S. *P r* : S. ☉ *R* : : t, < ☉ : t, < *P* ; that is,
As the Sine of the tenth Arch 25.48.38$\frac{7}{7}$ ———— 9.6388892

To the Sine of the ninth Arch 44.32.10$\frac{1}{7}$ ———— 9.8459410
So is the Tangent of the Angle *P* ☉ *Z* 38.58.26^{7} ——— 9.9079682

To the Tangent of the Hour 52.30 ———————— 10.1150200

Which reduced into Time and taken from twelve Hours, becauſe the Obſervation was made in the Forenoon, gives 8 h. 30 m. for the time of the firſt Obſervation in the Morning.

And if to this time 8 h. 30 m. 00 ſ. be added 1 h. 30 m. the difference of time between the two Obſervations, the Sum 10 h. 00 m. 00 ſ. will be the exact time in the Morning, when the 2*d* Obſervation was made.

To find the Azimuth at the time of the firſt Obſervation, it will be by the firſt *Caſe of Oblique Spheric.*

As S. Z. ☉ : S. *p* ☉ :: S. < *p* : S. < Z ; that is,

As the Co-ſine of the 1*ſt* Altitude 38.18.46½ —— 9 8946686

To the Co-ſine of the Declination 19:39.10¾ —— 9.9739342
So is the Sine of the Hour 52.32.00 —— 9.8994667

To the Sine of the the Azimuth 72.12.54 —— 9.9787323

Again, to find the Azimuth at the ſecond time of Obſervation, it will be,

As S. Z. ☉ : S. *p* ☉ :: S. < ☉ *p* Z ; S. *p* Z ☉ ; that is,

As the Co-ſine of the 2*d* Altitude 50.24.49¾ —— 9.8043018

To the Co-ſine of the Declination 19 39.10¾ —— 9.9739342
So is the Sine of the Hour from Noon 30.00.00 —— 9.6989700

To the Sine of the Azimuth at that time 47.38.24¾ —— 9.8686024

Whence it is apparent that the Latitude of the Place is 51 deg. 32 min. 00 ſec. North, the time when the firſt Obſervation was made was 8h. 30m. 00 ſ. when the Azimuth was South 72 deg. 12 min. 54 ſec. Eaſt, the time when the ſecond Obſervation was made, was 10 h. 00 m. 00 ſ. when the Azimuth was South 47 deg. 38 min. 24 ſec. ¾, and conſequently the difference of Azimuths is 24 deg. 34 min. 29 ſec. ¼.

Prob. VII.

Caſe 2.

Given two different Altitudes of the Sun made the ſame Day, together with the difference of the correſpondent Azimuths, and the Declination, to find the Hour of the Day when each Obſervation was made, and the Latitude of the Place.

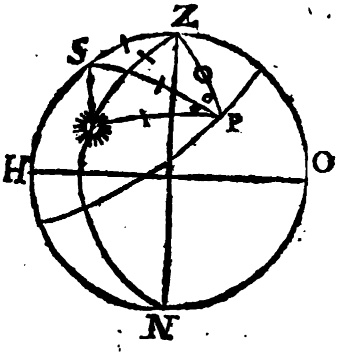
Example.

The Sun having 19 deg. 39 min. 10 ſec. ¾ North Declination, his Altitude was found to be 38 deg. 18 min. 46 ſec. ½ in the Forenoon, and ſome time after it was found to be 50 deg. 24 min. 49 ſec. ¾, during which time the Sun had altered his Azimuth 24 deg. 34

34 min. 29 ſec. $\frac{1}{4}$. I demand the Hour at each Obſervation, and the Latitude of the Place.

The Stereographic Solution.

1. Having drawn an Azimuth Circle *Z O N H*, the Horizon *H O*, and at Right-angles to it the Vertical Circle *Z N*, thro' the point Z draw the Azimuth Circle Z ☉ *N*, forming an Angle of 24 deg. 34 min. 29 ſec. $\frac{1}{4}$, with the Vertical Circle *H Z O*, equal to the difference of the Azimuths given.

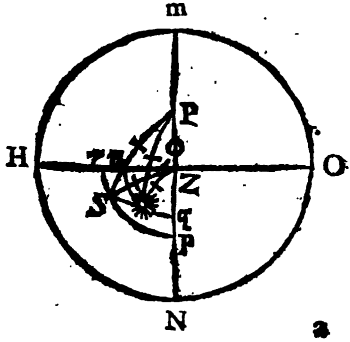


2. Set off the greater Altitude of the Sun from *H* to *S*, and the Complement of the leſſer Altitude from Z to ☉, by *Caſe* the 3*d* of *Problem* the 6*th* of *Section* the 2*d*, then will *S* and ☉ repreſent the Place of the Sun at each Obſervation.

3. About the points *S* and ☉ at the diſtance of 70 deg. 20 min. 49 ſec. $\frac{1}{4}$, the Complement of the Sun's Declination deſcribe the two Small Circles, by *Caſe* the 2*d* and 3*d* of *Problem* the 4*th*, of *Section* the 2*d*, to interſect each other in the point *P*.

4. Thro' the points *P* and *Z*, *P* and *S*; *P* and ☉, *S* and ☉, draw the great Circle *P* Z, *PS*, *P*☉, *S* ☉, and the thing is done; and the Angles Z *P* ☉, Z *P S*, the times from Noon, may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th* of *Section* the 2*d*, and the Angles *P* Z *S*, *P* Z ☉, the Azimuths, may be meaſured by *Caſe* the 2*d* of the ſame *Problem*; and the Arch Z *P* the Complement of the Latitude, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d*.

A due Conſideration of this *Problem* naturally leads us to the Plane of the *Horizon*, for the moſt eaſy Solution, and which is after the following manner.



1. Having drawn the Horizon *m O N H*, with the Semi-tangent of 51 deg. 41 min. 13 ſec. $\frac{1}{2}$, and 39 deg. 35 min. 10 ſec. $\frac{1}{4}$, the Complement of the greateſt and leaſt Altitude of the Sun, about the point Z as

a Center, draw the Small Circle *n* ⊙ *p*, and *r S q*, and make the Angle ⊙ Z *S*, equal to 24 deg. 34 min. 29 ſec. ½, equal to the difference between the obſerved Azimuths, then will the points *S* and ⊙ repreſent the two Places of the Sun, at the two different times of Obſervation.

2. At the diſtance of 70 deg. 26 min. 49 ſec. ¾, the Complement of the Sun's Declination, and about the points ⊙ and *S* as Poles, deſcribe the Small Circles ⊙ *P*, *S P*, to interſect each other in the point *P*, by *Caſe* the 3*d* of *Problem* the 4*th* of *Section* the 2*d*, and thro' the Interſection *P* and the Zenith Z, draw the Meridian *m P Z N*, and the thing is done; and the Angles Z *P S*, Z *P* ⊙, the times from Noon, may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th* of *Section* the 2*d*, the Angles *P* Z ⊙, *P* Z *S*, the Azimuths from the Meridian, may be meaſured by the *Line of Chords*, and the Arch of the Meridian *P* Z the Complement of the Latitude, may be meaſured by the *Line of Chords*.

Logarithmically, or by Calculation.

In the Oblique-angled Spherical Triangle *S* Z ⊙ are given, Z *S*, Z ⊙, the Complements of the two Altitudes, together with the Angle *S* Z ⊙ the Difference of the Azimuths, whence to find the Angle Z ⊙ *S*, it will be by the *9th Caſe* of *Oblique-angled Spherical Triangles*, having firſt let fall the Perpendicular *S R*.

As *R* : c s, < Z : : t, Z *S* : t, Z *R*; that is,

As the Radius —— 10.0000000

To the Co-ſine of the Differ. of the Arches 24.34.29½ — 9.9587639

So is the Co-tang. of the greateſt Altitude 50.24.49¾ 9.9174351

To the Tangent of the fourth Arch Z *R* 36.56.31¼ —— 9.8761990

But Z ⊙ — Z *R* = *R* ⊙; that is, if from the Complement of the leaſt Altitude 51 deg. 41 min. 13 ſec. ½, be taken the 4*th* Arch Z *R* there will remain a 5*th* Arch *R* ⊙, equal to 14 deg. 44 min. 42 ſec. ¼, whence to find the Angle Z ⊙ *S* it will be,

As S. ☉*R* : S. Z*R* : : t, < ☉*ZS* : t, <*Z*☉*S* ; that is,
As the Sine of the *5th* Arch ☉*R* 14° 44′ 42″⅜ ——— 9.4057207

To the Sine of the fourth Arch Z*R* 36° 56′ 31″ ½ — 9.7788788
So is the Tang. of the Dif. of the Arches 24° 34′ 29″ ½ — 9.6602060

To the Tang. of the <*Z*☉*S* a *6th* Arch 47° 11′ 55″¼ 10.0333641

And again, As cs, Z*R* : cs, ☉ *R* : : cs, *ZS* : cs, *Z*☉ ; that is,
As the Co-ſine of the *4th* Arch 36° 56′ 31″½ ——— 9.9026796

To the Co-ſine of the *5th* Arch 14° 44′ 42″⅜ ——— 9.9854568
So is the Sine of the greateſt Alt. 50° 24′ 49″¾ ——— 9.8868667

To the Co-ſine of a *7th* Arch S☉ 21° 10′ 25″½ ——— 9.9696439

Or the Baſe of the Equicrural Spherical Triangle *P*☉*S* ; the half of which is 10 deg, 35 min. 12 ſec.⅔, an eighth Arch or the Baſe of the Right-angled Spherical Triangle *P*☉♈, made by a Perpendicular *P* ♈, let fall from *P* upon the Side *S* ☉ ; wherefore, in the Spherical Triangle *P* ☉♈ Right-angled at ♈ are given, *P* ☉ equal to the Complement of the Sun's Declination, and ☉ ♈ before found, whence to find firſt, the Angle ☉ *P* ♈ half the Difference of the times between the two Obſervations, it will be by the *13th Caſe of Right-angled Spherical Triangles,*

As S. *P*☉ : *R* : : S. ☉ ♈ : S. ☉*P*♈ ; that is,
As the Co-ſine of the Sun's Declination 19.39.10¼ ——— 9.9739342

To the Radius ——————— 10.0000000
So is the the Sine of the *8th* Arch ☉ ♈ 10.35.12⅔ ——— 9.2641709

To the Sine of the Angle ♈ *P* ☉ 11.15.00 ——— 9.2902367

This being doubled, gives 22 deg. 30 min. .00 ſec. for the whole Angle *S P* ☉, equal to 2 h. 30 m. 00 ſ. the Difference of the times between the two Obſervations.

Again, to find the Angle *P* ☉ ♈ of the ſame Triangle *P* ☉ ♈, it will be by the *12th Caſe of Right-angled Spherical Triangles.*

As *R* : ct, ☉ *P* : : t, ☉ *R* : c s, *P* ☉ *R* ; that is,

As the Radius———————————————— 10,0000000

To the Tang. of the Sun's Declination 19·39·10$\frac{3}{4}$——— 9·5528217
So is the Tang of the 8*th* Arch ☉ ♈ 10·35·12$\frac{3}{4}$———— 9·2716273

To the Co ſine of a 9*th* Arch or <*P*☉ ♈ 86·10·21$\frac{5}{8}$—— 8·8244490

But the Angle *P* ☉ ♈—Angle *Z* ☉ *S* equal to the Angle *Z* ☉ *P* ; that is, if from the 9*th* Arch or Angle *P*,☉ ♈ equal to 86 deg. 10 min. 21 ſec. $\frac{5}{8}$, be taken the 6*th* Arch or Angle *Z* ☉ *S* 47 deg. 11 min. 55 ſec. $\frac{1}{4}$, there will remain 38 deg. 58 min. 26 ſec. $\frac{3}{8}$ for a 10*th* Arch, or the Angle of Poſition *P* ☉ Z, wherefore in the Spherical Triangle *P*☉Z are given, *P*☉ equal to 70 deg. 20 min 49 ſec. $\frac{1}{4}$ the Complement of the Sun's Declination Z ☉, equal to 51 deg. 41 min. 13 ſec. $\frac{1}{2}$, the Complement of the Sun's leaſt Altitude, and the Angle of Poſition *P*☉Z equal to 38 deg 58 min 26 ſec $\frac{3}{8}$, whence to find the Angles ☉*PZ* and ☉*ZP*, it will be by the 4*th Axiom of Spherical Triangles* in *Section* the 6*th*,

As cs, ½$\overline{P\odot+Z\odot}$: cs, *P*☉—Z☉ : : ct, ½<☉ : t, ½<s $\overline{P+Z}$.
That is,
As the Co-ſine of the ½Sum of the Sides *P*☉, Z☉, 61.1.c$\frac{3}{8}$ 9·6853381

To the Co-ſine of half their Difference 9·19.47$\frac{7}{8}$——9·9942154
So is the Co-tangent of half the Angle ☉ 19 29·13$\frac{3}{16}$—10·451646

To the T. of half the Sum of the Angles *P*+Z 80·8.33 10·7600419

And again, As S. ½ $\overline{P\odot+Z\odot}$: S. ½ $\overline{P\odot-Z\odot}$: : ct, ½ <☉ : t, ½<s $\overline{P+Z}$ That is,
As the S. of half the Sum of the Sides *P*☉, Z☉, 61.1.1$\frac{3}{8}$ 9·9418909

To the Sine of half their Difference 9.19.47$\frac{7}{8}$————9·2098361
So is the Co-tangent of half the Angle ☉ 19·29·13$\frac{3}{16}$——10·451646

To the Tang. of half the Diff. of the <s *P*—Z 27·38.33 9·7191098

Now the half Sum 80 deg 08 min. 33 ſec added to the half Difference 27 deg. 38 min. 33 ſec. will give the Angle *P Z* ☉ equal to 107 deg. 47 min. 06 ſec. the Supplement of 72 deg. 12 min. 54 ſec. the Azimuth from the South at the time of the Firſt Obſervation, and this leſſened by 24 deg. 34 min. 29 ſec. ¼, the Difference of the two Azimuths, will give 47 deg. 38 min. 24 ſec. ¾, for the Azimuth at the time of the Second Obſervation.

Again, if from the half Sum 80 deg. 08 min. 33 ſec. be taken the half Difference 27 deg. 38 min. 33 ſec. there will remain 52 deg. 30 min. 00 ſec. equal to 3 h. 30 m. 00 ſ. this taken from 12 Hours becauſe the Obſervation was made in the Morning, will give 8 h. 30 m 00 ſ. for the time in the Forenoon when the Firſt Obſervation was made: And if to this be added 1 h. 30 m. 00 ſ. the Difference of the times between the two Obſervations before found, the Sum 10 h. 00 m. 00 ſ. will be the time the Second Obſervation was made in the ſame Morning; wherefore to find the Latitude of the Place of Obſervation, it will be by the *1ſt Caſe of Oblique-angled Spherical Triangles,*

As S. < *P* : S. Z☉ : : S. < ☉ : S. *ZP*; that is,

As the Sine of the Hour at the 1ſt Obſ. 52.30.00 —— 9·8994667

To the S. of the correſpondent < of Poſition 38·58·26⅜ 9·7986283

So is the Co-ſine of the leaſt Altitude 38·18·46½ —— 9·8946686

To the Co-ſine of the Latitude 51.32.00 —— 9·7938302

Whence it appears that the time of the firſt Obſervation was at 8 h. 30 m. 00 ſ. when the Azimuth was South 72 deg. 12 min. 54 ſec. Eaſt, the time of the ſecond Obſervation was at 10 h. 00 min. 00 ſec. when the Azimuth was South 47 deg. 38 min. 24 ſec ¾ E. and the Latitude of the Place of Obſervation was 51 deg. 32 min. 00 ſec. North

Prob. VII.

Caſe 3.

Given two Altitudes of the Sun taken in the ſame Forenoon or Afternoon, together with the Alteration of Azimuth, and Difference between the times of the Obſervations, to find the Hour of the Day

Azimuth at each Obſervation, together with the Latitude of the Place and Declination of the Sun.

Example.

The Sun's Height with a good Quadrant was found to be 38 deg. 18 min. 46 ſec. ½, and in one Hour and a half the ſame Forenoon it was found to have increaſed 12 deg. 6 min. 3 ſec. ¾, during which time the Sun had made an Alteration of 24 deg. 34 min. 29 ſec. ¼ of Azimuth: I demand the Declination of the Sun, the Hour and Azimuth when each Obſervation was made, and the Latitude of the Place?

Anſwer.

The Declination is 19 deg. 39 min. 10 ſec. ¾ the time of the Day at the firſt Obſervation was 08 h. 30 m. 00 ſ. in the Morning, at which time the true Azimuth was South 72 deg. 12 min. 54 ſec. Eaſt, the time of the Day at the ſecond Obſervation was 10 h. 0m. 00 ſ. the ſame Morning, when the Azimuth was South 47 deg. 38 min. 24 ſec ¾ Eaſt, and the Latitude 51 deg. 32 min. 00 ſec. North.

Prob. VII.

Caſe 4.

Given the Altitude and Declination of the Sun, together with the alteration of Azimuth correſponding to a given time, to find the Hour of the Day, Azimuth of the Sun, and Latitude of the Place.

Example.

The Sun's Declination being 19 deg. 39 min. 10 ſec ¾ North, his Altitude was found in the Afternoon to be 50 deg. 24 min. 49 ſec ¾, and in the Space of one Hour and a half the ſame Afternoon, the Azimuth from the South was found to increaſe 24 deg. 34 min. 29 ſec. ¼, I demand the Latitude of the Place, the Hour of the Day, and Azimuth of the Sun at the time when the Sun's Height was taken.

Anſwer.

The Latitude of the Place was 51 deg. 32 min. 00 ſec. North the Hour of the Day was 2 in the Afternoon, and the Azimuth at that time was South 47 deg. 38 min. 24 ſec. ¾ Weſt.

Prob. VII.

Caſe 5.

Given two Altitudes taken on the ſame Day, together with the Hour of the Day when the firſt Obſervation was made, to find the Latitude of the Place and Azimuth of the Sun.

Example.

On the *7th* of *May* 1720, the Sun's Altitude at half an Hour paſt Eight in the Morning was found to be 38 deg. 18 min. 46 ſec. ½, and ſome time after the ſame Morning, the Height was obſerved to be 50 deg. 24 min. 49 ſec. ¾, I demand the Latitude of the Place, Hour and Azimuth, at the laſt time of Obſervation.

Anſwer.

The Latitude is 51 deg. 32 min. 00 ſec. the Hour is 10 h. 00 m. 00 ſec. and Azimuth South 47 deg. 38 min. 24 ſec. ¾ Eaſt.

Prob. VII.

Caſe 6.

Given the Sun's Altitude, and the Hour on the ſame Day, together with his alteration of Azimuth, to find the Latitude of the Place and Azimuth of the Sun.

Example.

On the *7th* of *May* 1720, the Sun's Altitude at Two in the Afternoon was found to be 50 deg. 24 min. 49 ſec. ¾, and during a certain Space of Time the Azimuth was found to increaſe 24 deg.

34 min. 29 ſec. ¼, I demand the Latitude of the Place, Hour of the Day and Azimuth of the Sun at the End of the time of Obſervation.

Anſwer.

The Latitude is 51 deg. 32 min. 00 ſec. the Hour is 3 h. 50 m. and the Azimuth is South 72 deg. 12 min. 54 ſec. Weſt.

A great variety of other *Caſes* might be formed from this *Problem*, but theſe that I have given ſhall ſuffice for this time.

In the ſix preceding *Caſes* the Declination of the Sun has been ſuppoſed the ſame at each Obſervation, when in reality it varies about 48 Seconds in 1½ Hour, in that part of the Ecliptic where the Sun is, that is, increaſing when the Sun is in the firſt Quadrant of the Ecliptic, between the time of the firſt and ſecond Obſervation; and decreaſing when the Sun is in the ſecond Quadrant of the Ecliptic, but the Difference is ſo ſmall, that for the Eaſe of Calculation it has been rejected; and altho it be impoſſible by any Inſtrument hitherto made, or that can be made to take the Suns Altitude and Azimuth to Seconds and parts of Seconds, yet it was neceſſary here to ſuppoſe it to be done, that the *Reader* may ſee how very exact the ſeveral Requiſites anſwer one to another in the ſeveral *Caſes*, when Care is taken in the Calculation; and for this End I compoſed a Table of *Sines*, *Tangents* and *Secants* on Purpoſe to the given Numbers.

The following Tables of the Sun's Place, conſtructed after the manner taught in the *10th Section* of *Part* the *1ſt*, and the correſpondent Right Aſcenſions and Declinations thence deduced by the help of the *1ſt Problem* of *Section* the *3d* of this *Part* are all fitted to the Meridian of *London*, and adapted to the Noon of each reſpective Day, and therefore to find the true Place of the Sun, his Declination or Right Aſcenſion to any other time of the Day, we muſt ſay, as 24 Hours is to the Difference between the Place of the Sun, his Declination or Right Aſcenſion the Noon of the given Day, and his Place, Declination, or Right Aſcenſion the Noon of the following Day, ſo is the time from Noon to a fourth Proportional; which being added to the Place of the Sun, his Declination or Right Aſcenſion at the Noon of the given Day, will give the true Place of the Sun, his Declination or Right Aſcenſion at the time propoſed; but if the Sun be in the ſecond or fourth Quadrants of the Ecliptic, then the fourth Proportional being ſubſtracted from the Declination of the Sun at the Noon of the given Day, will give his true Declination at the time propoſed.

Suppoſe for *Example*, that it were required to find the true Place of the Sun in the Ecliptic, his Declination and Right Aſcenſion at *London* on the firſt Day of *January* 1725, at 2 h. 35 m. 36 ſ. in the Afternoon, ſay, as 24 hours is to 1 deg. 01 min. 07 ſec. the Difference between 23 deg. 22 min. 02 ſec. the Place of the Sun in *Capricorn* the Noon of the Day given, and 23 deg. 23 min. 09 ſec. of *Capricorn* the Place of the Sun at Noon the Day following, or on the 2*d* of *January* ſo is 2 h. 35 m 36 ſ the time from Noon to 7 m. 34 ſ. which being added to 22 deg. 22 min. 02 ſec. the Place of the Sun on the firſt Day of *January*, will give 22 deg. 29 min. 36 ſec. for the true Place of the Sun on the firſt Day of *January* 1725, at 2 h 35 m 36 ſ. *P. M.* After the ſame manner the Sun will be found to have 21 deg. 36 min. 19 ſec. of South Declination, and 19 h. 37 m. 08 ſ. of Right Aſcenſion

And inaſmuch as the Tables are fitted to the Meridian of *London*, if the Place of the Sun, his Declination or Right Aſcenſion be required at any other Place that lyes under a different Meridian, the proportional Quantity to be applied to the Tabular Place, Declination, or Right Aſcenſion, in order to find the true Place, Declination and Right Aſcenſion, muſt be found after the ſame manner as in the former *Example*, by ſaying, as 24 hours to the Tabular Difference, ſo is the Difference of Longitude in Time to a fourth Proportional, which being applied to the Tabular Place, Right Aſcenſion or Declination, will give the true Place of the Sun, his Declination and Right Aſcenſion at the Place propoſed.

Thus the true Place of the Sun at *Port Royal* in *Jamaica*, which lies 5 h. 4 m. to the Weſtward of the Meridian of *London* on the 1ſt of *January* 1725 at Noon, will be found to be 22 deg. 34 min. 56 ſec. in *Capricorn*, his true Declination at that time being 21 deg. 35 min. 15 ſec. South, and his Right Aſcenſion 19 hor. 37 min. 35 ſec but at *Cheuſan* in *China* which lies 8 h, 6 m. to the Eaſtward of the Meridian of *London*, the Sun at Noon on the ſame firſt Day of *January* at Noon, will be found to be in 22 deg. 01 min. 25 ſec. in *Capricorn* having 21 deg. 40 min. 52 ſec. South Declination, and his Right Aſcenſion at the ſame time will be found to be 19 h. 35 m. 13 ſ.

And inaſmuch as the Place of the Sun and conſequently his Declination and Right Aſcenſion is found to vary annually, by reaſon of the odd 5 Hours and 49 Minutes of which the Solar conſiſts of, over and above the 365 Days; I have added Tables to ſhew the proportionable Increaſe or Decreaſe to be applied to the Tabular Place, *&c.* to give the true Place, *&c.* Thus, the true Place of the Sun on the firſt of *January* 1800 at *London*, will be found to be *Capricorn* 22 deg. 8 min. 41 ſec. his Declination at that time 21 deg. 39 min. 29 ſec. S. and his Right Aſcenſion 19 hor. 35 min. 42 ſec. and by applying the Quantities of the Variation the contrary way to their reſpective Titles, may the true Place of the Sun, be found for any time paſt, within the compaſs of the Tables.

TABLES

TABLES

OF THE

Sun's true Place

Every Day at Noon,
Under the *MERIDIAN* of
His Majeſty's Obſervatory.
Calculated from Mr. *FLAMSTEED*'s

Solar Numbers:

For the Years 1725, 1726, 1727, 1728.

TOGETHER

With his *Right Aſcenſion* in Time, and *Declination*
from the *Equator.*

WITH

Variations to make them ſerve for an Hundred
Years to come.

A New and Correct Table of the Sun's Place for the Year 1725, being the First Year after Leap-Year.

1725

Days	*Jan.* ♑			*February* ♒			*March* ♓			*April* ♈			*May* ♉			*June* ♊			Days
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	22	22	02	23	49	39	21	54	19	22	29	25	21	35	43	21	17	58	1
2	23	23	09	24	50	12	22	54	03	23	28	04	22	33	30	22	15	14	2
3	24	24	15	25	50	43	23	53	44	24	26	41	23	31	16	23	12	30	3
4	25	25	21	26	51	13	24	53	23	25	25	16	24	29	01	24	09	44	4
5	26	26	27	27	51	42	25	52	59	26	23	50	25	26	43	25	06	58	5
6	27	27	31	28	52	09	26	52	33	27	22	22	26	24	24	26	04	12	6
7	28	28	34	29	52	35	27	52	05	28	20	51	27	22	05	27	01	25	7
8	29	29	37	♓	52	59	28	51	34	29	19	18	28	19	44	27	58	37	8
9	♒	30	39	1	53	21	29	51	03	♉	17	44	29	17	22	28	55	50	9
10	1	31	40	2	53	41	♈	50	29	1	16	08	♊	14	58	29	53	02	10
11	2	32	42	3	54	00	01	49	53	2	14	30	01	12	33	♋	50	14	11
12	3	33	41	4	54	17	02	49	15	3	12	50	02	10	07	1	47	20	12
13	4	34	39	5	54	32	03	48	35	4	11	08	03	07	39	2	44	37	13
14	5	35	37	6	54	45	04	47	52	5	09	25	04	05	14	3	41	48	14
15	6	36	34	7	54	57	05	47	09	6	07	40	05	02	42	4	38	59	15
16	7	37	32	8	55	07	06	46	22	07	05	53	06	00	14	5	36	10	16
17	8	38	26	9	55	15	07	45	34	08	04	04	06	57	40	6	33	21	17
18	9	39	21	10	55	21	08	44	43	09	02	13	07	55	07	7	30	32	18
19	10	40	14	11	55	24	09	43	51	10	00	20	08	52	39	8	27	43	19
20	11	41	06	12	55	26	10	42	57	10	58	26	09	49	59	9	24	54	20
21	12	41	56	13	55	27	11	42	01	11	56	30	10	47	23	10	22	05	21
22	13	42	44	14	55	26	12	41	02	12	54	32	11	44	47	11	19	16	22
23	14	43	31	15	55	23	13	40	01	13	52	33	12	42	09	12	16	27	23
24	15	44	17	16	55	16	14	38	59	14	50	33	13	39	31	13	13	38	24
25	16	45	02	17	55	08	15	37	54	15	48	30	14	36	52	14	10	50	25
26	17	45	46	18	54	59	16	36	47	16	46	27	15	34	13	15	08	0	26
27	18	46	28	19	54	47	17	35	39	17	44	21	16	31	32	16	05	13	27
28	9	47	08	20	54	34	18	34	28	18	42	14	17	28	51	17	02	25	28
29	20	47	48				19	33	15	19	40	05	18	26	09	17	59	36	29
30	21	48	27				20	32	00	20	37	55	19	23	26	18	56	48	30
31	22	49	04				21	30	44				20	20	42				31

A New and Correct Table of the Sun's *Place for the Year* 1725, *being the Firſt Year after* Leap-Year.

1725

Days	*July.* ♋			*Auguſt* ♌			*September* ♍			*October* ♎			*November* ♏			*December* ♐			Days
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	19	54	00	19	33	38	19	33	54	19	4	50	20	6	13	20	31	23	1
2	20	51	14	20	31	19	20	32	26	20	4	23	21	6	44	21	32	28	2
3	21	48	27	21	29	01	21	31	00	21	3	59	22	7	17	22	33	34	3
4	22	45	42	22	26	45	22	29	37	22	3	36	23	7	51	23	34	40	4
5	23	42	57	23	24	30	23	28	15	23	3	16	24	8	27	24	35	48	5
6	24	40	13	24	22	16	24	26	56	24	2	57	25	09	05	25	36	56	6
7	25	37	29	25	20	04	25	25	38	25	2	40	26	09	44	26	38	05	7
8	26	34	45	26	17	53	26	24	23	26	2	25	27	10	24	27	39	13	8
9	27	32	03	27	15	44	27	34	09	27	2	13	28	11	05	28	40	22	9
10	28	29	20	28	13	36	28	21	57	28	2	03	29	11	48	29	41	31	10
11	29	26	39	29	11	30	29	20	48	29	1	55	♐	12	32	♑	42	42	11
12	♌	23	58	♍	09	26	♎	19	40	♏	1	48	01	13	18	01	43	52	12
13	01	21	18	01	07	23	01	18	34	01	1	44	02	14	05	02	45	00	13
14	02	18	39	02	05	22	02	17	31	02	1	42	03	14	54	03	46	12	14
15	03	16	01	03	03	22	03	16	30	03	1	41	04	15	44	04	47	22	15
16	04	13	24	04	01	24	04	15	30	04	1	44	05	16	35	05	48	33	16
17	05	10	47	04	59	28	05	14	34	05	1	47	06	17	28	06	49	44	17
18	06	08	11	05	57	33	06	13	38	06	1	52	07	18	21	07	50	54	18
19	07	05	35	06	55	40	07	12	45	07	1	59	08	19	16	08	52	05	19
20	08	03	01	07	53	48	08	11	54	08	2	08	09	20	12	09	53	16	20
21	09	00	28	08	51	59	09	11	06	09	2	19	10	21	8	10	54	26	21
22	09	57	56	09	50	11	10	10	19	10	2	32	11	22	6	11	55	37	22
23	10	55	25	10	48	25	11	09	34	11	2	46	12	23	5	12	56	47	23
24	11	52	55	11	46	41	12	08	51	12	3	[illegible]2	13	24	4	13	57	58	24
25	12	50	26	12	44	59	13	08	10	13	3	20	14	25	5	14	59	08	25
26	13	47	59	13	43	19	14	07	32	14	3	40	15	26	07	16	0	19	26
27	14	45	32	14	41	40	15	06	55	15	4	02	16	27	09	17	1	29	27
28	15	43	07	15	40	03	16	06	20	16	4	25	17	28	11	18	2	38	28
29	16	40	43	16	38	28	17	05	48	17	4	50	18	29	14	19	3	47	29
30	17	38	20	17	36	54	18	05	17	18	5	16	19	30	18	20	4	55	30
31	18	35	58	18	35	23				19	5	44				21	6	03	31

A New and Correct Table of the Sun's *Place for the Year* 1726, *being the Second Year after* Leap-Year.

1726

Days	*January* ♑			*February* ♒			*March* ♓			*April* ♈			*May* ♉			*June* ♊			Days
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	22	07	10	23	34	56	21	39	50	22	15	12	21	21	44	21	04	08	1
2	23	08	18	24	35	30	22	39	33	23	13	52	22	19	32	22	01	24	2
3	24	09	24	25	36	02	23	39	15	24	12	29	23	17	18	22	58	39	3
4	25	10	30	26	36	33	24	38	54	25	11	05	24	15	02	23	55	54	4
5	26	11	35	27	37	02	25	38	31	26	09	30	25	12	46	24	53	08	5
6	27	12	39	28	37	29	26	38	06	27	8	11	26	10	28	25	50	22	6
7	28	13	43	29	37	55	27	37	39	28	6	41	27	08	08	26	47	35	7
8	29	14	46	♓	38	19	28	37	10	29	5	09	28	05	48	27	44	48	8
9	♒	15	48	01	38	42	29	36	39	♉	3	35	29	03	26	28	42	00	9
10	01	16	50	02	39	03	♈	36	05	01	2	00	♊	01	03	29	39	12	10
11	02	17	51	03	39	22	01	35	30	02	00	22	10	58	38	♋	36	22	11
12	03	18	52	04	39	39	02	34	52	02	58	43	01	56	12	01	33	34	12
13	04	19	51	05	39	54	03	34	13	03	57	02	02	53	45	02	30	46	13
14	05	20	50	06	40	08	04	33	31	04	55	19	03	51	17	03	27	57	14
15	06	21	47	07	40	20	05	32	47	05	53	24	04	48	48	04	25	08	15
16	07	22	44	08	40	30	06	32	02	06	51	47	05	46	18	05	22	20	16
17	08	23	39	09	40	38	07	31	14	7	49	59	06	43	46	06	19	32	17
18	09	24	33	10	40	44	08	30	24	08	48	09	07	41	14	07	16	43	18
19	10	25	26	11	40	49	09	29	32	09	46	17	08	38	41	08	13	54	19
20	11	26	19	12	40	51	10	28	38	10	44	23	09	36	06	09	11	05	20
21	12	27	09	13	40	52	11	27	42	11	42	28	10	33	30	10	08	15	21
22	13	27	58	14	40	51	12	26	44	12	40	31	11	30	54	11	05	26	22
23	14	28	46	15	40	48	13	25	44	13	38	32	12	28	17	12	02	37	23
24	15	29	33	16	40	43	14	24	42	14	36	33	13	25	39	12	59	48	24
25	16	30	18	17	40	36	15	23	37	15	34	31	14	23	00	13	57	00	25
26	17	31	01	18	40	27	16	22	31	6	32	27	15	20	21	14	54	12	26
27	18	31	14	19	40	17	17	21	23	17	30	22	16	17	41	15	51	23	27
28	19	32	25	20	40	04	18	20	1[illegible]	18	28	15	17	14	59	16	48	35	28
29	20	33	05				19	19	01	19	26	07	18	12	18	17	45	47	29
30	21	33	43				20	17	47	20	23	56	19	09	35	18	42	59	30
31	22	34	20				21	16	21				20	06	52				31

A New and Correct Table of the Sun's *Place for the Year 1726, being the Second Year after* Leap-Year.

1726

Days	*July* ♋			*August* ♌			*September* ♍			*October* ♎			*November* ♏			*December* ♐			Days
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	19	40	12	19	19	48	19	19	43	18	50	21	19	51	32	20	16	33	1
2	20	37	25	20	17	21	20	18	15	19	49	54	20	52	03	21	17	38	2
3	21	34	39	21	15	03	21	16	50	20	49	29	21	52	35	22	18	43	3
4	22	31	53	22	12	46	22	15	25	21	49	06	22	53	09	23	19	50	4
5	23	29	08	23	10	35	23	14	02	22	48	46	23	53	45	24	20	57	5
6	24	26	23	24	08	17	24	12	42	23	48	27	24	54	22	25	22	05	6
7	25	23	39	25	06	05	25	11	25	24	48	11	25	55	00	26	23	13	7
8	26	20	55	26	03	54	26	10	09	25	47	56	26	55	40	27	24	21	8
9	27	18	12	27	01	44	27	08	55	26	47	43	27	56	21	28	25	30	9
10	28	15	29	27	59	36	28	07	4	27	47	32	28	57	03	29	26	40	10
11	29	12	47	28	57	30	29	6	32	28	47	24	29	57	47	♑	27	49	11
12	♌	10	06	29	55	25	♎	5	25	29	47	17	♐	58	32	1	28	59	12
13	01	07	25	♍	53	29	01	4	15	♏	47	12	01	59	18	2	30	09	13
14	02	04	45	01	51	20	02	3	15	01	47	09	03	00	07	3	31	19	14
15	03	02	07	02	49	20	03	2	12	02	47	08	04	00	57	4	32	30	15
16	03	59	30	03	47	22	04	01	13	03	47	10	05	1	46	5	33	41	16
17	04	56	54	04	45	25	05	00	15	04	47	13	06	2	40	6	34	51	17
18	05	54	18	05	43	30	05	59	19	05	47	18	07	3	34	7	36	02	18
19	06	51	43	06	41	36	06	58	26	06	47	24	08	4	28	8	37	12	19
20	07	49	09	07	39	48	07	57	35	07	47	33	09	5	24	9	38	23	20
21	08	46	36	08	37	54	08	56	45	08	47	43	10	6	20	10	39	34	21
22	09	44	04	09	36	06	09	55	57	09	47	55	11	7	18	11	40	44	22
23	10	41	32	10	34	20	10	55	12	10	48	09	12	8	16	12	41	55	23
24	11	39	02	11	32	35	11	54	28	11	48	25	13	9	16	13	43	05	24
25	12	36	33	12	30	52	12	53	47	12	48	43	14	10	16	14	44	16	25
26	13	34	05	13	29	11	13	53	07	13	49	02	15	11	18	15	45	27	26
27	14	31	38	14	27	31	14	52	30	14	49	23	16	12	19	16	46	37	27
28	15	29	12	15	25	54	15	51	54	15	49	45	17	13	22	17	47	47	28
29	16	26	47	16	24	18	16	51	21	16	50	10	18	14	25	18	48	56	29
30	17	24	23	17	22	45	17	50	50	17	50	36	19	15	28	19	50	05	30
31	18	22	01	18	21	13				18	51	03	—	—	—	20	51	12	31

A New and Correct Table of the Sun's *Place for the Year* 1727, *being the Third Year after* Leap-Year.

1727

Days	*January* ♑			*February* ♒			*March* ♓			*April* ♈			*May* ♉			*June* ♊			Days
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	21	52	20	23	20	13	21	25	21	22	00	50	21	07	45	20	50	16	1
2	22	53	27	24	20	48	22	25	05	22	59	40	22	05	33	21	47	34	2
3	23	54	34	25	21	21	23	24	46	23	58	19	23	03	20	22	44	45	3
4	24	55	40	26	21	52	24	24	26	24	56	55	24	01	05	23	42	0[illegible]	4
5	25	56	45	27	22	21	25	24	04	25	55	25	24	58	49	24	39	18	5
6	26	57	5	28	22	49	26	23	39	26	54	01	25	56	31	25	36	32	6
7	27	58	53	29	23	16	27	23	1[illegible]	27	52	3[illegible]	26	54	12	26	33	40	7
8	28	59	57	♓	23	40	28	22	44	28	51	00	27	51	52	27	30	58	8
9	♒	01	00	01	24	03	29	22	13	29	49	26	28	49	30	28	28	11	9
10	01	02	02	02	24	24	♈	21	40	♉	47	51	29	47	07	29	25	22	10
11	02	03	04	03	24	44	01	21	05	01	46	13	♊	44	43	♋	22	34	11
12	03	04	04	04	25	01	02	20	28	02	44	33	01	42	17	01	19	46	12
13	04	05	04	05	25	17	03	19	45	03	42	52	02	39	50	02	16	57	13
14	05	06	03	06	25	31	04	19	08	04	41	09	03	37	23	03	14	09	14
15	06	07	01	07	25	44	05	18	2[illegible]	05	39	25	04	34	54	04	11	20	15
16	07	07	58	08	25	54	06	17	37	06	37	35	05	32	25	05	08	31	16
17	08	08	53	09	26	03	07	16	52	07	35	52	06	29	54	06	05	42	17
18	09	09	48	10	26	10	08	16	03	08	34	03	07	27	22	07	02	5[illegible]	18
19	10	10	42	11	26	15	09	15	12	09	32	12	08	24	49	08	00	04	19
20	11	11	35	12	26	18	10	14	19	10	30	19	09	22	14	08	57	1[illegible]	20
21	12	12	26	13	26	19	11	13	24	11	28	24	10	19	39	09	54	25	21
22	13	13	15	14	26	16	12	12	2[illegible]	12	26	27	11	17	02	10	51	36	22
23	14	14	03	15	26	10	13	11	27	13	24	29	12	14	2[illegible]	11	48	47	23
24	15	14	50	16	26	12	14	10	2[illegible]	14	22	29	13	11	4[illegible]	12	45	50	24
25	16	15	35	17	26	0[illegible]	15	09	22	15	20	27	14	09	09	13	43	09	25
26	17	16	19	18	25	57	16	08	17	16	18	24	15	06	30	14	40	21	26
27	18	17	01	19	25	47	17	07	09	17	16	15	16	03	50	15	37	33	27
28	19	17	42	20	25	35	18	05	59	18	14	13	17	01	05	16	34	45	28
29	20	18	22				19	04	47	19	12	05	17	58	27	17	31	5[illegible]	29
30	21	19	00				20	03	3[illegible]	20	09	56	18	55	45	18	29	0[illegible]	30
31	22	19	27				21	02	1[illegible]				19	53	02				31

A New and Correct Table of the Sun's *Place for the Year* 1727*, being the Third Year after* Leap-Year.

1727

Days	*July*			*August*			*September*			*October*			*November*			*December*			Days
	♋			♌			♍			♎			♏			♐			
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	19	26	22	19	05	45	19	05	32	18	35	57	19	36	52	20	01	44	1
2	20	23	35	20	03	25	20	04	08	19	35	29	20	37	22	21	02	49	2
3	21	20	48	21	01	07	21	02	38	20	35	04	21	37	54	22	03	54	3
4	22	18	12	21	58	49	22	01	13	21	34	40	22	38	28	23	05	00	4
5	23	15	17	22	56	34	22	59	50	22	34	19	23	39	03	24	06	07	5
6	24	12	32	23	54	19	23	58	30	23	33	59	24	39	40	25	07	14	6
7	25	09	48	24	52	06	24	57	11	24	33	42	25	40	18	26	08	22	7
8	26	07	04	25	49	54	25	55	54	25	33	26	26	40	57	27	09	30	8
9	27	04	20	26	47	44	26	54	48	26	33	13	27	41	38	28	10	39	9
10	28	01	37	27	45	36	27	53	27	27	33	02	28	42	20	29	11	48	10
11	28	58	55	28	43	29	28	52	17	28	32	52	29	43	02	♑	12	57	11
12	29	56	14	29	41	22	29	51	08	29	32	45	♐	43	48	01	14	07	12
13	♌	53	34	♍	39	20	♎	50	02	♏	32	39	01	44	34	02	15	17	13
14	01	50	54	01	37	18	01	48	57	01	32	36	02	45	22	03	16	27	14
15	02	48	16	02	35	17	02	47	55	02	32	33	03	46	11	04	17	38	15
16	03	45	38	03	33	19	03	46	54	03	32	35	04	47	02	05	18	49	16
17	04	43	01	04	31	21	04	45	56	04	32	38	05	47	54	06	20	00	17
18	05	40	24	05	29	26	05	45	00	05	32	42	06	48	47	07	21	10	18
19	06	37	49	06	27	32	06	44	06	06	32	49	07	49	41	08	22	21	19
20	07	35	14	07	25	39	07	43	14	07	32	57	08	50	37	09	23	32	20
21	08	32	41	08	23	49	08	42	24	08	33	07	09	51	33	10	24	42	21
22	09	30	08	09	22	00	09	41	36	09	33	19	10	52	30	11	25	53	22
23	10	27	36	10	20	13	10	40	50	10	33	32	11	53	28	12	27	04	23
24	11	25	06	11	18	28	11	40	06	11	33	47	12	54	27	13	28	14	24
25	12	22	37	12	16	44	12	39	25	12	34	04	13	55	28	14	29	25	25
26	13	20	09	13	15	03	13	38	45	13	34	23	14	56	29	15	30	36	26
27	14	17	42	14	13	23	14	38	07	14	34	44	15	57	31	16	31	46	27
28	15	15	17	15	11	45	15	37	31	15	35	06	16	58	33	17	32	56	28
29	16	12	52	16	10	09	16	36	58	16	35	30	17	59	36	18	34	03	29
30	17	10	28	17	08	35	17	36	26	17	35	56	19	00	40	19	35	14	30
31	18	08	06	18	07	03				18	36	22				20	36	22	31

A New and Correct Table of the Sun's *Place for the Year* 1728, *being* Leap-Year.

1728

Days	*Jan.* ♑			*February* ♒			*March* ♓			*April* ♈			*May* ♉			*June* ♊			Days
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	21	37	30	23	05	30	22	10	31	22	45	28	21	51	35	21	33	44	1
2	22	38	37	24	06	05	23	10	18	23	44	07	22	49	22	22	30	59	2
3	23	39	43	25	06	38	24	09	58	24	42	44	23	47	08	23	28	14	3
4	24	40	49	26	07	09	25	09	36	25	41	19	24	44	52	24	25	29	4
5	25	41	55	27	07	39	26	09	12	26	39	52	25	42	35	25	22	43	5
6	26	43	00	28	08	07	27	08	46	27	38	23	26	40	16	26	19	56	6
7	27	44	04	9	08	34	28	08	18	28	36	52	27	37	56	27	17	09	7
8	28	45	07	♓	08	59	29	07	48	29	35	19	28	35	35	28	14	22	8
9	29	46	10	1	09	23	♈	07	15	♉	33	44	29	33	12	29	11	34	9
10	♒	47	13	2	09	45	01	06	41	1	32	07	♊	30	48	♋	08	46	10
11	1	48	14	3	10	05	02	06	04	2	30	29	01	28	22	1	05	58	1
12	2	49	15	4	10	23	03	05	26	3	28	48	02	25	56	2	03	09	12
13	3	50	15	5	10	40	04	04	45	4	27	06	03	23	28	3	00	20	13
14	4	51	14	6	10	54	05	04	02	5	25	22	04	21	00	3	57	32	14
15	5	52	11	7	11	07	06	03	18	6	23	37	05	18	30	4	54	43	15
16	6	53	08	8	11	18	07	02	31	07	21	49	06	15	59	5	51	54	16
17	7	54	04	9	11	28	08	01	43	08	20	00	07	13	27	6	49	04	17
18	8	54	59	10	11	35	09	00	52	09	18	09	08	10	54	7	46	15	18
19	9	55	52	11	11	41	09	59	59	10	16	16	09	08	20	8	43	34	19
20	10	56	45	12	11	44	10	59	05	11	14	21	10	05	45	9	40	44	20
21	11	57	36	13	11	46	11	58	08	12	12	25	11	03	09	10	37	54	21
22	2	58	26	14	11	46	2	57	09	13	10	27	12	00	32	11	35	04	22
23	13	59	14	15	11	44	13	56	08	14	08	28	12	57	55	12	32	14	23
24	15	00	01	16	1	40	14	55	04	15	06	27	13	55	17	13	29	24	24
25	16	00	47	17	11	34	15	53	59	16	04	24	14	52	38	14	26	34	25
26	17	01	31	8	11	26	16	52	52	17	02	19	15	49	58	15	23	45	26
27	18	02	14	19	11	17	17	51	43	18	00	14	16	47	17	16	20	56	27
28	19	02	56	20	11	05	18	50	32	18	58	06	17	44	36	17	18	07	28
29	20	03	36	21	10	52	19	49	19	19	55	57	18	41	44	18	15	19	29
30	21	04	[illegible]				20	48	04	20	53	47	19	39	11	19	12	21	30
31	22	04	54				21	46	47				20	36	28				31

A New and Correct Table of the Sun's *Place for the Year* 1728, *being* Leap Year.

1728

Days	*July:* ♋	*August* ♌	*September* ♍	*October* ♎	*November* ♏	*December* ♐	Days
	° ′ ″	° ′ ″	° ′ ′	° ′ ″	° ′ ″	° ′ ″	
1	20 09 44	19 49 25	19′ 49 55	19 20 02	20 22 41	20 47 58	1
2	21 06 57	20 47 06	20 48 28	20 20 36	21 23 12	21 49 04	2
3	22 04 11	21 44 49	21 47 03	21 20 12	22 23 46	22 50 09	3
4	23 01 25	22 42 33	22 45 40	22 19 51	23 24 20	23 51 16	4
5	23 58 40	23 40 19	23 45 19	23 19 31	24 24 57	24 52 23	5
6	24 55 56	24 38 06	24 43 00	24 19 13	25 25 34	25 53 31	6
7	25 53 11	5 35 54	25 41 43	25 18 57	26 26 13	26 54 39	7
8	26 50 28	26 33 43	26 40 28	26 18 43	27 26 54	27 55 48	8
9	27 47 45	27 31 35	27 39 15	27 18 31	28 27 36	28 56 57	9
10	28 45 03	28 29 27	28 38 04	28 18 21	29 28 19	29 58 06	10
11	29 42 22	29 27 22	29 36 55	29 18 13	♐ 29 03	♑ 59 16	12
12	♌ 39 42	♍ 25 18	♎ 35 18	♏ 18 07	01 29 49	02 00 26	12
13	01 37 02	01 23 15	01 34 42	01 18 03	02 30 37	03 01 36	13
14	02 34 23	02 21 15	02 33 39	02 18 02	03 31 26	04 02 47	14
15	03 31 46	03 19 15	03 32 38	03 18 02	04 32 16	05 03 58	15
16	04 29 08	04 17 18	04 31 38	04 18 04	05 33 08	06 05 08	16
17	05 26 32	05 15 22	05 30 41	05 18 08	06 34 00	07 06 19	17
18	06 23 57	06 13 27	06 29 47	06 18 14	07 34 54	08 07 30	18
19	07 21 22	07 11 35	07 28 54	07 18 21	08 35 49	09 08 41	19
20	08 18 48	08 09 44	08 28 03	08 18 31	09 36 45	10 09 51	20
21	09 16 15	09 07 55	09 27 15	09 18 42	10 37 42	11 11 02	21
22	10 13 44	10 06 07	10 26 28	10 18 55	11 38 40	12 12 13	22
23	11 11 13	11 04 22	11 25 44	11 19 10	12 39 39	13 13 23	23
24	12 08 43	12 02 38	12 25 01	12 19 26	13 40 39	14 14 34	24
25	13 06 14	13 00 56	13 24 20	13 19 45	14 41 40	15 15 45	25
26	14 03 46	13 59 16	14 23 42	14 20 05	15 42 41	16 16 55	26
27	15 01 20	14 57 38	15 23 06	15 20 27	16 43 44	17 18 05	27
28	15 58 54	15 56 01	16 22 32	16 20 50	17 44 46	18 19 14	28
29	16 56 30	16 54 27	17 22 00	17 21 16	18 45 50	19 20 23	29
30	17 54 07	17 52 54	18 21 30	18 21 42	19 46 54	20 21 31	30
31	18 51 45	18 51 24		19 22 11		21 22 39	31

A Table of the Variations of the Suns Place

		4		8		12		16		20		40		60		80		100	
D.	M	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″	′	″
Janu.	10	1	44	3	28	5	12	6	57	8	41	17	22	26	3	34	44	43	25
	20	1	44	3	29	5	14	6	58	8	43	17	26	26	9	34	52	43	35
	30	1	45	3	30	5	15	7	00	8	45	17	31	26	17	35	2	43	48
Feb.	9	1	46	3	31	5	17	7	03	8	59	17	37	26	26	35	15	44	04
	19	1	46	3	33	5	19	7	06	8	52	17	45	26	37	35	30	44	22
	29	1	47	3	34	5	22	7	09	8	46	17	52	26	48	35	45	44	41
Mar.	10	1	48	3	36	5	24	7	12	9	00	18	01	26	1	36	1	45	2
	20	1	49	3	38	5	27	7	16	9	04	18	09	26	14	36	18	45	23
	30	1	50	3	39	5	29	7	19	9	09	18	17	26	26	36	35	45	44
Apil	9	1	50	3	41	5	31	7	22	9	13	18	25	27	38	36	51	46	4
	19	1	51	3	42	5	34	7	25	9	16	18	33	27	49	37	5	46	22
	29	1	52	3	44	5	36	7	28	9	19	18	39	27	59	37	18	46	38
May	9	1	52	3	45	5	37	7	30	9	22	18	45	28	8	37	30	46	52
	19	1	53	3	46	5	39	7	32	9	25	18	49	28	14	37	39	47	04
	29	1	53	3	46	5	40	7	33	9	26	18	53	28	19	37	46	47	12
June.	8	1	53	3	47	5	40	7	34	9	27	18	55	28	22	37	50	47	17
	18	1	53	3	47	5	41	7	34	9	28	18	55	28	23	37	55	47	19
	28	1	53	3	47	5	40	7	34	9	27	18	55	28	22	37	49	47	17
July	8	1	53	3	46	5	40	7	33	9	26	18	53	28	19	37	45	47	12
	18	1	53	3	46	5	39	7	32	9	25	18	50	28	15	37	40	47	05
	28	1	52	3	45	5	37	7	30	9	23	18	45	28	8	37	31	46	54
Aug.	7	1	52	3	44	5	36	7	28	9	20	18	40	28	0	37	21	46	41
	17	1	51	3	43	5	34	7	26	9	17	18	34	27	51	37	9	46	26
	27	1	51	3	41	5	31	7	23	9	13	18	27	27	40	36	53	46	07
Sept.	6	1	50	3	40	5	29	7	19	9	09	18	19	27	28	36	37	45	47
	16	1	49	3	38	5	27	7	16	9	05	18	11	27	16	36	21	45	27
	26	1	48	3	36	5	25	7	13	9	01	18	2	27	3	36	5	45	06
Octob.	6	1	47	3	35	5	22	7	10	8	57	17	54	26	51	35	49	44	46
	16	1	46	3	33	5	20	7	06	8	53	17	46	26	39	35	33	44	26
	26	1	46	3	32	5	18	7	04	8	49	17	39	26	29	35	18	44	08
Nov.	5	1	45	3	30	5	16	7	01	8	46	17	33	25	19	35	5	43	52
	15	1	45	3	29	5	14	6	59	8	43	17	27	26	11	34	54	43	38
	25	1	44	3	28	5	13	6	57	8	41	17	23	26	4	34	45	13	27
Dec.	5	1	44	3	28	5	12	6	56	8	40	17	20	26	0	34	40	43	20
	15	1	44	3	28	5	11	5	55	8	39	17	19	26	58	34	37	43	17
	25	1	44	3	28	5	11	5	55	8	39	17	19	26	58	34	37	42	17
	31	1	44	3	28	5	12	5	56	8	40	17	19	26	59	34	39	42	10

A
NEW and *CORRECT*
TABLE
OF THE
Sun's Declination

For the Years 1725, 1726, 1727, 1728.

WITH

VARIATIONS

To make them ſerve for an Hundred
Years to come.

A New and Correct Table of the Sun's *Declination for the Year 1725, being the First Year after* Leap-Year.

1725

Days	*January*			*February*			*March*			*April*			*May*			*June*			Days
	S			S			S *			N			N			N			
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	21	37	24	13	26	11	03	12	59	08	46	04	18	11	4[illegible]	23	11	50	1
2	21	27	14	13	16	01	02	49	2[illegible]	09	07	51	18	26	39	23	15	23	2
3	21	16	37	12	55	39	02	25	42	09	29	29	18	41	16	23	18	31	3
4	21	05	38	12	35	05	02	02	03	09	50	5[illegible]	18	55	32	23	21	14	4
5	20	54	14	12	14	18	01	38	2	10	12	16	19	09	39	23	23	35	5
6	20	42	2[illegible]	11	53	19	01	14	40	10	33	25	19	23	09	23	25	29	6
7	20	30	15	11	32	11	00	50	58	10	54	2[illegible]	19	36	29	23	26	58	7
8	20	17	40	11	10	49	00	27	16	11	15	1[illegible]	19	49	26	23	28	04	8
9	20	04	42	10	49	19	00	03	3[illegible]	11	35	47	20	02	07	23	28	44	9
10	19	51	22	10	27	38	N.	20	06	11	56	13	20	14	26	23	29	00	10
11	19	37	40	10	05	48	00	43	47	12	16	26	20	26	23	23	28	51	11
12	19	23	36	09	43	48	01	07	25	12	36	27	20	38	01	23	28	16	12
13	19	09	11	09	21	40	01	31	02	12	56	17	20	49	16	23	27	17	13
14	18	54	25	08	59	23	01	54	35	13	15	53	21	00	11	23	25	54	14
15	18	39	18	08	36	59	02	18	07	13	35	17	21	10	45	23	24	05	15
16	18	23	51	08	14	26	02	41	36	13	54	27	21	20	56	23	21	49	16
17	18	08	04	07	51	47	03	05	02	14	13	22	21	30	44	23	19	14	17
18	17	50	57	07	29	01	03	28	24	14	32	06	21	40	10	23	16	12	18
19	17	35	31	07	06	09	03	51	43	14	50	35	21	49	14	23	12	45	19
20	17	18	46	06	43	12	04	14	56	15	08	48	21	57	57	23	08	54	20
21	17	01	44	06	20	06	04	38	05	15	26	47	22	06	15	23	04	39	21
22	16	44	24	05	56	57	05	01	10	15	44	31	22	14	11	23	00	00	22
23	16	26	47	05	33	43	05	24	05	16	01	58	22	21	44	22	54	56	23
24	16	08	52	05	10	25	05	47	03	16	19	10	22	28	53	22	49	28	24
25	15	50	41	04	47	03	06	09	50	16	36	06	22	35	35	22	43	37	25
26	15	32	13	04	23	37	06	32	31	16	52	45	22	42	01	22	37	22	26
27	15	13	30	04	00	08	06	55	05	17	09	09	22	47	59	22	30	43	27
28	14	54	32	03	36	35	07	17	33	17	25	14	22	53	34	22	23	42	28
29	14	35	18				07	39	52	17	41	02	22	58	45	22	16	17	29
30	14	15	50				08	02	04	17	56	32	23	03	31	22	08	30	30
31	13	56	07				08	24	08				23	07	52				31

A New and Correct Table of the Sun's *Declination for the Year* 1725, *being the First Year after* Leap-Year.

1725

Days	*July*			*August*			*September*			*October*			*November*			*December*			Days
	N			N			N *			S			S			S			
	°	'	"	°	'	"	°	'	"	°	'	"	°	'	"	°	'	"	
1	22	00	18	14	58	46	04	08	19	07	29	05	17	48	04	23	08	39	1
2	21	51	44	14	40	33	03	45	18	07	51	39	18	04	10	23	12	46	2
3	21	42	48	14	22	05	03	22	12	08	14	06	18	19	56	23	16	26	3
4	21	33	30	14	03	24	02	59	02	08	36	26	18	35	23	23	19	38	4
5	21	23	50	13	44	29	02	35	49	08	58	39	18	50	30	23	22	2[illegible]	5
6	21	13	47	13	25	21	02	12	32	09	20	44	19	05	17	23	24	32	6
7	21	03	23	13	06	00	01	49	14	09	42	42	19	19	43	23	26	25	7
8	20	52	39	12	46	26	01	25	52	10	04	30	19	33	48	23	27	45	8
9	20	41	32	12	26	40	01	02	28	10	26	10	19	47	32	23	28	36	9
10	20	30	05	12	06	43	00	39	04	10	47	40	20	00	55	23	28	59	10
11	20	18	16	11	46	34	00	15	38	11	09	00	20	13	54	23	28	53	11
12	20	06	08	11	26	14	S	07	50	11	30	12	20	26	32	23	28	19	12
13	19	53	40	11	05	42	00	31	18	11	51	12	20	38	48	23	27	17	13
14	19	40	51	10	45	00	00	54	47	12	12	01	20	50	40	23	25	47	14
15	19	27	43	10	24	08	01	18	16	12	32	29	21	02	08	23	23	47	15
16	19	14	15	10	03	06	01	41	41	12	53	05	21	13	13	23	21	18	16
17	19	06	28	09	41	55	02	05	11	13	13	20	21	23	55	23	18	25	17
18	18	46	23	09	20	22	02	28	38	13	33	23	21	34	10	23	15	01	18
19	18	31	59	08	59	02	02	52	04	13	53	11	21	44	02	23	11	10	19
20	18	17	17	08	37	24	03	15	26	14	12	46	21	53	29	23	06	51	20
21	18	02	15	08	15	37	03	38	48	14	32	08	22	02	30	23	02	06	21
22	17	46	58	07	53	42	04	02	07	14	51	16	22	11	06	22	56	49	22
23	17	31	22	07	31	38	04	25	23	15	10	09	22	19	16	22	51	08	23
24	17	15	29	07	09	28	04	48	36	15	28	47	22	26	59	22	45	00	24
25	16	59	19	06	47	11	05	11	46	15	47	10	22	34	17	22	38	22	25
26	16	42	53	06	24	46	05	14	50	16	05	17	22	41	09	22	31	18	26
27	16	26	11	06	02	17	05	57	53	16	23	07	22	47	33	22	23	42	27
28	16	09	13	05	39	40	06	20	48	16	40	42	22	53	30	22	15	53	28
29	15	51	59	05	16	57	06	43	40	16	57	59	22	59	00	22	07	31	29
30	15	34	30	04	54	10	07	06	25	17	14	59	23	04	03	21	58	43	30
31	15	16	46	04	31	18				17	31	42				21	49	28	31

A New and Correct Table of the Sun's Declination for the Year 1726, being the Second Year after Leap-Year.

1726

Days	*Jan.* S	*February* S	*March* S *	*April* N	*May* N	*June* N	Days
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	
1	21 39 48	13 41 02	03 18 43	08 40 45	18 08 07	23 10 54	1
2	21 29 44	13 20 56	02 55 07	09 02 35	18 23 05	23 14 3	2
3	21 19 14	13 00 37	02 31 27	09 24 15	18 37 45	23 17 48	3
4	21 08 20	12 40 05	02 07 47	09 45 46	18 52 07	23 20 38	4
5	20 57 22	12 19 21	01 44 06	10 07 07	19 06 10	23 23 07	5
6	20 45 29	11 58 25	01 20 24	10 28 18	19 19 54	23 25 04	6
7	20 33 14	11 37 15	00 56 42	10 49 19	19 33 17	23 26 49	7
8	20 20 45	11 16 0	00 33 00	11 10 09	19 46 21	23 27 50	8
9	20 07 53	10 54 33	00 09 18	11 30 49	19 59 06	23 28 37	9
10	19 54 39	10 32 55	N 14 23	11 51 17	20 11 29	23 28 58	10
11	19 41 02	10 11 07	00 38 03	12 11 33	20 23 32	23 28 55	11
12	19 27 02	9 49 09	01 01 42	12 31 37	20 35 23	23 28 27	12
13	19 12 43	9 27 03	01 25 19	12 51 30	20 46 35	23 27 34	13
14	18 58 01	9 04 49	01 48 53	13 11 10	20 57 34	23 26 16	14
15	18 43 00	8 42 26	02 12 26	13 30 37	21 08 13	23 24 34	15
16	18 27 37	8 19 55	02 35 56	13 49 50	21 18 29	23 22 27	16
17	18 11 55	7 57 18	02 59 22	14 08 50	21 28 23	23 19 54	17
18	17 55 53	7 34 33	03 22 45	14 27 36	21 37 55	23 16 58	18
19	17 39 33	7 11 42	03 46 06	14 46 08	21 47 06	23 13 37	19
20	17 22 53	6 48 47	04 09 29	15 04 25	21 55 53	23 09 53	20
21	17 05 54	6 25 44	04 32 30	15 22 28	22 04 17	23 05 43	21
22	16 48 38	6 02 35	04 55 36	15 40 15	22 12 18	23 01 09	22
23	16 31 05	5 39 22	05 18 36	15 57 46	22 19 56	22 56 11	23
24	15 13 14	5 16 05	05 41 30	16 15 02	22 27 11	22 50 50	24
25	15 55 06	4 52 43	06 04 19	16 32 02	22 34 03	22 45 06	25
26	15 36 03	4 29 18	06 27 02	16 48 46	22 40 31	22 38 56	26
27	15 18 04	4 05 50	06 49 38	17 05 00	22 46 35	22 32 22	27
28	14 59 05	3 42 18	07 12 06	17 21 22	22 52 15	22 25 26	28
29	14 39 53		07 34 27	17 37 15	22 57 32	22 18 07	29
30	14 20 31		07 56 41	17 52 50	23 02 25	22 10 24	30
31	14 00 55		08 18 48		23 06 51		31

A New and Correct Table of the Sun's Declination for the Year 1726, being the Second Year after Leap-Year.

Days	July.			August			September			October			November			December			Days
	N			N			N*			S			S			S			
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	22	02	18	15	03	09	04	13	54	07	23	34	17	44	07	23	07	34	1
2	21	53	51	14	44	58	03	50	53	07	46	11	18	00	18	23	11	48	2
3	21	45	00	14	26	34	03	27	46	08	08	39	18	16	08	23	15	35	3
4	21	35	47	14	07	57	03	04	38	08	31	01	18	31	40	23	18	53	4
5	21	26	11	13	49	05	02	41	28	08	53	15	18	46	52	23	21	43	5
6	21	16	15	13	30	00	02	18	11	09	15	23	19	01	43	23	24	07	6
7	21	05	57	13	10	42	01	54	52	09	37	23	19	16	15	23	26	02	7
8	20	55	17	12	51	11	01	31	31	09	59	13	19	30	25	23	27	28	8
9	20	44	15	12	31	19	01	08	08	10	20	55	9	44	14	23	28	26	9
10	20	32	52	12	11	33	00	44	44	10	42	27	19	57	42	23	28	56	10
11	20	21	10	11	31	27	00	21	19	11	03	51	20	10	47	23	28	57	12
12	20	09	06	11	31	10	0S	02	10	11	25	05	20	23	30	23	28	30	12
13	19	56	43	11	10	42	00	25	35	11	46	07	20	35	51	23	27	35	13
14	19	43	52	10	50	02	00	49	07	12	06	59	20	47	49	23	26	11	14
15	19	30	56	10	29	12	01	12	35	12	27	49	20	59	23	23	24	19	15
16	19	17	32	10	08	12	01	36	03	12	48	09	21	10	35	23	21	57	16
17	19	03	50	09	47	02	01	59	31	13	08	27	21	21	21	23	19	09	17
18	18	49	49	09	25	44	02	22	58	13	28	52	21	31	44	23	15	58	18
19	18	35	29	09	04	16	02	46	24	13	48	23	21	41	41	23	12	08	19
20	18	20	51	08	42	38	03	09	47	14	08	02	21	51	14	23	07	56	20
21	18	05	55	08	20	54	03	33	05	14	27	28	22	00	21	23	03	17	21
22	17	60	41	07	59	01	03	56	28	14	46	38	22	09	04	22	58	08	22
23	7	35	10	07	36	59	04	19	46	15	05	36	22	17	19	22	52	33	23
24	17	19	21	07	14	50	04	42	59	15	24	17	22	25	10	22	46	30	24
25	17	03	15	06	52	35	05	06	08	15	42	44	22	32	34	22	40	00	25
26	16	46	53	06	30	12	05	29	14	16	00	54	22	39	32	22	33	04	26
27	16	30	15	06	07	44	05	52	16	16	18	51	22	46	02	22	25	41	27
28	16	13	20	05	45	08	06	15	14	16	36	28	22	52	06	22	17	51	28
29	15	56	10	05	22	28	06	38	06	16	53	49	22	57	43	22	09	34	29
30	15	38	44	04	59	42	07	00	53	17	10	53	23	02	54	22	00	52	30
31	15	21	04	04	36	50				17	27	40				21	51	45	31

A New and Correct Table of the Sun's *Declination for the Year* 1727, *being the Third Year after* Leap-Year.

1727

Days	*Jan.* S	*February* S	*March* S *	*April* N	*May* N	*June* N	Days
	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	
1	21 42 12	13 45 53	03 24 25	08 35 28	18 04 26	23 09 58	1
2	21 32 12	13 25 50	03 00 49	08 57 19	18 19 29	23 13 43	2
3	21 21 49	13 05 34	02 37 12	09 19 02	18 34 14	23 17 03	3
4	21 11 02	12 45 06	02 13 32	09 40 35	18 48 41	23 19 59	4
5	20 59 45	12 24 25	01 49 50	10 01 58	19 02 48	23 22 31	5
6	20 48 13	12 03 32	01 26 09	10 23 12	19 16 36	23 24 37	6
7	20 36 13	11 42 27	01 02 27	10 44 15	19 30 05	23 26 19	7
8	20 23 50	11 21 12	00 38 45	11 05 08	19 43 13	23 27 36	8
9	20 11 03	10 59 47	00 15 03	11 25 53	19 56 02	23 28 28	9
10	19 57 54	10 38 10	N 08 38	11 46 21	20 08 30	23 28 55	10
11	19 44 22	10 16 24	00 32 19	12 06 40	20 20 38	23 28 58	11
12	19 30 25	9 54 30	00 55 58	12 26 46	20 32 26	23 28 36	12
13	19 16 14	9 32 26	01 19 35	12 46 42	20 43 53	23 27 49	13
14	19 01 37	9 10 14	01 43 10	13 06 24	20 54 58	23 26 37	14
15	18 46 40	8 47 52	02 06 43	13 25 54	21 05 41	23 25 00	15
16	18 31 28	8 25 24	02 30 14	13 45 11	21 16 03	23 22 59	16
17	18 15 45	8 02 49	02 53 42	14 04 14	21 26 02	23 20 34	17
18	17 59 48	7 40 05	03 17 05	14 23 04	21 35 39	23 17 44	18
19	17 43 32	7 17 15	03 40 26	14 41 40	21 44 55	23 14 29	19
20	17 26 56	6 54 20	04 03 41	15 00 01	21 53 47	23 10 49	20
21	17 10 02	6 31 18	04 26 53	15 18 06	22 02 17	23 06 45	21
22	16 52 51	6 08 12	04 50 01	15 35 57	22 10 24	23 02 19	22
23	16 35 22	5 45 00	05 13 06	15 53 33	22 18 08	22 57 26	23
24	16 17 35	5 21 43	05 35 58	16 10 53	22 25 28	22 52 10	24
25	15 59 31	4 58 23	05 58 48	16 27 57	22 32 26	22 46 30	25
26	15 41 13	4 34 59	06 21 34	16 44 44	22 38 59	22 40 27	26
27	15 22 37	4 11 32	06 44 12	17 01 15	22 45 10	22 33 59	27
28	15 03 46	3 48 00	07 06 48	17 17 29	22 50 56	22 27 09	28
29	14 44 39		07 29 04	17 33 25	22 56 17	22 19 55	29
30	14 25 18		07 51 20	17 49 05	23 01 15	22 12 18	30
31	14 05 43		08 13 28		23 05 49		31

A New and Correct Table of the Sun's *Declination for the Year* 1727, *being the Third Year after* Leap-Year.

1727

Days	*July*			*August*			*September*			*October*			*November*			*December*			Days
	N			N			N *			S			S			S			
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	22	04	18	15	07	29	04	19	28	07	18	06	17	40	10	23	06	28	1
2	21	55	56	14	49	23	03	56	28	07	40	42	17	56	24	23	10	45	2
3	21	47	11	14	31	03	03	33	24	08	03	13	18	12	19	23	14	42	3
4	21	38	03	14	12	27	03	10	16	08	25	37	18	27	56	23	18	08	4
5	21	28	33	13	53	39	02	47	05	08	47	53	18	43	13	23	21	04	5
6	21	18	42	13	34	37	02	23	49	09	10	03	18	58	09	23	23	35	6
7	21	08	28	13	15	23	02	00	31	09	32	03	19	12	45	23	25	37	7
8	20	57	53	12	55	56	01	37	11	09	53	55	19	27	00	23	27	09	8
9	20	46	57	12	36	16	01	13	49	16	15	41	19	40	55	23	28	15	9
10	20	35	51	12	16	24	00	50	25	10	37	16	19	54	28	23	28	51	10
11	20	24	03	11	56	20	00	26	59	10	58	40	20	07	39	23	28	59	11
12	20	12	03	11	36	06	00	03	32	11	19	56	20	20	28	23	28	39	12
13	19	59	45	11	15	40	S	19	57	11	41	02	20	32	54	23	27	51	13
14	19	47	06	10	55	03	00	43	25	12	01	56	20	44	57	23	26	33	14
15	19	34	07	10	34	16	01	06	53	12	22	40	20	56	37	23	24	49	15
16	19	20	48	10	13	18	01	30	21	12	43	12	21	07	53	23	22	35	16
17	19	07	11	09	52	11	01	53	49	13	03	33	21	18	46	23	19	53	17
18	18	53	15	09	30	55	02	17	17	13	23	40	21	29	14	23	16	43	18
19	18	38	59	09	09	28	02	40	42	13	43	35	21	39	18	23	13	06	19
20	18	24	25	08	47	53	03	04	07	14	03	10	21	48	56	23	09	00	20
21	18	09	34	08	26	10	03	27	30	14	22	46	21	58	11	23	04	27	21
22	17	54	25	08	04	20	03	50	50	14	42	01	22	06	59	22	59	25	22
23	17	38	57	07	42	20	04	14	07	15	01	01	22	15	22	22	53	56	23
24	17	23	12	07	20	13	04	37	21	15	19	46	22	23	18	22	48	00	24
25	17	07	10	06	58	06	05	00	33	15	38	17	22	30	48	22	41	38	25
26	16	50	52	06	35	39	05	23	39	15	56	31	22	37	52	22	34	48	26
27	16	34	18	06	13	12	05	46	42	16	14	30	22	44	31	22	27	31	27
28	16	17	27	05	50	38	06	09	41	16	32	13	22	50	40	22	19	47	28
29	16	00	20	05	27	58	06	32	35	16	49	38	22	56	23	22	11	37	29
30	15	42	59	05	05	18	06	55	22	17	06	47	23	01	41	22	03	02	30
31	15	25	21	04	42	28				17	22	37				21	54	01	31

A New and Correct Table of the Sun's *Declination for the Year* 1728, *being* Leap-Year.

1728

Days	*January*			*February*			*March*			*April*			*May*			*June.*			Days
	S			S			S *			N			N			N			
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	21	44	33	13	50	44	03	06	32	08	52	02	18	15	53	23	12	51	1
2	21	34	41	13	30	44	02	42	56	09	13	4[illegible]	18	30	42	23	16	18	2
3	21	24	2[illegible]	13	10	30	02	19	17	09	35	22	18	45	12	23	19	19	3
4	21	13	41	12	50	05	01	55	35	09	56	48	18	59	25	23	21	54	4
5	21	02	35	12	29	27	01	31	54	10	18	04	19	13	18	23	24	06	5
6	20	51	04	12	08	38	01	08	[illegible]	10	39	10	19	26	51	23	25	56	6
7	20	39	10	11	47	36	00	44	31	11	00	06	19	40	05	23	27	19	7
8	20	26	52	11	26	23	00	20	49	11	20	51	19	52	58	23	28	18	8
9	20	14	12	11	05	01	N.	02	53	11	41	25	20	05	31	23	28	51	9
10	20	01	08	10	43	26	00	26	34	12	01	46	20	17	44	23	29	00	10
11	19	47	41	10	21	43	00	50	13	12	21	56	20	29	36	23	28	43	11
12	19	33	53	09	59	50	01	13	51	12	41	55	20	41	08	23	28	03	12
13	19	19	43	09	37	48	01	37	27	13	01	40	20	52	18	23	26	55	13
14	19	05	12	09	15	37	02	01	01	13	21	13	21	03	07	23	25	[illegible]	14
15	18	50	20	08	53	18	02	24	32	13	40	33	21	13	34	23	23	31	15
16	18	35	08	08	30	52	02	48	01	13	59	40	21	23	39	23	21	09	16
17	18	19	35	08	08	18	03	11	26	14	18	32	21	33	21	23	18	27	17
18	18	03	42	07	45	36	03	34	47	14	37	11	21	42	42	23	15	18	18
19	17	47	30	07	22	49	03	58	04	14	55	36	21	51	40	23	11	44	19
20	17	31	0[illegible]	06	59	55	04	21	17	15	13	45	22	00	16	23	07	46	20
21	17	14	11	06	36	55	04	44	25	15	31	40	22	08	30	23	03	24	21
22	16	57	03	06	13	50	05	07	29	15	49	19	22	16	19	22	58	38	22
23	16	39	39	05	50	39	05	30	25	16	06	42	22	23	44	22	53	28	23
24	16	21	58	05	27	23	05	53	16	16	23	50	22	30	47	22	47	54	24
25	16	03	59	05	04	04	06	16	02	16	40	42	22	37	26	22	41	56	25
26	15	45	43	04	40	40	06	38	4[illegible]	16	57	17	22	43	4[illegible]	22	35	35	26
27	15	27	10	04	17	13	07	01	14	17	13	35	22	49	34	22	28	50	27
28	15	08	22	03	53	42	07	23	39	17	29	35	22	55	02	22	21	42	28
29	14	49	19	03	30	09	07	45	58	17	45	19	23	00	05	22	14	10	29
30	14	30	02				08	08	07	18	00	45	23	04	45	22	06	16	30
31	14	10	30				08	30	09				23	09	00				31

A New and Correct Table of the Sun's *Declination for the Year* 1728, *being* Leap Year.

1728

Days	*July.*			*August*			*September*			*October*			*November*			*December*			Days
	N			N			N*			S			S			S			
	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	°	′	″	
1	21	57	59	14	53	48	04	02	01	07	35	14	17	52	29	23	09	48	1
2	21	49	19	14	35	31	03	38	59	07	57	46	18	08	29	23	13	49	2
3	21	40	17	14	17	00	03	15	52	08	20	12	18	24	10	23	17	21	3
4	21	30	53	13	58	15	02	52	42	08	42	30	18	39	32	23	20	25	4
5	21	21	07	13	39	16	02	29	27	09	04	40	18	54	33	23	23	02	5
6	21	10	59	13	20	04	02	06	10	09	26	44	19	09	14	23	25	10	6
7	11	00	29	13	00	40	01	42	50	09	48	38	19	23	35	23	26	50	7
8	20	49	38	12	41	03	01	19	28	10	10	25	19	37	34	23	28	02	8
9	20	38	27	12	21	14	00	56	05	10	32	02	19	51	13	23	28	45	9
10	20	26	54	12	01	13	00	32	39	10	53	30	20	04	29	23	29	00	10
11	20	15	00	11	41	01	00	09	12	11	14	48	20	17	22	23	28	47	11
12	20	02	46	11	20	37	S	14	16	11	35	55	20	29	55	23	28	05	12
13	19	50	11	11	00	04	00	37	45	11	56	53	20	42	03	23	26	55	13
14	19	37	17	10	39	28	01	01	13	12	17	40	20	53	49	23	25	16	14
15	19	24	03	10	18	24	01	24	41	12	38	15	21	05	12	23	23	10	15
16	19	10	30	09	57	18	01	48	08	12	58	38	21	16	10	23	20	35	16
17	18	56	38	09	36	04	02	11	36	13	18	49	21	26	44	23	17	32	17
18	18	42	28	09	14	41	02	35	02	13	38	47	21	36	54	23	14	01	18
19	18	27	58	08	53	08	02	58	26	13	58	32	21	46	39	23	10	02	19
20	18	13	11	08	31	27	03	21	49	14	18	04	21	55	59	23	05	35	20
21	17	58	06	08	09	38	03	45	11	14	37	22	22	04	53	23	00	41	21
22	17	42	43	07	47	41	04	08	29	14	56	25	22	13	22	22	55	19	22
23	17	27	02	07	25	36	04	31	44	15	15	15	22	21	25	22	49	29	23
24	17	11	04	07	03	23	04	54	55	15	33	49	22	29	02	22	43	13	24
25	16	54	59	06	41	05	05	18	03	15	52	07	22	36	12	22	36	30	25
26	16	38	20	06	18	39	05	41	06	16	10	10	22	42	56	22	29	19	26
27	16	21	34	05	56	06	06	04	07	16	27	57	22	49	12	22	21	42	27
28	16	04	32	05	33	27	06	27	02	16	45	26	22	55	02	22	13	39	28
29	15	47	14	05	10	45	06	49	52	17	02	38	23	00	25	22	05	09	29
30	15	29	40	04	47	59	07	12	35	17	19	33	23	05	21	21	56	23	30
31	13	11	51	04	25	01				17	36	11				21	46	53	31

A Table of the Variation of the Suns Declination

		4		8		12		16		20		40		60		80		100	
D.	*M.*	'	"	'	"	'	"	'	"	'	"	'	"	'	"	'	"	'	"
Janu.	10	S	23	S	45	1S	08	1S	31	1	S54	3S	48	5	42	7	S37	9S	31
	20	0	29	0	57	1	26	1	55	2	24	4	48	7	12	9	36	12	00
	30	0	34	1	09	1	43	2	18	2	49	5	38	8	27	11	15	14	04
Feb.	9	0	38	1	15	1	53	2	31	3	08	6	17	09	25	12	34	15	43
	19	0	41	1	21	2	02	3	42	3	23	6	46	10	09	13	32	16	55
	29	0	42	1	25	2	07	2	49	3	32	7	04	10	36	14	08	17	40
Mar.	10	A	43	1A	26	2A	09	2A	52	3	35	7A	11	10A	46	14	21	17A	57
	20	0	43	1	25	2	08	2	50	3	33	7	06	10	39	14	13	17	46
	30	0	41	1	22	2	03	2	45	3	26	6	52	10	18	13	43	17	09
Apil	9	0	39	1	17	1	56	2	34	3	13	6	26	9	39	12	52	16	05
	19	0	35	1	09	1	44	2	19	2	54	5	48	8	42	11	37	14	31
	29	0	30	1	00	1	30	2	00	2	29	4	59	7	28	9	58	12	28
May	9	0	24	0	48	1	12	1	36	1	59	3	59	5	58	7	58	9	58
	19	0	17	0	34	0	51	1	08	1	25	2	50	4	15	5	39	7	04
	29	0	09	0	19	0	28	0	37	0	46	1	33	2	19	3	05	3	52
June	8	0	01	0	02	0	03	0	05	0	05	0	11	0	16	0	22	0	28
	18	S	07	S	14	S	21	S	28	0	35	1S	10	1S	45	2S	20	2S	55
	28	0	15	0	30	0	45	1	07	1	14	2	28	3	42	4	57	6	11
July	8	0	22	0	44	1	06	1	28	1	50	3	41	5	31	7	21	09	12
	18	0	28	0	57	1	25	1	54	2	22	4	45	7	07	9	29	11	52
	28	0	34	1	07	1	41	2	15	2	49	5	38	8	27	11	16	14	05
Aug.	7	0	38	1	16	1	54	2	32	3	10	6	20	09	30	12	39	15	49
	17	0	41	1	22	2	03	2	44	3	25	6	50	10	15	13	39	17	04
	27	0	43	1	26	2	09	2	52	3	35	7	10	10	45	14	20	17	55
Sept.	6	0	44	1	27	2	11	2	55	3	38	7	17	10	55	14	33	18	12
	16	A	43	1A	27	2A	10	2A	53	3	37	7A	14	10A	51	14A	27	18A	04
	26	0	42	1	24	2	06	2	48	3	30	7	00	10	30	13	59	17	29
Octob.	6	0	39	1	19	1	58	2	38	3	17	6	34	9	51	13	09	16	27
	16	0	36	1	12	1	48	2	24	2	59	5	59	8	58	11	58	14	58
	26	0	31	1	02	1	33	2	05	2	36	5	13	7	49	10	25	13	02
Nov.	5	0	26	0	51	1	17	1	42	2	08	4	16	6	24	8	32	10	40
	15	0	19	0	38	0	57	1	16	1	35	3	10	4	45	6	19	07	54
	25	0	12	0	23	0	35	0	46	0	58	1	56	2	54	3	52	04	50
Dec.	5	0	04	0	08	0	12	0	15	0	18	0	37	0	55	1	14	1	33
	15	S	04	S	09	S	13	S	17	0	22	0S	44	1	S6	1S	27	1S	49
	25	0	12	0	24	0	36	0	48	1	00	2	01	3	01	4	01	5	02
	31	0	16	0	33	0	49	1	06	1	22	2	45	4	07	5	30	6	53

A
NEW and *CORRECT*
TABLE
OF THE
Sun's Right Aſcenſion

For the Years 1725, 1726, 1727, 1728.

WITH

VARIATIONS

To make them ſerve for an Hundred
Years to come.

A New and Correct Table of the Sun's *Right Aſcenſion for the Year* 1725, *being the firſt Year after* Leap-Year.

1725

Days	*Jan.*			*February*			*March*			*April*			*May*			*June*			Days
	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	
1	19	36	40	21	44	37	23	30	17	01	23	10	03	16	38	05	22	07	1
2	19	40	59	21	48	32	23	33	56	01	26	50	03	20	34	05	26	16	2
3	19	45	18	21	52	26	23	37	36	01	30	31	03	24	30	05	30	25	3
4	19	49	35	21	56	19	23	41	15	01	34	13	03	28	26	05	34	34	4
5	19	53	52	22	00	12	23	44	54	01	37	55	03	32	22	05	38	43	5
6	19	58	07	22	04	04	23	48	33	01	41	36	03	36	20	05	42	52	6
7	20	02	24	22	07	56	23	52	11	01	45	18	03	40	19	05	47	01	7
8	20	06	39	22	11	46	23	55	49	01	49	00	03	44	18	05	51	11	8
9	20	10	53	22	15	36	23	59	27	01	52	43	03	48	17	05	55	20	9
10	20	15	06	22	19	25	00	03	05	01	56	27	03	52	17	05	59	29	10
11	20	19	18	22	23	14	00	06	43	02	00	11	03	56	16	06	03	39	11
12	20	23	30	22	27	02	00	10	20	02	03	55	04	00	17	06	07	48	12
13	20	27	41	22	30	49	00	13	17	02	07	40	04	04	18	06	11	57	13
14	20	31	52	22	34	36	00	17	35	02	11	25	04	08	20	06	16	07	14
15	20	36	02	22	38	22	00	21	12	02	15	11	04	12	22	06	20	16	15
16	20	40	11	22	42	07	00	24	50	02	18	58	04	16	25	06	24	25	16
17	20	44	29	22	45	52	00	28	29	02	22	45	04	20	29	06	28	34	17
18	20	48	26	22	49	37	00	32	07	02	26	32	04	24	33	06	32	43	18
19	20	52	32	22	53	21	00	35	45	02	30	20	04	28	37	06	36	51	19
20	20	56	38	22	57	04	00	39	23	02	34	09	04	32	42	06	40	59	20
21	21	00	43	23	00	47	00	43	01	02	37	57	04	36	47	06	45	17	21
22	21	04	46	23	04	30	00	46	39	02	41	46	04	40	52	06	49	15	22
23	21	08	49	23	08	12	00	50	17	02	45	37	04	44	57	06	53	13	23
24	21	12	50	23	11	54	00	53	55	02	49	28	04	49	03	06	57	39	24
25	21	16	51	23	15	35	00	57	34	02	53	19	04	53	11	07	01	37	25
26	21	20	52	23	19	16	01	01	12	02	57	11	04	57	18	07	05	43	26
27	21	24	52	23	22	56	01	04	51	03	01	03	05	01	24	07	09	49	27
28	21	28	59	23	26	37	01	08	30	03	04	56	05	05	32	07	13	55	28
29	21	32	48				01	12	10	03	08	50	05	09	40	07	18	00	29
30	21	36	45				01	15	50	03	12	44	05	13	49	07	22	05	30
31	21	40	41				01	19	30				05	17	58				31

A New and Correct Table of the Sun's *Right Aſcenſion for the Year* 1725, *being the firſt Year after* Leap-Year.

1725

Days	*July*			*Auguſt*			*September*			*October*			*November*			*December*			Days
	h	′	″	h	′	″	h	′	″	h	′	″	h	′	″	h	′	″	
1	07	26	10	09	27	57	11	21	49	13	10	24	15	10	35	17	18	45	1
2	07	30	14	09	31	43	11	25	16	13	14	06	15	14	40	17	23	10	2
3	07	34	17	09	35	29	11	28	51	13	17	49	15	18	47	17	27	36	3
4	07	38	21	09	39	14	11	32	26	13	21	33	15	22	55	17	32	02	4
5	07	42	23	09	42	59	11	36	02	13	25	17	15	27	03	17	36	27	5
6	07	46	25	09	46	44	11	39	38	13	29	02	15	31	11	17	40	53	6
7	07	50	37	09	50	28	11	43	14	13	32	47	15	35	20	17	45	19	7
8	07	54	27	09	54	10	11	46	50	13	36	33	15	39	31	17	49	45	8
9	07	58	27	09	57	55	11	50	25	13	40	20	15	43	41	17	54	12	9
10	08	02	27	10	01	36	11	54	01	13	44	07	15	47	53	17	58	40	10
11	08	06	26	10	05	18	11	57	37	13	47	54	15	52	00	18	03	06	11
12	08	10	24	10	09	00	12	01	12	13	51	43	15	56	19	18	07	32	12
13	08	14	22	10	12	41	12	04	47	13	55	32	16	00	34	18	11	59	13
14	08	18	20	10	16	21	12	08	24	13	59	22	16	04	49	18	16	26	14
15	08	22	08	10	20	01	12	11	40	14	03	12	16	09	05	18	20	53	15
16	08	16	15	10	23	41	12	15	37	14	07	03	16	13	20	18	25	19	16
17	08	30	11	10	27	21	12	19	13	14	10	55	16	17	38	18	29	45	17
18	08	34	06	10	31	01	12	22	50	14	14	49	16	21	57	18	34	21	18
19	08	38	01	10	34	39	12	26	28	14	18	43	16	26	16	18	38	37	19
20	08	41	55	10	38	18	12	30	07	14	22	37	16	30	35	18	43	02	20
21	08	45	49	10	41	56	12	33	44	14	26	33	16	34	55	18	47	27	21
22	08	49	42	10	45	34	12	37	22	14	30	29	16	39	15	18	51	52	22
23	08	53	34	10	49	11	12	41	01	14	34	25	16	43	36	18	56	17	23
24	08	57	26	10	52	48	12	44	39	14	38	23	16	47	58	19	00	41	24
25	09	01	17	10	56	25	12	48	19	14	42	22	16	52	29	19	05	05	25
26	09	05	07	11	00	02	12	51	59	14	46	22	16	56	43	19	09	28	26
27	09	09	37	11	03	39	12	55	29	14	50	22	17	01	06	19	13	51	27
28	09	12	46	11	07	16	12	59	19	14	54	23	17	05	39	19	18	13	28
29	09	16	34	11	10	52	13	03	00	14	58	24	17	09	55	19	22	35	29
30	09	20	22	11	14	28	13	06	41	15	02	27	17	14	29	19	26	56	30
31	09	21	10	11	18	04				15	06	31				19	31	16	31

A New and Correct Table of the Sun's Right Aſcenſion for the Year 1726, *being the Second Year after* Leap-Year.

1726

Days	*Jan.*			*February*			*March*			*April*			*May*			*June*			Days
	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	
1	19	35	36	21	43	00	23	30	22	01	22	17	03	15	41	05	21	06	1
2	19	39	35	21	47	35	23	33	02	01	25	57	03	19	37	05	25	15	2
3	19	44	14	21	51	30	23	36	42	01	29	38	03	23	33	05	29	24	3
4	19	48	33	21	55	24	23	40	22	01	33	19	03	27	29	05	33	34	4
5	9	52	50	21	59	16	23	44	01	01	37	01	03	31	25	05	37	43	5
6	19	57	07	22	03	08	23	47	39	01	40	42	03	35	22	05	41	52	6
7	20	01	23	22	06	59	23	51	27	01	44	24	03	39	26	05	46	01	7
8	20	05	37	22	10	50	23	54	55	01	48	06	03	43	19	05	50	10	8
9	20	09	51	22	14	40	23	58	33	01	51	49	03	47	18	05	54	20	9
10	20	14	04	2[illegible]	18	29	00	02	11	01	55	33	03	51	18	05	58	30	10
11	20	18	17	22	22	18	00	05	50	01	59	17	03	55	18	06	02	39	11
12	20	22	30	22	26	07	00	09	28	02	03	01	03	59	18	06	06	48	12
13	20	46	42	22	29	54	00	13	05	02	06	45	04	03	19	06	10	57	13
14	20	30	53	22	33	49	00	16	43	02	10	31	04	07	21	06	15	07	14
15	20	35	02	22	37	26	00	20	21	02	14	17	04	11	23	06	19	16	15
16	20	39	10	22	41	12	00	23	58	02	18	04	04	15	26	06	23	25	16
17	20	4[illegible]	18	22	44	57	00	27	36	02	21	50	04	19	30	06	27	35	17
18	20	47	27	22	48	42	00	31	14	02	25	37	04	23	34	06	31	43	18
19	20	51	32	22	52	26	00	34	52	02	29	24	04	27	38	06	35	51	19
20	20	55	38	22	56	10	00	38	30	02	33	13	04	31	43	06	39	58	20
21	20	59	43	22	59	53	00	42	08	02	37	12	04	35	49	06	44	07	21
22	21	03	47	23	03	36	00	45	46	02	40	53	04	39	53	06	48	15	22
23	21	07	50	23	07	19	00	49	24	02	44	42	04	43	58	06	52	23	23
24	21	11	52	23	11	01	00	53	02	02	48	32	04	48	05	06	56	30	24
25	21	15	53	22	14	42	00	56	40	02	52	23	04	52	11	07	00	37	25
26	21	19	53	23	18	23	01	00	19	02	56	14	04	56	18	07	04	44	26
27	21	23	54	23	22	03	01	03	58	03	00	07	05	00	25	07	08	50	27
28	21	27	52	3	25	43	01	07	38	03	04	00	05	04	32	07	12	56	28
29	21	31	50				01	11	17	03	07	53	05	08	41	07	17	01	29
30	21	35	48				01	14	56	03	11	46	05	12	50	07	21	06	30
31	21	39	44				01	18	37				05	16	58				31

A New and Correct Table of the Sun's *Right Aſcenſion for the Year* 1726, *being the Second Year after* Leap-Year

1726

Days	*July*			*Auguſt*			*September*			*October*			*November*			*December*			Days
	h	′	″	h	′	″	h	′	″	h	′	″	h	′	″	h	′	″	
1	07	25	11	09	27	02	11	20	47	13	09	30	15	09	36	17	17	40	1
2	07	29	15	09	30	48	11	24	23	13	13	13	15	13	49	17	22	05	2
3	07	33	19	09	34	34	11	27	59	13	16	56	15	17	46	17	26	33	3
4	07	37	22	09	38	20	11	31	35	13	20	40	15	21	54	17	[illegible]	57	4
5	07	41	24	09	42	05	11	35	10	13	24	23	15	26	02	17	35	22	5
6	07	45	26	09	45	50	11	38	46	13	28	07	15	30	10	17	39	48	6
7	07	49	28	09	49	35	11	42	22	13	31	52	15	34	19	17	44	14	7
8	07	53	29	09	53	18	11	45	57	13	35	38	15	38	29	17	48	41	8
9	07	57	30	09	57	00	11	49	33	13	39	25	15	42	41	17	53	08	9
10	08	01	30	10	00	42	11	53	08	13	43	12	15	46	53	17	57	35	10
11	08	05	29	10	04	24	11	56	44	13	47	00	15	51	05	18	02	02	11
12	08	09	27	10	08	06	12	00	19	13	50	48	15	55	18	18	06	28	12
13	08	13	25	10	11	48	12	03	55	13	54	37	15	59	32	18	10	55	13
14	08	17	23	10	15	29	12	07	32	13	58	26	16	03	46	18	15	22	14
15	08	21	21	10	19	09	12	11	08	14	02	16	16	08	02	18	19	49	15
16	08	25	18	10	22	49	12	14	45	14	06	07	16	12	18	18	24	15	16
17	08	29	14	10	26	29	12	18	21	14	10	00	16	16	36	18	28	41	17
18	08	33	19	10	30	08	12	21	57	14	13	53	16	20	54	18	33	07	18
19	08	37	05	10	33	46	12	25	35	14	17	46	16	25	13	18	37	32	19
20	08	40	59	10	37	24	12	29	13	14	21	40	16	29	32	18	41	58	20
21	08	44	53	10	41	02	12	32	51	14	25	35	16	33	52	18	46	23	21
22	08	48	46	10	44	40	12	36	29	14	29	31	16	38	12	18	50	48	22
23	08	52	37	10	48	18	12	40	08	14	33	28	16	42	33	18	55	13	23
24	08	55	49	10	51	55	12	43	47	14	37	26	16	46	55	18	59	37	24
25	09	00	20	10	55	32	12	47	26	14	41	24	16	51	17	19	04	01	25
26	09	04	11	10	59	09	12	51	05	14	45	23	16	55	39	19	08	24	26
27	09	08	02	11	02	46	12	54	45	14	49	23	17	00	02	19	12	47	27
28	09	11	51	11	06	23	12	58	26	14	53	24	17	04	26	19	17	09	28
29	09	15	49	11	09	59	13	02	07	14	57	25	17	08	50	19	21	31	29
30	09	19	29	11	13	35	13	05	48	15	01	28	17	12	15	19	25	53	30
31	09	23	16	11	17	11				15	95	32				19	30	14	31

A New and Correct Table of the Sun's Right Aſcenſion for the Year 1727, being the Third Year after Leap-Year.

1727

Days	*Jan.*			*February*			*March*			*April*			*May*			*June*			Days
	h	′	″	h	′	″	h	′	″	h	′	″	h	′	″	h	′	″	
1	19	34	34	21	42	43	23	28	30	01	21	24	03	14	44	05	20	06	1
2	19	38	54	21	46	38	23	32	10	01	25	04	03	18	39	05	24	15	2
3	19	43	13	21	50	33	23	35	50	01	28	44	03	22	35	05	28	25	3
4	19	4[illegible]	31	21	54	36	23	39	29	01	32	25	03	26	31	05	32	34	4
5	19	51	48	21	58	19	23	43	09	01	36	07	03	30	28	05	36	43	5
6	19	56	04	22	02	11	23	46	47	01	39	48	03	34	26	05	40	52	6
7	20	00	20	22	06	02	23	50	26	01	43	30	03	38	24	05	45	01	7
8	20	04	36	22	09	53	23	54	04	01	47	13	03	42	23	05	49	10	8
9	20	08	50	22	13	44	23	57	42	01	50	56	03	46	22	05	53	19	9
10	20	13	03	22	17	34	00	01	19	01	54	39	03	50	21	05	57	39	10
11	20	17	16	22	21	23	00	04	57	01	58	44	03	54	20	06	01	38	11
12	20	21	29	22	25	11	00	08	35	02	02	07	03	58	20	06	05	48	12
13	20	25	41	22	28	59	00	12	13	02	05	51	04	02	21	06	09	57	13
14	20	29	52	22	32	46	00	15	50	02	09	37	04	06	22	06	14	07	14
15	20	34	02	22	36	32	00	19	27	02	13	23	04	10	25	06	18	17	15
16	20	38	11	22	40	18	00	23	05	02	17	09	04	14	28	06	22	26	16
17	20	42	18	22	44	03	00	26	43	02	20	55	04	18	32	06	26	34	17
18	20	46	26	22	47	48	00	30	22	02	24	52	04	22	36	06	30	43	18
19	20	50	33	22	51	32	00	33	59	02	28	29	04	26	40	06	34	51	19
20	20	54	39	22	55	15	00	37	37	02	32	17	04	30	44	06	39	00	20
21	20	58	44	22	58	58	00	41	15	02	36	07	04	34	49	06	43	08	21
22	21	02	48	23	22	41	00	44	53	02	39	57	04	38	54	06	47	16	22
23	21	06	51	23	46	24	00	48	31	02	43	47	04	42	59	06	51	24	23
24	21	10	5	23	10	06	00	52	09	02	47	37	04	47	05	06	55	31	24
25	21	14	55	23	13	48	00	55	47	02	51	27	04	51	11	06	59	37	25
26	21	18	55	23	17	29	00	59	26	02	55	18	04	55	28	07	03	44	26
27	21	22	56	23	21	09	01	03	05	02	59	10	04	59	25	07	07	50	27
28	21	26	54	23	24	50	01	06	44	03	03	03	05	03	33	07	11	56	28
29	21	30	52				01	10	23	03	06	55	05	07	41	07	16	02	29
30	21	34	50				01	14	03	03	10	50	05	11	49	07	20	07	30
31	21	38	46				01	17	43				05	15	57				31

A New and Correct Table of the Sun's *Right Aſcenſion for Year* 1727, *being the Third Year after* Leap-Year.

1727

Days	*July*			*Auguſt*			*September*			*October*			*November*			*December*			Days
	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	
1	07	24	12	09	26	08	11	19	55	13	08	36	15	08	37	17	16	35	1
2	07	28	16	09	29	54	11	23	31	13	12	18	15	12	41	17	21	00	2
3	07	32	19	09	33	40	11	27	06	13	16	01	15	16	47	17	25	26	3
4	07	36	22	09	37	25	11	30	42	13	19	44	15	20	54	17	[illegible]	52	4
5	07	40	25	09	41	10	11	34	17	13	23	28	15	25	02	17	34	18	5
6	07	44	28	09	44	55	11	37	53	13	27	13	15	29	10	17	38	44	6
7	07	48	30	09	48	40	11	41	29	13	30	58	15	33	18	17	34	10	7
8	07	52	31	09	52	24	11	45	05	13	34	44	15	37	29	17	47	37	8
9	07	56	31	09	56	06	11	48	41	13	38	30	15	41	40	17	52	04	9
10	08	00	31	09	59	49	11	52	16	13	42	16	15	45	53	17	56	30	10
11	08	04	31	10	03	40	11	55	51	13	46	04	15	50	94	18	00	57	11
12	08	08	29	10	07	12	11	59	27	13	49	52	15	54	16	18	05	23	12
13	08	12	29	10	10	54	12	03	03	13	53	41	15	58	39	18	09	49	13
14	08	16	27	10	14	35	12	06	39	13	57	31	16	02	44	18	14	16	14
15	08	20	24	10	18	15	12	10	16	14	01	21	16	06	59	18	18	42	15
16	08	24	22	10	21	55	12	13	52	14	05	11	16	11	16	18	23	10	16
17	08	28	17	10	25	35	12	17	28	14	09	03	16	15	33	18	27	37	17
18	08	32	13	10	29	14	12	21	05	14	12	56	16	19	52	18	32	02	18
19	08	36	08	10	32	52	12	24	42	14	16	49	16	24	11	18	36	27	19
20	08	40	02	10	36	32	12	28	20	14	20	42	16	28	29	18	40	52	20
21	08	43	56	10	40	10	12	31	58	14	24	38	16	32	48	18	45	15	21
22	08	47	49	10	43	47	12	35	36	14	28	33	16	37	09	18	49	44	22
23	08	51	11	10	47	25	12	39	15	14	32	30	16	41	29	18	54	08	23
24	08	55	35	10	51	03	12	42	54	14	36	28	16	45	50	18	58	34	24
25	08	59	25	10	54	40	12	46	33	14	40	27	16	50	12	19	02	58	25
26	09	03	15	10	58	17	12	50	12	14	41	26	16	54	35	19	07	21	26
27	09	07	05	11	01	54	12	53	52	14	48	25	16	58	58	19	11	4[illegible]	27
28	09	10	56	11	05	30	12	57	32	14	52	25	17	03	22	19	16	06	28
29	09	14	44	11	09	06	13	01	13	14	56	27	17	07	46	19	20	2[illegible]	29
30	09	18	33	11	12	43	13	04	54	15	00	29	17	12	10	19	24	5[illegible]	0
31	09	22	21	11	16	19				15	04	33				19	29	11	31

A New and Correct Table of the Sun's *Right Aſcenſion for Year* 1728, *being* Leap-Year.

1728

Days	*Jan.*			*February*			*March*			*April*			*May*			*June*			Days
	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	
1	19	33	30	21	41	45	23	31	16	01	24	10	03	17	42	05	23	15	1
2	19	37	50	21	45	41	23	34	58	01	27	50	03	21	37	05	27	25	2
3	19	42	09	21	49	36	23	38	36	01	31	32	03	25	34	05	31	34	3
4	19	4[illegible]	27	21	53	30	23	42	15	01	35	14	03	29	31	05	35	42	4
5	19	50	42	21	57	23	23	45	54	01	38	36	03	33	28	05	39	51	5
6	19	55	02	22	03	15	23	49	32	01	42	37	03	37	27	05	44	00	6
7	19	59	18	22	05	06	23	53	10	01	46	19	03	41	26	05	48	09	7
8	20	03	33	22	08	57	23	56	48	01	50	01	03	45	25	05	52	19	8
9	20	07	48	22	12	48	00	00	26	01	53	44	03	49	24	05	56	29	9
10	20	12	02	22	16	38	00	04	04	01	57	28	03	53	23	06	00	39	10
11	20	16	16	22	20	27	00	07	42	02	01	12	03	57	23	06	04	48	11
12	20	20	28	22	24	16	00	11	19	02	04	56	04	01	24	06	08	57	12
13	20	24	40	22	28	04	00	14	57	02	08	42	04	05	25	06	13	06	13
14	20	28	51	22	31	50	00	18	34	02	12	28	04	09	27	06	17	16	14
15	20	33	01	22	35	36	00	22	12	02	16	14	04	13	29	06	21	29	15
16	20	37	11	22	39	22	00	25	50	02	20	00	04	17	32	06	25	34	16
17	20	41	20	22	43	07	00	29	29	02	23	47	04	21	36	06	29	43	17
18	20	45	27	22	46	53	00	33	07	02	27	34	04	25	40	06	33	52	18
19	20	49	33	22	50	38	00	36	44	02	31	23	04	29	44	06	38	09	19
20	20	53	38	22	54	22	00	40	23	02	35	11	04	33	49	06	42	08	20
21	20	57	44	22	58	05	00	44	00	02	39	01	04	37	54	06	46	16	21
22	21	01	49	23	01	48	00	47	38	02	42	51	04	41	59	06	50	24	22
23	21	05	52	23	05	31	00	51	16	02	46	41	04	46	05	06	54	32	23
24	21	09	55	23	09	13	00	54	54	02	50	32	04	50	12	06	58	39	24
25	21	13	56	23	12	54	00	58	34	02	54	23	04	54	19	07	02	44	25
26	21	17	57	23	16	35	01	02	13	02	58	14	04	58	26	07	06	51	26
27	21	21	57	23	20	15	01	05	52	03	02	07	05	02	32	07	10	47	27
28	21	25	57	23	23	56	01	09	31	03	06	00	05	06	40	07	15	03	28
29	21	29	55	23	27	36	01	13	10	03	09	54	05	10	48	07	19	08	29
30	21	33	52				01	16	50	03	13	47	05	14	57	07	23	13	30
31	21	37	49				01	20	30				05	19	06				31

A New and Correct Table of the Sun's *Right Aſcenſion for the Year* 1728, *being* Leap Year.

1728

Days	*January*			*February*			*March*			*April*			*May*			*June*			Days
	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	h	'	"	
1	07	27	17	09	28	59	11	22	38	13	11	24	15	11	41	17	19	56	1
2	07	31	21	09	32	45	11	26	14	13	15	07	15	15	47	17	24	22	2
3	07	35	24	09	36	31	11	29	49	13	18	51	15	19	54	17	28	48	3
4	07	39	27	09	40	16	11	33	16	13	22	35	15	24	01	17	33	13	4
5	07	43	30	09	44	01	11	37	01	13	26	19	15	28	09	17	37	49	5
6	07	47	32	09	47	46	11	40	37	13	30	03	15	32	17	17	42	05	6
7	07	51	33	09	51	30	11	44	13	13	33	49	15	36	28	17	46	31	7
8	07	55	33	09	55	12	11	47	49	13	37	35	15	40	40	17	50	57	8
9	07	59	34	09	58	55	11	51	24	13	41	21	15	44	51	17	55	24	9
10	08	03	33	10	02	37	11	54	59	13	45	08	15	49	03	17	59	56	10
11	08	07	33	10	06	19	11	58	36	13	48	56	15	53	15	18	04	19	11
12	08	11	31	10	10	00	12	02	11	13	52	45	15	57	29	18	08	45	12
13	08	15	28	10	13	41	12	05	47	13	56	34	16	01	43	18	13	12	13
14	08	19	25	10	17	22	12	09	23	14	00	24	6	05	57	18	17	39	14
15	08	23	23	10	21	02	12	12	59	14	04	15	16	10	12	18	22	05	15
16	08	27	20	10	24	42	12	16	36	14	08	07	16	14	30	18	26	31	16
17	08	31	16	10	28	21	12	20	13	14	12	00	16	18	49	18	30	58	17
18	08	35	11	10	32	09	12	23	50	14	15	53	16	23	08	18	35	23	18
19	08	39	05	10	35	39	12	27	28	14	19	47	16	27	27	18	39	48	19
20	08	42	49	10	39	17	12	31	05	14	23	41	16	31	46	18	44	14	20
21	08	46	53	10	42	55	12	34	43	14	27	36	16	36	06	18	48	39	21
22	08	50	45	10	46	33	12	38	22	14	31	33	16	40	27	18	53	04	22
23	08	54	37	10	50	10	12	42	01	14	35	31	16	44	48	18	57	29	23
24	08	58	29	10	53	47	12	45	40	14	39	29	16	49	09	19	01	52	24
25	09	02	20	10	57	24	12	49	19	14	43	28	16	53	31	19	06	16	25
26	09	06	10	11	01	01	12	52	59	14	47	27	16	57	54	19	10	40	26
27	09	10	00	11	04	38	12	56	39	14	51	27	17	02	17	19	15	03	27
28	09	13	40	11	08	14	13	00	19	14	55	28	17	06	41	19	19	25	28
29	09	17	37	11	11	50	13	04	07	14	59	30	17	11	06	19	23	47	29
30	09	21	25	11	15	26	13	07	42	15	03	34	17	15	31	19	28	07	30
31	09	25	13	11	19	03				15	07	38				19	32	27	31

A Table of the Variation of the Suns Right Aſcenſion

D. M	4	8	12	16	20	40	60	80	100
	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
Janu. 10	0 07	0 14	0 21	0 28	0 35	1 11	1 47	2 23	2 58
20	0 07	0 14	0 21	0 28	0 35	1 10	1 46	2 21	2 56
30	0 07	0 14	0 21	0 28	0 34	1 09	1 43	2 18	2 52
Feb. 9	0 07	0 14	0 21	0 28	0 34	1 08	1 41	2 15	2 49
19	0 07	0 14	0 21	0 28	0 33	1 06	1 40	2 13	2 46
29	0 07	0 14	0 21	0 28	0 33	1 06	1 38	2 11	2 44
Mar. 10	0 07	0 14	0 21	0 28	0 33	1 06	1 39	2 12	2 45
20	0 07	0 14	0 21	0 28	0 33	1 07	1 41	2 15	2 48
30	0 07	0 14	0 21	0 88	0 34	1 09	1 43	2 18	2 52
Apil 9	0 07	0 14	0 21	0 28	0 35	1 11	1 46	2 22	3 57
19	0 07	0 14	0 21	0 28	0 36	1 13	1 49	2 26	3 02
29	0 07	0 15	0 22	0 30	0 37	1 15	1 52	2 30	3 07
May 9	0 08	0 16	0 23	0 31	0 38	1 17	1 56	2 35	3 13
19	0 08	0 16	0 24	0 32	0 40	1 20	2 01	2 41	3 21
29	0 08	0 16	0 24	0 32	0 41	1 22	2 03	2 44	3 25
June 8	0 08	0 16	0 24	0 32	0 41	1 22	2 03	2 44	3 25
18	0 08	0 16	0 24	0 32	0 41	1 22	2 03	2 44	3 25
28	0 08	0 16	0 24	0 32	0 40	1 21	2 01	2 42	3 22
July 8	0 08	0 16	0 24	0 32	0 39	1 19	1 59	2 39	3 18
18	0 08	0 16	0 23	0 31	0 38	1 17	1 55	2 34	3 12
28	0 07	0 15	0 22	0 30	0 37	1 14	1 51	2 28	3 05
Aug. 7	0 07	0 14	0 21	0 29	0 36	1 12	1 47	2 23	2 59
17	0 07	0 14	0 21	0 28	0 35	1 10	1 45	2 20	2 55
27	0 07	0 14	0 21	0 28	0 34	1 09	1 43	2 18	2 52
Sept. 6	0 07	0 14	0 21	0 28	0 34	1 08	1 41	2 15	2 49
16	0 07	0 14	0 21	0 28	0 33	1 07	1 40	2 14	2 47
26	0 07	0 14	0 21	0 28	0 33	1 06	1 40	2 13	2 46
Octob. 6	0 07	0 14	0 21	0 28	0 34	1 08	1 41	2 15	2 49
16	0 07	0 14	0 21	0 28	0 34	1 09	1 43	2 18	2 52
26	0 07	0 14	0 21	0 28	0 35	1 10	1 45	2 20	2 55
Nov. 5	0 07	0 14	0 22	0 29	0 36	1 12	1 47	2 23	2 59
15	0 07	0 15	0 22	0 30	0 37	1 14	1 50	2 27	3 04
25	0 07	0 15	0 22	0 30	0 37	1 15	1 52	2 30	3 07
Dec. 5	0 07	0 15	0 22	0 30	0 37	1 15	1 53	2 31	3 08
15	0 07	0 15	0 22	0 30	0 37	1 15	1 52	2 30	3 07
25	0 07	0 15	0 22	0 30	0 37	1 14	1 52	2 29	3 06
31	0 07	0 15	0 22	0 30	0 36	1 13	1 49	2 26	3 03

Sect. IV.

Containing Methods for Calculating the Longitude Latitude, Right Aſcenſion, and Declination of the Moon, *or any* Fixed Star *or* Planet, *together with the Times of their Riſing, and Setting, their Amplitudes, Azimuths, Hour of the Night,* &c.

DEFINITIONS.

1. CONceive a Great Circle to paſs thro' the Poles of the Ecliptic and the Moon or any Fixed Star, and the Arch of the Ecliptic contained or intercepted between the firſt point of *Aries* and the point of the Ecliptic thro' which the Great Circle paſſes, is called the Longitude.

2. And the Arch of the Great Circle intercepted between the Ecliptic and the Moon or Fixed Star is called the Latitude; and if the Moon or Star be placed to the Northward of the Ecliptic it is called North Latitude, but if the Moon or Star is ſcituated on the South ſide of the Ecliptic it is called South Latitude.

3. Conceive a Great Circle to paſs thro' the Poles of the World and the Moon, or any Fixed Star, and the Arch of the Equator contained between the firſt point of *Aries* and the Interſection of this Circle with the Equator, is called the Right Aſcenſion.

4. And the Arch of this Great Circle intercepted between the Equator and the Moon or Fixed Star is called the Declination; and if the Star or Moon be on the North ſide of the Equator, it is called North Declination, but if either of them be ſcituated to the Southward of the Equator it is called South Declination.

Hence it is manifeſt, that all the Fixed Stars that lye between the Equator and the Northern half of the Eliptic, or the Moon, or any other Planet when they happen to be within this Space, have North Declination and South Latitude; and on the contrary, all the Stars or the Moon (when it ſo happens) that lye between the Equator and the Southern half of the Ecliptic, have South Declination but North Latitude.

For *Example*, the Southern Eye of the ***Bull Aldebaran***, which is placed in 5 deg. 57 min. of *Gemini*, has 15 deg. 55 min. 23 ſec. of North Declination, and 5 deg. 29 min. 49 ſec. of South Latitude; and on the contrary, the firſt of the two Stars in the following Horn of *Capricorn* which is placed in 29 deg. 57 min. 19 ſec. of *Capricorn*, has 13 deg. 19 min. 30 ſec. of South Declination, but 7 deg. 1 min. 31 ſec. of North Latitude.

N. B. Theſe Places of the fixed Stars, as well as thoſe that I ſhall make Uſe of in this *Section*, are fitted to the Year *1726*; that is, 36 Years after the commencement of Mr. *Flamſteeds* Catalogue.

Theſe Things tho' obvious to any that are but moderately skilled in *Aſtronomy*, are ſometimes apt to confound Young Beginners, and therefore I thought them neceſſary to be explained.

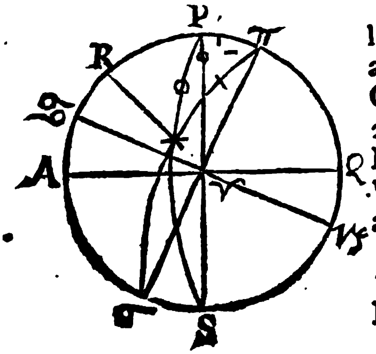
Prob. I.

Given the Latitude and Longitude of a fixed Star or Planet, to find the Right Aſcenſion and Declination.

Example.

The Bright Star of *Aries* being in *Taurus* 3 deg. 49 min. 18 ſec. and having 9 deg. 57 min. 12 ſec. North Latitude, I demand its Right Aſcenſion and Declination, or diſtance from the North Pole, the Obliquity of the Ecliptic or diſtance between the Poles of the Equinoctial and Ecliptic being fixed at 23 deg. 29 min.

The Stereographic Solution upon the Plane of the Solſtitial Colure.



1. Having drawn the Solſtitial Colure *P Q S A*, the Equator *A Q*, and at Right-angles to it the Equinoctial Colure *P* ♈ *S*, ſet off the Chord of 23 deg. 29 min. the obliquity of the Ecliptic from *A* to ♋ or from *Q* to ♑, and draw the Ecliptic ♋ ♈ ♑, and at Right-angles to it its Axis π ♈ σ.

2. Thro' the point π the Pole of the Ecliptic, draw the Circle of Longitude

π ✱ σ

π ✱ q, forming an Angle of 56 deg. 10 min. 42 ſec. equal to the diſtance of the Stars Place from the firſt point of *Cancer*, with the Solſtitial Colure π ♋ σ, by *Caſe* the 3*d* of *Prob.* the 9*th*; of *Sect.* the 2*d* of *Part* the 4*th*, and lay off 81 deg. 02 min. 48 ſec. the diſtance of the Star from the North Pole of the Eclíptic, or Complement of the Latitude in this Circle from π to ✱, by *Caſe* the 3*d* of *Problem* the 6*th* of the ſame *Section*, then will ✱ repreſent the Place of the firſt Star of *Aries* in the *Projection*.

3. Thro' the Poles of the Equator *P* and *S* and the point ✱, draw the Circle of Right Aſcenſion *P* ✱ *S*, by *Problem* the 1*ſt* of *Section* the 2*d* of *Part* the 4*th*, and the thing is done; and the Angle ✱ *P* ♋ the Right Aſcenſion from the Solſtitial Colure, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d* of *Part* the 4*th*, and the Arch *P* ✱, the diſtance from the North Pole, or Complement of the Declination, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th* of the ſame *Section*.

Logarithmically, or by Calculation.

In the Oblique-angled Spherical Triangle *P* π ✱ are given, *P* π the diſtance between the Poles of the Equator and Ecliptic, equal to 23 deg. 29 min. 00 ſec. the Angle *P* π ✱ the Stars Longitude from the Solſtitial Colure, equal to 56 deg. 10 min. 42 ſec. and the Arch π ✱, equal to the Complement of the Stars Latitude, equal to 80 deg. 02 min. 48 ſec. whence to find firſt *P* ✱ the Complement of the Stars Declination, it will be by *Caſe* the 11*th* of *Oblique-angled*, &c. (having firſt let fall the Perpendicular ✱ *R*.)

As *R* : c s, *P* π ✱ : : t, π ✱ : t, π *R*; that is,

As the Radius —————————— 10 0000000

To the Sine of the Longitude from ♈ 33.49.18 —— 9.7455508
So is the Co-tangent of the Latitude 9.57.12 ——— 10 7557545

To the Tangent of a 4*th* Arch π *R* 72.30.03$\frac{3}{4}$ ——— 10 5013053

But π *R*—*P* π =*P R*, that is, if from the fourth Arch equal to 72 deg. 30 min. 03 ſec. $\frac{3}{4}$, be taken the diſtance between the two Poles 23 deg. 29 min. 00 ſec. there will remain *P R* a fifth Arch, equal to 49 deg. 01 min. 03 ſec. $\frac{3}{4}$, whence to find the Declination it will be, As c s, ♈ *R* : c s, *P R* : : c s, ♈ ✱ : c s, *P* ✱ ; that is,

As the Co-ſine of the fourth Arch 72.30.03¾ ———— 9.4781168

To the Co-ſine of the fifth Arch 49.01.03¾ ———— 9.8167885
So is the Sine of the Latitude 9.57.12 ———— 9.2376592

To the Sine of the Declination 22.08.50 ½ ———— 9.5763309

Which is North, becauſe the Stars Place is in the Northern half of the Ecliptic, and the Latitude North.

Again, to find the Right Aſcenſion, it will be by the latter part of the *9th Caſe* of *Oblique*, &c.

As S. *P R* : S. ♈ *R* : : t, *P* ♈ ✱ : t, *A P* ✱ ; that is,

As the Sine of the fifth Arch *P R* 49.01.03¾ ———— 9.8778965

To the Sine of the fourth Arch ♈ *R* 72.30 03¾ ———— 9.9794220
So is the Co-tang. of the Longitude from *Aries* 33.49.18 10.1739321

To the Co-T. of the Right Aſcen. from ♈ 27.56.17¼ 10.2754576

Whence it appears, that according to Mr *Flamſteed*'s Catalogue of the fixed Stars, that in the Year 1726, the Right Aſcenſion of the Bright Star of *Aries* will be 27 deg. 56 min. 17 ſec. ¼, and its Northern Declination 22 deg. 08 min. 50 ſec. ½.

After the ſame manner, and according to the Rules given in the *1ſt Caſe* of the *1ſt Problem* of the former *Section*, may the Right Aſcenſion and Declination of the Moon or any fixed Star or Planet be determined, from the Latitude and Longitude firſt given, and to exerciſe the Young Student I ſhall ſubjoin a few *Examples*.

Example 1.

What is the Right Aſcenſion and Declination of *Aldebaran*, or the Southern Eye of the *Bull*, the Star being in *Gemini* 5 deg. 57 min. 00 ſec. and having 5 deg. 29 min. 49 ſec. South Latitude.

Anſwer

The Right Aſcenſion is 65 deg. 02 min. 57 ſec. the Declination 15 deg. 55 min. 23 ſec. North.

Example 2.

What is the Right Aſcenſion and Declination of the Head of *Andromeda*, the Star being in *Aries* 10 deg. 28 min. 53 ſec. and having 25 deg. 41 min. 01 ſec. North Latitude.

Anſwer.

The Right Aſcenſion is 358 deg. 33 min. 20 ſec. and Declination 27 deg. 34 min. 12 ſec. North.

Example 3.

I demand the Right Aſcenſion and Declination of the *Lyons Heart*, the Star being in *Leo* 26 deg. 01 min. 20 ſec. and having 00 deg. 26 min. 38 ſec. of North Declination.

Anſwer.

The Right Aſcenſion is 148 deg. 26 min. 00 ſec. and the Declination 13 deg. 17 min 04 ſec. North.

Example 4.

What is the Right Aſcenſion and Declination of the *Great Dog Star Syrius*, its Place being in *Cancer* 10 deg. 19 min. 01 ſec. and having 39 deg. 32 min. 08 ſec. South Latitude.

Anſwer.

The Right Aſcenſion is 98 deg. 16 min. 32 ſec. and Declination 16 deg. 20 min. 59 ſec. South.

Example 5.

What is the Right Aſcenſion and Declination of the Bright Star in the *Harp*, the Place being in *Capricorn* 11 deg. 27 min. 18 ſec. and having 61 deg. 45 min. 31 ſec. North Latitude.

Anſwer.

The Right Aſcenſion is 276 deg. 54 min. 03 ſec. ½, and Declination 38 deg. 33 min. 02 ſec. North.

Example 6.

What is the Right Aſcenſion and Declination of the Right Shoulder of *Orion*, the Star being in *Gemini* 24 deg. 55 min. 06 ſec. and having 16 deg. 04 min. 26 ſec. South Latitude.

Anſw.

Anſwer.

The Right Aſcenſion is 85 deg. 04 min. 33 ſec. and Declination 07 deg. 19 min. 15 ſec. North.

Prob. I.

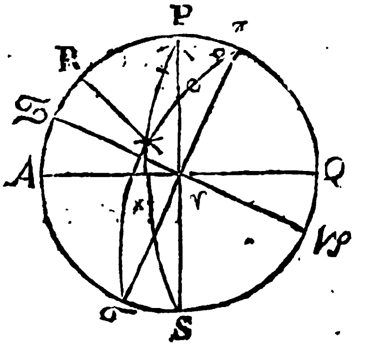
Caſe 2.

Given the Right Aſcenſion and Declination of a fixed Star or Planet, to find its Longitude and Latitude, the Obliquity of the Ecliptic, being fixed at 23 deg. 29 min. 00 ſec.

Example.

The Right Aſcenſion of the Bright Star of *Aries* being 27 deg. 56 min. 17 ſec. and its Declination 22 deg. 08 min. 50 ſec. North. I demand its Longitude and Latitude.

The Stereographic Solution.



1. Having drawn the Solſtitial Colure *P Q S A*, the Equator *A Q*, the Solſtitial Colure *P S*, the Ecliptic ♋ ♑, and its Axis π σ, as in the former *Caſe*, thro' the Poles *P* and *S* draw the Circle of Right Aſcenſion *P* ✱ *S*, forming an Angle of 62 deg. 03 min. 03 ſec. equal to the Complement of the Right Aſcenſion of the Star from *Aries*, by *Caſe* the 3*d* of *Problem* the 9*th* of *Part* the 4*th*, and ſet off 67 deg. 51 min. 10 ſec. the diſtance of the Star from the North Pole, or Complement of the Declination, in this Circle from *P* to ✱, by *Caſe* the 3*d* of *Problem* the 6*th* of the ſame *Section*, then will ✱ repreſent the Place of the firſt Star of *Aries* in this Projection.

2. Thro' the Poles of the Ecliptic π and σ and the point ✱, draw the Circle of Longitude π ✱ σ, and the thing is done; and the Angle ✱ π ♈ the Longitude from the firſt point of *Aries*, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d* of *Part* the 4*th*, and the Arch ✱ *x* the Latitude in this *Caſe*, or the Arch π ✱ its Complement, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th* of the ſame *Section*.

Logarithmically, or by Calculation.

In the Oblique-angled Spherical Triangle *P π ** are given, *P π* the conſtant diſtance between the Poles of the World and Ecliptic equal to 23 deg. 29 min. the Angle *R P ** the Complement of the Stars Right Aſcenſion from the firſt point of *Aries*, equal to 62 deg. 03 min. 41 ſec. and the Arch *P ** the diſtance of the Star from the North Pole, or Complement of its Declination, in this *Caſe* equal to 67 deg. 51 min 07 ſec. whence to find firſt the Arch *π **, the Stars diſtance from the Pole of the Ecliptic *π*, or Complement of the Latitude, it will be by *Caſe* the 11*th* of *Oblique-angled*, &c. (having firſt let fall the Perpendicular ** R*.)

As *R* : c s, *π* P * : : t, *P ** : t, *P R* ; that is,

As the Radius————————————10.0000000

To the Sine of the Right Aſcen. from *Aries* 27.56.17 —9.6707251
So is the Co-tangent of the Declination 22.08.50 ——10.3903861

To the Tangent of a fourth Arch *P R* 49.01.04½———10.0611112

But *P R*+*P π* =*R π*; that is, if to the fourth Arch *P R*, equal to 49 deg. 01 min. 04 ſec. ½, be added *P π* the conſtant diſtance between the two Poles, equal to 23 deg. 29 min. 00 ſec. the Sum 72 deg. 30 min. 04 ſec. ½, wil give *R π* for a fifth Arch, whence to find the Latitude it will be,

As the c s, *P R* : c s, *R π* : : c s, *P ** : c s, *π **; that is,

As the Co-ſine of the 4*th* Arch *PR* 49.01.04½————9.8167866

To the Co-ſine of the 5*th* Arch *P π* 72.30.04½———9.4781118
So is the Sine of the Declination 22.08.50——————9.5763272

To the Sine of the Latitude 9.57.11½ ————9.2376524

Which is North, becauſe the Stars Right Aſcenſion is leſs than 180 deg. 00 min. 00 ſec. and its Declination North.

Again to find the Longitude, it will be by the latter part of the 9th *Caſe* of *Oblique*, &c.

As the S. ♈ R : S. *PR* : : t, *AP* ✱ : t, ♋ ♈ ✱; that is,

As the Sine of the fifth Arch ♈ *R* 72.30.04$\frac{1}{2}$ ——— 9.9794295

To the Sine of the 4*th* Arch *P R* 49.01.04$\frac{1}{3}$ ——— 9.8778978

So is the Co-tang. of the Right Aſc. from ♈ 27.56.17 10.2754592

To the Co-tang. of the Longit. from ♈ 33.49.18$\frac{2}{3}$ —— 10.1739275

Which gives 3 deg. 49 min. 18 ſec. $\frac{2}{3}$ in *Taurus*, for the Place of the Star.

After the ſame manner, and by the help of the Rules given in the 2*d Caſe* of the 1*ſt Problem* of the laſt *Section*, may the Latitude of the Moon or any fixed Star, and its Place in the Ecliptic be determined, from the Right Aſcenſion and Declination firſt given, as will appear from the following *Examples*.

Example 1.

What is the Latitnde and Place of *Spica Virginis*, the Right Aſcenſion being 197 deg. 42 min. 07 ſec. and its Declination 9 deg. 43 min. 23 ſec. South.

Anſwer.

The Stars Place is in *Libra* 20 deg. 01 min. 22 ſec. and its Latitude 2 deg. 01 min. 59 ſec. South.

Example 2.

What is the Latitude and Longitude of the Bright Star in the *Eagle*, its Right Aſcenſion being 294 deg. 19 min. 56 ſec. and its Declination 8 deg. 10 min. 01 ſec. North.

Anſwer.

The Stars Place is in *Capricorn* 27 deg. 53 min. 24 ſec. and its Latitude is 29 deg. 19 min. 11 ſec. North.

Example 3.

What is the Longitude and Latitude of *Procyon*, or the Bright Star in the *Litle Dog*, its Right Aſcenſion being 111 deg. 14 min. 01 ſec. and its Declination 05 deg. 54 min. 26 ſec. North.

Anſw.

Anſwer.

The Stars Place is in *Cancer* 22 deg. 00 min. 21 ſec. and its Latitude 15 deg. 57 min. 55 ſec. South.

Example 4.

What is the Longitude and Latitude of the Bright Foot of *Orion*, called *Regel*; whoſe Right Aſcenſion is 75 deg. 20 min. 46 ſec. and its Declination 8 deg. 33 min. 00 ſec. South.

Anſwer:

The Stars Place is in *Gemini* 13 deg. 00 min. 00 ſec. and its Latitude is 31 deg. 10 min. 11 ſec. South.

Example 5.

What is the Longitude and Latitude of *Capella*, or the *Goat Star*, its Right Aſcenſion being 74 deg. 07 min. 14 ſec. and its Declination 45 deg. 41 min. 19 ſec. North.

Anſwer.

The Place of the Star is in *Gemini* 18 deg. 01 min. 41 ſec. and its Latitude 22 deg. 51 min. 47 ſec. North.

Example 6.

What is the Place and Latitude of *Arcturus*, its Right Aſcenſion being 210 deg. 47 min. 42 ſec. ½, and Declination 20 deg. 38 min. 43 ſec. ½ North.

Anſwer.

The Stars Place is in *Libra* 20 deg. 23 min. 52 ſec. and its Latitude is 30 deg. 57 min. 00 ſec. North.

These two *Caſes* being the principal and only uſeful *Caſes* of this *Problem*, I ſhall only propoſe the remaining Eight *Caſes*, and give their *Anſwers*, and leave the *Operations* to Exerciſe the Learner.

Caſe 3.

Given the Right Aſcenſion and Declination of a Star, together with its Latitude, to find the Longitude and greateſt Obliquity of the Ecliptic.

Example.

The Right Aſcenſion of the Southern Eye of the *Bull Aldebaran* being 65 deg. 02 min. 57 ſec. its Declination 15 deg. 55 min. 23 ſec. and its Latitude 5 deg. 29 min. 49 ſec. South. I demand the Longitude and Obliquity of the Ecliptic.

Anſwer.

The Star is in *Gemini* 5 deg. 57 min. 00 ſec. and the Obliquity of the Ecliptic is 23 deg. 29 min. 00 ſec.

Caſe 4.

Given the Longitude, Latitude and Declination of a Star, to find the Right Aſcenſion and Obliquity of the Ecliptic.

Example.

The Northen *Horn* of the *Bull* being in 18 deg. 43 min. 56 ſec. of *Gemini*, its Latitude 5 deg. 21 min. 34 ſec. North, and its Declination 28 deg. 20 min. 33 ſec. North. I demand its Right Aſcenſion, and the Obliquity of the Ecliptic.

Anſwer.

The Right Aſcenſion is 77 deg. 13 min. 49 ſec. and the Obliquity of the Ecliptic is 23 deg. 29 min. 00 ſec.

Caſe 5.

Given the Right Aſcenſion of a Star; its Latitude, and the Obliquity of the Ecliptic, to find the Longitude and Declination.

Example.

What is the Longitude and Declination of the *Right Shoulder of Orion*, its Latitude being 16 deg. 04 min. 26 ſec. South, its Right Aſcenſion 85 deg. 04 min. 33 ſec. and the greateſt Obliquity of the Ecliptic 23 deg. 29 min. 00 ſec.

Anſwer.

The Star is in *Gemini* 24 deg. 55 min. 00 ſec. and its Declination 16 deg. 04 min. 26 ſec. South.

But if the Latitude of the Star be greater than *66* deg. 31 min. 00 ſec. as happens to all the Stars that are ſcituated between the Poles of the World and the Ecliptic, then the *Caſe* becomes Ambiguous; and the ſame Data may be common to two Stars.

Caſe 6.

Given the Right Aſcenſion and Longitude of a Star, together with the Obliquity of the Ecliptic, to find the Latitude and Declination.

Example.

The Bright Foot of *Orion* called *Regel,* being in *Gemini* 13 deg. 00 min. 00 ſec. and its Right Aſcenſion being 75 deg. 20 min. 46 ſec. I demand its Latitude and Declination, the greateſt Obliquity of the Ecliptic being 23 deg. 29 min. 00 ſec.

Anſwer.

The Latitude is 31 deg. 10 min. 11 ſec. South, and the Declination is 8 deg. 33 min. 00 ſec. South.

Caſe 7.

Given the Latitude and Declination of a Star, together with the Obliquity of the Ecliptic, to find its Longitude and Right Aſcenſion.

Example.

What is the Longitude and Right Aſcenſion of the *Great Dog Star Syrius,* its Latitude being 39 deg. 32 min. 08 ſec. South, its Latitude *16* deg. 20 min. *29* ſec. South, the greateſt Obliquity being fixed at 23 deg. 29 min. 00 ſec.

Anſwer.

The Star is in *Cancer* 10 deg. 19 min. 01 ſec. and its Right Aſcenſion is 98 deg. 16 min. 32 ſec.

Caſe 8.

Given the Longitude and Declination of a Star, together with the Obliquity of the Ecliptic, to find the Right Aſcenſion and Latitude.

Example.

What is the Right Aſcenſion and Latitude of the Bright Star in the *Harp*, its Place being 11 deg. 27 min. 18 ſec. in *Capricorn*, and its Declination 51 deg. 26 min. 58 ſec. North.

Anſwer.

The Right Aſcenſion is 276 deg. 54 min. 03 ſec. and the Latitude 61 deg. 45 min. 31 ſec. North.

When the Complement of the Stars Declination is leſs than the Obliquity of the Ecliptic, as happens to all the Stars that are placed within the *Artic* and *Antartic Circle*, then the *Solution* is doubtful, and will give two different points in the *Heavens*.

Caſe 9.

Given the Longitude and Latitude of a Star, together with its Right Aſcenſion, to find the Stars Declination and the Obliquity of the Ecliptic.

Example.

The *Lyons Heart* called *Regulus*, being in *Leo* 26 deg. 01 min. 20 ſec. and having 148 deg. 26 min. 00 ſec. of Right Aſcenſion, and 00 deg. 26 ſec. 38 ſec. of Northern Latitude. I demand its Declination and the Obliquity of the Ecliptic.

Anſwer.

Its Declination is 13 deg. 17 min. 04 ſec. North, and the Obliquity of the Ecliptic is 23 deg. 29 min. 00 ſec.

Caſe 10.

Given the Right Aſcenſion, Declination, and Longitude of a Star, to find the Latitude and Obliquity of the Ecliptic.

Example.

The Head of *Andromeda* being in *Aries* 10 deg. 28 min. 53 ſec. its Right Aſcenſion being 358 deg. 33 min. 20 ſec. and having 27 deg. 34 min. 12 ſec. North Declination. I demand its Latitude and the Obliquity of the Ecliptic.

Anſwer.

Its Latitude is 25 deg. 41 min. 01 ſec. North, and the Obliquity of the Ecliptic 23 deg. 29 min. 00 ſec.

And altho' the *3d*, *4th*, *9th*, and *10th Caſes* are doubtful or not doubtful, (according as the given Numbers when compared one with another, are greater or leſſer than each other) conſidered as a Spherical Triangle, yet as applied in this *Problem* they are all limited, becauſe the diſtance between the two Poles or Obliquity of the Ecliptic, is always a conſtant Quantity.

By comparing together the Places of the fixed Stars, deduced from Obſervations made at diſtant Times, it has been found that the Stars move with a conſtant, regular, uniform Motion in Circles Parallel to the Ecliptic, whence it comes to paſs that they alter their Longitudes equally, and always keep the ſame Latitude; this Annual Motion of the fixed Stars has been determined by Mr. *Flamſteed*, to be 50 Seconds of a Degree, and conſequently each Star increaſes its Longitude One Degree in 72 Years; hence the firſt Star of *Aries* which according to Mr. *Flamſteed's* Catalogue in the Year 1700, was in *Aries* 28 deg. 59 min. 20 ſec. 387 Years before *Chriſt*, that is, about 247 Years before *Hipparchus's* Time; it was in the firſt point of *Aries*, and at the beginning of the *19th Century* it will be in *Taurus*, 22 min. 40 ſec. and at the end of 25920 Years, it will return to the ſelf ſame place of the *Heavens* it was at the Commencement of that Time, always keeping at the diſtance of 82 deg. 51 min. 02 ſec. according to the *Flamſteedian* Catalogue from the Northern Pole of the Ecliptic.

After the ſame manner it will be found that the Pole Star which in the Year 1690 was in *Gemini* 24 deg. 14 min. 41 ſec. having 66 deg. 04 min. 11 ſec. North Latitude, at the diſtance of 414 Years, or in the Year 2104, will be upon the *Summer Solſtice*; at which Time its diſtance from the North Pole will be but 26 min. 41 ſec. and that at the end of 12960 Years after that, or in the Year 15064 (if the World continue ſo long) it will be gotten upon the *Winter Solſtice*; when its diſtance from the North Pole will be 47 deg. 24 min. 49 ſec.

Hence the Longitude and Latitude of a fixed Star being given, its place in the *Heavens* at any Time may be eaſily found, and thence its Right Aſcenſion and Declination, by the *2d Caſe* of the preceding *Problem*.

This equable and uniform Motion of the fixed Stars in Circles parallel to the Ecliptic, muſt neceſſarily by Reaſon of the Inclination of the Planes of the Ecliptic and Equator, cauſe an unequal and irregular Motion of the ſame Star with regard to the Equator; and muſt neceſſarily cauſe an unequal Variation, of both Right Aſcenſion and Declination of every fixed Star, and conſequently of the ſame Star at different periods of Time, and therefore *Aſtronomers* to render their Catalogues as ready for Uſe as might be, were wont to Calculate the Right Aſcenſion and Declination of ſome of the Principle fixed Stars, at the diſtance of 50 or 100 Years from their places at that Time, and by ſuppoſing the Stars that lay nearly in the ſame part of the *Heavens* with theſe, to have the ſame Variation of Right Aſcenſion and Declination, they could readily pronounce the new Right Aſcenſion and Declination of any of the Stars in their Catalogue, without the trouble of a new Calculation; but this Method after that period of Time requiring a new Calculation, and in the mean Time rendering it ſomething difficult and troubleſome to Perſons not well skill'd in theſe Matters, to find out proper Stars to compare the ſmall Stars with, and at the beſt but uncertain; Mr. *Flamſteed* whoſe utmoſt View was to render every thing that he undertook as eaſy and ready for Uſe as might be, and whoſe happy Genius always rendered his good Endeavours ſucceſsful, Contrived and Calculated Tables to ſhew the Variation of Declination and Right Aſcenſion of every point of the *Heavens*, ſuppoſing it to have altered one Degree of Longitude, and has to every fixed Star in his Catalogue, added the Variation of Right Aſcenſion and Declination of that Star, during the time that it alters its Longitude one Degree, or according to the forementioned Period of one Degree in 72 Years, and by entering the ſame Table after the Expiration of that time, with the new Longitude and the ſame Latitude of the Star, the Variation of Right Aſcenſion and Declination of the ſame Star may be had for the next 72 Years, whence the Right Aſcenſion and Declination of the ſame Star being once known or given, its Right Aſcenſion for any time, either paſt or to come, may be had by Inſpection only, without the help of any new Calculation; which Contrivance of Mr. *Flamſteed*, together with the great Degree of Exactneſs to which he has determined the Places of all the fixed Stars and their Number, has rendered his Catalogue the largeſt, exacteſt, and moſt uſeful Catalogue that the World ever ſaw.

And altho' the Variations of the Right Aſcenſion and Declination are unequal, yet as the Stars are carried about the Pole of the Ecliptic in a progreſſive Motion, the Right Aſcenſion of every Star muſt conſtantly increaſe, while all thoſe Stars that lye between *Capricorn* and *Cancer*, conſtantly approach nearer to the North Pole, and conſequently receed farther from the South Pole; as on the contrary, all the Stars that are ſcituated between *Cancer* and *Capricorn*, continually receed from the North-Pole and approach nearer to the South Pole; as is obvious to any one who has but the leaſt Idea of the *Heavens*; whence it comes to paſs that tho' the Stars alway retain the ſame Latitude, yet their Declinations conſtantly vary, and a Star that has North Declination may come to have South Declination, as a Star that is on the Northern Side of the Equator, in proceſs of time may arrive on the South Side.

Prob. II.

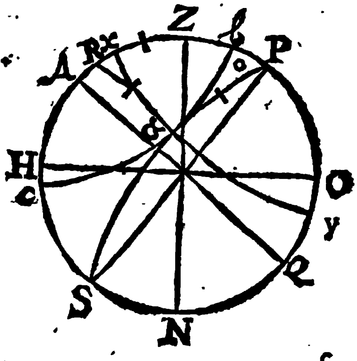
Given the Meridional Altitude of an unknown Star or Planet, and its diſtance from a known Star, to find the unknown Stars Right Aſcenſion, Declination, Longitude and Latitude.

Example.

At the *Royal Obſervatory* at *Greenwich*, on the 11*th* of *February* in the Year 1690, the Meridian Altitude of *Regulus* or the *Lyons Heart* (Allowance being made for the Refraction) was found to be 51 deg. 58 min. 50 ſec. and on the 7*th* of *April* in the Year 1682, its diſtance from *Arcturus* was obſerved to be 59 deg. 46 min. 50 ſec. and in Antecedence. I demand the Right Aſcenſion, Declination, *&c.* of *Regulus*, the Right Aſcenſion of *Arcturus* in the Year 1690, being 210 deg. 22 min. 30 ſec. and its diſtance from the Northern Pole 69 deg. 11 min. 00 ſec.

The eaſieſt *Conſtruction* is after the following manner:

Conſtruction.



1. Draw the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *ZN*.

2. Set off the Latitude of the *Obſervatory* 51 deg. 29 min. 30 ſec. from *O* to *P*, and draw the Axis *PS*, and at Right-angles to it the Equator *AQ*.

3. Set off 51 deg. 58 min. 50 ſec. the Meridian Altitude of *Regulus*

from *H* to *R*, and about the point *R* or place of *Regulus* as a Pole, deſcribe a Small Circle *b c*, (by *Caſe* the 2*d* of *Problem* the 4*th*, of *Section* the 2*d* of *Part* the 4*th*) at the diſtance of 59 deg. 46 min. 50 ſec. equal to the diſtance between the two Stars.

4. About the Pole *P* deſcribe the Small Circle *α y*, (by the ſame *Caſe*) at the diſtance of *Arcturus* from the Pole, equal to 69 deg. 11 min. 00 ſec. where this interſects the former Small Circle in the point *α*, it will give the place of *Arcturus*, thro' which point and the Poles *P* and *S*, if a Circle of Right Aſcenſion *P α S* be drawn, the thing is done; and the Angle *A P α* the difference of Right Aſcenſion between *Regulus* and *Arcturus*, may be meaſured by *Caſe* the 2*d* of *Problem* the 10*th*, of *Section* the 2*d* of *Part* the 4*th*, and the Arch *A R* the Declination of *Regulus*, in this *Caſe* North, may be meaſured by the Line of *Chords*.

The Solution by Calculation.

1. To find the Declination of the Star, or its diſtance from the Pole.

From the Meridian Altitude *H R* equal to 51 deg. 58 min. 50 ſec. take *H A*, the Complement of the Latitude of the *Obſervatory*, equal to 38 deg. 31 min. 30 ſec. the Remainder 13 deg. 27 min. 20 ſec. will be the Stars Declination, in this *Caſe* North, (becauſe the Stars Meridional Altitude is more than the Complement of the Latitude) and this taken from 90 Degrees, will leave 76 deg. 32 min. 00 ſec. for its diſtance from the Pole. Or,

To Z *R* the Stars Meridional Zenith-diſtance, equal to the Complement of the Meridional Altitude, equal to 38 deg. 01 min. 10 ſec. add Z *P*, equal to the Complement of the Latitude of the Place, equal to 38 deg. 31 min. 30 ſec. the Sum 76 deg. 32 min. 40 ſec. equal to *P R*, will be the Stars diſtance from the Pole, the ſame as before found; whence to find the Difference between the Right Aſcenſions.

In the Oblique-angled Spherical Triangle *R P α* are given, *R P* 76 deg. 32 min. 40 ſec. the diſtance of *Regulus* from the Pole, the Arch *P α* 69 deg. 11 min. 00 ſec. the diſtance of *Arcturus* from the Pole, and *R α* 59 deg. 46 min. 50 ſec. the conſtant diſtance between the two Stars, whence to find the Angle *R P α*, the difference between the Right Aſcenſion, from half the Sum of the three Arches equal to 102 deg. 45 min. 15 ſec. take the conſtant diſtance between the two Stars or Arch *R α*, 59 deg. 46 min. 50 ſec. and the Remainder will be 42 deg. 58 min. 25 ſec. then according to the 11*th Caſe* of *Oblique-angled Spherical Triangles.*

To the Arith. Comp. of the Sine *P r* equal to 76.32.40 0.0120878
Add the Arith. Comp. of the Sine of *P α* 69.11.00———0.0293174
Alſo the Sine of the half Sum 102.45.15 ——————9.9891499
And the Sine of the Remainder 42.58.25——————9.8335687

Half the Sum of theſe four Sines—————— 19.8641238
will be the Co-ſine of 31.13.10 which being doubled ————
will give 62.26.20 for ——————————9.9320619

The Difference of the Right Aſcenſions between the two Stars or Angle *r P α*; this therefore being ſubſtracted from the Right Aſcenſion of *Arcturus* 210 deg. 22 min. 30 ſec. becauſe *Arcturus* is in conſequentia of *Regulus*, will leave 147 deg. 56 min. 10 ſec. for the Right Aſcenſion of the *Lyon's Heart*.

Wherefore according to this Calculation, the Right Aſcenſion of *Regulus* in the Month of *February* in the Year 1690, is 147 deg. 56 min. 10 ſec. and its diſtance from the North Pole 76 deg. 32 min. 40 ſec. and after the ſame manner may the Right Aſcenſion of any other Star, from its Meridional Altitude and Diſtance from a known fixed Star be found, whence the Longitude and Latitude may be inveſtigated by the 2*d Caſe* of the preceding *Problem.*

Here follow a few *Examples* to Exerciſe the Learner.

Example 1.

On the 10*th* of *February* 1678, at 17h. 26m. at the *Obſervatory* at *Greenwich*, the diſtance between *Arcturus* and the Northermoſt of the two preceding Stars in the *Square* of the *Great Bear* was found to be 53 deg. 48 min. 45 ſec. in Antecedence, and on the 14*th* of *February* 1690, the greateſt Meridian Altitude of the ſame Star in the *Great Bear* was obſerved 78 deg. 03 min. 20 ſec. the Star then appearing to the Northward of the Zenith, at which time the Right Aſcenſion of *Acturus* was 210 deg. 22 min. 30 ſec. and its diſtance from the North Pole 69 deg. 11 min. 00 ſec. I demand the Right Aſcenſion, Declination, &c. of the Star in the *Back* of the *Great Bear.*

Anſwer.

The Right Aſcenſion is 161 deg. 02 min. 30 ſec. the Declination 63 deg. 25 min. 10 ſec. the Longitude *Leo* 10 deg. 49 min. 85 ſec. the Latitude 49 deg. 40 min. 05 ſec. North.

Example 2.

On the *9th* of *March* 1688, at *Greenwich*, the diſtance between *Arcturus* and the *Tail* of the *Lyon* was found by Obſervation to be 35 deg. 21 min. 40 ſec. the *Lyons Tail* being in Antecedence to *Arcturus*, and on the *9th* of *April* 1690, the Meridian Altitude of the *Lyons Tail* was determined to be 54 deg. 49 min. 05 ſec. at which time the Right Aſcenſion of *Arcturus* was 210 deg. 22 min. 30 ſec. and its diſtance from the North Pole 69 deg. 11 min. 00 ſec. I demand the Right Aſcenſion and Declination, *&c.* of the *Lyons Tail*.

Anſwer.

The Right Aſcenſion is 173 deg. 18 min. 10 ſec. the Declination 73 deg. 42 min. 25 ſec. North, the Longitude *Virgo* 17 deg. 19 min. 14 ſec. and the Latitude 12 deg. 16 min. 51 ſec North.

Example 3.

On the *17th* of *February* 1678, at *Greenwich*, the diſtance between the Southermoſt of the two preceding Stars in the *Square* of the *Great Bear* and *Arcturus*, was found to be 51 deg. 33 min. 10 ſec. the Star *Arcturus* being in Conſequence, and on the *1ſt* of *July* 1690, the Meridian Altitude of *Arcturus* was found by Obſervation to be 59 deg. 20 min. 30 ſec. at which time the Right Aſcenſion of the preceding Star in the *Bears Side* was 160 deg. 42 min. 30 ſec. and its diſtance from the North Pole 31 deg. 58 min. 00 ſec. I demand the Right Aſcenſion and Declination of *Arcturus*.

Anſwer

The Right Aſcenſion of *Arcturus* is 210 deg. 22 min. 30 ſec. its diſtance from the North Pole 69 deg. 11 min. 00 ſec. its Longitude *Libra* 19 deg. 53 min. 52 ſec. and its Latitude 30 deg. 57 min. 00 ſec. North.

Example 4.

On the *9th* of *March* 1688, the diſtance between the Joynt of the Southern *Wing* of the *Virgin* and *Arcturus* was meaſured and found to be 39 deg. 42 min. 10 ſec. the Star in the *Virgins Wing* being in Antecedence, and on the *13th* of *November* 1690, the Meridian Altitude of the Star in the *Virgins Wing* was taken and found to be 42 deg. 01 min. 25 ſec. the Right Aſcenſion of *Arcturus* at that time being 210 deg. 22 min. 30 ſec. and its diſtance from the Northern Pole 69 deg. 11 min. 00 ſec. I demand the Right Aſcenſion, Declination, &c. of the Star in the *Virgins Wing*.

Anſwer.

The Right Aſcenſion is 173 deg. 37 min. 10 ſec. the Declination 03 deg. 29 min. 55 ſec. North, the Longitude in *Virgo* 22 deg. 46 min. 14 ſec. and Latitude 00 deg. 47 min. 47 ſec.

Example 5.

On the 19*th* of *October* 1697, at *Greenwich*, the diſtance between the Left *Shoulder* of *Auriga* and *Aldebaran*, or the South *Eye* of the *Bull*, was found by Obſervation to be 33 deg. 24 min. 30 ſec. the *Bulls Eye* being in Antecedence, and on the ſame Night the Meridian Altitude of the Left *Shoulder* of *Auriga* was found to be 83 deg. 23 min. 30 ſec. at which time the Right Aſcenſion of *Aldebaran* was 64 deg. 39 min. 08 ſec. and its diſtance from the Northern Pole 74 deg. 10 min. 47 ſec. I demand the Right Aſcenſion, *&c.* of the Left *Shoulder* of *Auriga*.

Anſwer.

The Right Aſcenſion of the Left *Shoulder* of *Auriga* is 84 deg. 21 min. 15 ſec. its Declination 44 deg. 52 min. 16 ſec. North, its Longitude is *Gemini* 25 deg. 42 min. 12 ſec. and its Latitude 21 deg. 28 min. 20 ſec. North.

Example 6.

On the 19*th* of *October* 1697, at the ſame Place, the diſtance between *Pollux* and the ſame Star in the *Shoulder* of *Auriga*, was found to be 26 deg. 59 min. 15 ſec. the Left *Shoulder* of *Auriga* being in Antecedence of *Pollux*, and the ſame Evening the Meridian Altitude of the ſame Star was found to be 83 deg. 23 min. 30 ſec. at which time the Right Aſcenſion of *Pollux* was 111 deg. 41 min. 28 ſec. and its Declination 28 deg. 42 min. 55 ſec. North. I demand the Right Aſcenſion and Declination of the Left *Shoulder* of *Auriga*.

Anſwer.

The Right Aſcenſion is 84 deg. 21 min. 15 ſec. its Declination 44 deg. 52 min. 16 ſec. North, its Longitude *Gemini* 25 deg. 42 min. 12 ſec. *&c.* its Latitude 21 deg. 28 min. 08 ſec. North.

Prob. III.

The Altitude of an unknown Star or Planet taken at any time, together with its diſtance from a known fixed Star being given, to find the Right Aſcenſion, Declination, Longitude and Latitude of the unknown Star or Planet.

Example.

In the Year 1688, on the *9th* of *March*, at 9 h. 12 min. *P. M.* the Altitude of the Star in the Extremity of the Left *Wing* of the *Virgin* was found to be 33 deg. 40 min. 25 ſec. and at the ſame time her diſtance from *Arcturus* was meaſured 39 deg. 42 min. 10 ſec. in Antecedence, the Right Aſcenſion of *Arcturus* at that time being 210 deg. 21 min. 06 ſec. and its diſtance from the North Pole 69 deg. 10 min. 26 ſec. I demand the Latitude, Longitude, Right Aſcenſion and Declination of the Star in the Joynt of the Left *Wing* of the *Virgin*.

Conſtruction.

The Sun's Right Aſcenſion at the time of the Obſervation was 00 h. 00 m. 43 ſec. this taken from the Right Aſcenſion of *Arcturus* 14 h. 01 m. 24 ſ. will leave 14 h. 00 m. 41 ſ. for the time of the Culmination of *Arcturus*, from which taking away 9 h. 12 m. 00 ſ. the time given, the Remainder 4 h. 48 m. 41 ſec. equal to 72 deg. 10 min. 15 ſec. will be the Angle $ZP\alpha$, or difference of Right Aſcenſion between *Arcturus* and the *Mid Heaven*.

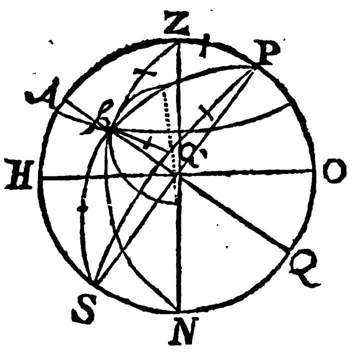
Or,

To the Sun's Right Aſcenſion 00 h. 00 m. 43 ſec. add the given Hour 9 hou. 12 m. 00 ſ. the Sum 9 h. 12 m. 43 ſ. will give the Right Aſcenſion of the *Mid-Heaven* which taken from the Right Aſcenſion of *Arcturus* 14 hou. 01 min. 24 ſec. will leave 4 h. 48 m. 41 ſ. equal to 72 deg. 10 min. 15 ſec for the Difference. Wherefore,

1. Having drawn the Meridian *ZONH*, the Horizon *HO*, and at Right-angles to it the Prime Vertical *ZN*, ſet off the Latitude of the Place 51 deg. 28 min. 30 ſec. from *O* to *P*, and draw the Axis *PS*, and at Right-angles to it the Equator *AQ*.

2. Thro' the Poles *P* and *S* draw the Circle of Right Aſcenſion $P\alpha S$, forming an Angle of 72 deg. 10 min. 15 ſec. equal to the difference of the Right Aſcenſion of *Arcturus* and the *Mid-Heaven*

by *Caſe* the 3*d* of *Problem* the 9*th* of *Section* the 2*d* of *Part* the 4*th*, and lay off 69 deg. 10 min 26 ſec. the diſtance of *Arcturus* from the Pole, from *P* to α, by *Caſe* the 3*d* of *Problem* the 6*th* of the ſame *Section*, then will α repreſent the place of *Arcturus*.



3. About the point α as a Pole, deſcribe a Small Circle at the diſtance between the two Stars, equal to 39 deg 42 min. 10 ſec. by *Caſe* the 3*d* of *Problem* the 4*th*, of *Section* the 2*d* of *Part* the 4*th*, and about the point Z as a Pole, deſcribe a Small Circle at the diſtance of 56 deg. 19 min. 35 ſec. the Complement of the Altitude of the unknown Star, by the 2*d Caſe* of the ſame *Problem*, where theſe two Small Circles interſect as in β, it will give the place of the unknown Star, and the Angle β *p* α the difference of the Right Aſcenſion between *Arcturus* and the unknown Star, may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th*, of *Section* the 2*d* of *Part* the 4*th*, and the Arch *P* β the diſtance of the Star from the North Pole, or Complement of the Declination, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th* of the ſame *Section*.

Logarithmically, or by Calculation.

1. In the Oblique-angled Spherical Triangle Z *P* α are given Z*P*, equal to the Complement of the Latitude of the Places, equal to 38 deg 31 min. 30 ſec. *P* α equal to the diſtance of *Arcturus* from the Pole, equal to 69 deg 10 min. 26 ſec and the Angle Z *P* α equal to 7 deg. 10 min 15 ſec equal to the difference of the Right Aſcenſion between *Arcturus* and the *Mid Heaven* ; whence to find firſt Z α equal to the Zenith diſtance of *Arcturus*, it will be by the 10*th Caſe* of *Oblique-angled Spherical Triangles* (having firſt conceived a Perpendicular to be let fall from Z upon *P S*, and which is omitted to avoid Confuſion)

As R : c s, Z *P* α : : t, Z *P* : t, of a 4*th* Arch ; that is,

As the Radius ———————— 10.0000000
To the Co-ſine of the Diff of *R A* 72.10 15 ———— 9 4859767
So is the Co-tangent of the Latitude 51 28.30 ———— 9.9009941
To the Tangent of a 4*th* Arch 13.41.$58\frac{3}{4}$ ———— 9.3869708

This foutrh Arch 13 deg. 41 min. 58 ſec. ¾, taken from the diſtance of *Arcturus* from the Pole, 69 deg. 10 min. 26 ſec. will leave 55 deg. 28 min. 27 ſec ¼ for a fifth Arch. Wherefore again,

As c s, 4*th* Arch : c s, 5*th* Arch : : c s, *P* Z : c s, α Z ; that is,
As the Co-ſine of the 4*th* Arch 13.41.58¾ ——— 9.9874655

To the Co-ſine of the 5*th* Arch 55 28 27¼ ——— 9.7534121
So is the Sine of the Latitude 51.28.30 ——— 9.8933936

To the Co-ſine of Z α the Zenith diſtance 62.50.43¼ — 9.6593402

Again, to find the Angle Z α *P*, it will be by the latter part of the 9*th Caſe* of *Oblique-angled*, *&c.*

As the S. 5*th* Arch : S. 4*th* : : t, <Z *P* α : t, <Z α *P*; that is,
As the Sine of the 5*th* Arch 55.28.27¼ ——— 9.9158594

To the Sine of the 4*th* Arch 13.41.58¾ ——— 9.3744419
So is the Tangent of Diff. of Right Aſcenſion 70.10.15 10.4926481

To the Tangent of the Angle Z α *P* 41.47.23¾ ——— 9.9512306

2. In the Oblique-angled Spherical Triangle Z β α are given Z β, the Complement of the Altitude of the unknown Star, 56 deg. 19 min. 25 ſec. β α the diſtance of the two Stars, 39 deg. 42 min. 10 ſec. and Z α the Zenith diſtance of *Arcturus* before found, 62 deg. 50 min. 43 ſec. ½, whence to find the Angle Z α β, (by *Caſe* the 11th of *Oblique-angled Spherical Triangles*) from half the Sum of the three Sides equal to 79 deg. 26 min. 14 ſec. ⅛, take the Zenith diſtance of the unknown Star 56 deg. 19 min. 35 ſec. and the Remainder will be 23 deg. 06 min. 39 ſec. ⅛; then,

To the Arith. Comp. of the Sine of β α 39.42.10 ——— 0.1946320
Add the Arith. Comp. of the Sine of Z α 62.50.43¼ ——— 0.0507184
Alſo the Sine of the half Sum 79.26.14⅛ ——— 9.9925780
And the Sine of the Remainder 23.06.39⅛ ——— 9.5938524

Half the Sum of theſe four Sines ——— 19.8317808

Will be the Co-ſine of 34.31.46 ——— 9.9158904

Whi

Which being doubled, will give 69 deg. 03 min. 32 ſec. for the Angle $Z\alpha\beta$, this therefore added to the Angle $Z\alpha P$ before found, equal to 41 deg. 47 min. 23 ſec. $\frac{3}{4}$, will give 110 deg. 50 min. 55 ſec. $\frac{3}{4}$, for the Angle $P\alpha\beta$.

3 In the Oblique-angled Spherical Triangle $P\alpha\beta$ are given, $P\alpha$ the diſtance of *Arcturus* from the Pole 69 deg. 10 min. 26 ſec. $\beta\alpha$ the diſtance between the two Stars, 39 deg. 42 min. 10 ſec. and the Angle contained $P\alpha\beta$ before found, equal to 110 deg. 50 min. 55 ſec. $\frac{3}{4}$, whence to find firſt the Angle $\beta P\alpha$, the Difference of Right Aſcenſion between *Arcturus* and the unknown Star, it will be by the *9th Caſe* of *Oblique-angled*, &c. having firſt imagined a Perpendicular let fall from β upon $P\alpha$ produced.

As R : c s, $P\alpha\beta$: : t, $\beta\alpha$: t, *6th* Arch ; that is,

As the Radius ———————————— 10.0000000

To the Co-ſine of $P\alpha\beta$ 110.50.55$\frac{3}{4}$ ———————— 9.5513321
So is the Tangent of $\beta\alpha$ the Diſt. *&c.* 39.42.10 ——— 9.9192339

To the Tangent of a *6th* Arch 16.27.45$\frac{1}{2}$ ———————— 9.4705660

This *6th* Arch 16 deg. 27 min. 45 ſec. $\frac{1}{2}$, added to $P\alpha$ the diſtance of *Arcturus* from the Pole 69 deg. 10 min. 26 ſec. will give 85 deg. 38 min. 11 ſec. $\frac{1}{2}$ for a ſeventh Arch, whence to find the Angle $\beta P\alpha$, it will be

A S. *7th* Arch : S. *6th* Arch : : t, $\beta\alpha p$ t : t, $\beta P\alpha$; that is,

As the Sine of the *7th* Arch 85.38.11$\frac{1}{2}$ ———————— 9.9987393

To the Sine of the *6th* Arch 16.27.45$\frac{1}{2}$ ———————— 9.4523810
So is the Tangent of the Angle $\beta\alpha P$ 110.50.55$\frac{3}{4}$ —— 10.4192579

To the Tang. of the Diff. of Right Aſcenſion 36.43.58$\frac{1}{2}$ 9.8728996

This Angle 36 deg. 43 min. 58 ſec. $\frac{1}{2}$, taken from 210 deg. 21 min. 00 ſec. the Right Aſcenſion of *Arcturus*, becauſe the Star is in Antecedence to *Arcturus*, will leave 173 deg. 37 min. 01 ſec. $\frac{1}{2}$, for the Right Aſcenſion of the unknown Star.

Again, to find its diſtance from the North Pole, it will be by the latter part of the *10th Caſe* of *Oblique-angled*, &c.

As cs, of the *6th* Arch : c s, *7th* Arch : : c s, $\beta\alpha$: c s, $P\beta$; that is,

As the Co-ſine of the *6th* Arch 16.27.45½ ———————9.9818210

To the Co-ſine of the *7th* Arch 85.38.11½ ——————8.8812891
So is the Co-ſine of the Arch of diſtance 39.42.10 ——— 9.8861344

To the Co-ſine of the diſtance from the Pole 86.30 2 8.7856025

Whence it appears that the Right Aſcenſion of the Star in the Joynt of the Left Wing of the *Virgin* is 173 deg. 37 min. 01 ſec. and its diſtance from the North Pole 86 deg. 30 min. 02 ſec. and conſequently its Declination is 3 deg. 29 min. 58 ſec. North.

The Right Aſcenſion and Declination being thus obtained, the Longitude and Latitude may be found by the *1ſt Prob.* of this *Section.*

If inſtead of the Hour of the Night the Altitude of *Arcturus* had been given, then in the Oblique-angled Spherical Triangle $ZP\alpha$ are given ZP the Complement of the Latitude of the Place 51 deg. 28 min. 30 ſec. the Complement of the Declination, or diſtance from the Pole of *Arcturus* 69 deg. 10 min. 20 ſec. and $Z\alpha$ the Complement of the Altitude of *Arcturus* 27 deg. 09 min. 16 ſec. ¾ before found; whence by the *11th Caſe* of *Oblique-angled Spherical Triangles*, the Angle $P\alpha Z$ will be found to be 41 deg. 47 min. 23 ſec. ¾. And again,

In the Oblique-angled Spherical Triangle $Z\beta\alpha$ are given, $Z\alpha$ the Complement of the Altitude of *Arcturus*, 27 deg. 09 min. 16 ſec. ¾, $Z\beta$ the Complement of the Altitude of the unknown Star, 33 deg. 40 min. 25 ſec. and the Arch $\beta\alpha$ the diſtance between the two Stars 39 deg. 42 min. 10 ſec. whence by the *11th Caſe* of *Oblique-angled Spherical Triangles* the Angle $Z\alpha\beta$ will be found to be 69 deg. 03 min. 32 ſec. which being added to the Angle $P\alpha Z$ before found, will give 110 deg. 50 min. 55 ſec. ¾, for the Angle P : Laſtly, in the Triangle $P\alpha\beta$ are given, $P\alpha$ the diſtance of *Arctura* from the North Pole 69 deg. 10 min. 26 ſec. the Arch $\beta\alpha$ the conſtant diſtance between the two Stars, and the contained Angle P before found 110 deg. 50 min. 55 ſec. ¾, whence by the *10th Ca* of *Oblique*, &c. the Arch $P\beta$ the diſtance of the unknown Star fro the North Pole, will be found to be 86 deg. 30 min. 02 ſec. and the *9th Caſe* of the ſame, the Angle $\beta P\alpha$ the Difference between t Right Aſcenſion of *Arcturus* and the unknown Star will be found be 36 deg. 43 min. 58 ſec. ½, this therefore taken from the Right Aſcenſion of *Arcturus* 210 deg. 21 min. 00 ſec. becauſe *Arcturus* is in Conſequence of the unknown Star, will leave 173 deg. 37 min. 01 ſec. for the unknown Stars Right Aſcenſion, whence the Longitude and Latitude may be inveſtigated by the firſt *Problem*.

Example 2.

In the Year 1682, on the 7*th* of *April*, at 9 h. 41 m. 00 ſec. at *Greenwich*, the Altitude of the *Lyon's Heart* was found by Obſervation to be 45 deg. 40 min. 55 ſec. its diſtance at that time from *Arcturus* being 59 deg. 46 min. 00 ſec. in Antecedence, now the Right Aſcenſion of *Arcturus* at that time being 210 deg. 16 min. 54 ſec. and its diſtance from the North Pole 69 deg. 08 min. 43 ſec. I demand the Latitude, Longitude, Right Aſcenſion and Declination of the *Lyon's Heart Regulus*.

Anſwer:

The Right Aſcenſion of the *Lyon's Heart Regulus* at that time was 147 deg. 44 min. 43 ſec. its Declination 11 deg. 31 min. 41 ſec. North, its Place was in *Leo* 25 deg. 24 min. 40 ſec. and its Latitude 00 deg. 26 min. 38 ſec. North.

Example 3.

At 9 h. 46 m. 00 ſ. on the 7*th* of *April*, in the Year 1682, the Altitude of the *Lyon's Tail* was found by Obſervation to be 54 deg. 51 min. 35 ſec. its diſtance from *Arcturus* being 35 deg. 21 min. 40 ſec. in Antecedence, the Right Aſcenſion of *Arcturus* at that time being 210 deg. 16 min. 54 ſec. and its diſtance from the Northern Pole 69 deg. 08 min. 43 ſec. I demand the Right Aſcenſion Declination, *&c.* of the Star in the *Lyon's Tail*.

Anſwer.

The Right Aſcenſion is 173 deg. 11 min. 58 ſec. its diſtance from the North Pole 73 deg. 39 min. 47 ſec. and conſequently its Declination is 16 deg. 20 min. 13 ſec. North, its Place is in *Virgo* 17 deg. 12 min. 34 ſec. and its Latitude 12 deg. 16 min. 51 ſec. North.

Example 4.

In the ſame Year on the ſame Day, and at the ſame Place, at 9 h. 37 m. 00 ſ. the Altitude of the middlemoſt Star of the three n the *Neck* of the *Lyon*, was found to be 56 deg. 13 min. 00 ſec. and its diſtance from *Arcturus* 55 deg. 19 min. 25 ſec. in Antecedence, now the Right Aſcenſion of *Arcturus* being 210 deg. 16 min. 54 ſec. and its diſtance from the North Pole 69 deg. 08 min. 43 ſec. I demand the Right Aſcenſion, Declination, *&c.* of the unknown or middlemoſt of the three Stars in the *Neck* of the *Lyon.*

Anſwer.

The Right Aſcenſion is 150 deg. 35 min. 14 ſec. its diſtance from the North Pole 68 deg. 34 min. 35 ſec. and conſequently its Declination is 21 deg. 25 min. 25 ſec. North, its Place is in *Leo* 25 deg. 08 min. 25 ſec. and its Latitude 8 deg. 47 min. 27 ſec. North.

Example 5.

In the Year 1678, on the 10*th* of *February*, at 17 h. 44 m. the Altitude of the Southermoſt of the two preceding Stars in the *Square* of the *Great Bear* was found to be 45 deg. 56 min. 20 ſec. and its diſtance from *Arcturus* 51 deg. 33 min. 10 ſec. in Antecedence, now the Right Aſcenſion of *Arcturus* at that time being 210 deg. 14 min. 06 ſec. and its diſtance from the North Pole 69 deg. 07 min. 34 ſec. I demand the Right Aſcenſion, Declination, *&c.* of the ſuppoſed unknown, or Southermoſt of the two preceding Stars in the *Square* of the *Great Bear.*

Anſwer.

The Right Aſcenſion is 160 deg. 31 min. 13 ſec. its diſtance from the North Pole 31 deg. 54 min. 14 ſec. and conſequently its Declination is 58 deg. 05 min. 46 ſec. its Place is in *Leo* 14 deg. 54 min. 12 ſec. and its Latitude 45 deg. 06 min. 16 ſec. North.

Example 6.

In the ſame Year on the ſame Day, and at the ſame Place, at 17 h. 34 m. the Altitude of the Star in the Root of the *Great Bears Tail*, was found to be 57 deg. 20 min. 35 ſec. and its diſtance from *Arcturus* 43 deg. 46 min. 50 ſec. now the Right Aſcenſion of *Arcturus* at that time being 210 deg. 14 min. 06 ſec. and its diſtance from the North Pole 69 deg. 07 min. 34 ſec. I demand the Right Aſcenſion, Declination, *&c.* of the Star in the *Root* of the *Great Bears Tail.*

Anſwer.

Its Right Aſcenſion is 179 deg. 50 min. 50 ſec. its diſtance from the North Pole, is 31 deg. 09 min. 16 ſec. and conſequently its Declination is 58 deg. 50 min. 44 ſec. North, its Place is in *Leo* 26 deg. 30 min. 40 ſec. and its Northern Latitude 51 deg. 39 min. 36 ſec.

Prob. IV.

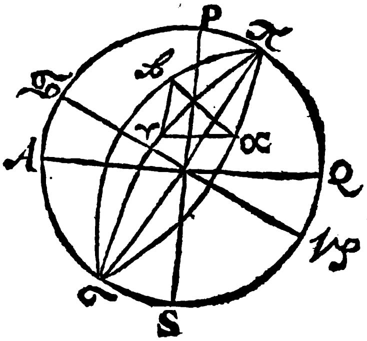
Given the Longitude and Latitude of two fixed Stars, together with the diſtance of a third fixed Star or Planet, from each of them to find out the Longitude and Latitude of that fixed Star or Planet.

Example.

On the 10*th* of *February*, 1678, at 17 h. 44 m. at the *Royal Obſervatory* at *Greenwich*, the diſtance between *Arcturus* and the Southermoſt of the two preceding Stars in the *Square* of the *Great Bear*, (commonly called the *Guards* or *Pointers*) was meaſured by a large *Sextant*, and found to be 51 deg. 33 min. 10 ſec. and on the 7*th* of *April* 1682, at 9 h. 41 m. the diſtance of *Arcturus* from the *Lyon's Heart Regulus* was meaſured by the ſame Inſtrument and found to be 59 deg. 46 min. 00 ſec. now the *Lyon's Heart* being in *Leo* 25 deg. 24 min. 40 ſec. and having 00 deg. 26 min. 38 North Latitude, at the time of the laſt Obſervation, and the Southermoſt of the two preceding Stars in the Square of the *Great Bear* being at the ſame time in *Leo* 14 deg. 57 min. 32 ſec. and having 45 deg. 06 min. 16 ſec. North Latitude. I demand the Longitude and Latitude of *Arcturus*.

The Stereographic Solution upon the Plane of the Solſtitial Colure.

1. Having drawn the Solſtitial Colure *P Q S A*, the Equator *A Q*, and at Right-angles to it the Equinoctial Colure *P S*, ſet off the Obliquity of the Ecliptic 23 deg. 29 min. 00 ſec. from *A* to ♋, or from *Q* to ♑, and draw the Ecliptic ♋ ♑, and at Right-angles to it its Axis *π σ*.



2. Thro' the Poles of the Ecliptic *π* and *σ*, draw the Great Circle *π β σ*,

forming an Angle with the Solſtitial Colure of 44 deg. 57 min 30 ſec. equal to the Difference of Longitude between the Solſtitial point ♋, and the Southermoſt of the two preceding Stars in the Square of the *Great Bear*, by *Caſe* the 3*d* of *Problem* the 9*th*, of *Section* the 2*d* of *Part* the 4*th*, and lay off upon it from π to β 44 deg. 53 min. 44 ſec. the Complement of the Latitude of the ſame Star, by *Caſe* the 3*d* of *Problem* the 6th of the ſame *Section*, then will β repreſent the Place of this Star in this Projection; alſo thro' the ſame Poles π and σ, draw the Great Circle π γ σ, forming an Angle with the Solſtitial Colure of 55 deg. 24 min. 40 ſec. equal to the Difference of Longitude, or Angle which the Circle of Longitude paſſing thro' the *Lyon*'s *Heart Regulus*, forms with the Solſtitial Colure, by *Caſe* the 3*d* of *Prob.* the 9*th*, of *Section* the 2*d* of *Part* the 4*th*, and lay off upon it from π to γ, 86 deg. 33 min. 22 ſec. the Complement of the Latitude of *Regulns*, by *Caſe* the 3*d* of *Problem* the 6*th*, of the ſame *Section*, then will γ repreſent the Place of the *Lyon*'s *Heart* in this Projection.

3. About the point β draw a SmallCircle at the diſtance of 51deg. 33min. 10 ſec. equal to the diſtance between theSouthern of the two preceding Stars in the Square of the *Great Bear*, by *Caſe* the 3*d* of *Problem* the 4*th*, of *Section* the 2*d* of *Part* the 4*th*; alſo about the point γ, draw a Small Circle at the diſtance of 59 deg. 46 min. 00 ſec. equal to the diſtance between the *Lyon*s *Heart* and *Arcturus*, by the ſame *Caſe*, and where theſe two Small Circles interſect each other as in the point α, it will give the Place of *Arcturus* in this Projection, thro' which point and the Poles of the Ecliptic π and σ, if the Great Circle π α σ be drawn, by the 1*ſt* *Problem* of *Section* the 2*d* of *Part* the 4*th*, the thing is done; and the Angle ♋ π α the Difference of Longitude between the Solſtitial point ♋ and *Arcturus*, may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th*, of *Section* the 2*d* of *Part* the 4*th*, and the Arch π α, the diſtance of *Arcturus* from the North Pole of the Ecliptic, or Complement of its Latitude, may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th* of the ſame *Section*.

Logarithmically, or by Calculation.

Becauſe the *Lyon*'s *Heart* is in Conſequence of the Southermoſt of the two preceding Stars in the Square of the *Great Bear*, from the Place of *Regulus* ♌, 25 deg. 24 min. 40 ſec. take the Place of the Southermoſt of the two preceding Stars in the Square of the *Great Bear* ♌, 14 deg. 57 min. 32 ſec. and the Reſidue 10 deg. 27 min. 08

08 ſec. equal to the Difference of Longitude between the two known Stars, will be equal to the Angle in the Figure $\beta\pi\gamma$, wherefore in the Oblique-angled Spherical Triangle $\beta\pi\gamma$ are given, $\beta\pi$, 44 deg. 53 min. 44 ſec. the Complement of the Latitude of the Southermoſt of the two preceding Stars in the Square of the *Great Bear* $\pi\gamma$, 89 deg. 33 min. 22 ſec. equal to the Complement of the Latitude of *Regulus*, and the Angle $\beta\pi\gamma$ 10 deg. 27 min. 08 ſec. equal to the Difference of Longitude between the two Stars, whence to find 1*ſt*, $\beta\gamma$, the diſtance between the two known Stars, it will be by the 10*th Caſe* of *Oblique angled*, &c. having firſt imagined a Perpendicular to fall from the point β upon the Side $\pi\gamma$.

As R : c s, $<\beta\pi\gamma$: : t, $\beta\pi$: t, of a 4*th* Arch ; that is.
As the Radius ——————————— 10.0000000

To the Co-ſine of the Diff. of Longit. $\beta\pi\gamma$ 10.27.08 —— 9.9927331
So is the Co-tang. of the Lat. of the Star β 45.06.16 —— 9.9984166

To the Tangent of a 4*th* Arch 44.24.58½ —————— 9.9911497

Which fourth Arch 44 deg. 24 min. 58 ſec. ½, taken from the Complement of the Latitude of the Star *Regulus*, equal to 89 deg. 33 min. 22 ſec. will leave 45 deg. 08 min. 23 ſec. ½ for a fifth Arch

And again,

As c s, 4*th* Arch : c s, 5*th* Arch : : c s, $\beta\pi$: c s, $\beta\gamma$; that is,
As the Co-ſine of the 4*th* arch 44 24.58½ —————— 9.8538650

To the Co-ſine of the 5*th* arch 45.08.23½ ———— 9.8484222
So is the Sine of the Latitude of the Star β 45.06.16 —— 9.8502752

To the c s, of the diſt. betw. the 2 Stars $\beta\gamma$ 45.36.26¼ 9.8448324

And again for the Angle $\beta\gamma\pi$, it will be by the latter part of the 9*th Caſe* of *Oblique-angled Spherical Triangles*,

As the S. 5*th* arch : S. 4*th* arch : : t, $<\beta\pi\gamma$: t, $<\beta\gamma\pi$; that is,
As the Sine of the fifth arch 45.08.23½ —————— 9.850542

To the Sine of the fourth arch 44.24.58½ ———— 9.8450150
So is the Tangent of the Angle $\beta\pi\gamma$ 10 27.08 ———— 9.2659414

To the Tangent of the Angle $\beta\gamma\pi$ 10.19.22½ ———— 9.2604139

2. In the Oblique-angled Spherical Triangle $\beta\gamma\alpha$ are given, $\beta\alpha$, equal to 51 deg. 33 min. 10 ſec. the diſtance between the Southermoſt of the two preceding Stars in the Square of the *Great Bear* and *Arcturus*, $\alpha\gamma$, the diſtance between *Arcturus* and the Star in the *Lyon's Heart Regulus*, equal to 59 deg. 46 min. 00 ſec. and $\beta\gamma$, the diſtance between *Regulus* and the Southermoſt of the two preceding Stars in the Square of the *Great Bear*; whence to find the Angle $\beta\gamma\alpha$, by the 11*th Caſe* of *Oblique-angled Spherical Triangles*, from 78 deg. 27 min. 48 ſec. half of the Sum 156 deg. 55 min. 36 ſec. of the three Sides, take $\beta\alpha$, the diſtance between the Southermoſt of the two Stars in the *Great Bear* and *Arcturus*, 51 deg. 33 min. 10 ſec. and the Remainder will be 26 deg. 54 min. 30 ſec. *Then*,

To the Arith. Comp. of the Sine of $\alpha\gamma$ 59.46.00 —— —— 0.0634953
Add the Arth. Comp. of the Sine of $\beta\gamma$ 45.36.26 —— 0.1459603
Alſo the Sine of the half Sum 78.27 48 —— —— 9.9911360
And the Sine of the Remainder 26.54.38 —— —— 9.6557135

Half the Sum of theſe four Sines —— —— —— 19.8563051

Will be the Co-ſine of $32.03.23\frac{1}{4}$ —— —— —— 9.9281526

Which being doubled, will give 69 deg. 03 min. 32 ſec. for the Angle $\beta\gamma\alpha$; from which therefore taking the Angle $\beta\gamma\pi$ before found, equal to 10 deg. 19 min. 22 ſec. $\frac{1}{2}$, the Remainder will give 53 deg. 47 min. 24 ſec. for the Angle $\pi\gamma\alpha$.

3. In the Oblique-angled Spherical Triangle $\pi\gamma\alpha$ are given, $\pi\gamma$, the diſtance of the *Lyons Heart Regulus* from the Pole of the Ecliptic π, or Complement of its Latitude 89 deg. 33 min. 32 ſec. $\gamma\alpha$ the diſtance between the *Lyons Heart* and *Arcturus* 59 deg. 46 min. 00 ſec. and the contained Angle $\pi\gamma\alpha$ before found equal to 53 deg. 47 min. 24 ſec. whence to find the Angle $\gamma\pi\alpha$, or the Difference of Longitude between the *Lyon's Heart* and *Arcturus*, it will be by the 9*th Caſe* of *Oblique-angled*, &c. having firſt imagined a Perpendicular let fall from α upon the Arch or Side πR.

As R : c s, $<\pi\gamma\alpha$: : t, $\gamma\alpha$: t, of a 6*th* Arch; that is,

As the Radius —— —— —— 10.0000000
To the Co-ſine of the Angle $\pi\gamma\alpha$ 53.47.24 —— —— 9.7714012
So is the Tangent $\gamma\alpha$ the diſtance &c. 59 46.00 —— 10.2344857
To the Tangent of a 6*th* arch $45.23.18\frac{1}{2}$ —— —— 10.0058869

This ſixth arch 45 deg. 23 min. 18 ſec. ½, ſubſtracted from $\pi\gamma$, the Complement of the Latitude of the *Lyon's Heart*, equal to 89 deg. 33 min. 22 ſec. will give 44 deg 10 min. 03 ſec. ½, for a *7th* arch, whence to find the Angle $\gamma\pi\alpha$ it will be,

As S. *7th* arch : S. *6th* arch : t, $<\pi\gamma\alpha$: t, $<\gamma\pi\alpha$; that is,

As the Sine of the *7th* arch 44.10.03½ ———— 9.8430833

To the Sine of the *6th* Arch 45.23:18½ ———— 9.8524097
So is the Tangent of the $<\pi\gamma\alpha$ 53.47.24 ———— 10.1353956

To the Tangent of the $<\gamma\pi\alpha$ 54.22.29 ———— 10.1447220

This Angle $\gamma\pi\alpha$ 54 deg. 22 min. 29 ſec. or the arch of the Ecliptic $\alpha\gamma$, added to the Place of the *Lyon's Heart Leo* 25 deg. 24 min. 40 ſec. becauſe *Arcturus* is in Conſequence of *Regulus* will give *Libra* 29 deg. 47 min. 09 ſec. for the Place of *Arcturus* at the time of the laſt Obſervation.

Again, to find $\pi\alpha$, its diſtance from the Pole of the Ecliptic π, or Complement of its Latitude, it will be by the latter part of the *9th Caſe* of *Oblique*, &c.

As c s, *6th* arch : c s, *7th* arch : : c s, $\gamma\alpha$: c s, $\pi\alpha$; that is,

As the Co-ſine of the *6th* arch 45.23.18½ ———— 9.8465205

To the Co-ſine of the *7th* arch 44.10.03½ ———— 9.8557035
So is the Co-ſine of the diſtance $\gamma\alpha$ 59.46.00 ———— 9.7020190

To the Sine of Latitude 30.56.58½ ———— 9.7112020

Whence it appears, that at the time of the laſt Obſervation, which was in *April* 1682, that the Star *Arcturus* was in *Libra* 19 deg. 47 min. 09 ſec. having 30 deg. 56 min. 58 ſec. ½ North Latitude.

The Longitude and Latitude being thus obtained, the Right Aſcenſion and Declination may be found by the *2d Caſe* of the *1ſt Problem*.

In the firſt Triangle $\beta\pi\gamma$, where the two Sides $\beta\pi$, $\pi\gamma$, and the contained Angle $\beta\pi\gamma$ are given, if inſtead of the Angle $\beta\gamma\pi$: the Angle $\pi\beta\gamma$ had been found, and in the ſecond Triangle $\beta\alpha\gamma$, where the three Sides $\beta\alpha$, $\beta\gamma$, and $\gamma\alpha$, are given, if inſtead of the Angle βγα the Angle αβγ had been found, then the Difference of the Angles πβα and αβγ would have been the Angles πβα, and conſequently in the third Triangle πβα, there would have been given the two Sides πβ, and βα; and the contained Angle πβα, whence the Side πα the diſtance of *Arcturus* from the North Pole of the Ecliptic might have been found, by the **10th** ***Caſe*** of *Oblique*, &c. and the Angle βπα, the Difference of Longitude between the Star β and *Arcturus*, by the ***9th Caſe*** of the ſame, and being added to the Longitude of the Star β, becauſe the Star β is Antecedent to α, will give the Place of α in the Ecliptic, or its Longitude from the firſt point of *Aries*.

Example 2.

Let us now ſuppoſe the Southermoſt of the two preceding Stars in the Square of the *Great Bear* to be the third unknown Star, whoſe diſtance from *Arcturus* is 51 deg. 33 min 10 ſec. (*Arcturus* being in *Libra* 19 deg. 47 min. 9 ſec. and having 30 deg. 57 min. 00 ſec. North Latitude) and from the *Lyon's Heart Regulus* 45 deg. 36 min. 25 ſec. *Regulus* being in *Leo* 25 deg. 24 min. 40 ſec. and having 00 deg. 26 min. 38 ſec. North Latitude, and let it be required to find the Longitude and Latitude of the unknown Star in the Square of the *Great Bear*.

1. In the Oblique-angled Spherical Triangle γπα are given, γπ, equal to 89 deg. 33 min. 22 ſec. and πα 59 deg. 03 min. 00 ſec. the diſtances of *Regulus* and *Arcturus* from the North Pole of the Ecliptic, and the contained Angle γπα 54 deg. 22 min 32 ſec. the Difference of Longitude between *Regulus* and *Arcturus*, whence by the 10*th Caſe* of *Oblique-angled*, &c. the Arch γα, the diſtance between *Regulus* and *Arcturus* will be found to be 59 deg. 46 min. 00 ſec. and by the 9*th Caſe* of the ſame, the Angle πγα will be found to be 53 deg. 47 min 24 ſec.

2. In the Oblique angled Spherical Triangle βαγ are given, βα 51 deg. 33 min. 10 ſec. βγ 45 deg 36 m. 25 ſ. and γα 59 deg. 46 min. 00 ſec. the mutual diſtances between the three fixed Stars, whence by the 11*th Caſe* of *Oblique-angled*, &c. the Angle βγα will be found to be 64 deg. 06 min. 46 ſec. from which taking away the Angle πγα, 53 deg. 47 min. 24 ſec. before found, there will remain the Angle βγπ 10 deg. 19 min. 22 ſec.

3. In the Oblique-angled Spherical Triangle γβπ are given, γπ 89 deg. 33 min. 22 ſec. the Complement of the Latitude of *Reg*
and γβ 45 deg. 36 min. 25 ſec. its diſtance from the unknown St

together with the contained Angle $\beta\gamma\pi$ 10 deg. 19 min. 22 ſec whence by the 10*th Caſe* of *Oblique-angled*, &c. the Side $\pi\beta$ the Complement of the unknown Stars Latitnde, will be found to be 44 deg. 53 min. 44 ſec. and by the 9*th Caſe* of the ſame, the Angle $\beta\pi\gamma$ the Differenee of Longitude between *Regulus* and the unknown Star, 10 deg. 27 min. 08 ſec. this therefore ſubſtracted from the Place of *Regulus*, *Leo* 25 deg. 24 min. 40 ſec. (becauſe the unknown Star is Antecedent to *Regulus*, will leave *Leo* 14 deg. 57 min. 32 ſec. for the Place of the unknown Star, or Southermoſt of the two preceding Stars in the Square of the *Great Bear*.

Example 3.

Let us now ſuppoſe the *Lyon's Heart* to be the third unknown Star, whoſe diſtance from the Southermoſt of the two preceding Stars in the Square of the *Great Bear* is 45 deg. 36 min. 25 ſec. (the Star in the *Great Bear* being in *Leo* 14 deg. 57 min. 32 ſec. and having 45 deg. 06 min. 16 ſec. North Latitude) and from *Arcturus* to be 59 deg. 46 min. 00 ſec. *Arcturus* being in *Libra* 19 deg. 47 min. 09 ſec. and having 30 deg. 57 min. 00 ſec. North Latitude, and let it be required to find the Latitude and Longitude of *Arcturus*.

1. In the Oblique-angled Spherical Triangle $\beta\pi\alpha$ are given, $\pi\beta$, 44 deg. 53 min. 44 ſec. and $\pi\alpha$, 59 deg. 03 min. 00 ſec. the Complements of the Latitudes of the two known Stars, alſo the contained Angle $\beta\pi\alpha$, 64 deg. 49 min. 40 ſec. the Difference of Longitude between the two Stars, whence by the 10*th Caſe* of *Oblique-angled*, &c. the Arch $\beta\alpha$ the diſtance between *Arcturus* and the unknown Star, will be found to be 51 deg. 33 min. 10 ſec. and by the 9*th Caſe* of the ſame, the Angle $\pi\alpha\beta$ will be found to be 54 deg. 39 min. 00 ſec.

2. In the Oblique-angled Spherical Triangle $\beta\gamma\alpha$ are given, $\beta\alpha$, 51 deg. 33 min. 10 ſec. $\beta\gamma$, 45 deg. 36 min. 25 ſec. and $\gamma\alpha$, 59 deg. 46 min. 00 ſec. the mutual diſtances between the three Stars, whence by the 11*th Caſe* of *Oblique-angled*, &c. the Angle $\beta\alpha\gamma$, will be found to be 55 deg. 10 min. 05 ſec. which therefore added to the Angle $\beta\alpha\pi$, 54 deg. 39 min. 00 ſec. before found, will give 109 deg. 49 min. 05 ſec. for the Angle $\pi\alpha\gamma$.

3. In the Oblique-angled Spherical Triangle $\pi\alpha\gamma$ are given, $\pi\alpha$, the Complement of the Latitude of *Arcturus*, 59 deg. 03 min. 00 ſec. and $\alpha\gamma$, its diſtance from the unknown Star *Regulus*, 59 deg.

46 min. 00 ſec. and the contained Angle $\pi\alpha\gamma$, 109 deg. 49 min. 05 ſec. whence by the 10*th* *Caſe* of *Oblique-angled*, &c. the Side $\pi\gamma$, the Complement of the Latitude of the unknown Star *Regulus*, will be found to be 89 deg. 33 min. 22 ſec. and by the 11*th* *Caſe* of the ſame, the Angle $\gamma\pi\alpha$, the Difference of Longitude between *Arcturus* and the unknown Star *Regulus*, will be found to be 54 deg. 22 min. 32 ſec. which therefore taken from the Place of *Arcturus*, *Libra* 19 deg. 47 min. 12 ſec. will leave *Leo* 25 deg. 24 min. 40 ſec. for the Place of the unknown Star *Regulus*.

Or,

Having from the Side $\pi\alpha$, 59 deg. 03 min. 00 ſec. and $\pi\beta$, 44 deg. 53 min. 44 ſec. and the contained Angle $\beta\pi\alpha$, 64 deg. 49 min. 40 ſec. found out the Angle $\pi\beta\alpha$, 82 deg. 19 min. 42 ſec. and from the three Sides $\beta\alpha$, 51 deg. 33 min. 10 ſec. $\beta\gamma$, 45 deg. 36 min. 25 ſec. and $\gamma\alpha$, 59 deg. 46 min. 00 ſec. of the Triangle $\beta\gamma\alpha$ found out, the Angle $\alpha\beta\gamma$ 82 deg. 57 min. 58 ſec. the Sum of theſe two Angles 165 deg. 17 min. 40 ſec. will be the Angle $\pi\beta\gamma$; ſo that in the Triangle $\pi\beta\gamma$ are given, $\pi\beta$, 44 deg. 53 min. 44 ſec. the Side $\beta\gamma$, 45 deg. 36 min. 25 ſec. and the Angle $\pi\beta\gamma$, 165 deg. 17 min. 40 ſec. whence by the 10*th* *Caſe* of *Oblique-angled*, &c. the Side $\pi\gamma$, the Complement of the Latitude of the unknown Star will be found again to be 89 deg. 33 min. 22 ſec. and by the 9*th* *Caſe*, the Angle $\beta\pi\gamma$, the Difference of Longitude between the Star in the *Great Bear* and *Regulus*, will be found to be 10 deg. 27 min. 08 ſec. this therefore added to the Place of the known Star in the *Great Bear*, *Leo* 14 deg. 57 min. 32 ſec. becauſe the known Star is Antecedent to *Regulus*, will give *Leo* 25 deg. 24 min. 40 ſec. for the Place of the unknown Star *Regulus*.

In the firſt of theſe *Examples*, the unknown fixed Star is in Conſequence of, or has more Longitude than either of the fixed Stars, from whence it was taken; and conſequently the Difference of Longitude found muſt be added.

In the ſecond *Example* the unknown fixed Star is Antecedent to, or has leſs Longitude than either of the fixed Stars from whence it was taken, and therefore the Difference of Longitude found muſt be ſubſtracted, and in the 3*d* *Example* the unknown Star falling between the two known Stars, will have greater Longitude than one and leſs than the other, and conſequently the Difference of Longitude found, muſt be either added or ſubſtracted, according as the known Star from whence the Difference is computed, is either in Antecedence

Ante-

or Conſequence of the unknown Star; and theſe being all the Varities that can ariſe in this *Caſe*, from the different Poſitions of the unknown Star, in reſpect to the two known Stars, I ſhall only ſubjoin for the ſake of the Induſtrious *Reader*, a few *Examples* of Diſtances of each of the Planets taken from ſome noted fixed Stars, together with their Places deduced from them.

Example 1.

On the 30*th* of *March* 1688, at 11 h. 40 m. at the *Royal Obſervatory* at *Greenwich*, the diſtance of *Saturn* from the *Lyon*'s *Heart Regulus*, was found by Obſervation to be 56 deg. 18 min. 15 ſec. *Regulus* at that time being in *Leo* 25 deg. 29 min. 40 ſec. and having 0 deg. 26 min. 38 ſec. North Latitude, and at the ſame time the diſtance of *Saturn* from *Arcturus* was found to be 28 deg. 13 min. 00 ſec. *Arcturus* at that time being in *Libra* 19 deg. 52 min. 12 ſec and having 30 deg. 57 min. 00 ſec. North Latitude. I demand the Latitude and Longitude of *Saturn* at the time of the Obſervation.

Anſwer.

The Place of *Saturn* is *Libra* 21 deg. 47 min. 49 ſec. and Latitude 2 deg. 46 min. 51 ſec. North.

Example 2.

On the 28*th* of *Auguſt* 1688, at 8 h. 16 m. at the ſame Place, the diſtance of *Jupiter* from the preceding Shoulder of *Aquarius*, was meaſured and found to be 32 deg. 48 min. 40 ſec. the preceding Shoulder of *Aquarius* being at that time in *Aquarius* 19 deg. 03 min. 03 ſec. and having 8 deg. 38 min. 43 ſec. North Latitude, and at the ſame time the diſtance of *Jupiter* from the following of the two Stars in the Tail of the *Eagle*, was found to be 36 deg. 45 min. 15 ſec. the following of the two Stars in the Tail of the *Eagle* being in *Capricorn* 15 deg. 27 min. 14 ſec. and having 36 deg. 13 min. 48 ſec. North Latitude. I demand the Latitude and Longitude of *Jupiter*, at the time of the Obſervation.

Anſwer.

Jupiter is in *Capricorn* 17 deg. 25 min. 06 ſec. having 00 deg. 30 min. 30 ſec. North Latitude.

Example 3.

On the 29*th* of *September* 1687, at 6 h. 14 m. at the ſame Place, the diſtance of *Mars* from the following Star of the three in the Head of *Sagittary*, was found to be 35 deg. 41 min. 15 ſec. the Star being then in *Capricorn* 11 deg. 54 min. 54 ſec. and having 01 deg. 28 min. 59 ſec. North Latitude, and at the ſame time the diſtance of *Mars* from the Bright Star in the *Eagle* was found to be 37 deg. 53 min. 30 ſec. the Bright Star in the *Eagle* being then in *Capricorn* 27 deg. 21 min. 34 ſec. and having 29 deg. 19 min. 11 ſec. North Latitude I demand the Latitude and Longitude of *Mars* at the time of the Obſervation.

Anſwer.

The Place of *Mars* was in *Aquarius* 17 deg. 17 min. 12 ſec. and its Latitude 3 deg. 26 min. 22 ſec. South.

Example 4.

On the 11*th* of *February* 1686, at 6 h. 16 m. the diſtance of *Venus* from the Head of *Andromeda*, was found to be 24 deg. 18 min. 20 ſec. the Head of *Andromeda* at that time being in 09 deg. 55 min. 33 ſec. of *Aries*; and having 25 deg. 41 min. 01 ſec. North Latitude, and at the ſame time the diſtance of *Venus* from *Palilicium* or the Southern Eye of the *Bull* was found to be 46 deg. 54 min. 40 ſec. *Palilicium* at that time being in *Gemini* 5 deg. 23 min. 40 ſec. and having 5 deg. 29 min. 49 ſec. South Latitude. I demand the Latitude and Longitude of *Venus* at the time of the Obſervation.

Anſwer.

The Place of *Venus* was at that time in *Aries* 19 deg. 07 min. 45 ſec. and her Latitude 3 deg. 2 min. 36 ſec. North.

Example 5.

On the 14*th* of *April* 1680, at 8 h. 27 m. *Mercury* was diſtant from *Procyon* or the *Little Dog* Star, 55 deg. 47 min. 30 ſec. *Procyon* at that time being in *Cancer* 21 deg. 22 min. 01 ſec. having 15 deg. 57 min. 55 ſec. South Latitude, and at 8 h. 15 m. the ſame Evening *Mercury* was found to be diſtant from the Northern Horn of the *Bull*, or the Southern Foot of *Auriga* 22 deg. 11 min. 55 ſec. the North Horn of the *Bull* at that time being in *Gemini* 18 deg. 05

min. 36 ſec. and having 5 deg. 21 min. 34 ſec. North Latitude. demand the Latitude and Longitude of *Mercury* at the time of the Obſervation.

Anſwer.

Mercury at that time was in *Taurus* 26 deg. 02 min. 42 ſec. having 2 deg. 46 min. 30 ſec. North Latitude.

Example 6.

On the 27*th* of *January* 1688, at 6 h. 44 m. 15 ſ. the diſtance of the neareſt Limb of the *Moon* from *Mirach*, or the Bright Star in the Girdle of *Andromeda*, was found to be 27 deg. 13 min. 05 ſec. *Mirach* at that time being in *Aries* 26 deg. 01 min. 04 ſec. and having 25 deg. 56 min. 19 ſec. North Latitude, and at the ſame time the diſtance of the remote Limb of the *Moon* from *Palilicium* or *Aldebaran*, was found to be 41 deg. 13 min. 25 ſec. *Palilicium* being at that time in *Gemini* 5 deg. 25 min. 20 ſec. and having 5 deg. 29 min. 49 ſec. South Latitude. I demand the Latitude and Longitude of the *Moon* at the time of the Obſervation.

Anſwer.

The Moon was at that time in *Aries* 24 deg. 34 min. 30 ſec: and her Latitude was 1 deg. 30 min. 48 ſec. South.

Prob. IV.

Caſe 2.

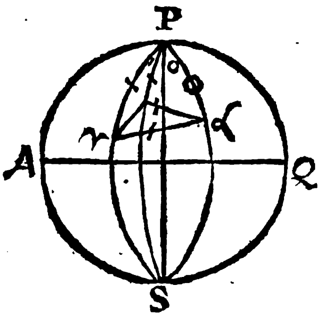
Given the Right Aſcenſion and Declination of two fixed Stars, together with the diſtance of a third fixed Star or Planet, from each of them, to find out the Right Aſcenſion and Declination of that third Star or Planet.

Example.

On the 10*th* of *February* 1678, at 17 h. 44 m. *Arcturus* was diſtant from the Southermoſt of the two preceding Stars in the Square of the *Great Bear* 51 deg. 33 min. 10 ſec. and on the 7*th* of *April* in the Year 1682, at 9 h. 41 m. *Arcturus* was diſtant from *Regulus* 59 deg. 46 min. 00 ſec. now the Right Aſcenſion of the Southermoſt of the two preceding Stars in the Square of the *Great Bear* being at the time of the latter Obſervation 160 deg. 34 min. 59 ſec. and ha-

having 58 deg. 04 min. 30 ſec. North Declination, and the Right Aſcenſion of *Regulus* at the ſame time being 147 deg. 50 min. 33 ſec. and its Northern Declination 13 deg. 29 min. 31 ſec. I demand he Right Aſcenſion and Declination of *Arcturus.*

The Stereographic Solution.



Having drawn the Solſtitial Colure *A P Q S,* the Equator *A Q,* the Axis *P S,* thro' the Poles *P* and *S,* draw the Circles of Right Aſcenſion *P* γ *S, P* β *S,* the firſt forming an Angle of 57 deg. 50 min. 33 ſec. the ſecond an Angle of 70 deg. 34 min. 50 ſec. with the Solſtitial Colure, equal to the difference of Right Aſcenſions between the two Stars and the Solſtitial point ♋, by *Caſe* the 3*d* of *Problem* the 9*th,* of *Section* the 2*d* of *Part* the 4*th,* and lay off 76 deg. 30 min. 29 ſec. in the 1*ſt* from *P* to γ, and 31 deg. 55 min. 30 ſec. in the 2*d* from *P* to β, the diſtance of each of the Stars from the North Pole, by *Caſe* the 3*d* of *Problem* the 6*th* of the ſame *Section,* theſe points γ and β, will repreſent the Places of the two Stars.

2. About the points γ and β, deſcribe Small Circles at the diſtance of 59 deg. 46 min. 00 ſec. and 51 deg. 33 min 10 ſec. by *Caſe* the 3*d* of *Prob.* the 4*th* of *Sect.* the 2*d,* and where theſe interſect each other as in the point α, it will give the Place of *Arcturus,* and *P* α will give the diſtance of *Arcturus* from the Pole and the Angle *A P* α the Right Aſcenſion from *Cancer,* and to find each of them by Calculation.

1. In the Oblique-angled Spherical Triangle γ *P* β are given, *P* γ, 76 deg. 30 min. 29 ſec. *P* β, 31 deg. 55 min. 30 ſec. the diſtances of the two known Stars from the Pole, and the Angle γ *P* β, 12 deg. 44 min. 26 ſec. the Difference of their Right Aſecenſions, whence by the 10*th Caſe* of *Oblique, &c.* the Arch of Diſtance γ β, will be found to be 45 deg. 36 min. 25 ſec. and by the 9*th Caſe* of the ſame, the Angle *P* γ β will be found to be 9 deg. 23 min. 35 ſec

2. In the Oblique-angled Spherical Triangle γ β α are given, the mutual diſtances between the three Stars, *viz.* β γ, 45 deg. 36 min. 25 ſec. γ α, 59 deg. 46 min. 00 ſec. and β α, 51 deg. 33 min. 1 ſec. whence by the 11*th Caſe* of *Oblique,* &c. the Angle β γ α, w

b

be found to be 64 deg. 06 min. 46 ſec. this added to the Angle $P\gamma\beta$ before found, equal to 9 deg. 23 min. 35 ſec, will give 73 deg. 30 min. 21 ſec. for the Angle $P\gamma\alpha$.

3. In the Oblique-angled Spherical Triangle $P\gamma\alpha$ are given, $P\gamma$, equal to 76 deg. 30 min. 29 ſec. $\gamma\alpha$, equal to 45 deg. 36 min. 25 ſec. and the contained Angle $P\gamma\alpha$, equal to 73 deg. 30 min. 21 ſec. whence by the 10*th Caſe* of *Oblique*, &c. the Arch $P\alpha$ the diſtance of *Arcturus* from the North Pole, or the Complement of its Declination will be found to be 69 deg. 08 min. 41 ſec. ¼, and by the 9*th Caſe*, the Angle $\gamma P\alpha$ the Difference of the Right Aſcenſion between *Regulus* and *Arcturus* will be found to be 62 deg. 26 min. 16 ſec. ¼, this therefore added to the Right Aſcenſion of *Regulus* 147 deg. 50 min. 33 ſec. becauſe *Regulus* precedes *Arcturus*, will give 210 deg. 16 min 49 ſec. ¼, for the Right Aſcenſion of *Arcturus* at the time of the Obſervation.

After the ſame manner may the Right Aſcenſion and Declination of any unknown Star, Planet or Comet be determined, from the ſame Data, and all the Varieties ariſing from the different Poſitions of the unknown Stars, with reſpect to the known, are accounted for after the ſame manner as in the former *Caſe*, as the *Reader* will readily apprehend, if he compare the two Conſtructions together, and imagine the Poles π and σ, in the former to repreſent the Points P and S in the latter, and the Ecliptic in the firſt, to ſtand for the Equator in the latter.

By the help of theſe *Problems* and a Knowledge of the Place of any one fixed Star, we are taught, how to find the Places of all the reſt; and 'tis by the Aſſiſtance of the preceding *Problem* chiefly, that *Aſtronomers* have rectified the Places of all the fixed Stars, and determined the Places of the *Moon* and the reſt of the *Planets*; upon which the Tables of their ſeveral Motions are founded, but Obſervations of this kind requiring Three Perſons at leaſt to aſſiſt, beſides a long train of Calculations, before their Latitudes, Longitudes, Right Aſcenſions, and Declinations, can be obtained, Mr. *Flamſteed* ſet himſelf about making a Meridional Arch, which tho' it had been before attempted by others, and particularly by the famous *De La Hire* at *Paris*, (as he himſelf tells us) without Succeſs; yet Mr. *Flamſteed*'s unwearied Endeavours and conſtant Application proceeding from, and ſupported by an exceeding Love to this Science, made him ſurmount all Difficulties, and in the Space of Fourteen Months, at his own Expence, he erected a Meridional MURAL ARCH of near Eight

Foot Radius, and about 140 Degrees in the Limb, furniſhed with Teleſcopical Sights, *&c.* by which Inſtrument in the Space of a few Years, he has not only rectified all the Places of the fixed Stars obſerved before his time, but has added at leaſt 2000 new fixed Stars to his Catalogue, which were never before taken Notice of by any other Perſon, and has alſo deduced about 1500 Places of the *Moon* and near 1000 of the other *Planets*, from Obſervations made with the ſame Inſtrument, from which Stock of Obſervations, together with a great Number made by a large *Sextant*, for the Space of 15 Years before, he has formed new Tables of the *Moon's Motions* as well as of the other *Planets*, by which Tables the Places of the *Moon* and the reſt of the *Planets* will be exhibited to much greater degrees of Exactneſs than by any Tables that have ever yet appeared Abroad, a Work hitherto much deſired, and which will be Publiſhed with all convenient Speed.

The principal End of mentioning this Inſtrument (and of which a large Account how it was framed and fixed, may be ſeen in the *Preface* to his *Hiſtoria Cæleſtis*) is to recomend its Uſe to all ſuch as have leaſure and Inclination to view the *Heavens*; ſince by this Inſtrument alone one Perſon may make more Obſervations, and that with greater Certainty in One Nights time, than Three Perſons can do with any other in Three Nights; beſides the Right Aſcenſion and Declination of a fixed Star is had from the Obſervation, by Addition or Subſtraction only, which from Obſervations made with the *Sextant* requires four Calculations, as the *Reader* may ſee, if he Conſults the 2*d Caſe* of the 4*th Problem*.

For ſince the difference of the Meridional Altitudes of the Stars gives the Difference of their Declinations, and the difference of the times of their Tranſits over the Meridian, reduced into Equatorial Time gives the Difference of their Right Aſcenſions; if the Difference between the Meridional Heights of the *Moon*, &c. and any Star whoſe Place is known be added to, or ſubſtracted from, (according as the *Caſe* requires) the Declination of the known Star, the Sum or Difference will give the Declination of the *Moon*, &c.

In like manner, if the Difference between the times of their Tranſits over the Meridian be added to, or ſubſtracted from the Right Aſcenſion of the given Star, the Sum or Difference will give the Right Aſcenſion of the *Moon*, &c.

And again, inaſmuch as in the ſame Place, at the ſame time, and at the ſame Alitude, the Refractions are conſtantly the ſame; if a Star whoſe Place is known be choſen in the ſame Parallel of Declination nearly with the *Moon* or any unknown Star or Planet, there is no need of making any Allowance for the Refractions, ſince the Errors in both are the ſame; which is a thing of no ſmall Advantage, ſince as is well known, the Refraction do not only differ conſiderably in different Places, but alter in the ſame Place at different times of the Year, eſpecially when the Stars are near the Horizon, which muſt neceſſarily cauſe great Errors in the Places deduced, if due Allowance be not made for it; and this Method of deducing the Right Aſcenſions and Declinations from their obſerved Differences has this farther Advantage, that if the Inſtrument ſhould not be ſo perfectly placed in the Plane of the Meridian as it ſhould be, nor ſo well adjuſted to the Plane of the Horizon as it ought to be; yet the Errors ariſing even from this are ſo very ſmall, as not to amount to any real Quantity, as every one muſt know who duly conſiders the Thing.

As the Degree of the Equator that tranſits the Meridian with the Sun is called his Right Aſcenſion, inaſmuch as it is the ſame Degree of the Equator that aſcends with him in a Right Poſition of the Sphere, ſo the Degree of the Equator that tranſits the Meridian with any Star, is called the Right Aſcenſion of that Star.

And inaſmuch as the Earth revolves about her own Axis, or which is the ſame thing, inaſmuch as the Heavens ſeem to be carried about with an Equable Motion, in a Space of Time called a Natural Day or 24 Hours, during which time the Equator makes an Inter-Revolution. Hence it is,

1. That during the Space of an Hour there will tranſit the Meridian 15 degrees of the Equator, (for $\frac{1}{24}$ of 360 equal to 15 degrees) and in the Space of 1 minute of Time, there will paſs over the Meridian 15 minutes of the Equator, *&c.* and conſequently the Number of deg. and minutes being known, the Quantity of Time is known alſo, and the contrary.

2. That the Arch of the Equator intercepted between the points of the Equator that tranſit the Meridian, with the Sun, Moon, or any Star, or which is the ſame thing, the difference of their Right Aſcenſions meaſures out the Space of Time between the Tranſit of the Sun, Moon, or any Star, and this being reduced into Time by allowing 15 degrees of the Equator to an Hour of Time, *&c.* it will give the exact Hour, Minute, *&c.* of the time of the Tranſit of the Sun, Moon, or any Star, over the Meridian. Wherefore,

To find the time of the Moon, or any Stars coming to the Meridian.

Subſtract the Right Aſcenſion of the Sun from the Right Aſcenſion of the Moon or of any Star, and the Remainder converted into Time (if need be) ſhews the Hour and Minute, *&c.* of the Moon or Stars coming to the Meridian; but if it happen that the Right Aſcenſion of the Sun exceed the Right Aſcenſion of the Moon or fixed Star, then to the Stars Right Aſcenſion, add 24 Hours, and proceed as before.

Example 1.

Let it be required to find at what Time *Arcturus* will come to the Meridian the 1*ſt* of *April* in the Year 1726.

The Right Aſcenſion of the Sun at that time is 1 h. 24 m. 06 ſ. and the Right Aſcenſion of *Arcturus* 14 h. 03 m. 11 ſ. at the ſame time. Wherefore,

	h.	m.	ſ.
From the Right Aſcenſion of *Arcturus*	14	03	11
Take the Right Aſcenſion of the Sun	1	24	06
The Remainder gives the Hour of the Stars Southing	12	39	05

Example 2.

Let it be required to find at what time the ſame Star *Arcturus* will tranſit the Meridian on the 25*th* of *January* 1727.

Becauſe the Sun's Right Aſcenſion at that time is 21 h. 17 h. 22 ſ. and conſequently greater than the Right Aſcenſion of *Arcturus*. Therefore,

	h.	m.	ſ.
To the Right Aſcenſion of *Arcturus*	14	03	11
Add 24 Hours	24	00	00
From the Sum	38	03	11
Take the Sun's Right Aſcenſion at the time propoſed	21	17	22
The Remainder will give the time of the Stars coming to the Meridian	16	45	49

Which according to the vulgar Way of Computation, is on the 26*th* of *January*, at 4 h. 45 m. 49 ſ. in the Morning.

In finding the time of the Culmination, the Right Aſcenſion of the Sun muſt not be taken as it is at Noon, but as near the time of the Stars Southing as poſſible, for otherwiſe the Error in the former *Example* would amount to near three minutes of Time.

After the ſame manner may the time of the Culmination of the ſame or any other Star, or of the Moon or any other Planet be determined, at any time propoſed; and all other Methods differing from this are falſe and erroneous.

Hence and from the Latitude of the Place, and the Declination of the Star, may be found the times of its Riſing and Setting, its Amplitude, the time when it will appear due Eaſt and Weſt, *&c.* as will be exemplified in the following *Problems.*

Prob. V.

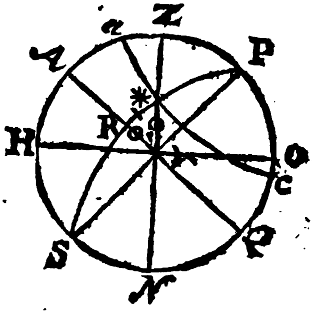
Given the Latitude of the Place, the Day of the Month, Right Aſcenſion and Declination of a Star, to find the Star's Amplitude, the time of its Riſing and Setting, and continuance above or below the Horizon.

Example.

Let it be required to find the Amplitude, time of Riſing, Setting, *&c.* of *Arcturus* at *London,* in the Latitude of 51 deg. 32 min. N. on the 1*ſt* of *April* in the Year 1726, the Right Aſcenſion of *Arcturus* at that time being 14 h. 03 m. 11 ſ. and its diſtance from the North Pole, or Complement of its Declination 69 deg. 21 min. 16 ſec. ½.

The Stereographic Solution.

1. In the adjacent Figure where the Circle *ZONH* repreſents the Meridian, *HO* the Horizon, and *ZN* the Prime Vertical, ſet off 51 deg. 32 min. 00 ſec. the Latitude of the Place from *O* to *P*, as in the Sun, and draw the Six a Clock Hour Circle *PS*, and at Right-angles to it the Equator *AQ*.



2. About the Pole *P*, at the diſtance of 69 deg. 21 min. 16 ſec. ½, the diſtance of *Arcturus* from the Pole or Complement of its Declination, deſcribe the Small Circle *a* ✱ *c*, this will repreſent the Parallel of the Stars diurnal Courſe, or viſible Path that *Arcturus* ſeems to deſcribe in the *Heavens*, and where this Parallel cuts the Horizon as in the point ✱, it ſhews the Place where the Star Riſes and Sets.

3. Thro' ✱ the Place of the Star, and the Poles *P* and *S*, draw the Horary Circle *P* ✱ *S*, and the thing is done, and the Arch of

the Equator *A R* equal in Quantity to the Arch of the Parallel *a* ✱, contained between the Place of the Star at his Riſing, and the Meridian, and therefore equal to half the time of the Stars continuance above the Horizon, may be meaſured by *Caſe* the *2d* of *Prob.* the *7th*, of *Section* the *2d* of *Part* the *4th*, and the Arch of the Horizon ♈ ✱ the Amplitude, or diſtance that the Star Riſes or Sets, in this *Caſe* to the Northward of the Eaſt or Weſt, may be meaſured by the ſame, the firſt of theſe, *viz.* the Semi-duration being applied to the time of the Stars Southing, will give the time of its Riſing and Setting.

Logarithmically, or by Calculation.

In the Right-angled Spherical Triangle ♈ ✱ *R*, Right-angled at *R* are given, the Angle ✱ ♈ *R* equal to the Arch *O Q*, the Complement of *P O* the Latitude, the Arch ✱ *R* the Declination of *Arcturus*, whence to find ♈ ✱ the Amplitude, it will be by the *10th Caſe* of *Right-angled Spherical Triangles*,

As the S. < ✱ ♈ *R* : *R* : : S. ✱ *R* : S. ♈ ✱; that is,
As the Co-ſine of the Latitude of the Place 51.32.00 —— 9.7938317

To the Radius —— —— —— —— —— 10 0000000
So is the Sine of the Stars Declination 20.38.43½ —— 9.5472621

To the Sine of the Amplitude 34.31,38¾ —— —— 9 7534304

Which being always the ſame way with the Declination, is therefore Northerly, and conſequently the Star riſes 34 deg. 31 min. 38 ſec. to the Northward of the Eaſt point of the Horizon, and ſets 34 deg 31 min. 38 ſec. to the Northward of the Weſt, that is according to the *Nautical Phraſe*, *Arcturus* in the Latitude of 51 deg. 32 min. 00 ſec. North or South, riſes in the N. E. by N. point of the Horizon nearly, and ſets in the N. W. by N. point.

And ſince the Declination of the Stars with regard to Common Uſe, may be ſaid to be conſtantly the ſame, inaſmuch as the Stars that are placed near the Equinoctial Colure, don't alter their Declination ſcarce the one third part of a Minute in a Year, and thoſe that are placed near the Solſtitial Colure ſcarce the ſame Quantity in a Hundred Years, and the other intermediate Stars in Proportion, it follows, that the ſame Star in the ſame Place has conſtantly the ſame Amplitude, and during the whole Year riſes and ſets in the ſame points of the Horizon nearly.

2. In the ſame Triangle ♈ * *R*, to find the Aſcenſional Difference ♈ *R*, it will be by the *9th Caſe of Right-angled Spherical Triangles.*

As t,<*♈*R* : *R* : : t, * *R* : S.♈ *R*; that is,

As the Co-tangent of the Latitude of the Place	51.32.00	9.9000865
To the Radius		10.0000000
So is the Tangent of the Stars Declination	20.38.43½	9.5760881
To the Sine of the Aſcenſional Difference	28 18.36½	9.6760016

This Aſcenſional Difference or Arch of the Equator ♈ *R*, equal to 28 deg. 18 min. 36 ſec. ½, added to the Arch ♈ *R* equal to 90 deg. 00 min. 00 ſec. or 6 Hours, gives 118 deg. 18 min. 36 ſec. ½ for the Arch of the Equator *AR*, and this being reduced into Time, gives 7 deg. 53 min. 14 ſec ½ for half or the time the Stars continuance above the Horizon, and therefore being added to the time of the Stars Southing, will give the time of its ſetting, but ſubſtracted will leave the time of its riſing.

It has been already determined, that *Arcturus* paſſes over the Meridian on the 1*ſt* of *April* 1726, at 12 h. 39 m. 05 ſec. Wherefore,

	h.	m.	ſ.
From the time of the tranſit of *Arcturus* over the Mer.	12	39	05
Take the Semi-viſible Arch of duration	7	53	14½
It leaves the time of the riſing of *Arcturus* the ſame Day	4	45	50½

Again,	h.	m.	ſ.
To the time of the tranſit of *Arcturus* over the Merid.	12	39	05
Add the ſame viſible Arch of duration	7	53	14½
It gives the time of the ſetting	20	32	19½

Which according to the vulgar Way of Reckoning is the 2*d* of *April*, at 8 h. 32 min. 19 ſec. ½ in the Morning.

And becauſe the Declination of the ſame Star is very nearly the ſame during the whole Year, it follows, that in the ſame Place the Aſcenſional Difference will be nearly the ſame; and conſequently the Stars Semi-viſible Arch, or half the time of the Stars continuance above the Horizon in the ſame Place, will be very nearly the ſame, and therefore to find the time of the ſame Stars riſing and ſetting at any other Day of the ſame Year, we have nothing more to do than to find the time of the ſame Stars coming to the Meridian.

For *Example*, Suppoſe it were required to find at what time the ſame Star *Arcturus* will riſe and ſet at *London*, on the 25*th* of *January*, 1727.

It has been already determined in Page 458*th*, that on the 25*th* of *January Arcturus* will tranſit over the Meridian of *London* at 16h. 45 m. 49 ſ. Wherefore, to find the time of its riſing the ſame Day.

	h	m.	ſ.
From the time of the tranſit of *Arcturus* ———————	16	45	49
Take its Semi-continuance, *&c.* before found ———	7	53	14½
It leaves the time of the riſing the ſame Day ———	8	52	34½

And for the ſetting.

	h.	m.	ſ.
To the time of the tranſit of *Arcturus* ———————	16	45	49
Add the time of its Semi-continuance, *&c.* ———	7	53	14½
It gives the time of its ſetting ———————————	24	39	03½

Which according to the *Aſtronomical* Way of Reckoning is *January* the 26*th*, at 0 h. 39 m. 03 ſ. ½, but according to the common Way, is *January* the 26*th* at 0 h. 39 m. 03 ſ. ½ in the Afternoon.

And inaſmuch as in the ſame Place the Semi-duration of the ſame Stars continuance above the Horizon is very nearly the ſame, it follows, that the diurnal difference of the times of the ſame Stars riſing and ſetting in the ſame place, is equal to the diurnal difference of the times of the tranſit of the ſame Star over the Meridian; and this depending upon the different Right Aſcenſions of the Sun, ariſing from his diurnal Motion thro' the Ecliptic, it follows, that the diurnal difference of the times of the ſame Stars riſing, culminating, and ſetting, in the ſame Place, is equal to the diurnal difference of the Right Aſcenſion of the Sun nearly at the ſame Place, and becauſe the Sun by Mean Motion moves over 59 min. 08 ſec. of the Ecliptic in one Natural Day, it follows, that the diurnal difference of the Sun's Right Aſcenſion is nearly equal to 59 min. 08 ſec. (the Exceſs or Defect being ſo ſmall, that in the preſent *Caſe* it may be wholly neglected) and this being equal in Time to 3 h. 56 m. 32 ſ. it follows, that the ſame Star in the ſame Place, riſes, culminates, and ſets every Day

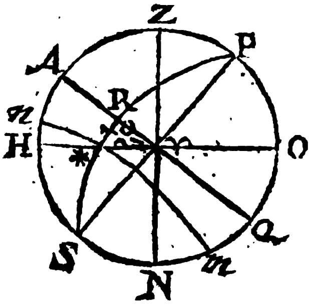
3 h. 56 m. 32 ſ. later than it did the preceding Day, that is, about an Hour in the Space of 15 Days and a quarter nearly.

And becauſe the Sine of the Aſcenſional Difference is to the Radius, as the Tangent of the Stars Declination to the Co-tangent of the Latitude of the Place, it follows, that when the Stars Declination is equal to the Complement of the Latitude, and on the ſame ſide of the Equator with the Place, that the Aſcenſional Difference becomes equal to the Radius, or 90 d. 00 m. 00 ſ. equal to 6 Hours, and conſequently the time of the Semi-duration of the Star above the Horizon becomes equal to 12 Hours; whence it follows, that that Star never riſes or ſets but only touches the Horizon in the loweſt point; and hence it is that all the Stars that lye within 51 deg. 32 min. 00 ſec. of the North Pole, never ſet at *London*, but are always above the Horizon, and viſible when ever the Sun is below; and on the contrary, all the Stars that lye within 51 deg. 32 min. 00 ſec. of the South Pole, never aſcend the Horizon of *London*, but are always below and inviſible; and *vice verſa*, as all the Stars that lye within 51 deg. 32 min. 00 ſec. of the Southern Pole, are always viſible to the Inhabitants in the Latitude of 51 deg. 32 min. 00 ſec. South, when the Sun is abſent, ſo alſo all thoſe Stars that are ſcituated within 51 deg. 32 min. 00 ſec. of the North Pole, are always inviſible; and the ſame Law follows in all Latitudes whatſoever.

Example 2.

Let it be required to find at what time the *Dog Star Syrius* will Riſe, Culminate, and Set at *London*, on the 18*th* of *December* 1726, the Right Aſcenſion of *Syrius* at that time being 6 h. 33 m. 06 ſ. and the Declination 16 deg. 20 min. 59 ſec. South.

The Stereographic Solution.



In the adjacent Figure where *ZONH* repreſents the Meridian, *HO* the Horizon, *ZN* the Prime Vertical, *PS* the Axis, and *AQ* the Equator, about the point *P* draw the Small Circle *n* ✱ *m*, at the diſtance of 106 deg. 20 min. 59 ſec. the diſtance of *Syrius* from the North Pole, and thro' the Interſection ✱ with the Horizon, draw the Horary Circle *P* ✱ *S*, then in the Spherical Triangle

γ ✱ *R*

♈ * R are given, the Angle ♈ the Complement of the Latitude, and * R the Declination of *Syrius*; whence by the former Method of Calculation, the Amplitude ♈ * will be found to be 26 deg. 54 min. 13 ſec. and the Aſcenſional Difference ♈ R 21 deg. 40 min. 02 ſec. equal to 1 h. 26 m. 40 ſ. this taken from ♈ A equal to 90 deg. 00 min. 00 ſec. or 6 Hours, will leave 68 deg. 19 min. 58 ſec. equal to 4 h. 33 m. 20 ſ. equal to the Arch A R, or the half time of the Stars continuance above the Horizon.

The Right Aſcenſion of the Sun at the time given is 18 h. 35 m. 18 ſ. from this taking the Right Aſcenſion of *Syrius* 6 h. 33 m. 06 ſ. there will remain 12 h. 02 m. 12 ſ. for the time of the Southing, and this leſſened by 4 h. 33 m. 20 ſ. the half time of the continuance of the Star above the Horizon, will give 7 h. 58 m. 52 ſ. for the time of the Stars riſing, but increaſed by the ſame Arch of Semi-duration, will give 16 h. 35 m. 32 ſ. for the time of the Stars ſetting; whence it appears,

That the *Dog Star Syrius* on the 18*th* of *December* 1727, riſes at 7 h. 28 m. 52 ſ. in the Evening, at the diſtance of 26 deg. 54 min. 13 ſec. to the Southward of the Eaſt point of the Horizon, culminates at 12 h 02 m. 12 ſ. and ſets at 16 h. 35 m. 32 ſec. or 4 h. 35 m. 32 ſ. next Morning, at the diſtance of 26 deg. 54 min. 13 ſec. to the Southward of the Weſt point, after it has continued 9 h. 06 m. 40 ſ. above the Horizon.

The Semi-Arch of duration being thus found, the time of its riſing, culminating, and ſetting at any other time propoſed, may be obtained as in the former *Example*, by Addition and Subſtraction only.

From the Stars Right Aſcenſion and Aſcenſional Difference, the Oblique Aſcenſion and Deſcenſion may be obtained, after the ſame manner as the Suns was, in Page the 26*th*, *&c.*

The Arch of half continuance of the Star above the Horizon may be inveſtigated, without regard being had to the Aſcenſional Difference.

For in the Triangle * P O in the firſt *Example*, in Page 459*th*, and in the Triangle H S * in the ſecond *Example*, in Page the 463*d*. are given in each P O, or H S the Latitude of the Place, and P * or S *, the diſtance of the Star from the neareſt Pole, whence the Angle * P O in the firſt equal to the Arch of the Equator * Q, and in the ſecond the Angle A S * equal to the Arch of the Equator A *, may be eaſily had by the 12*th Caſe* of *Right-angled Spherical Triangles*, by ſaying, as the Radius is to the Tangent of the Latitude, ſo is the Tangent of the Declination, to the Co-ſine of the Arch of half the time of the continuance above the Horizon, if the Latitude of the Place and Declination of the Star are contrary, as in the 2*d Example*, but to the Co-ſine of Supplement of half the times of continuance, if the Latitude of the Place and Declination of the Star are both the ſame way as in the firſt *Example*, this half time of continuance applied to the time of the Southing as before, will give the time of the riſing, ſetting, *&c.* as the following *Examples* will make appear, if the *Reader* will be at the Pains to work them.

Example 1.

Let it be required to find the Amplitude, time of Riſing, Culminating, Setting, *&c.* of *Aldebaran* at *London*, on the 20*th* of *November* 1726, the Right Aſcenſion of *Aldebaran* at that time being 2 h. 20 m. 12 ſ. and its Declination 15 deg. 55 min. 23 ſec. North.

Anſwer.

The Amplitude is 26 deg. 10 min. 10 ſec. ½ North, the time of the Riſing at 6 h. 47 m. 21 ſ. ½, the time of the Southing at 14 h. 11 m. 32 ſ. and the time of the Setting at 21 h. 35 m. 42 ſ. ½, after it has continued above the Horizon 14 h. 48 m. 21 ſ.

Example 2.

Let it be required to find the Amplitude, time of Riſing, Culminating, Setting, *&c.* of the *Lyon's Heart Regulus* at *London*, on the 6*th* of *February* 1727, the Right Aſcenſion of *Regulus* at that time being 9 h. 53 m. 44 ſ. and its Declination 13 deg. 07 min. 04 ſec. North.

Anſwer.

The Amplitude is 21 deg. 40 min. 42 ſec. North, the time of the Riſing at 4 h. 39m. 36 ſ. the time of the Culmination at 11 h. 48 m. 45 ſ. and the time of the Setting at 18 h. 57 m. 54 ſ. after it has continued 14 h. 18 m. 18 ſ. above the Horizon.

Example 3.

What is the Amplitude of the *Scorpions Heart*, and at what time will it Riſe, Culminate, and Set at *London*, on the 15*th* of *May* 1727, its Right Aſcenſion being 16 h. 12 m. 39 ſ. and its Declination 24 deg. 28 min. 31 ſec. South.

Anſwer.

Its Amplitude is 41 deg. 45 min. 37 ſec. South, the time of its Riſing 8 h. 20 m. 35 ſ. ½, the time of its Southing 12 h. 00 m. 46 ſ. and the time of its Setting 15 h. 40 m. 56 ſ. ½, after it has continued above the Horizon 7 h. 20 m. 21 ſ.

Example 4.

What is the Amplitude of *Caſtor*, and at what time will it riſe, culminate, and ſet, *&c.* at *London*, on the *1ſt* of *December* 1726, its Right Aſcenſion being 7 h. 17 m. 03 ſ. and its Declinatoin 32 deg 27 min. 09 ſec. North.

Anſwer.

Its Amplitude is 59 deg. 36 min. 43 ſec. North, the time of its riſing is at 4h. 24m. 29 ſ. ½, the time of its culminating at 13h. 57m. 10 ſ. and the time of its ſetting at 23 h. 29 m. 50 ſ. after it has continued above the Horizon 19 h. 05 m. 21 ſ.

Example 5.

What is the Amplitude of *Spica Virginis*, and at what time does it riſe, culminate, and ſet at *London*, on the *25th* of *March* 1727, its Right Aſcenſion being 13 h. 10 m. 48 ſ. ½, and its Declination 9 deg. 43 min. 23 ſec. South.

Anſwer.

Its Amplitude is 15 deg. 45 min. 12 ſec. South, the time of its riſing at 2 h. 57 m. 51 ſ. ½, the time of its Southing at 9 h. 47 m. 40 ſec. ½, and the time of its ſetting at 16 h. 37 m. 29 ſ. ½, after it has continued 13 h. 39 m. 38 ſ. above the Horizon.

Example 6.

What is the Amplitude of *Syrius*, and at what time will it riſe, culminate, and ſet at *London*, on the *20th* of *November* 1726, its Right Aſcenſion being 6 h. 33 h. 06 ſ. and its Declination 16 deg. 20 min. 59 ſec. South.

Anſwer.

Its Amplitude is 26 deg. 54 min. 13 ſec. South, the time of its riſing at 9 h. 28 m. 02 ſ. the time of its Southing at 14 h. 1 m. 21 ſ.

and the time of its ſetting at 18 h. 34 m. 42 ſ. after it has continued 9 h. 6 m. 40 ſ. above the Horizon.

If it were required to find at what time the ſame Star *Syrius*, will riſe, culminate, and ſet, *&c* on the ſame Day in the Latitude of 51 deg. 32 min. 00 ſec. South.

Becauſe the Latitude and Declination are both South, the Semi-viſible Arch of the Star will be 7 h. 26 m. 40 ſ. and conſequently it will riſe at 6 h. 34 m. 42 ſ. culminate at 14 h. 01 m. 22 ſ. and ſet at 21 h 28 m. 02 ſ. and the ſame Law muſt be obſerved in all the other Stars.

After the ſame manner, may the time of the riſing and ſetting of any of the ſuperior Planets, as *Saturn*, *Jupiter* and *Mars*, be determined, but to pronounce the exact time of the riſing and ſetting of any of the inferior Planets, as *Venus*, *Mercury*, or the *Moon*, eſpecially the latter, their places in the *Heavens* muſt be found as near as poſſible, to the times when thoſe Appearances will happen; for by Reaſon of their ſwift Motion thro' the *Zodiac*, with reſpect to the fixed Stars, their variations of Right Aſcenſion and Declination are ſo great, during one diurnal Revolution, that unleſs proper Allowances be made for them, the Errors in the times will be very conſiderable; and to ſet the young Beginner right in his Methods of Inveſtigation, I ſhall give an *Example* of the Moon, ſhe being the ſwifteſt of all the Planets in general.

Let it be required to find the time of the riſing, culmination, and ſetting of the Moon at *London*, on the 25*th* Day of *December* 1721.

On the 25*th* Day of *December* 1721, at Noon, according to Mr. *Flamſteeds Lunar Tables* in his *Doctrine* of the *Sphere*, printed in the Year 1680, which Book will be ſoon Re-publiſhed with his New and Correct Tables, the Moon will be in *Leo* 16 deg. 08 min. 54 ſec. having 3 deg. 00 min. 00 ſec. North Latitude, whence according to the 1*ſt Problem* of this *Section*, her Right Aſcenſion will be 139 deg. 38 min. 10 ſec. equal to 9 h. 18 m. 33 ſ. and her diſtance from the North Pole, or Complement of her Declination 71 deg. 06 min. 50 ſec.

At the ſame Time and by the ſame Tables, the Sun will be found to be in *Capricorn* 14 deg. 57 min. 44 ſec. whence according to the 1*ſt Problem* of the former *Section*, his Right Aſcenſion will be found to be 19 h. 04 m. 59 ſ.

Wherefore to find the time of the Moon's Southing,

From the Right Aſcenſion of the Moon 9 h. 18 m. 33 ſ. *plus* 24 Hours, take the Right Aſcenſion of the Sun 19 h. 04 m. 59 ſ. the Re-

Remainder 14 h. 13 m. 34 ſ. will be the time of the Moons Culmination.

Again by the *1ſt Caſe* of the *2d Problem* of the former *Section*, the Moons Aſcenſional Difference will be found to be 1 h. 42 m. 01 ſ. this added to 6 hours becauſe her Declination is North, gives 7 h. 42 m. 01 ſ. for her Semi-viſible Arch, or half the time of her continuance above the Horizon, this therefore being ſubſtracted from the time of her Culmination 14 h. 13 m. 34 ſ. will leave 6h. 31 m. 33 ſ. for the time of her riſing, and the ſame Semi-viſible Arch of 7 h. 42 m. 01 ſ. being added to the time of her Culmination 14 h. 13 m. 34 ſ will give 21 h. 55 m. 35 ſ. for the time of her ſetting; theſe times for Diſtinction ſake I call the Mean Times of her riſing, culmination, and ſetting, whence to obtain the True, becauſe of the great velocity of the Moon, which by mean Motion between Noon and the time of the mean Tranſit is 7 deg. 51 min. and ſo proportionally between Noon and the mean Times of her riſing and ſetting.

To each of theſe times let the place of the Sun and Moon again be Calculated, and by the ſame Tables the Moon at 14h. 13m. 34ſ. the time of the mean Culmination will be found to be in *Leo* 23 deg. 11 min. 58 ſec. having 3 deg. 30 min. 45 ſec. of North Declination; whence by the firſt *Problem* of this *Section*, her Right Aſcenſion will be found to be 146 deg. 45 min. 05 ſec. equal to 9h. 47m. 00 ſ. ½, and her Declination North 17 deg. 07 min. 28 ſec.

The Place of the Sun at 14h. 13 m. 34 ſ. is *Capricorn* 15 deg. 14 min. 03 ſec. whence his Right Aſcenſion according to the *2d Problem* of the former *Section* will be 19 h. 7 m. 34 ſ. ½.

Wherefore to find the true time of the Culmination.

From the Right Aſcenſion of the Moon, in this *Caſe* 9 h. 47 m. 00 ſec. ½ *plus* 24 Hours, take the Right Aſcenſion of the Sun 19 h. 07 m. 34 ſ. ½, the Remainder 14 h. 39 m. 26 ſ. will be the true time of the Southing.

The Place of the Moon at 6 h. 31 m. 33 ſ. the mean time of her riſing will be found to be in *Leo* 19 deg. 22 min. 32 ſec. and her Latitude North 3 d. 14 m. 28 ſ. whence by the *1ſt Problem* of this *Section*, her Right Aſcenſion will be 142 deg. 52 min. 19 ſec. equal to 9 h. 31 m. 29 ſ. and her Declination 18 deg. 06 min. 47 ſec. North, and conſequently by the *2d Problem* of the laſt *Section*, her Aſcenſional Difference will be found to be 1 h. 37 m. 15 ſ. this therefore added to 6 Hours, becauſe her Declination is North, will give 7 h. 37 m. 15 ſ. for the difference between the Right Aſcenſion of the Moon and the Mid-Heaven at the time of her riſing, and therefore being ſubſtracted from the Right Aſcenſion of the Moon at that time equal to 9 h. 31 m. 29 ſ. will give 1 h. 54 m. 14 ſ. for the Right Aſcenſion of the Mid-Heaven, or degree of the Equator under the Meridian at the time of the Moon's riſing.

The Sun at 6 h 31 m. 33 ſ. is in *Capricorn* 15 deg 14 min. 21 ſec. whence his Right Aſcenſion will be found to be 19 hou. 06 min. 10 ſec. wherefore to find the exact time of the Moons riſing.

From the Right Aſcenſion of the Mid-Heaven before found, 1 h. 54m. 14ſ. ſubſtract the Sun's Right Aſcenſion 19h. 06m. 10ſ. the Remainder 6 h. 48m. 04 ſ. will be the true time of the Moons riſing, and by the *2d Problem* of the former *Section*, from her Declination 18 deg. 06 min. 47 ſec. North, and the Latitude of the Place 51 deg. 32 min. 06 ſec. her Amplitude will be found to be 29 deg. 59 min. 10 ſec. North.

Again, at 21 h. 55 m. 35 ſ. the mean Time of the Moon's ſetting, the Sun will be found to be in *Capricorn* 15 deg. 53 min. 45 ſec. whence his Right Aſcenſion will be found to be 19 h. 8 m. 59 ſ. and at the ſame time the Moon's Place will be found to be in *Leo* 27 deg. 01 min. 21 ſec. ſhe having at that time 3 deg. 46 min. 13 ſec. North Latitude; whence by the *1ſt Problem* of this *Section* her Right Aſcenſion will be 150 deg. 34 min. 11 ſec. equal to 10 h. 02 m. 17 ſ. and her Declination 16 deg. 03 min. 57 ſec. North, and conſequently by the *2d Problem* of the former *Section*, her Aſcenſional Difference will be 1 h. 25 m. 01 ſ. which being added to ſix Hours, will give 7 h. 25 m. 01 ſ. for the Difference of the Right Aſcenſion of the Moon and the Mid-Heaven at the time of her riſing: this therefore added to the Moons Right Aſcenſion at that time 10 h. 02 m. 17 ſ. will give 17 h. 27 m. 18 ſ. for the Right Aſcenſion of the Mid-Heaven, from which taking away the Right Aſcenſion of the Sun before found equal to 19 h. 08m. 59 ſ. (24 h. being firſt added to the Right Aſcenſion of the Mid-Heaven) the Remainder 22 h. 18 m. 19 ſ. will be the true time of the Moons ſetting.

And becauſe her Declination at that time is 16° 3′ 57″ N. her Amplitude will be found by the *2d Prob.* of the laſt *Sect.* to be 26° 24′ 57″ N.

Whence it appears, that upon the *25th* day of *December* 1721, the will riſe at 6 h. 48 m. 04 ſ. 29 deg. 59 min. 10 ſec. to the Northward of the Eaſt, that ſhe will tranſit the Meridian at 14 h. 39 m. 26 ſ. and ſet at 22 h. 18 m. 19 ſ. at the diſtance of 26 deg. 24 min 57 ſec. to the Northward of the Weſt point of the Horizon, after ſhe has continued above the Horizon 15 h. 30 m. 15 ſ. during which time the difference of her Amplitude ariſing from the Alteration of Declination is 3 deg. 34 min. 13 ſec. and the difference of time between the riſing and the tranſit greater than theSpace of time between the tranſit and the ſetting, by 12 m. 29 ſ.

If greater Exactneſs be required, the Places of the Sun and Moon muſt be Re-calculated to the times laſt found, and the whole Operation be repeated over again; but the Correction in this *Caſe* would be ſo very ſmall that it may be totally rejected.

For tho in the Space of 14 hours and a quarter, the time of the mean Southing in the former *Example*, the Difference of Right Aſcenſion ariſing from the Exceſs of the Moons diurnal Motion above the Sun's, (which according to meanMotion is 12¼ degrees in a *Day*) be 0 h. 25 m. 53 ſ. yet in the ſpace of half an Hour, it does not amount to a minute of Time, and conſequently in a leſs Space of time than that, it in a manner vaniſhes.

And 'tis upon this Account that in computing the time of the riſing and ſetting of the Sun, no regard is had to his diurnal meanMotion which is 59 min. 08 ſec. that is, about 15 minutes in 6 hours, or a Semi-Equatorial Day, for the difference of Declination, upon which the riſing and ſetting depends, when the Sun is near the Soſtice, in this Space of time cauſes ſo ſmall an Alteration in the Aſcenſional Difference that it is next to nothing, and at the time when the Sun is near the Equinox, tho' the Variation is then the greateſt, yet in all Places within 60 Degrees of the Equator, the Error never amounts to more than a minute of Time; and as it gives the time of the riſing of theSun ſomething ſooner than it ought to do, when theSun is in the firſt and third Quadrants of the Ecliptic, and the contrary when he is in the ſecond and fourthQuadrants, ſo it gives the time of the ſetting ſooner and later than it ought to do, according as the Sun is in the firſt and third, or ſecond and fourth Quadrants, and conſequently makes the Length of the Day and Night nearly the ſame, as what it ought to be, and the Difference is ſo ſmall, as the Induſtrious *Reader* may ſee, if he will be at the Pains to compute the Sun's Declination, and thence his Aſcenſional Difference, *&c.* at the time of his riſing and ſetting, that for all common Uſes it may ſafely be rejected, tho in the ſtricteſt Sence it ought to be conſidered.

For the ready obtaining the time of the Southing of the Moon, or any of the Planets, *Aſtronomers* form Tables ſhewing the Right Aſcenſion to every Degree and Minute of the Ecliptic with Tables of PROSTAPHÆRESIS to every Degree of Latitude within the

Zo

Zodiac (which may be readily computed by the help of the *1ſt Problem* this of *Section*) ſhewing how much muſt be added to, or ſubſtracted from the ſimple Right Aſcenſion, according as the Latitude is North or South, to obtain the true Right Aſcenſion; and to obtain the times of their riſings and ſetting, they compute Tables ſhewing the Degrees of the Equator that aſcend and deſcend with any point of the Ecliptic, in the ſame Place, with Tables of *Proſtaphæreſis*, ſhewing how much muſt be added to, or ſubſtracted from the ſimple Oblique Aſcenſion or Deſcenſion, according to the Quantities of the Latitude, to obtain the true Oblique Aſcenſion and Deſcenſion; as alſo Tables for obtaining the true Amplitude, ſo that by the help of a good *Ephemerides* of the Planets Motions and theſe Tables, the true times of the Riſings, Culminations, Settings, and Amplitudes of the Planets may be obtained by Inſpection only.

In the Calculation of the times of the Riſing and Setting of the Moon in the former *Example*, there has no regard been had to her horizontal Parallax, or the Refraction of the Medium thro' which the Rays muſt paſs; but the times there deduced are the true times when the Center of the Moon arrives at the Eaſtern and Weſtern Sides of the Horizon; but in order to obtain the true times of the viſible and apparent Aſcent and Deſcent of her Center above or below the Horizon, regard muſt be had to both her horizontal Parallax and Refraction; for as by the Refraction the Planets appear at a leſs diſtance from the Zenith than they really are, as will be made appear in the latter part of this *Section*, ſo by the Refraction they are made to appear at a greater, and conſequently by compounding of theſe two together, we have the true diſtance of the Moon from the Zenith, or her true Altitude above or Depreſſion below the Mathematical or True Horizon, when ſhe begins to appear or diſappear.

Let it therefore be required to find in the preſent *Caſe*, the true times when the Center of the Moon will viſibly appear upon the Eaſtern Limb of the Horizon, or the true time when her Center will apparently Riſe, her horizontal Parallax being 54 min. 50 ſec. and Refraction 33 min. 45 ſec.

Now becauſe as ſhe is depreſſed by Parallax, and rais'd by Refraction, their Effects will be always contrary, therefore, if from the horizontal Parallax 54 min. 50 ſec. be taken the horizontal Refraction 33 min. 45 ſec. the Remainder 21 min. 05 ſec. will be the true Altitude of the Moon above the Horizon, when her Center firſt begins to appear, and conſequently to find the Difference of the

Right Aſcenſions of the Moon and the Mid-Heaven, at the time of her apparent Riſing we have given, the Latitude of the Place 51 deg. 32 min. the Moons Declination before found 18 deg. 07 min. 46 ſec. and the true Altitude of the Moon 21 min. 05 ſec. whence by the 11th *Caſe* of *Oblique-angled Spherical Triangles*, the Horary Arch will be found to be 113 deg. 39 min. 44 ſec. equal in Time to 7h. 34 m. 39 ſ. which taken from the Right Aſcenſion of the Moon at that time 9 h. 31 m. 29 ſ. will give 1 h. 56 m. 50 ſ. for the Right Aſcenſion of the Mid-Heaven at the time required, from which deducting the Sun's Right Aſcenſion at that time 19 h. 06 m. 10 ſ. we ſhall have 6 h. 50 m. 40 ſ. for the true time when the Moon's Center appears to aſcend the viſible Horizon, later than the time of her real Aſcent above the true Horizon before deduced by 2 min. 36 ſec.

After the ſame manner may the time of her apparent Setting be found, and if the Altitude before found be increaſed or diminiſhed by her horizontal Semi-Diameter, which in the preſent *Caſe* is 14 m. 52 ſ. and the Operation repeated we ſhall have the true times of the apparent Aſcent or Deſcent of the Moon's Limbs above or below the Horizon.

As the Effects of the Parallax and Refraction are always contrary, if theſe happen to be equal, then the true time of the Riſing and Setting of the Center or either Limb, will be the true times of the apparent Riſing or Setting of the Center or the ſame Limb, but if the Refraction exceed the Parallax, then the difference ſhews the Depreſſion of the Moon below the true Horizon, when the Moon firſt begins to appear, and in this *Caſe* it muſt be added to 90 deg. 00 min. 00 ſec. to obtain the true Zenith Diſtance.

From this Error of Altitude ariſing from the horizontal Parallax and Refraction there will ariſe an Error in the Amplitude, and in order to obtain the apparent Amplitude, or viſible Diſtance that the Moon will riſe from the Eaſt point of the Horizon, we have given as before, the Latitude 51 deg 32 min. 00 ſec the Declination of the Moon 18 deg. 06 min. 46 ſec. and her Zenith Diſtance 89 deg. 38 min. 55 ſec. whence her true Azimuth from the Meridian will be found by the 11th *Caſe of Oblique-angled Spherical Triangles*, to be 60 deg. 31 min. 24 ſec. and conſequently her apparent Amplitude, or viſible diſtance that ſhe Riſes from the Eaſt will be 29 deg. 28 min. 36 ſec. leſs than is given by the former Calculation by 30 min. 34 ſec.

This Method of allowing for the Parallax and Refraction, in computing the true Times of the riſing and ſetting of the Sun, Moon and Stars is not taken Notice of as I can remember, by the Writers upon this Subject; and tho' the Difference ariſing from it be but ſmall, yet as the great End of all our Enquiries is to come as near the Truth as may be, and that the exact time of the Moon's riſing and ſetting may be requiſite ſometimes, in order to pronounce whether an Eclipſe of the Moon or an Occultation be viſible or not at a given Place; I judged it would not be unacceptable to the Ingenious *Reader*.

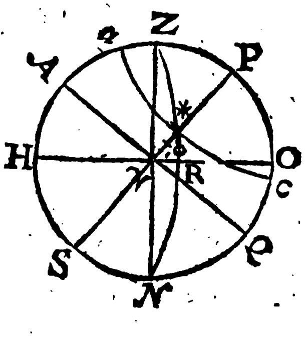
Prob. VI.

Given the Latitude of the Place, the Declination of a Star, and its Right Aſcenſion, together with the Day of the Month, to find the Stars Altitude and Azimuth, and the time when it will appear upon the Six a Clock Hour Circle.

Example.

Let it be required to find at what time *Arcturus* will appear upon the Six a Clock Hour Circle, and what will be its Height and Azimuth at *London*, in the Latitude of 51 deg. 32 min. 00 ſec. North, on the firſt of *April* in the Year 1726, the Right Aſcenſion of *Arcturus* at that time being 14 h. 03 m. 11 ſ. and its Declination 20 deg: 38 min. 43 ſec. North.

The Stereographic Solution.



Having drawn the Meridian *ZONH*, the Horizon *HO*, the Prime Vertical Z *N*, the Axis *P S*, the Equator *AQ*, and the Parallel of the Stars Declination *a* * *c*, according to the Directions given in the laſt *Problem*, and thro' the Interſection * of the Parallel *a* * *c*, and the Six a Clock Hour Circle *P***S*, and having drawn the Vertical Circle *Z***N*, then may **R* the Altitude of *Arcturus* be meaſured by *Caſe* the 3*d* of *Prob.* the 7*th*, of *Sect* the 2*d* of *Part* the 4*th*, and *R O* the Azimuth from the Meridian may be meaſured by *Caſe* the 2*d* of the ſame *Problem.*

Logarithmically, or by Calculation

In the Right-angled Spherical Triangle ♈ *R* ✱ Right-angled at *R* are given, ♈ ✱ the Declination of *Arcturus* 20 deg. 38 min. 43 ſec. North, and the Angle ✱ ♈ *R* equal to the Arch *P O*, equal to the Latitude of the Place 51 deg. 32 min. 00 ſec. whence to find the ✱ *R* the Altitude of *Arcturus*, it will be by *Caſe* the 5*th* of *Right-angled*, &c.

As *R* : S. < ✱ ♈ *R* : : S. ♈ ✱ : S. ✱ *R*; that is,

As the Radius ——————————— 10.0000000

To the Sine of the Latitude of the Place 51.32.00 ——— 9.8937452
So is the Sine of the Declination 20.38.43 ——— 9.5472593

To the Sine of the Altitude 16.01.30$\frac{3}{4}$ ——————— 9.4410045

2. To find the Azimuth from the Meridian, it will be by *Caſe* the 4*th* of *Right-angled*, &c.

As *R* : c.s, < ✱ ♈ *R* : : t, ✱ ♈ : t, ♈ *R*; that is,

As the Radius ——————————— 10.0000000

To the Co-ſine of the Latitude of the Place 51.32.00 — 9.7938317
So is the Tangent of the Declination 20.38.43 ——— 9.5760849

To the Co-tang. of the Azim. from the Merid. 76.28.33$\frac{1}{4}$ 9.3699166

And becauſe as has been already determined, the Star *Arcturus* will appear upon the Meridian on the 1*ſt* of *April* at 12h. 39m. 05ſ. if from 12 h. 39 m. 05 ſ. be taken 6 Hours, the Remainder 6 h. 39 min. 05 ſec. ſhews the time when the Star will appear upon the Six a Clock Hour Circle in the Eaſtern Semicircle, and if to 12 h. 39 m. 05 ſ. be added 6 Hours, the Sum gives the time when the ſame Star will appear upon the Six a Clock Hour Circle in the Weſtern Semicircle.

Whence it appears, that on the 1*ſt* of *April* 1726, the Star *Arcturus* will be upon the Eaſtern Side of the Six a Clock Hour Circle at 6 h. 39 m. 05 ſ. when its Azimuth will be North 76 deg. 48 min. 33 ſec. Eaſt, or Eaſt by North nearly; and again, upon the Weſtern Side of the ſame Circle at 18 h. 39 m. 05 ſ. when its Azimuth will be North 76 deg. 48 min. 33 ſec. Weſt, or Weſt by North nearly; at each of which times its Altitude will be 16 deg. 01 min. 31 ſec.

And becauſe the Right Aſcenſion and Declination of the ſame Star remains nearly the ſame, the Altitude and Azimuth of that Star upon the Six a Clock Hour Circle will be nearly the ſame, and conſequently the diurnal Difference of the times will be equal to the diurnal Difference of the Sun's Right Aſcenſion.

Example 2.

At what time will *Aldebaran* appear upon the Six a Clock Hour Circle at *London*, on the 20*th* of *November* 1726, and what will be its Height and Azimuth at that time, the Right Aſcenſion of *Aldebaran* being 4 h. 20 m. 12 ſ. and its Declination 15 deg. 55 min. 23 ſec. North.

Anſwer.

It will appear upon the Eaſtern part of the Six a Clock Hour Circle at 6 h. 11 m. 32 ſ. when its Azimuth will be Eaſt 10 deg. 03 min. 46 ſec. North, and upon the Weſtern part at 18 h. 11 m. 32 ſ. when its Azimuth will be Weſt 10 deg. 03 min. 46 ſec. North, at each of which times its Altitude will be 12 deg. 24 min. 14 ſec. ½.

Example 3.

At what time will *Regulus* appear upon the Six a Clock Hour Circle at *London*, on the 6*th* of *February* 1727, and what will be its Height and Azimuth at that time, the Right Aſcenſion of *Regulus* being 9 h. 53 m. 44 ſ. and its Declination 13 deg. 17 min 04 ſec. North.

Anſwer:

It will appear upon the Eaſtern part at 6 h. 11 m. 15 ſ. when its Azimuth will be North 81 deg. 38 min. 41 ſec. Eaſt, and upon the Weſtern part at 18 h. 11 m. 15 ſ. when its Azimuth will be North 81 deg. 38 min. 41 ſec. Weſt, at each of which times its Height will be 10 deg. 21 min. 53 ſec. ½.

Example 4.

At what time will *Caſtor* appear upon the Six a Clock Hour Circle at *London*, *December* the 1*ſt*, and what will be its Height and Azimuth at that time, the Right Aſcenſion of *Caſtor* being 7 h. 17 m. 03 ſ. and its Declination 32 deg. 27 min. 09 ſec. North.

Anſwer.

Caſtor will appear upon the Eaſtern part of the Six a Clock Hour Circle at 4h. 02m. 50ſ its Azimuth at that time being North 68 deg. 25 min. 04 ſec. Eaſt, and upon the Weſtern part at 16 h. 2 m. 50 ſ. when its Azimuth will be North 68 deg. 25 min. 04 ſec. Weſt, at each of which times its Altitude will be 24 deg. 50 min. 37 ſec.

Example 5.

At what time will the Bright Star of the *Eagle* be upon the Six a Clock Hour Circle at *London*, on the 15*th* of *May* 1727, and what will be its Height and Azimuth at that time, the Right Aſcenſion of the Star being 19 deg. 37 min. 20 ſec. and its Declination 8 deg. 10 min. 1 ſec. North.

Anſwer

The Bright Star of the *Eagle* will be upon the Eaſtern part of the Six a Clock Hour Circle at 8 h. 36 m. 05 ſ. when its Azimuth will be North 84 deg. 53 min. 55 ſec. Eaſt, and upon the Weſtern part at 23 deg. 50 min. 45 ſec. when the Azimuth will be Weſt 5 deg. 6 min 5 ſec. North, at each of which times its Altitude will be 6 deg. 23 min. 10 ſec.

Example 6.

At what time will the Head of *Andromeda* be upon the Six a Clock Hour Circle at *London*, on the 10*th* of *September* 1726, and what will be its Height and Azimuth, the Right Aſcenſion of the Star at that time being 23 h. 54 m. 13 ſ. and its Declination 27 deg. 34 min. 12 ſec. North.

Anſwer.

It will be upon the Eaſtern part at 6 h. 00 m. 41 ſ. when its Azimuth will be North 72 deg. 00 min. 24 ſec. Eaſt, and upon the Weſtern part at 18 h. 00 m. 41 ſ. when its Azimuth will be North 72 deg. 00 min 24 ſec. Weſt, at each of which times its Altitude will be 21 deg. 14 min. 50 ſec.

Example 7.

At what time will the Bright Star in the *Harp* be upon the Six a Clock Hour Circle at *London*, on the 10*th* of *September* 1726, and what will be its Height and Azimuth at that time, the Right Aſcencenſion of the Star being 18 h. 27 m. 36 ſ. and its Declination 38 deg. 33 min. 02 ſec. North.

Anſwer.

It will be upon the Eaſtern part of the Six a Clock Hour Circle at 11 h. 27 m. 18 ſ. when its Azimuth will be North 63 deg. 37 min. 56 ſec. Eaſt, and upon the Weſtern part at 23 h. 27 m. 18 ſ. when its Azimuth will be North 63 deg. 37 min. 56 ſec. Weſt, at each of which times its Altitude will be 29 deg. 12 min. 21 ſec.

Prob. VII.

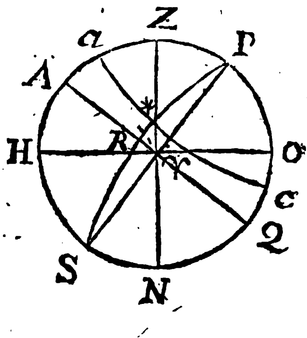
Given the Latitude of the Place, the Day of the Month, the Declination and Right Aſcenſion of a Star, to find the Altitude of the Star, and the time when it will appear due Eaſt or Weſt.

Example.

At what time will *Arcturus* appear upon the Prime Vertical, or due Eaſt or Weſt at *London*, on the *1ſt* Day of *April* 1726, and what will be its Height, the Right Aſcenſion of *Arcturus* being at that time 14 h. 03 m. 11 ſ. and its Declination 20 deg. 38 min. 43 ſec.

The Stereographic Solution.

Having drawn the Meridian *ZONH*; the Horizon *HO*, the Prime Vertical *ZN*, the Six a Clock Hour Circle *PS*, the Equator *AQ*, and the Parallel of the Stars Declination *a* ✱ *c*, thro' the Interſection of the Parallel with the Prime Vertical *ZN*, draw the Hour Circle *P* ✱ *S*, then may ♈ ✱ the Altitude and *A* ✱ the time from the Stars Culmination, or ſidereal Noon, be meaſured by *Caſe* the *2d* of *Problem* the *7th*, of *Section* the *2d* of *Part* the *4th*.



Logarithmically, or by Calculation.

In the Right-angled Spherical Triangle ♈ *R* ✱ Right-angled at *R* are given, ✱ *R* the Declination of *Arcturus* 20 deg. 38 min. 43 ſec. ½, and the Angle ✱ ♈ *R* equal to the Arch *AZ*, equal to the

Arch *PO*, equal to the Latitude of the Place 51 deg. 32 min. 00 ſec. whence to find * ♈ the Altitude, it will be by the 10*th Caſe* of *Right-angled Spherical Triangles.*

As S. <*R* ♈ * : *R* : : S. * *R* : S. * ♈ ; that is,

As the Sine of the Latitude of the Place 51.32.00 —— 9.8937451

To the Radius —————— 10.0000000

So is the Sine of the Stars Declination 20.38.43 —— 9.5472593

To the Sine of the Height 26.45.49 —— 9.6535141

2. To find the Time, it will be by the 9*th Caſe* of *Right-angled Spherical Triangles*,

As the t, <*R* ♈ * : *R* : : t, * *R* : S ♈ *R* ; that is,

As the Tangent of the Latitude of the Place 51.32.00 — 10.0999135

To the Radius —————— 10.0000000

So is the Tangent of the Stars Declination 20.38.43 —— 9.5760849

To the Co-ſine of the time from the Culmin. 72.34.54½ 9.4761714

Which being reduced into Time, makes 4 h. 50 min. 19 ſec. this therefore taken from the time of the Stars Southing 12 h 39 m. 05 ſ. will leave 7 h. 48 m. 45 ſ. ½, for the time when the Star will appear due Eaſt, and the ſame Horary Arch of 4 h. 50 m. 19 ſ. being added to the time of the Culmination 12 h. 39 m. 05 ſ. will give 17 h. 29 m. 24 ſ. for the time when the Star will appear due Weſt.

And inaſmuch as the Right Aſcenſion and Declination of the ſame Star continues nearly the ſame, the Height of the ſame Star upon the Prime Vertical in the ſame Place, will be always nearly the ſame; and the Difference of the times of its appulſe, will be equal to the Difference of the times of its Culmination, which is equal to the diurnal Difference of the Sun's Right Aſcenſion.

Example 2.

At what time will *Aldebaran* appear due Eaſt and Weſt at *London*, on the 20*th* of *November* 1726, and what will its Altitude be at that time, the Right Aſcenſion of *Aldebaran* being 4 h. 20 m. 12 ſ. and its Declination 15 deg. 55 min. 23 ſec. North.

Anſw.

Anſwer.

It will appear due Eaſt at 7 h. 03 m. 56 ſ. and due Weſt at 17h. 19 min. 08 ſ. at each of which times its Altitude will be 20 deg. 30 min. 40 ſec.

Example 3.

At what time will *Regulus* appear due Eaſt and Weſt at *London*, on the *6th* of *February* 1727, and what will be its Altitude at that time, the Right Aſcenſion of *Regulus* being 9 h. 53 m. 44 ſ. and its Declination 13 deg. 17 min. 04 ſec. North.

Anſwer.

It will appear due Eaſt at 6 h. 54 m. 30 ſ. and due Weſt at 17h. 28 m. 00 ſ. at each of which times its Altitude will be 17 deg. 03 min. 59 ſec.

Example 4.

At what time will *Caſtor* appear upon the Prime Vertical at *London*, on the 1ſt of *December* 1726, and what will be its Altitude at that time, the Right Aſcenſion of *Caſtor* being 7 h. 17 m. 03 ſ. and its Declination 32 deg. 27 min. 09 ſec. North.

Anſwer,

Caſtor will appear due Eaſt at 6 h. 4 m. 13 ſ. and due Weſt at 14 h. 01 min. 27 ſec. at each of which times its Altitude will be 43 deg. 15 min. 44 ſec.

Example 5.

At what time will the Bright Star of the *Eagle* be upon the Prime Vertical at *London* on the 15th of *May* 1727, and what will be its Altitude at that time, the Right Aſcenſion of the Star being 19 h. 37 m. 20 ſ. and and its Declination 8 deg. 10 min. 01 ſec. North.

Anſwer.

The Bright Star of the *Eagle* will be on the Eaſtern Azimuth at 3 h. 2 m. 16 ſ. and it will appear due Weſt at 14 h. 09 m. 54 ſ. at each of which times its Altitude will be 10 deg. 27 min. 12 ſec.

Example 6.

At what time will the Head of *Andromeda* be upon the Prime Vertical at *London*, on the 10*th* of *September* 1726, and what will be its Altitude at that time, the Right Aſcenſion of the Star being 23 h. 54 m. 13 ſ. and its Declination 27 deg. 34 min. 12 ſec. North.

Anſwer.

The Head of *Andromeda* will appear due Eaſt at 7h. 37m. 21ſ. and due Weſt at 16 h. 21 m. 17 ſ. at each of which times its Altitude will be 36 deg. 14 min. 16 ſec.

Example 7.

At what time will the Bright Star in the *Harp* be upon the Prime Vertical at *London*, on the 10*th* of *September* 1726, and what will be its Altitude at that time, the Right Aſcenſion of the Star being 18h. 27 m. 25 ſ. and its Declination 38 deg. 30 min. 02 ſec. North.

Anſwer.

It will appear due Eaſt at 14h. 04m. 25 ſ. and be upon the Weſt Azimuth at 20 h. 50 m. 11 ſ. at each of which times its Altitude will be 52 deg. 44 min. 42 ſec.

Prob. VIII.

Given the Latitude of the Place, the Day of the Month, and Altitude of a Star, together with its Right Aſcenſion and Declination, to find the Hour of the Night and Azimuth of the ſame Star.

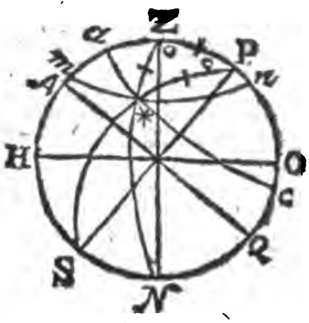
Example.

In the Latitude of 51 deg. 32 min. 00 ſec. North, on the 1*ſt* of *April* 1726, the Altitude of *Arcturus* being 44 deg. 30 min. 20 ſec. I demand the Hour of the Night and Azimuth of the Star, its Right Aſcenſion being 14 h. 03 m. 11 ſ. and its Declination 20 deg. 38 min. 43 ſec. North.

The Stereographic Solution.

Having drawn the Meridian *ZONH*, the Horizon *H O*, the Prime Vertical *Z N*, the Six a Clock Hour Circle *P S*, the Equator *A Q*. and the Parallel of the Stars Declination *a* ✱ *c*, as in the former

Problems; about the Zenith Z, draw the Small Circle *m* * *n*, at the diſtance of 45 deg. 29 min. 40 ſec. equal to the Complement of the Stars Altitude, where this interſects the Parallel of Declination as in the point *, it gives the Place of *Arcturus*.



2. Thro' the point * and the Poles *P* and *S*, draw the Horary Circle *P* * *S*, alſo thro' the ſame point * and the Zenith and Nadir, draw the Vertical Circle Z * *N* and the thing is done; and the Angle *H* Z * the Azimuth from the Meridian, may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th*, of *Section* the 2*d* of *Part* the 4*th*, and the Horary Angle Z *P* *, may be meaſured by the ſame *Caſe*.

Logarithmically, or by Calculation.

In the Oblique-angled Spherical Triangle Z *P* * are given, Z *P*, the Complement of the Latitude 51 deg. 32 min. 00 ſec. Z *, the Complement of the Altitude 44 deg. 20 min. 30 ſec. and *P* * the diſtance of *Arcturus* from the North Pole 69 deg. 21 min. 17 ſec. whence 1*ſt* to find the Horary Angle Z *P* *, or Difference of Times between the Culmination and the Obſervation, (by the 11*th Caſe* of *Oblique-angled Spherical Triangles*.)

From 76 deg. 39 min. 28 ſec. ½, half the Sum 153 deg. 18 min. 57 ſec. of the Complement of the Latitude, of the Zenith diſtance of the Star, and its diſtance from the North Pole, ſubſtract the Stars Zenith diſtance, or Complement of its Altitude, 45 deg. 29 min. 40 ſec. and keep the Difference 31 deg. 09 min. 48 ſec. ½. Then,

To the Arith. Comp. of the Co-ſine of the Lat. 51.32—0.2061683
Add the Arith. Comp. of the Co-ſine of the Decl. 20.38.43 0.0288257
Alſo the Sine of the half Sum 76.39.28½———— —— 9.9881172
And the Sine of the Remainder 31.09.48½———— 9.7138948

Half the Sum of theſe four Sines ———— 19.9370060

Will be the Co-ſine of 21.33.30 ———— 9.9685030

Which

Which being doubled will give 43 deg. 07 min. 01 ſec. for the Horary Angle *Z P* ✱, equal in Time to 2 h. 52 m. 28 ſ. whence to find the Hour of the Night.

If the Star be in the Eaſtern Semicircle or Aſcending,

From the time of the Tranſit of *Arcturus* on the 1*ſt* of *April* 1726, equal to 12 h. 39 m. 05 ſ. take the Horary Angle *Z P* ✱, equal to 2 h. 52 m. 28 ſ. the Remainder 9 h. 46 m. 37 ſ. will give the Hour of the Night. But,

If the Star be in the Weſtern Semicircle or deſcending,

To the time of the Southing of *Arcturus* 12 h. 39 m. 05 ſ. add the Horary Angle *Z P* ✱, equal to 2 h. 52 m. 28 ſ. the Sum 15 h. 31 m. 33 ſ. will be the Hour of the Night required.

2. To find the Angle *P Z* ✱ the Azimuth from the Meridian, it will be by the 1*ſt Caſe* of *Oblique-angled*, &c.

As S. Z ✱ : S. *P* ✱ : : S. < Z *P* ✱ : S. < *P Z* ✱ ; that is,

As the Co-ſine of the Stars Altitude 44.30.20 ——— 9.8532007

To the Co-ſine of the Stars Declination 20.38.43 —— 9.9711743
So is the Sine of the Horary Angle 43.07.01 ——— 9.8347432

To the Sine of the Azimuth from the Merid. 63.44.46 9.9527168

Wherefore if the Star be in the Eaſtern Semicircle, the Azimuth is South 63 deg. 44 min. 46 ſec. Eaſt, or S. E. by E. half E. nearly; but if the Star be in the Weſtern Semicircle, the Azimuth is South 63 deg. 44 min. 46 ſec. Weſt, or S. W. by W. half Weſt nearly,

Or,

The Azimuth of the Star might have been directly inveſtigated by the help of the 11*th Caſe* of *Oblique-angled Spherical Triangles*, and thence the Hour of the Night, after the manner taught in the 2*d Caſe* of the *6th Problem* of the former *Section*, and all the Varieties in that *Caſe* ariſing from the different Denominations of the Latitude of the Place, and Declination of the Sun, are applicable to the Stars, reſpect being had to the time of the Stars coming to the Meridian; and univerſally, all the various *Caſes* of the ſeveral *Problems* in the former *Section*, and the ſeveral Conſequences thence reſulting are applicable to the fixed Stars; *mutatis mutandis*, provided that in the room of the time of the Sun's paſſing over the Meridian, the commencement of the *Solar* or *Aſtronomical* Day, you make Uſe of the time of the Stars Southing, or its tranſit over the Meridian.

Ex-

Example 2.

What will be the Hour of the Night, and Azimuth of *Aldebaran* at *London*, on the 20*th* of *November* 1726, when *Aldebaran* ſhall have 44 deg. 40 min. 30 ſec. of Altitude in the Eaſtern Semicircle, its Right Aſcenſion at that time being 4 h. 20 m. 12 ſ. and the Declination 15 deg. 55 min. 23 ſec. North.

Anſwer.

The Hour is 9 h. 50 m. 23 ſ. and the Azimuth South 51 deg. 21 min. 17 ſec. Eaſt.

Example 3.

What will be the Hour of the Night, and Azimuth of *Syrius* at *London*, in the Latitude of 51 deg. 32 min. 00 ſec. on the 18*th* of *December* 1726, when *Syrius* ſhall have 18 deg. 30 min. 55 ſec. of Altitude in the Weſtern Semicircle, the Right Aſcenſion of *Syrius* at that time being 6 h. 33 m. 06 ſ. and its Declination 16 deg. 20 min. 59 ſec. South.

Anſwer.

The Hour of the Night is 13 h. 44 m. 54 ſ. ½, or 1 h. 44 m. 54 ſ. ½ the Morning of the following Day, and the Azimuth South 26 deg. 00 min. 22 ſec. Weſt.

Example 4.

What will be the time of the Night, and Azimuth of *Regulus* or the *Lyons Heart* at *London*, in the Latitude of 51 deg 32 min. 00 ſec. North, on the 6*th* of *February* 1727, when his Altitude ſhall be 43 deg. 50 min. 10 ſec. in the Eaſtern Semicircle, the Right Aſcenſion of *Regulus* being at that time 9 h. 53 m. 44 ſ. and its Declination 13 deg. 17 min. 04 ſec. North.

Anſwer.

The Hour of the Night is 10 h. 2 m. 43 ſ. ½, and the Azimuth South 45 deg. 51 min. 27 ſec. ½ Eaſt.

Example 5.

What will be the Hour of the Night, and Azimuth of the Star *Caſtor*, on the 1*ſt* of *December* 1726, at *London*, in the Latitude of 51 deg. 32 min. 00 ſec. when *Caſtor* ſhall have 60 deg. 01 min. 30

ſec. of Altitude in the Eaſtern Semicircle, the Right Aſcenſion of *Caſtor* at that time being 7 h. 17 m. 03 ſ. and the Declination 32 deg 27 min. 09 ſec. North.

Anſwer.

The Hour is 7 h. 55 m. 31 ſ. and Azimuth South 62 deg. 57 min. 59 ſec. Weſt.

Example 6.

What will be the Hour of the Night, and Azimuth of *Spica Virginis* at *London*, in the Latitude of 51 deg. 32 min. 00 ſec. North, on the 25*th* of *March* 1727, when the Altitude of the Star ſhall be 24 deg. 30 min. 20 ſec. in the Eaſtern Semicircle, the Right Aſcenſion of *Spica* at that time being 13 h. 10 m. 48 ſ. ½, and its Declination 9 deg. 43 min. 23 ſec. North.

Anſwer.

The Hour will be 10 h. 00 m. 17 ſ. and the Azimuth South 29 deg. 17 min. 18 ſec. Eaſt.

Example 7.

What will be the Hour of the Night, and Azimuth of the *Scorpions Heart* at *London*, on the 15*th* of *May* 1727, when the Altitude of the *Scorpions Heart* ſhall be 12 deg. 30 min. 30 ſec. in the Eaſtern Semicircle, the Right Aſcenſion of the *Scorpions Heart* at that time being 16 h. 12 m. 39 ſ. and its Declination 24 deg. 28 min. 31 ſec. South.

Anſwer.

The Hour will be 10 h. 52 m. 08 ſec. and the Azimuth South 15 deg. 57 min. 49 ſec. Eaſt.

This *Problem* and the 1*ſt* and 2*d Example* of the 2*d Caſe* of the 6*th Problem* of the laſt *Section*, are of great Uſe, both to the *Navigator* and *Aſtronomer*; to the *Navigator* in ſhewing him how to find the true Azimuth of the Sun or Star, which being compared with the Azimuth obſerved by the *Compaſs*, as will be ſhewn in its proper Place, will ſhew him the true Variation of the *Compaſs*, or the Angle formed by the magnetic and true Meridian: And to the *Aſtronomer*, in teaching him how to find the true Hour of the Day or Night, which being compared with the times given by the Clock or Watch, will ſhew their Errors; or how much either of them are faſter or ſlower than the true time; whence the true times of the

Tranſit of the Moon over the Fixed Stars, the Beginning or End of Solar and Lunar Eclipſes, or the exact time when any remarkable *Phænomenon* happens is readily had; and in order to account for any little Errors that may ariſe from the Imperfection of the Movement, Care ought to be taken to correct the *Clock* or *Watch*, both before and after any remarkable Appearances happen, that ſo the little Inequalities that may have crept into the Movement during that ſmall part of time, if any there be, may be allowed for with the more certainty.

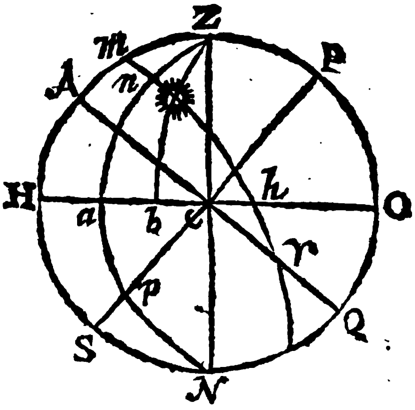
In the Calculation of the Tranſits of the Moon over the Fixed Stars, which are of admirable Uſe for aſſigning the Difference of Longitude between thoſe Places where they can be obſerved, it is neceſſary to know what Degree of the Ecliptic is under the Meridian, and what is the higheſt point or *90th* Degree from the aſcending or deſcending point, which is called the *Nonageſime* or *Nonageſimal* point, and what are the Altitudes of each of theſe above the Horizon, and what Angle the Vertical Circle forms with the Ecliptic in any point of it, or with the Circle of Longitude paſſing thro' the Moon or any fixed Star having Latitude, which is called the Parallactic Angle, at any given time.

Let it therefore be required to find on the *10th* of *June* 1721, at 3 h. 44 m. 21 ſ. apparent Time, when according to Mr. *Flamſteed's* *Solar Tables*, the Sun enters into the firſt point of *Cancer* or upon the *Summer Solſtice*, what is the Longitude of the *Mid-Heaven* and *Nonageſime*, and conſequently what points of the Ecliptic are then riſing and ſetting, and what are the Altitudes of each of the former above the Horizon, or their diſtance from the Vertex, and what is the Angle formed by the Vertical Circle (at that time paſſing thro' the Sun) and the Ecliptic, and the true Altitude of the Sun at *London*, in the Latitude of 51 deg. 32 min. 00 ſec. North.

The Solution.

To the Right Aſcenſion of the Sun in this *Caſe* 90 deg. 00 min. 00 ſec. add the time paſt from Noon 3 h. 44 m. 21 ſec. equal to 56 deg. 5 min. 15 ſec. the Sum 146 deg. 05 min. 15 ſec. will be the Right Aſcenſion of the Mid-Heaven or Degree of the Equator that is then under the Meridian. Wherefore,

1. Having drawn the Meridian *Z O N H*, the Horizon *H O*, the Prime Vertical *Z N*, the Six a Clock Hour Circle *P S*, and the Equator *A Q*, ſet off the Semi-tangent of 56 deg. 05 min. 15 ſec. the Exceſs of the Right Aſcenſion of the Mid-Heaven above a Quadrant from



from *c* to ♈, then will ♈ repreſent the firſt point of *Aries* in this Projection.

2. Thro' the point ♈ draw the Great Circle *m n* ♈, forming an Angle of 23 deg. 29 min. 00 ſec. with the Equator (by *Caſe* the 2*d* of *Problem* the 9*th*, of *Section* the 2*d* of *Part* the 4*th*) this will repreſent the Ecliptic, and the Arch ♈ *m* (which may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d* of *Part* the 4*th*) will be the Longitude of the Mid-Heaven from the firſt point of *Aries*, and the Arch of the Meridian *H m* (which may be meaſured by the *Line of Chords*) will be the Altitude; and to find each of them by *Calculation*.

In the Spherical Triangle *m* ♈ *A* Right-angled at *A* are given, *A* ♈, the Right Aſcenſion of the Mid-Heaven from the firſt point of *Aries*, equal to 146 deg. 05 min. 15 ſec. and the Angle *m* ♈ *A*, the greateſt Obliquity of the Ecliptic, equal to 23 deg. 29 min. 00 ſec. whence, firſt to find the Arch ♈ *m*, the Longitude of the point *A*, it will be by the 2*d Caſe* of the 1*ſt Problem* of the former *Section*,

As the *R* : c s, < *m* ♈ *H* : : c t, ♈ *A* : c t, ♈ *m* ; that is,

As the Radius ——————————— 10.0000000

To the Co-ſine of the greateſt Obliquity 23.29.00 —— 9.9624527
So is the Co-tangent of the Right Aſcenſion of the Mid-Heaven 146.05.15 ——————— } 10.1724441

To the Co-tang. of the Longit. of the Mid-Heaven from the firſt point of *Aries* 143.45.30 ——— } 10.1348968

And conſequently the point of the Ecliptic then culminating is *Leo* 23 deg. 45 min. 30 ſec.

Again, to find the Declination of that point, it will be by the ſame *Caſe* of the former *Section*, As the *R* : S. ♈*A* : : t, <*m* ♈*A* : t, *Am*; that is,

As the Radius	10.0000000
To the S. of the Rt. Aſc. of the Mid-Heaven 146.5.15	9.7465767
So is the Tangent of the greateſt Obliquity of the Ecliptic 23.29.00	9.6379563
To the Tangent of the Declination 13.37.32½	9.3845330

Which being added to the Arch *HA*, equal to the Complement of the Latitude, equal to 38 deg, 28 min. 00 ſec. becauſe the Latitude of the Place, and Declination of the Mid-Heaven are both North, will give 52 deg. 05 min. 27 ſec ½ for the Arch *Hm*, or Altitude of the Mid-Heaven, whoſe Complement 37 deg. 54 min. 32 ſec. ½, equal to the Arch *Zm*, will give its diſtance from the Zenith Z.

Again, to find the Angle *A m* ♈, that the Ecliptic forms with the Meridian in the Mid-Heaven *m*, it will be by the 2*d Caſe* of the 1*ſt Problem* of the former *Section*.

As the *R* : S. < *A* ♈*m* : : c s, *A* ♈ : c s, <♈ *mA*; that is,

As the Radius	10.0000000
To the Sine of the greateſt Obliquity 23.29.00	9.6004090
So is the Co-ſine of the Right Aſcenſion of the Mid-Heaven 146.05.15	9.9190208
To the Co-ſine of the Angle of the Ecliptic and Meridian 70.41.21	9.5194298

2. Thro' the Zenith and Nadir and the Pole of the Ecliptic *p*, draw the Great Circle *Z n p N*, this will cut the Ecliptic at Right-angles in the point *n*, the *Nonageſimal* or higheſt point, or 90*th* Degree from the point of the Ecliptic *h*, that at that time is Riſing or Setting, and the Arch of the Ecliptic ♈ *n* (which may be meaſured by *Caſe* the 3*d* of *Problem* the 7*th*, of *Section* the 2*d* of *Part* the 4*th*) will be the Longitude of this point, and the Arch *a n* of the Vertical Circle or in this *Caſe* Circle of Longitude (and which may be meaſured by the ſame *Caſe*) will be its Altitude, and to find each of them by Calculation.

In the Spherical Triangle *m* Z *n*, Right-angled at *n* are given, Z*m*, the Complement of the Altitude of the Mid-Heaven, 37 deg. 54 min. 27 ſec. ½, and the Angle formed by the Ecliptic and Meridian Z *m n*, equal to 70 deg, 41 min. 20 ſec. whence to find firſt the Arch Z *n* the Zenith Diſtance, or Complement of the Altitude of the *Nonageſime*, it will be by the 5*th Caſe of Right-angled Spherical Triangles*,

As the *R* : S. Z *m* : : S. < Z *m n* : S. Z *n*; that is,

As the Radius	10.0000000
To the Co-ſine of the Alt. of the Mid-Heav. 52.5.32½	9.7884444
So is the S. of the Angle of the Eclip. and Mer. 70.41.20	9.9748509
To the Co-ſine of the Alt. of the *Nonageſime* 54.33.43	9.7632953

Again, to find *m n* the diſtance of the *Nonageſime* from the Mid-Heaven, it will be by the 4*th Caſe of Right-angled Spherical Triangles*,

As the *R* : c s, < Z *m n* : : t, Z *m* : t, *m n*; that is,

As the Radius	10 0000000
To the Co-ſine of the Angle of the Ecliptic and Meridian 70.41.20	9.5194298
So is the Co-tangent of the Altitude of the Mid-Heaven 52.05.32½	9.8913663
To the Tangent of the diſtance of the *Nonageſime* from the Mid-Heaven 14.26 26	9.4107961

Which Arch of 14 deg. 26 min. 26 ſec. taken from the Longitude of the Mid-Heaven *Leo* 23 deg. 45 min. 30 ſec. becauſe the Right Aſcenſion of the Mid-Heaven is more than 90 Degrees, and leſs than 270 deg. 00 min. will give *Leo* 9 deg. 19 min. 04 ſec. for the Longitude of the *Nonageſime* at this time.

3. From the point ♈ lay off 90 deg. 00 min. 00 ſec. the Sun's Longitude in the Ecliptic from ♈ to ☉ (by *Caſe* the 3*d* of *Proble* the 6*th*, of *Section* the 2*d* of *Part* the 4*th*) this point ☉ will give th Place of the Sun; and if thro' the Zenith and Nadir and the poi ☉, the Vertical Circle Z ☉ *N* be drawn, the Arch ☉ *b* will be th Sun's true Altitude, and (which may be meaſured by *Caſe* the

Problem the 7*th*, of *Section* the 2*d* of *Part* the 4*th*) the Angle Z☉*N* (which may be meaſured by *Caſe* the 3*d* of *Problem* the 10*th* of the ſame *Section*) the Parallactic Angle, and to find each of them by Calculation.

In the Spherical Triangle Z *n* ☉ Right-angled at *n*, are given, Z *n* the diſtance of the *Nonageſime* from the Zenith, or Complement of its Altitude equal to 35 deg. 26 min. 17 ſec. and the Arch of the Ecliptic *n* ☉, equal to 39 deg. 09 min. 04 ſec. equal to the diſtance of the Sun from the *Nonageſime*, equal to the Difference between *Leo* 9 deg. 19 min. 04 ſec. the Longitude of the *Nonageſime*, and the Place of the Sun in the firſt point of *Cancer*; whence to find the Sun's true Altitude, it will be by the 1*ſt Caſe of Right-angled Spherical Triangles*,

As the *R* : c s, Z *n* : : c s, ☉ *n* : c s, Z ☉; that is,

As the Radius ———————————— 10.0000000

To the Sine of the Altitude of the *Nonageſime* 54.33.43 9.9110216
So is the Co-tangent of the diſtance of the Sun from the *Nonageſime* 39.19 04 ———————— 9.8885410

To the Sine of the Sun's true Altitude 39.04.26 ——— 9.7995626

Again, to find the Parallactic Angle Z ☉ *n*, it will be by the 2*d Caſe* of *Right-angled Spherical Triangles*,

As the S. *n* ☉ : *R* : : t, Z *n* : t, Z ☉ *n*; that is,

As the Sine of the diſtance of the Sun from the *Nonageſime* 39.19.04 ———————— 9.8018295

To the Radius ———————————— 10.0000000
So is the Co-tang. of the Alt. of the *Nonageſime* 54.33.43 9.8522744

To the Tangent of the Parallactic Angle 48.19.22 — 10.0504449

So that on the 10*th* of *June* at 3 h. 44 m. 21 ſ. when the Sun enters upon the *Summer* Solſtice at *London*.

	°	'	''
The Right Aſcenſion of the Mid-Heaven is	146	05	15
The point of the Ecliptic then under the Merid. is *Leo*	23	45	30
Its Altitude above the Horizon is	52	05	27
And conſequently its diſtance from the Zenith is	37	54	33
The Altitude of the *Nonageſimal* point of the Ecliptic is	54	33	43
And conſequently its Zenith Diſtance is	35	26	17
The diſtance of the *Nonageſime* from the Mid-Heaven	14	26	26
And conſequently the *Nonageſimal* point it ſelf is *Leo*	29	19	04

Hence,

The point of the Ecliptic then riſing is *Taurus*	9	19	04
The point of the Ecliptic then Setting is *Scorpio*	9	19	04
The Sun's true Altitude is	39	04	26
And conſequently his Zenith diſtance is	50	55	34
And the Parallactic Angle is	48	19	12

Hence we are taught a ready Way to find when any Star Riſes and Sets with the Riſing Sun, which is called its *Coſmical* Riſing and Setting; or when the ſame or any other Star Riſes and Sets with the Setting Sun, which is called its *Achronical* Riſing and Setting: For as every Star Riſes with that point of the Ecliptic that has the ſame Oblique Aſcenſion with it, and conſequently at the ſame time with the Sun when he poſſeſſes that Degree of the Ecliptic, ſo every Star that has the ſame Oblique Deſcenſion with any point of the Ecliptic, Sets with that point of the Ecliptic, and at the ſame time with the Sun when he is in the ſame point; hence the *Coſmical* or *Achronical* Riſing and Setting of any fixed Star or Planet is eaſily had, for the Right Aſcenſion and Declination of a Star being known, its Aſcenſional Difference, and Oblique Aſcenſion or Deſcenſion may be readily found, and thence the Right Aſcenſion of the Mid-Heaven, and its Longitude together with its Diſtance from the *Nonageſime*, whence the *Nonageſime* it ſelf, and conſequently the points in the Ecliptic then Aſcending or Deſcending, and which the Sun muſt poſſeſs, to cauſe the *Coſmical* or *Achronical* Riſing or Setting of the given Star.

Suppoſe for *Example*, it were required to find when the *Dog Star Syrius*(which is the brighteſt of all the fixed Stars that appears at preſent in our Hemiſphere) will Riſe and Set *Coſmically* or *Achronically* at *London*, in the Latitude of 51 deg. 32 min: 00 ſec. in the Year 1726, when the Right Aſcenſion of *Syrius* will be 98 deg. 16 min. 32 ſec and its Declination 16 deg. 20 min. 59 ſec. South.

From the Declination of *Syrius* 16 deg. 20 min. 59 ſec. and the Latitude of the Place 51 deg. 32 min. 00 ſec. the Aſcenſional Difference of *Syrius* by the help of the *5th Problem*, will be found to be 21 deg. 40 min. 02 ſec. this added to the Right Aſcenſion 98 deg. 16 min. 32 ſec. becauſe the Latitude of the Place is North, and the Declination of the Star South, will give 119 deg. 56 min. 34 ſec. for the Stars Oblique Aſcenſion, from which taking 90 Degrees or a Quadrant, we ſhall have 29 deg. 56 min. 34 ſec. for the Right Aſcenſion of the Mid-Heaven, at the time of the Stars Riſing. Hence the Mid-Heaven it ſelf will be found to be in *Taurus* 2 deg. 7 min. 49 ſec. its Declination 12 deg. 14 min. 08 ſec. and conſequently its diſtance from the Vertex 39 deg. 17 min. 52 ſec. and the Angle formed by the Ecliptic and Meridian 69 deg. 48 min. 01 ſec. whence the Diſtance of the MidHeaven from the *Nonageſime*, will be found to be 15 deg. 46 min. 49 ſec. and the *Nonageſime* it ſelf in *Taurus* 17 deg. 54 min. 38 ſec. and conſequently the point of the Ecliptic then Aſcending will be *Leo* 17 deg. 54 min. 38 ſec. which is the Place of the Sun about the 31*ſt* Day of *July*, and conſequently the Bright Star *Syrius* Riſes *Coſmically* at *London* on the 31*ſt* Day of *July* 1726.

Again, if the time of its *Coſmical* Setting be required,

From the Right Aſcenſion of the Star 98 deg. 16 min. 32 ſec. take its Aſcenſional Difference 21 deg. 40 min. 32 ſec. the Remainder 76 deg. 36 min. 00 ſec. will be the Stars Oblique Deſcenſion, to which adding 90 Degrees or a Quadrant, the Sum 166 deg. 36 min. 00 ſec. will be the Right Aſcenſion of the Mid-Heaven at the time of the Stars Setting.

Hence the Place of the Mid-Heaven will be found to be in *Virgo* 15 deg. 26 min. 54 ſec. its Declination 5 deg. 44 min. 45 ſec. North; and conſequently the diſtance of the Mid-Heaven from the Zenith, will be 45 deg. 47 min. 15 ſec. and the Angle formed by the Ecliptic and Meridian 67 deg. 11 min. 30 ſec. whence according to the former manner of Calculation, the diſtance of the *Nonageſime* from the Mid-Heaven will be found to be 21 deg. 43 min. 27 ſec. the *Nonageſime* it ſelf in *Leo* 23 deg. 43 min. 27 ſec. and conſequently the point of the Ecliptic then Deſcending will be *Taurus* 23 deg. 43 min. 27 ſec. whence the oppoſite or Aſcending point will be *Scorpio* 23 deg. 43 min. 27 ſec. which is the point the Sun muſt poſſeſs to Riſe at the ſame time, and which is the Place of the Sun, about the *5th* of *November* in the Morning, when the Bright Star *Syrius* will Set *Coſmically* at *London* in the Year 1726.

The *Coſmical* Riſing and Setting of a Star being known, the *Achronical* Riſing and Setting of the ſame Star is given; for inaſmuch as the point of the Ecliptic anſwering to *Leo* 17 deg. 54 min. 38 ſec. Riſes with *Syrius* at *London*, and is therefore called its *Coſmical* Riſing, the point of the Ecliptic oppoſite to this and anſwering to *Aquarius* 17 deg. 54 min. 38 ſec. muſt deſcend below the Horizon at the ſame time, and is the point the Sun muſt poſſeſs, to cauſe the Star *Syrius* to Riſe *Achronically*; and this being the Place of the Sun about the *26th* of *January*, it follows, that on the *26th* of *January* 1726, the Bright Star *Syrius* will Riſe *Achronically* at *London*.

Again, inaſmuch as the point of the Ecliptic anſwering to *Scorpio* 23 deg. 43 min. 27 ſec. Aſcends above the Horizon at the time that *Syrius* Deſcends; and is therefore the point the Sun muſt poſſeſs, to cauſe the *Coſmical* Setting; the point of the Ecliptic oppoſite to this and anſwering to *Taurus* 23 deg. 43 min. 27 ſec. is the point of the Ecliptic that Deſcends at the ſame time with *Syrius*, and is the Place of the Sun at the time of the *Achronical* Setting; and this happening about the *3d* of *May*, it follows, that on the *3d* of *May* 1726, the Bright Star *Syrius* will Set *Achronically* at *London*.

Whence it follows, that the time of the *Coſmical* Riſing and Setting of a Star being given, the time of the *Achronical* Riſing and Setting of the ſame Star is given alſo; and on the contrary, the time of the *Achronical* Riſing and Setting of a Star being known, the time of the *Coſmical* Riſing and Setting of the ſame Star is known alſo: And inaſmuch as every Star that lyes near the Ecliptic Riſes and Sets at the ſame time nearly with that point of the Ecliptic; hence it is that all the Stars that have not very great Latitude that Riſe *Coſmically*, Set *Achronically*, and all Stars that Set *Coſmically* Riſe *Achronically*; and the contrary: whence it is that all thoſe Stars that Set nearly with, or a little after the Sun, are ſaid to Set *Achronically*, as all thoſe Stars that Riſe with, or a little before the Sun, are ſaid to Riſe *Coſmically*.

There are other Methods leſs operoſe for inveſtigating the *Coſmical* and *Achronical* Riſing, Setting, *&c.* but theſe that I have made Uſe of are the moſt natural, general and eaſy.

From the preceding Calculation it is manifeſt,

1. That from the time of the *Coſmical* Setting *November* the *5th*, to the time of the *Achronical* Riſing *January* the *26th*, during the Space of 82 Days the Star is viſible above the Horizon, from the time of its Riſing to the time of its Setting; and on the contrary,

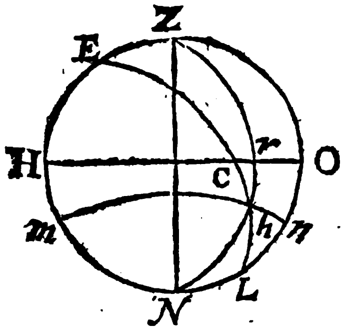
from the time of its *Achronical* Setting *May* the 3*d*, to the time of its *Coſmical* Riſing *July* the 31*ſt*, during the Space of 89 Days the Star is altogether inviſible; and never appears above the Horizon from the Setting of the Sun to the Riſing.

2. That from the time of its *Coſmical* Riſing *July* the 31*ſt*, when it firſt begins to appear on the Eaſtern Limb of the Horizon before Sun Riſe, to the time of its *Achronical* Setting *May* the 3*d*, when it appears upon the Weſtern Limb of the Horizon juſt after Sun Set, during the Space of 276 Days it conſtantly appears above the Horizon at ſome part of the Night or other.

But becauſe of its nearneſs to the Sun at the time of its *Coſmical* Riſing and *Achronical* Setting it is ſo far Eclipſed by the ſuperior Light of the Sun, as to become inviſible; to determine therefore the time when a Star that has been in Conjunction with the Sun, is ſo far left by him as to Emerge out of his Rays and become viſible; and on the contrary, to pronounce the time when a Star that has been viſible is ſo far overtaken as to be ſwallowed up in the Solar Rays, is by *Aſtronomers* termed to find het times of the *Heliacal* Riſing and Setting of that Star.

It is found by Obſervation that the ſmalleſt fixed Stars are not viſible, till the Hemiſphere is wholly free from the Solar Rays, that is, not till after the End of the Evening Twilight, and before the Begining of Morning Twilight; which happens when the Sun is 18 Degrees below the Horizon; but that Stars of the firſt Magnitude may be ſeen when the Sun is depreſſed 12 Degrees, and that others become viſible at proportional Depreſſions between 18 and 12 Degrees, according to their different Magnitudes; that is, Stars of the 6*th* Magnitude become viſible when the Sun is depreſſed 17 Degrees, Stars of the 5*th* Magnitude at 16, thoſe of the 4*th* at 15 Degrees, thoſe of the 3*d* at 14 Degrees, and thoſe of the 2*d* at 13 Degrees: Let it therefore be required to find when the Bright Star *Syrius* Riſes and Sets *Heliacally* at *London*, in the Latitude of 51 deg. 32 min. 00 ſec. North, it being a Star of the firſt Magnitude, and conſequently viſible according to the former Law in the Limb of the Horizon, when the Sun is depreſſed 12 Degrees below it.

1. In the adjacent Figure, Let *Z O N H* repreſent the Meridian, *H O* the Horizon, *E C* a portion of the Ecliptic, and *C* the Place of the Sun in the Ecliptic at the time when the Star Riſes *Coſmically*, and which by the former Method of Calculation is found to be *Leo's* 17 deg. 54 min. 38 ſec.



2. About the point *N* draw the Small Circle *m.c n* at the diſtance of 78 Degrees the Complement of the Stars Depreſſion at the time of its *Heliacal* Riſing, where this cuts the Ecliptic in the point *h*, it will give the Place of the Sun at that time, and to find its diſtance from the *Coſmical* point *C*, thro' the Zenith and Nadir and the point *h*, draw the Vertical Circle *Z r h*; then in the Spherical Triangle *C r h*, Right-angled at *r* are given, *r h*, the Sun's Depreſſion below the Horizon equal to 12 Degrees, and the Angle *r C h* formed by the Ecliptic and the Horizon, equal to the Angle *E C H*, equal to the height of the *Nonageſime* before found 53 deg. 31 min. 49 ſec. whence to find the Hypothenuſe *C h*, the diſtance between the *Coſmical* and *Heliacal* points of the Ecliptic, it will be by the 10*th Caſe* of *Right-angled Spherical Triangles*,

As S < *r C h* . *R* : : S. *r h* . S. *C h* ; that is,

As the Sine of the Altitude of the *Nonageſime* 53.31.49—9.9053485

To the Radius———————————— 10.0000000

So is the Sine of the Depreſſion 12.00.00———— 9.3178789

To the Sine of the Arch *C h* 14.59.01———— 9.4125304

Which being added to the Place of the Sun *Leo* 17 deg. 54 min. 38 ſec. at the time of the *Coſmical* Riſing gives *Virgo* 2 deg. 53 min. 39 ſec. for the Place of the Sun at the time of the *Heliacal* Riſing, and this being the point of the Ecliptic that the Sun poſſeſſes about the 15*th* of *Auguſt*; it follows, on the 15*th* of *Auguſt Syrius* will begin to appear after its Conjunction with the Sun, and will be ſeen in the Eaſtern Limb of the Horizon, a little before Sun Riſe.

Again, to find the time of its *Heliacal* Setting.

In the ſame Figue let *C* repreſent the Place of the Sun in the Ecliptic at the time of its *Achronical* Setting, then will *h* repreſent the Place of the Sun at the time of its *Heliacal* Setting, therefore in the Spherical Triangle *C r h*, Right-angled at *r* are given, the Side *r h* 12 deg. 00 min. 00 ſec. the Sun's Depreſſion as before; and the Angle *r C h* equal in this *Caſe* to 48 deg. 38 min. 45 ſec., the Altitude of the *Nonageſime* at the time of the *Achronical* Setting, whence to find the Arch of the Ecliptic *Ch* the diſtance between the *Achronical* and *Heliacal* points of the Ecliptic at the Setting, it will be by the ſame *Caſe*,

As the S. ∠ *r C h* : *R* : : S. *r h* : S. *C h*; that is,

As the Sine of Altitude of the *Nonageſime* 48.38.45 ——9.8754316

To the Radius —————————— 10 0000000

So is the Sine of the Depreſſion 12.00.00 ———— 9.3178789

To the Sine of the Arch of diſtance *C h* 16.04.48 ——9.4424473

Which taken from the Place of the Sun in *Taurus* 23 deg. 43 min. 27 ſec. at the time of its *Ghronical* Setting, will leave *Taurus* 07 deg. 38 min. 39 ſec. for the Place of the Sun at the time of the *Heliacal* Setting, and this being the point of the Ecliptic that the Sun poſſeſſes about the 16*th* of *April*, it follows, that on the 16*th* of *April* the Bright Star *Syrius* will be ſeen in the Weſtern Edge of the Horizon ſome little time after Sun Set, and after that will diſappear to the time of its *Heliacal* Riſing; whence it is manifeſt, that from the 16*th* of *April* to the 15*th* of *Auguſt*, during the Space of 121 Days the Bright Star *Syrius* is inviſible to the Inhabitants in the Latitude of 51 deg. 32 min. 00 ſec. and that during the remaining part of the Year, that is for the Space of 245 Days, *viz.* from the 15*th* of *Auguſt* to the 16*th* of *April* it may be ſeen above the Horizon between Sun Set and Sun Riſe, and that more or leſs according as the Star is nearer too, or farther from the Oppoſition.

After the ſame manner may the time of the *Heliacal* Riſings and Settings of any of the Planets be found, proper Allowances being made for their Depreſſion, according to their different Brightneſſes, which according to the Rules laid down by *Aſtronomers*, is for *Mars* and *Saturn* about 11 Degrees, for *Jupiter* and *Mercury* 10 Degrees, and for *Venus* 5 Degrees; tho when *Venus* is at her greateſt Elongation from the Sun ſhe may be ſeen with the naked Eye any time of the Day.

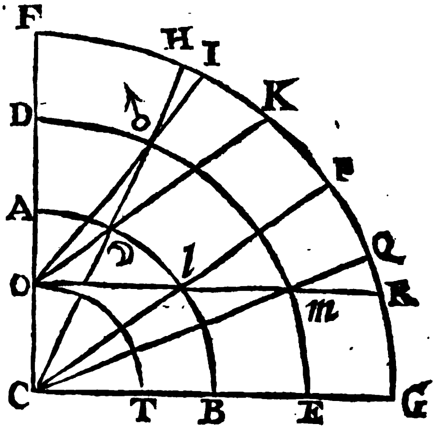
The Knowledge of theſe Poetical Riſings and Settings of the Stars, as they are uſually called, and of which frequent mention is made by the Ancient *Poets*, *Hiſtorians*, and *Philoſophers*, in their ſeveral Writings, (and therefore neceſſary to be underſtood, in order to clear up ſeveral Paſſages contained therein) were of great eſteem among the *Ancients*, and were very uſeful to them in adjuſting the times ſet a-part for their *Religious* and *Civil* Uſes, and for the *Seaſons* proper for the ſeveral Parts of *Husbandry*; for as their Knowledge of the Motions of the Heavenly Bodies was not ſufficient to adjuſt the true Length of the Year, but that ſometimes they happened longer and ſometimes ſhorter than at other times; and as the returns of the *Seaſons* depend upon the approach of the Sun, to the *Tropics* and *Equinoctial* points, ſo they made Uſe of theſe Riſings and Settings as Criteria to determine the Commencement of the *Seaſons* proper for any Uſe, but in theſe latter Times, ſince Mankind is arrived to ſuch a Perfect Knowledge of the Motions of the Sun and the other Cœleſtial Bodies, as to pronounce the Length of the Year, the Commencement of the New and Full Moons, the Times of the Conjunctions of the ſeveral Cœleſtial Bodies, to the greateſt Degree of Exactneſs imaginable, the Uſe of theſe are in a great meaſure laid aſide, and good *Calendars* are made to ſupply their Places.

As the Star which Riſes *Heliacally* with the Sun is called the Morning Star, inaſmuch as it ſhews at its diſappearance when the Apparent Day is really began, ſo the Star that Sets *Heliacally* is called the Evening Star, inaſmuch as it ſhews by its appearance when the Apparent Day ends, and the Night properly begins; and 'tis at the firſt appearance of this Star that the *Jewiſh Sabbath* properly begins and ends.

From a Knowledge of the Times of the *Heliacal* Riſing and Setting of the Planet *Jupiter*, we are taught how to find the time when *Jupiter* is ſo far influenced by the Solar Rays as to become inviſible, and conſequently the time when the Eclipſes of his Satellites begin and ceaſe to be viſible; and which are of very great Uſe in aſſigning the Differences of Longitude between thoſe Places where they may be obſerved.

As the true Place of the Sun, Moon, *&c.* is that point in the Heavens which is pointed out, by a Line drawn from the Center of the Earth, thro' the Center of the Sun, Moon, *&c.* ſo the viſible or apparent Place of the Sun, Moon, *&c.* is that point in the Heavens which is pointed out, by a Line drawn from the Eye of the Obſerver thro' the Center of the Sun, Moon, *&c.* and the diſtance between thoſe two Places is called the Parallax; ſo that the Parallax of Altitude of the Sun, Moon, *&c.* is an Arch of a Vertical Circle paſſing thro' the Sun, Moon, *&c.* and intercepted between the viſible and true Place.

Thus



Thus in the adjacent Figure, where *C* repreſents the Center, *OT* a Quadrant of the Earths Superficies, *O* the Place of the Obſerver, and *A* ☽ *B* a Quadrant of the Orb of one of the Planets, ſuppoſe the Moon, it is manifeſt, that as the Line *C* ☽ *H* drawn from the Center of the Earth, thro' ☽ the Center of the Moon at ☽, gives the true Place of the Moon in *H*, ſo the Line *O* ☽ *K* drawn from the Obſerver thro' the Center of the Moon, will give the viſible Place at *K*, and conſequently that the Angle *O* ☽ *C* equal to the Exceſs of the Angle *FO* ☽, the viſible Zenith Diſtance of the Moon, or Complement of her Altitude above the Angle *FC* ☽, the true Zenith Diſtance or Complement of the true Altitude, equal to the Angle *H* ☽ *K*, will be the Parallax of the Moon in the point ☽; whence it follows,

1. That when the Planet is in the Zenith at the point *A* or *D*, the Parallax vaniſhes: For the Line *COAD* in this *Caſe*, which is drawn from the Center of the Earth at *C*, thro' the Planet at *A* or *D* coincides, or is the ſame with the Line *OAD*, drawn from the Obſerver thro' the ſame point *A* or *D*.

2. That the nearer the Planet is to the Earth, the greater is its Parallax at the ſame Diſtance from the Zenith or at the ſame Altitude; and the contrary.

For let *D* ♂ *m E* repreſent a Quadrant of the Orb of *Mars*, and let us ſuppoſe the Planet *Mars* to be in the point ♂, where the Line drawn from the Center of the Earth thro' the Moon, cuts the Orb of the Planet *Mars*, and let the Line *O* ♂ *I* from the Place of the Obſerver thro' the Planet be drawn, then will the Angle *O* ♂ *C* equal to the Angle *H* ♂ *I*, be the Parallax of *Mars* in the point ♂, when the Planet ♂ has the ſame true Zenith Diſtance *FCH* or Altitude *HCG*, with the Moon in the point ☽; now becauſe in the two Triangles *C* ☽ *O* and *C* ♂ *O* the Angle at *C* is common, it follows, that the Angle *O* ☽ *C* the Parallax of the Moon in the point ☽, is greater than the Angle *C* ♂ *O* the Parallax of *Mars* in the point ♂, by the Quantity of the Angle *δ O* ☽; and this will happen in whatſoever part of the Orb the point ☽ be taken, except in the Zenith *A*.

3. That the Sines of the Parallaxes of Altitude of the ſame Planet at different diſtances from the Vertex, are directly as the Radius to the Sines of the viſible diſtance of the Planet from the Zenith, or as the Co-ſines of their viſible Altitudes above the Horizon; for in the Triangle *C* ☽ *O* the Sine of the Parallax of Altitude *C* ☽ *O*, is to the Sine of the Angle ☽ *OA*, the viſible diſtance of the Moon from the Zenith, as the Side *CO* the Semi-diameter of the Earth is to *C* ☽ the mean diſtance of the Moon from the Earths Center; and in the Triangle *C l O*, the Sine of the Parallax of Altitude *C l O* is to the Sine of the Angle *A O l* the viſible diſtance of the Moon from the Zenith *A*, as the Side *CO* the Earths Semi-diameter, is to *C l* equal to *C* ☽, the Moon's diſtance from the Center of the Earth; wherefore, becauſe the two laſt Terms in each Proportion are the ſame, the two firſt Terms in each will be directly proportional; that is, the Sine of the Parallax in the point *l* will be to the Sine of the Parallax in the point ☽, as the Sine of the diſtance of the point *l* from the Zenith *A*, to the Sine of the diſtance of the point ☽ from the ſame point *A*; whence it follows,

1. That the nearer the Planets are to the Zenith, the leſs is their Parallax of Altitude; and on the contrary, the farther any of the Planets is removed from the Zenith, the greater is its Parallax of Altitude; and conſequently when the Planets are in the Horizon, their Parallax will be the greateſt; and becauſe at that time their diſtance from the Zenith is equal to a Quadrant or 90 Deg. and that the Sine of 90 Degrees is equal to the Radius, it follows,

That as the Radius is to the Sine of the viſible diſtance of any Planet from the Zenith, or to the Co-ſine of its Altitude above the Horizon, ſo is the Sine of the horizontal Parallax, to the Sine of its Parallax at that Altitude; and conſequently the horizontal Parallax being given, the Parallax at any Altitude is likewiſe given; and on the contrary, the Parallax at any Altitude being given, the horizontal Parallax may be eaſily known.

4. From the Projection it is manifeſt, that the nearer the Moon or any other Planet that has a Senſible Parallax, approaches the Zenith the nearer it is to the Obſerver, and the greater is its apparent Semi-diameter: For in the Triangles *C O l* or *C O* ☽, the Sides *O C* *C l* and *O C*, *C* ☽ remaining the ſame, the leſſer the contained Angle *O C m*, *O C* ☽, the leſſer will be the oppoſite Side *O m*, *O* ☽; till at

laſt when the Planet arrives at the Zenith in the point *A*, it is nearer to the Obſerver placed in the point *O*, then it is at the Horizon *m*, by almoſt the whole Semi-diameter of the Earth *O C*.

Whence it is manifeſt, that the apparent Diameters of the Planets are leaſt in the Horizon, and greateſt in the Zenith; and are to each other at the ſeveral intermediate diſtances as the Sines of thoſe reſpective diſtances from the Zenith.

5. The Parallax of the Moon or of any Planet at any Altitude being known, the diſtance of the Moon or of that Planet from the Earth is eaſily found; and on the contrary, the diſtance of the Planet from the Earth being known or given, the Parallax at any Altitude is eaſily had: Thus in the Triangle *C O l*, if the Moon's horizontal Parallax or the Angle *C l O*, be ſuppoſed 1 degree or 60 minutes, her diſtance from the Center of the Earth will be found to be 57.2987 of the Earths Semi-diameters; for as the Sine of the Angle *C l O* of one Degree is to the Radius, ſo is the Semidiameter *OC* of the Earth, in this *Caſe* ſuppoſed Unity, to *C l* the diſtance of the Moon from the Center in the ſame parts; and hence we are taught how to find the Parallax of Altitude of the Moon, and conſequently her viſible Altitude from the horizontal Parallax (which is always given in the Tables) and the true diſtance of the Moon from the Zenith, or her Altitude above the Horizon. Suppoſe for *Example*, the Moon at ☽, and it were required to find the viſible Altitude of the Moon ☽ *O l*, her horizontal Parallax *O l C*, being 59 min. 45 ſec. and her true Zenith diſtance *A C* ☽ 65 deg. 45 min. 20 ſec. or her true Altitude ☽ *C B* 24 deg. 14 min. 40 ſec.

1. By the former Method of Calculation her diſtance from the Earth *C* ☽ will be found to be 57.543 Semi-diameters of the Earth, wherefore in the Triangle *O* ☽ *C* are given, *C O* the Semi-diameter of the Earth equal to Unity, the Side *C* ☽ the mean diſtance of the Moon in the ſame parts, equal to 57.543, and the contained Angle *O C* ☽, the Moons true diſtance from the Zenith, equal to 65 deg. 45 min 20 ſec. whence by the *4th Caſe* of *Oblique-angled Plane Triangles*, the Angle *O* ☽ *C* the Moons Parallax of Altitude will be found to be 54 min. 51 ſec. ¾, the Angle *A O* ☽ her viſible Zenith diſtance *66* deg. 40 min. 11 ſec. ¾, and conſequently her viſible Altitude or the Angle ☽ *O l*, will be found to be 23 deg. 19 min. 48 ſec. ½.

The Knowledge of the Parallaxes are of the greateſt Uſe in *Aſtronomy*, ſince by them the true Magnitudes, Diſtances and Periods of the heavenly Bodies are exactly known, but the manner of determining them from Obſervations being a little foreign to my preſent Deſign, I ſhall refer the *Reader* to the ſeveral excellent *Treatiſes* that have been wrote upon them, till a more convenient Opportunity offers it ſelf.

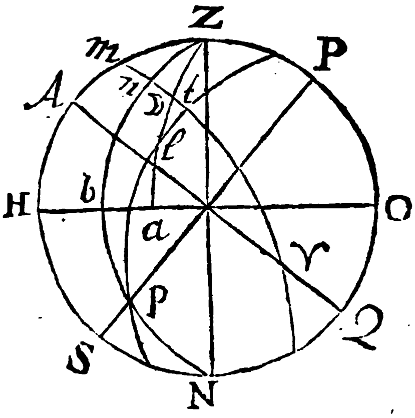
From the Parallax of Altitude or Angle of Depreſſion, by the means of which the Planets appear at a greater diſtance from the Zenith than otherwiſe they would do, there will ariſe a Parallax of Longitude or of Latitude, or of both; and conſequently a Parallax of Right Aſcenſion or Declination, or of both.

Thus if the Planet happen to be in the Ecliptic, and the Ecliptic become a Vertical Circle, as it will do at thoſe times, and at thoſe Places where the Latitude of the Place becomes equal to the Declination of the point of the Ecliptic which the Planet poſſeſſes at the given time, and both the ſameway, then the Planet being depreſſed in the Ecliptic it ſelf, there can be no Parallax of Latitude, but the Parallax of Altitude will become the Parallax of Longitude, and be equal to it. For *Example*, ſuppoſe the Moon in *Leo* 00 deg. 03 min. 31 ſec. and upon her Aſcending Node, when her horizontal Parallax is 55 min. 32 ſec. and which happens upon the *2d* of *January* 1721, at 15 h. 41 m. 09 ſ. becauſe at the Height of 40 deg. 47 min. 05 ſec. her Parallax of Altitude is 42 min. 29 ſec. $\frac{1}{2}$, her viſible or apparent Place if ſhe be on the Eaſtern Side of the *Nonageſime*, will be *Leo* 00 deg. 46 min. 00 ſec. $\frac{1}{2}$, but if ſhe be on the Weſt Side of the *Nonageſime*, her viſible Place will be *Cancer* 29 deg. 21 min. 02 ſec.

If the Planet be in the *Nonageſime*, or upon that Vertical Circle that paſſes thro' the Poles of the Ecliptic, and conſequently cuts the Ecliptic at Right-angles, becauſe in this *Caſe* the Planet is depreſſed in the Circle of Longitude, the Parallax of Longitude ceaſes, and the Parallax of Latitude becomes equal to the Parallax of Altitude: Thus for *Example*, at *London* on the *2d* of *January* 1721, at 13 h. 00 min. 03 ſec. when the Moon is in *Cancer* 28 deg. 44 min. 57 ſec. having 00 deg. 08 min. 16 ſec. South Latitude, and upon the *Nonageſime*, at which time her true Altitude being 57 deg. 56 min. 09 ſec. and her horizontal Parallax 55 min. 32 ſec. her Parallax of Altitude will be 29′ 53″, and conſequently her viſible Latitude will be 00 deg. 38 min. 09 ſec. South; for the Parallax depreſſes the ſame way as the Planet appears, that is, if they appear to the Southward of the Zenith, the Parallax depreſſes them to the Southward; and conſequently diminiſhes their Latitude if it be North, and increaſes it if it be South; and on the contrary, if the Planets appear to the Northward of the Zenith, the Parallax increaſes their Latitude if it be North, but diminiſhes if it be South: All which are ſo obvious that they need no further Explanation.

But if the Planet appear neither in the Ecliptic nor upon the *Nonageſime*, or in the Vertical Circle paſſing thro' it; that is, if the Vertical Circle paſſing thro' the Planet cuts the Ecliptic at Oblique-angles, then the Parallax of Altitude will cauſe a Parallax both of Latitude and Longitude, and how both may be obtained will appear by the following *Example*.

On the 2*d* of *January* 1721, at 15 h. 41 m. 09 ſ. apparent time, according to Mr. *Flamſteed's Lunar Tables*, publiſhed in his *Doctrine of the Sphere*, the Moon will be in *Leo* 3 min. 31 ſec. and upon her aſcending Node; let it therefore be required to find her Parallax of Longitude and Latitude, and conſequently her viſible Place and viſible Latitude, her horizontal Parallax at that time being 55 min. 32 ſec.



On the 2*d* of *January* 1721, at 15 h. 41 m. 09 ſ. the Sun will be in *Capricorn* 24 deg. 01 min. 47 ſec: and conſequently his Right Aſcenſion according to the former Methods of Inveſtigation will be found to be 295 deg. 55 min. 30 ſec. equal to 19 h. 43 m. 42 ſ. to this therefore adding 15 h. 41 m. 09 ſ. the time paſt from the preceding Noon, equal to 235 deg. 17 min. 15 ſec. we ſhall have the Right Aſcenſion of the Mid-Heaven, or Degree then under the Meridian, equal to 171 deg. 12 min. 45 ſec. or 11 h. 24 m. 51 ſ.

Whence the Degree of the Ecliptic then Culminating, will be found to be *Virgo* 20 deg. 25 min. 59 ſec. its Declination *A m* 3 deg. 47 50 ſec. ½, N. and conſequently its Altitude *H m* will be 42 deg. 15 min. 50 ſec. ½ and its Diſtance from the Zenith Z *m* 47 deg. 8 min. 12 ſec. and the Angle Z *m n* formed by the Ecliptic and Meridian 66 deg. 48 min. 30 ſec.

Hence the Altitude of the *Nonageſime n* will be found to be 47 deg. 08 min. 12 ſec. and its diſtance from the Vertex Z *n* 42 deg. 51 min. 48 ſec its diſtance from the Mid-Heaven 23 deg. 25 min. 43 ſec. ½, and conſequently the *Nonageſime* its ſelf will be in *Leo* 27 deg. 00 min. 15 ſec. ½.

And becauſe the Moon's true Place is in *Leo* 00 deg. 03 min. 31 ſec. her diſtance from the *Nonageſime* will be 26 deg. 56 min. 44 ſec. ½, to the Weſtward of the *Nonageſime* or in Antecedence, whence the Moon's true Altitude at that time *a* ☽, will be found to be 40 deg. 47 min. 05 ſec. her Zenith diſtance Z ☽ 49 deg. 12 min. 55 ſec. and the Parallactic Angle Z ☽*n* 63 deg. 58 min. 30 ſec.

And becauſe her horizontal Parallax is 55 min. 32 ſec. her Parallax at the Altitude of 40 deg. 47 min. 05 ſec. will be 42 min. 29 ſec. ½. Wherefore,

Set off 42′ 29″ ½, in the Vertical Circle Z ☽*a* paſſing thro' the Moon from ☽ the true Place of the Moon to *l*, then will *l* be the Moon's viſible Place, and thro' the point *l* and the Pole of the Ecliptic *p* draw the Circle of Longitude *p l t*, then will *p l* be her viſible diſtance from the South Pole of the Ecliptic, or Complement of her Latitude ♈*t* her viſible Longitude; and conſequently ☽ *t* her Parallax of Longitude, and *l t* her Parallax of Latitude; to find each of which Trigonometrically, in the Right-angled Spherical Triangle ☽ *l t* Right-angled at *t* are given, ☽ *l* the Moon's Parallax of Altitude 42 min. 29 ſec ½, and the Angle *l* ☽ *t* equal to the parallactic Angle Z ☽ *n* equal to 63 deg. 58 min. 30 ſec. whence to find 1*ſt l t* the Parallax of Altitude, it will be by the 5*th Caſe* of *Right-angled Spherical Triangles*,

As *R* : S. ☽ *l* : : S. <*l* ☽ *t* : S. *l t*; that is,

As the Radius ———————————— 10 0000000

To the Sine of the parallactic Angle 63.58.30 ———— 9.9535677
So is the Sine of the parallax of Altitude 42.29½ —— 8.0919898

To the Sine of the parallax of Latitude 38.11 ———— 8.0455575

Which becauſe the Moon was upon her aſcending Node, and conſequently has no true Latitude, gives 00 deg. 38 min. 11 ſec. for her viſible Latitude South.

Again, to find ☽ *t* her Parallax in Longitude, it will be by the 4*th Caſe* of *Right-angled Spherical Triangles*, As *R* : cs, < *l* ☽ *t* : : t, ☽ *l* : t, ☽ *t* ; that is,

As the Radius———————————————— 10.0000000

To the Co-ſine of the parallactic Angle 63.58.30———9.6422304
So is the Tangent of the Parallax of Altitude 42.29.½ ——8 0920220

To the Tang. of the Parallax of Longitude 18.39½——7.7342524

Which taken from the Moon's true Longitude *Leo* 00 deg. 03 min. 31 ſec. becauſe the Moon is on the Weſtern Side of the *Nonageſime*, gives *Cancer* 29 deg. 44 min. 52 ſec. ½ for the Moon's viſible Place.

But if the Moon had been on the Eaſtern Side of the *Nonageſime*, then her true Place would have preceded her viſible, and conſequently the Parallax muſt have been added to the true Place, to have obtained the viſible.

The Parallactic Triangle ☽ *l t*, tho' really Spherical, yet by reaſon of its exceeding ſmallneſs, may for the Eaſe of Calculation be conſidered as a Plane Triangle, ſince the Anſwers to all Intents and Purpoſes are the ſame, as will appear by the following Calculation.

The Parallax of Altitude is 42 min. 29 ſec. ½, which being reduced into Seconds makes 2549.5 for the Length of the Side ☽ *l*; wherefore by the 1ſt *Caſe* of *Right-angled Plane Triangles*, to find the Side *l t* the Parallax of Latitude it will be,

As *R* : < *l* ☽ *t* : : S. ☽ *t* : *l t* ; that is,

As the Radius ———————————— 10.0000000

To the Sine of the parallactic Angle 63.58.30 ————9.9535677
So is the Logar. of the Par. of Alt. in Seconds 2549.5 3.4064550

To the Logar. of the Par. of Latitude in Seconds 2291.6 3.3600227

Which being reduced into Minutes and Seconds, by dividing by 60, makes 38 min. 11 ſec. for the Parallax of Latitude as before found.

And for the Side ☽ *t* the Parallax of Longitude, it will be by the ſame *Caſe* of *Right-angled Plane Triangles*, As *R* : <*l* ☽ *t* : : c s, *l* ☽ : *l* ☽ ; that is,

As the Radius	10 0000000
To the Co-ſine of the parallactic Angle 63.58.30	9.6422304
So is the Log of the Par. of Altitude in Seconds 2549.5	3.4064550
To the Log. of the Par. of Longit. in Seconds 1118.6	3.0486854

Which being reduced into Minutes and Seconds makes 18 min. 38 ſec. $\frac{6}{10}$, the ſame as was found by the former Calculation.

But if the Moon have any Senſible Latitude, it will render the Calculation a little more intricate, as the *Reader* will ſee by the Subſequent *Example*.

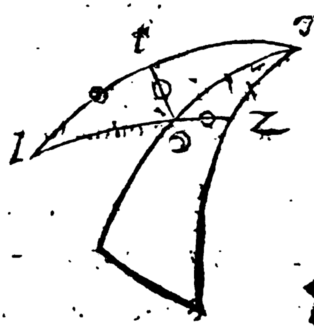
Let it be required to find the Parallax of Latitude and Longitude of the Moon at *London*, on the *22d* of *June* 1721, at 9h. 30m. apparent time, when her true Place is in *Libra* 26 deg. 27 min. 18 ſec. and her true Latitude 5 deg. 15 min. 19 ſec. North, ſhe being at that time near the Northern Limit, and her horizontal Parallax 59 min. 45 ſec.

The Sun on the *22d* of *June* 1721, at 9 h. 30 m. is in *Cancer* 11 deg. 39 min. 53 ſec. and conſequently his Right Aſcenſion is 102 deg. 41 min. 06 ſec. $\frac{1}{2}$, to which adding the time paſt from Noon 9 h. 30 min. 00 ſec. equal to 142 deg. 30 min. 00 ſec. the Sum 245 deg. 11 min. 06 ſec. $\frac{1}{2}$ will be the Right Aſcenſion of the Mid-Heaven at that time, whence the Angle formed by the Ecliptic and the Meridian will be found to be 80 deg. 22 min. 22 ſec. the point of the Ecliptic then under the Meridian *Sagittary*, 7 deg. 01 min. 08 ſec. and its diſtance from the Equator 21 deg. 31 min. 19 ſec. to the Southward, which therefore added to the diſtance of the Equator from the Vertex 51 deg. 32 min. 00 ſec: will give 73 deg. 03 min. 19 ſec. for the diſtance of the Mid-Heaven from the Zenith, or being taken from 38 deg. 28 min. 00 ſec. the Height of the Equator, will leave 16 deg. 56 min. 41 ſec. for the Altitude of the Mid-Heaven, or diſtance between the Pole of the Ecliptic and the Zenith

Hence the Altitude of the *Nonageſime* will be found to be 19 deg 25 min. 07 ſec. and its Diſtance from the Mid-Heaven 28 deg 45 min. 43 ſec. $\frac{1}{2}$ which therefore taken from the Longitude of the Mid-Heaven *Sagittary* 7 deg. 01 min, 08 ſec. becauſe the Mid-Heaven is between *Cancer* and *Capricorn*, will leave *Scorpio* 8 deg. 1

m

min. 24 ſec. ½ for the Longitude of the *Nonageſime* at the time given Wherefore,



If from the Longitude of the *Nonageſime Scorpio* 08 deg. 15 min. 24 ſec. be taken the true Place of the Moon *Libra* 26 deg. 27 min. 18 ſec., the Remainder 11 deg. 48 min. 06 ſec. will give the Difference of Longitude between the *Nonageſime* and the Moon, equal to the Angle Z π ☽, whence to find Z ☽, the true diſtance of the Moon from the Zenith, or Complement of her Altitude, and the Angle Z ☽ π, the Complement of the parallactic Angle; in the Oblique-angled Spherical Triangle Z π ☽ are given, Z π, the diſtance of the Vertex from the Pole of the Ecliptic, equal to the Altitude of the *Nonageſime*, equal to 19 deg. 25 min. 07 ſec. the Side π ☽, equal to the Complement of the Moons true Latitude, equal to 84 deg. 44 min. 01 ſec. and the contained Angle ☽ π Z, equal to the Difference of Longitude between the Moon and the *Nonageſime*; whence to find firſt, the Complement of the parallactic Angle π ☽ Z, it will be by the *9th Caſe* of *Oblique-angled Spherical Triangles*, having firſt imagined a Perpendicular to fall from the point Z upon the oppoſite Side π ☽.

As R : cs, ∠ Z π ☽ : : t, Z π : t, of a *4th* Arch; that is,

As the Radius ——— 10.0000000

To the Co-ſine of the Angle Z π ☽ 11.48.06 ——— 9.9907213

So is the Tangent of the Arch π Z 19.25.07 ——— 9.5471847

To the Tangent of a *4th* Arch 19.02.16½ ——— 9.5379060

Which taken from the Complement of the Moons true Latitude 84 deg. 44 min. 01 ſec. will leave 65 deg. 41 min. 44 ſec. for a fifth Arch. Wherefore,

As the Sine of the *5th* Arch 65.41.44 ——— 9.9596959

To the Sine of the *4th* Arch 19.02.16½ ——— 9.5134756

So is the Tangent of the Angle Z π ☽ 11.48.06 ——— 9.3200242

To the Tangent of the Angle π ☽ Z 04.16.36½ ——— 8.8738039 true Latitude being 4 deg. 36 min. 15 ſec. South, her diſtance from the North Pole of the Ecliptic, will be 94 deg. 36 min. 15 ſec. hence and from the Altitude of the *Nonageſime* 32 deg 24 min. 57 ſec. her true Altitude above the Horizon will be found to be 21 deg. 28 min. 59 ſec. and the parallactic Angle 70 deg. 21 min. 43 ſec.

Her horizontal Parallax is 56 min. 37 ſec. whence her Parallax of Altitude will be found to be 53 min. 00 ſec. and conſequently her Parallax in Latitude will be 49 min. 55 ſec. which added to her true Latitude 4 deg 36 min. 15 ſec. becauſe it is South, will give 5 deg. 26 min. 10 ſec. for her viſible Latitude South: Alſo her Parallax in Longitude will be found to be 17 min. 49 ſec. which being added to her true Place, becauſe ſhe is on the Eaſtern Side of the *Nonageſime*, will give *Taurus* 8 deg. 19 min. 11 ſec. for her viſible Place; and conſequently the viſible Difference of Longitude between her and the Star is 17 min. 49 ſec. in Conſequence.

At 6 h 33 m. *P. M.* the Sun is in *Scorpio* 11 deg. 17 min. 52 ſec. and conſequently his Right Aſcenſion is 218 deg. 51 min. 30 ſec. this added to 6 h. 33 m. the time from Noon, equal to 98 deg. 15 min. 00 ſec. makes 317 deg. 06 min. 30 ſec. for the Right Aſcenſion of the Mid-heaven at that time, whence the Mid-heaven it ſelf will be found to be in *Aquarius* 14 deg. 38 min. 00 ſec. the Angle formed by the Ecliptic and the Meridian 75 deg. 01 min. 32 ſec. and the Declination of the Mid-Heaven 16 deg. 18 min. 23 ſec. S. and conſequently its diſtance from the Zenith 68 deg. 00 min. 23 ſec. whence the diſtance of the *Nonageſime* from the Mid-Heaven will be found to be 35 deg. 51 min. 37 ſec. in Conſequence; the Mid-Heaven it ſelf in *Piſces* 20 deg. 29 min. 37 ſec. and its Altitude 27 deg. 31 min. 20 ſec.

The Moon's true Place is in *Taurus* 07 deg 35 min. 14 ſec. whence the Difference of Longitude between her and the *Nonageſime* is 47 deg. 05 min. 37 ſec. at which time her true Latitude being 4 deg. 37 min. 12 ſec. South, her diſtance from the viſible Pole of the Ecliptic is 94 deg. 37 min. 12 ſec. hence and from the Altitude of the *Nonageſime* 27 deg. 31 min. 20 ſec. her true Altitude will be found to be 13 deg. 37 min. 14 ſec. and the parallactic Angle 69 deg. 58 min. 13 ſec.

Her horizontal Parallax being 56 min. 37 ſec. her Parallax of Altitude will be found to be 55 min. 18 ſec. and conſequently her Parallax in Longitude will be 18 min 53 ſec. which being added
he true Place *Taurus* 07 deg 35 min. 14 ſec. will give her viſible
in *Taurus* 7 deg. 54 min. 7 ſec. which taken from her viſible
Place

Place before found *Taurus* 8 deg. 19 min. 11 ſec. will leave 25 min 04 ſec. for the viſible motion of the Moon in Longitude, between the times of Calculation 6 h. 33 m. and 7 h. 22 m. 08 ſec. during the Space of 49 min. 08 ſec.

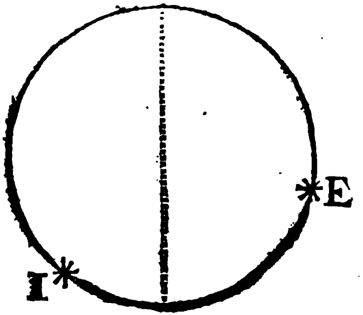
Whence to find the mean Time of the viſible Conjunction, it will be as 25 min. 04 ſec. the Difference between the viſible Places of the Moon at 6 h. 33 m. and at 7 h. 22 m. 08 ſ. the time of the true Conjunction to 49 min. 08 ſec. the Difference between the times; ſo is 17 min. 49 ſec. the viſible diſtance of the Moon from the Conjunction, at the time of the true Conjunction, equal to her Parallax in Longitude at that time, to 34 min. 55 ſec. the interval of Time between the true and viſible Conjunction, which being ſubſtracted from 7 h. 22 m. 08 ſ. the time of the true Conjunction, will leave 6 h. 47 m. 13 ſ. for the mean time of the viſible Conjunction, to which adding the Equation of Days 16 m. 07 ſ. the Sum 7 h. 3 m. 20 ſ. will be the apparent time of the viſible Conjunction.

From the parallactic Angle 69 deg. 58 min. 13 ſec. and the Parallax of Altitude 55 deg. 18 min. the Parallax of Latitude will be found to be 51 min. 57 ſec. this added to her true Latitude 4 deg. 37 min. 12 ſec. South, will give 5 deg. 29 min. 9 ſec. for her viſible Latitude South, and comparing this with her viſible Latitude 5 deg. 26 min. 10 ſec. before determined, it appears that her viſible Alteration of Latitude between the times of Calculation, is 2 m. 59 ſ. increaſing; whence to find the Moon's viſible Latitude at the time of the viſible Conjunction it will be, as 49 min. 08 ſec. the Difference between the times 6 h. 33 m. and 7 h. 22 min. 08 ſec. to 2 min. 59 ſec. the Difference of Latitude, ſo is 34 min. 55 ſec. the Difference between the true and viſible Conjunction, to 2 min. 07 ſec. the proportional Alteration of Latitude, which being added to 5 deg. 26 min. 10 ſec. the viſible Latitude of the Moon at the time of the true Conjunction, will give 5 deg. 28 min. 17 ſec. for the Moon's viſible Latitude South, at the time of the viſible Conjunction.

The Stars viſible Latitude is 5 deg. 35 min. 33 ſec. South, and conſequently the Center of the Moon paſſes 7 min. 16 ſec. to the Northward of the Star.

The Moon's horizontal Semidiameter is 15 min. 22 ſec. whence her apparent Semidiameter at 21 deg. 28 min. 59 ſec. of Altitude, will be found to be 15 min. 27 ſec. ½, hence and from the Difference of Latitude between the Star and the Center of the Moon, the Immerſion of the Star under the Moon's Body will be found to be at 6h. 36 m. 40ſ. at the diſtance of 138 deg. 2 min.½, from the Vertex of the Moon on the Eaſtern Side at the point *I*, equal to the Supplement of the Difference between the firſt Angle of Incidence 61 deg. 57 min. 38 ſec. and 20 deg. 00 min. 2 ſec. the Complement of the parallactic Angle 69 deg. 59 min. 58 ſec. at that time.

The Moon's apparent Diameter at the Altitude of 13 deg. 37 min. 14 ſec. is 15 min. 25 ſec.½, hence the time of the Emerſion from under the Moon's Body will be found to be at 7 h. 30 m. 04 ſec. after it has continued under the Body of, or been covered by the Moon for the Space of 53 min. 24 ſec. and will appear on the Weſtern Limb of the Moon at the point *E*, at the diſtance of 98 deg. 32 min. from the Vertex, equal to the Supplement of 81 deg. 28 min. the Sum of the ſecond Angle of Incidence 61 deg. 53 min. 40 ſec. and 19 deg. 34 min. 27 ſec. the Complement of the parallactic Angle 70 deg. 25 min. 33 ſec. at that time; as in the following Figure.



Again, on the 16*th* of *January* 1722, at 14 h. 00 m. 00 ſ. P. *M.* the Moon will be in Conjunction with the Southern Horn of the *Bull*, whoſe Place according to Mr. *Flamſtead*'s Catalogue, is *Gemini* 20 deg. 54 min. 35 ſec. having 02 deg. 14 min. 24 ſec. South Latitude; at which time the Sun being in *Aquarius* 7 deg. 56 min. 51 ſec. his Right Aſcenſion will be found to be 310 deg. 22 min. 20 ſec. to which adding the time of the Conjunction 14 h. 00 m. 00 ſ. equal to 210 deg 00 min. 00 ſec. the Sum 160 deg. 22 min. 20 ſec will be the Right Aſcenſion of the Mid-Heaven at that time; hence the Mid-Heaven it ſelf will be found to be in *Virgo* 8 deg. 45 min. 08 ſec. its Declination 8 deg. 18 min. 11 ſec. ½ North, and conſequently its diſtance from the Zenith 43 deg. 13 min. 48 ſec. ½, and the Angle formed by the Ecliptic and the Meridian 67 deg. 57 min. 20 ſec. hence the diſtance of the *Nonageſime* from the Mid-Heaven will be found to be 19 deg. 26 min. 03 ſec. in Antecedence, and conſequently the Mid-Heaven it ſelf is *Leo* 19 deg. 19 min. 05 ſec. and its Altitude 50 deg. 35 min. 26 ſec. ½.

The Moon's true Place is in *Gemini* 20 deg. 54 min. 35 ſec. whence the Difference of Longitude between the Moon and the *Nonageſime* is 58 deg. 24 min. 30 ſec. at which time her true Latitude being 1 deg. 48 min. 25 ſec. South, her diſtance from the viſible Pole of the Ecliptic will be 91 deg. 48 min. 25 ſec. whence and from the Altitude of the *Nonageſime* 50 deg. 35 min. 26 ſec. ½, her true Altitude above the Horizon will be found to be 22 deg. 36 min. 54 ſec. and the parallactic Angle 44 deg. 31 min. 29 ſec.

The Moon's horizontal Parallax is 55 m. 15 ſ. whence her Parallax of Altitude will be found to be 51 m. 19 ſ. and conſequently her Parallax in Latitude will be 35 min. 59 ſec. which added to her true Latitude 1 deg. 48 min. 25 ſec. South, will give her viſible Latitude 2 deg. 24 min. 24 ſec. alſo her Parallax in Longitude will be found to be 36 min. 35 ſec. which being ſubſtracted from her true Place *Gemini* 20 deg. 54 min. 35 ſec. becauſe ſhe is on the Weſtern Side of the *Nonageſime*, will leave *Gemini* 20 deg. 18 min. for her viſible Place at that time, and conſequently the viſible Difference of Longitude between her and the Star is 36 min. 35 ſec. in Antecedence.

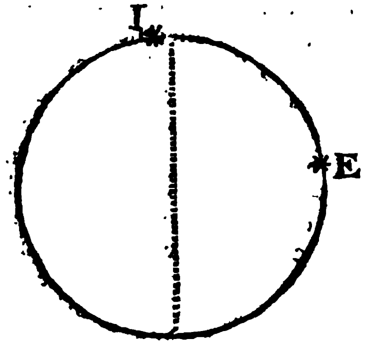
At 16 h. 00 m. 00 ſ. *P. M.* the ſame Day, the Moon will be in *Gemini* 21 deg. 55 min. 27 ſec. having 1 deg. 43 min. 21 ſec. South Latitude, at which time the Sun being in *Aquarius* 8 deg. 1 min. 53 ſec. the viſible Place of the Moon by the former Method of Calculation will be found to be *Gemini* 21 deg. 20 min. 20 ſec. and her viſible Latitude 2 deg. 25 min. 45 ſec. South, whence to find the time of the viſible Conjunction it will as 62 min. 20 ſec. the viſible motion of the Moon in Two Hours, to 2 h. ſo is 36 min 35 ſec. the viſible Difference of Longitude between the Moon and the Star, to 1 h. 10 min. 26 ſec. the Interval of Time between the true and viſible Conjunction; which being added to the time of the true Conjunction 14 h. 00 m. 00 ſ. becauſe the Moon is in Antecedence to the Star, will give 15 h. 10 m. 26 ſ. for the mean Time of the viſible Conjunction, from which taking away the Equation of Days, 13 min. 29 ſec. there will remain 14 h. 56 m. 57 ſ. for the apparent time of the viſible Conjunction.

And for the viſible Latitude at the time of the viſible Conjunction it will be as 2 Hours to 1 min. 21 ſec. the viſible Difference of Latitude, ſo is 1 h. 10 m. 26 ſ. the Difference of time between the true and viſible Conjunction, to 47 min. the proportional Alteration of Latitude, which being added to 2 deg. 24 min. 24 ſec. the viſible Latitude of the time at the true Conjunction, will give 2 deg. 25 min.

25 min: 11 ſec. for the viſible Latitude South, at the time of the viſible Conjunction.

The Stars viſible Latitude is 2 deg. 14 min. 24 ſec. South, and conſequently the Center of the Moon paſſes 10 min. 47 ſec. to the Southward of the Star.

The Moon's horizontal Semidiameter is 14 min. 59 ſec. whence her apparent Diameter at 22 deg. 36 min. of Altitude, will be found to be 15 min. 04 ſec. hence and from the Difference of Latitude between the Moon and the Star, the Emerſion will be found to be at 14 h. 36 m. 58 ſec. at the diſtance of 2 deg. 5 min. to the Eaſtward of the Vertex of the Moon, and the Immerſion at 15 h. 16 m. 52 ſ. when the Star will appear on the Weſtern Limb of the Moon at the diſtance of 89 deg. 27 min. from the vertical point of the Moon, as is exhibited in the following Figure.



After the ſame manner may the time when the Moon will cover any of the Planets be determined, or that the Moon or any of the Planets will paſs by, or approach ſo near to any fixed Star, as to come within the compaſs of a *Telleſcope*; ſo that the time of their viſible Conjunction and the diſtance of their Centers may be pronounced, by the help of a *Micrometer*, which latter Appearances are called the Appulſes of the Moon or Planets, to the fixed Stars, and are of equal Benefit and Advantage to the Seamen, with the Occultations before Inveſtigated.

An exact Knowledge of the times when Appearances of this kind do happen, are of very great Uſe in finding the true Place of the Moon and Planets, and the Difference of Longitude between thoſe Places where they can be obſerved, as has been ſhewn in its proper Place: and His late Majeſty King *Charles* the Second, was ſo well aſſured of this, that one of the principal Ends for which he Built the *Royal Obſervatory* at *Greenwich*, was with a View to obtain a good and correct Catalogue of the fixed Stars, in order to obtain good Tables of the *Motions of the Moon* and of the reſt of the Planets, and render Predictions of this kind more certain this that great *Aſtronomer* and moſt ſkilful and diligent *Obſervat* the late Reverend Mr. *Flamſteed*, appointed by the *Royal Founder* himſelf, has after 30 Years indefatigable Pains and conſtant Application to the *Heavens*, happily accompliſhed; and has left behind him the largeſt and moſt exact Catalogue that ever the World was enrich'd with, containing about 3500 Fixed Stars, of which near 1000 lye within the Zodiack, to the great Honour of the *Britiſh* Nation and the laſting Reputation of the *Author*; a Treaſure that will render his Name valuable to the lateſt Poſterity; and will perpetuate his Memory ſo long as Men ſhall view the *Heavens*, or Ships ſail upon the *Ocean*.

By the ſame Method as we compute the Time when any fixed Star will be Eclipſed by the Moon, and its Emerſion from her, or the diſtance of the Center from it, when not covered by her, may the Beginning, Middle, End, or Quantity of a Solar or Lunar Eclipſe be determined; regard being had to the horizontal Parallax of the Sun, his Semi-diameter and hourly Motion; but for this I ſhall at preſent refer the *Reader* to the ſeveral *Treatiſes* that have been Wrote upon this Subject, and particularly to the *Doctrine of the Sphere*, where he will find theſe things performed by a New Method of Conſtruction, intirely free from the perplexed and intricate Conſideration of Parallaxes.

As the generous and unprejudic'd *Reader*, will readily conclude, that the frequent mention made of Mr. *Flamſteed*, cannot proceed from any ſelfiſh View, (ſince no Advantage can ariſe from praiſing or flattering the Dead) but from a great Eſteem, grounded upon a juſt Knowledge of the Merrits of ſo great a Man, ſo the ſame generous way of reaſoning, will neceſſarily lead him to excuſe the following Account of what he left behind him; and the rather, in that it only ſupplies a vacant Space, and may be a Satisfaction to ſome People who are true lovers of this *Science*; and who may be willing to know what Helps in time may be afforded them.

THE firſt Volume contains the Obſervations of Mr. *William Gaſcoign*, (the firſt Inventor of the Way of meaſuring Angles in a *Teleſcope*, by the help of Screws; and the firſt that apply'd *Teleſcopial Sights* to *Aſtronomical Inſtruments*) taken at *Middleton*, near *Leeds* in *Yorkſhire*, betwixt the Years 1638 and 1643; excerpted from his Letters to Mr. *Crabtree*, with ſome of Mr. *Crabtrees* Obſervations of the ſame Years: as alſo Obſervations of the Sun and Moon's Diameters, Configurations and Elongations of *Jupiter's* Satellites from him, ſmall Diſtances of fixed Stars, with Appulſes of the Moon and Planets to them, obſerv'd with a *Teleſcope* and *Micrometer* at *Derby*, by Mr. *Flamſteed*, betwixt the Years 1670 and 1675; with the larger intermutual Diſtances of fixed Stars and of the Planets from them, Eclipſes of the *Sun*, *Moon* and *Jupiters* Satelites, Spots on the *Sun*, *Comets* and *Refractions* taken with a *Sextant* of near Seven Foot Radius, a voluble *Quadrant*, and the above-mentioned Inſtruments, betwixt the Years 1673 and 1689, at His Majeſty's *Obſervatory*; ranked under proper Heads, and ſubjoin'd the Places of the *Moon*, *Saturn*, *Jupiter*, *Mars*, *Venus* and *Mercury*, deduced from the Obſervations, and alſo neceſſary Tables to be uſed with them.

The Second Volume contains his Obſervations (made with a *Mural Arch* of near 7 Foot Radius, and 140 Degrees in the Limb) of the Meridional Zenith-diſtances of the fixed Stars, Sun, Moon, and the other Planets, with the time of their Tranſits over the Meridian, together with Obſervations of the Sun and Moon's Diameters, Eclipſes of the Sun, Moon, and of Jupiters Satellites, Variations of the *Compaſs*, from 1689 to the end of the Year 1719, *&c.* Tables ſerving to render the Calculation of the Stars and Planets Places from the Obſervations eaſy and expeditious; to which is added near 2000 Places of the Moon, (at the Oppoſitions, Quadratures, and on her Limits, *&c.* with the Places of *Saturn*, *Jupiter*, *Mars*, *Venus*, and *Mercury* deriv'd from the above-mentioned Obſervations.

The Third Volume contains a Catalogue of the Right Aſcenſions, Diſtances from the Pole, Longitudes, Latitudes and Magnitudes of about 3500 fixed Stars, with the Variations of the Right Aſcenſions, and Polar Diſtances, whilſt they change their Longitudes 1 Degree; whereby their Right Aſcenſions and Diſtances from the Pole may be determined for 200 Years paſt or to come ſufficiently exact, large Tables, by which the Right Aſcenſions and Diſtances from the Pole of the Stars and Planets being given, their Longitudes and Latitudes may be found by Inſpection. To this Volume will be prefix'd a very large *Preface*, containing an Account of all the *Aſtronomical Obſervations* made before his own Time; with a Deſcription of the Inſtruments made Uſe of: As alſo an Account of his own Obſervations and Inſtrmuments, together with *Ptolomy's* Catalogue of 1026 fixed Stars in *Greek*, with a new *Latin* Verſion of it and *Uleg-beig*'s Places, annexed on the *Latin* Page, with the Corrections. Small Catalogues

the *Arabs*, *Tycho Brahe*'s of about 780 fixed Stars, in a proper
rder: the *Landsgrave* of *Heſſes* of 386; *Hevelius*'s of 1534 in a
oper Order: Dr. *Halley*'s Catalogue of about 300 of the South-
n fixed Stars, reduced to Right Aſcenſion, Diſtance from the
ole, Longitude, Latitude, with Variations of Right Aſcenſion
d diſtance from the Pole; done from his Diſtances and Mr. *Flam-
eds* genuin Catalogue, in which the Stars Places are reduced to
e Year 1726.

Charts of all the Conſtellations viſible in our Hemiſphere, conſtruct-
l after a new Method of his own; wherein the Ancient Figures are
ſtored and the Stars laid down from the above-mentioned Catalogue,
ith large *Planiſpheres* both of the Northern and Southern Conſtella-
ons; alſo *new Aſtronomical Tables* deduced from his own Obſerva-
ons, for finding the Places of the *Luminaries* and *Planets*; one Vo-
me of *Acta*; a Volume of *Aſtronomical Lectures* read at *Greſham
llege*, with ſeveral other Tracts; a Volume of *Letters* of the
arned and beſt reputed *Mathematicians*, (in which are many
luable Things) particular of Monſieur *Caſſini*, Sen. Monſ. *Hevelius*,
lonſ. *Blanchini*, Sir *Jonas Moor*, Sir *Chriſtopher Wren*, Dr. *Wallis*,
lr. *Mollyneux*, Dr. *Gregory*, Mr. *Caſwell*, Monſ. *Romer*, Mr. *Olden-
rg*, Mr. *Collins*, Mr. *Townley*, Sir *Iſaac Newton*, Dr. *Halley*, Mr.
arp, and others of leſs Note.

TABLE, *containing the* Right Aſcenſions, Declinations, Longitudes *and* Latitudes *of above Sixty Eminent Fixed* Stars, *deduced from the* Flamſteedian Obſervations, *and fitted to the Year* 1726.

A

The Names of the Fixed Stars.

IN the Breaſt of *Caſſiopeia Scheder*
The Bright Star in the Tail of the *Whale*
The Knot of the String of the *Fiſhes*
The Bright Star of *Aries*
In the Jaw of the *Whale Mandibula*
The Head of *Meduſa Algol*
The bright Side of *Perſeus*
The Southern Eye of the *Bull Aldebaran*
The *Goat* Star *Capella*
The bright Foot of *Orion Regel*
The Northern Horn of the *Bull*
The preceding Shoulder of *Orion*
The Southern Horn of the *Bull*
The middlemoſt in the Belt of *Orion*
The laſt in the Girdle of *Orion*
The following Shoulder of *Orion*
The Dog Star *Syrius*
Caſtor, or the Head of the Northermoſt Twin
Procyon, or the Little Dog Star
Pollux, or the Head of the Southermoſt Twin
The Heart of *Hydra*
The *Lyon*'s Heart *Regulus*
The Southermoſt of the two preceding Stars in the *Square* of the *Great*
In the Baſe of the Cup
The Northermoſt of the two preceding Stars in the *Square* of the *Great*
Juſt below the bottom of the Cup
The Tail of the *Lyon Deneb*
The Southermoſt of the two preceding Stars in the Square of the *Great*
In the Bill of the *Crow*
In the preceding Wing of the *Crow*
The Northermoſt of the two following in the Square of the *Great Bear*
In the following Wing of the *Crow*
In the Foot of the *Crow*
The firſt in the Tail of the *Great Bear*
In the Northern Wing of the *Virgin Vindemiatrix*
The *Virgin*'s *Spike*
The middlemoſt of the three in the Tail of the *Great Bear*
The laſt but one in the Tail of *Hydra*
The laſt of the three in the Tail of the *Great Bear*
In the following Shoulder of the *Centaur*

ight Aſcenſion.			Declination.			Denomination.		Longitude.			Latitude.			Denomination.	Magnitude.
D.	M.	S.	D.	M.	S.			D.	M.	S.	D.	M.	S.		
6	18	14	55	02	08	N	♉	4	00	00	46	35	54	N	2 3
7	26	47	19	29	49	S	♓	28	43	02	20	46	52	S	3
6	59	42	01	25	20	N	♈	25	32	33	09	05	10	S	3
7	56	17	22	08	48	N	♉	03	49	18	09	57	12	N	2
1	59	38	02	59	18	N		10	29	15	12	36	59	S	2
2	36	25	39	52	38	N		22	20	42	22	23	47	N	2 3
6	14	15	48	51	27	N		28	16	04	30	05	20	N	2
5	02	57	15	55	23	N	H	05	57	00	05	29	49	S	1
4	07	14	45	41	19	N		18	01	41	22	51	47	N	1
5	20	46	08	33	00	S		13	00	00	31	10	11	S	1
7	13	49	28	20	33	N		18	43	56	05	21	34	N	2
7	37	10	06	04	18	N		17	07	33	16	51	30	S	2
0	19	12	20	56	20	N		20	57	28	02	14	24	S	3
0	34	23	01	24	49	S	♊	19	37	44	24	33	23	S	2
1	44	34	02	07	01	S		20	51	45	25	20	17	S	2
5	04	33	07	19	15	N		24	55	28	16	04	26	S	1
8	16	32	16	26	59	S	♋	10	19	01	39	32	08	S	1
9	15	42	32	27	09	N		16	25	20	10	03	48	N	1 2
1	14	01	05	54	26	N		22	00	21	15	57	55	S	1 2
2	07	34	28	39	25	N		19	26	09	06	39	27	N	2
8	31	18	07	29	23	S	♌	23	27	59	22	24	32	S	2
8	26	00	13	17	04	N		26	01	20	00	26	38	N	1
1	16	20	57	50	44	N		15	34	12	45	06	16	N	2
1	37	22	16	51	06	S		19	56	02	22	43	47	S	3
1	37	47	63	13	52	N		11	20	00	49	40	05	N	2
4	34	01	21	20	16	S	♍	24	45	35	25	37	33	S	4
3	46	03	16	05	43	N	♍	17	49	14	12	16	51	N	1 2
4	49	43	55	13	05	N	♌	26	36	38	47	07	20	N	2
8	34	54	23	11	37	S	♎	08	24	35	21	44	26	S	4
0	26	59	16	01	07	S		06	55	58	14	29	00	S	3
0	27	34	58	34	48	N		27	10	40	41	29	36	N	2 3
3	56	29	14	59	06	S		09	39	13	12	09	47	S	3
5	08	06	12	25	54	N	♎	06	07	40	16	12	40	N	3
0	00	55	21	52	49	S		13	33	25	18	01	54	S	3
2	30	42	57	28	04	N	♍	05	03	05	54	19	35	N	2
	42	07	09	43	23	S	♎	20	01	22	02	01	59	S	1
	43	57	56	22	18	N	♍	11	48	23	56	22	47	N	2
	02	04	21	43	20	S		23	12	30	13	43	18	S	3
	11	35	50	42	04	N		23	03	40	54	24	30	N	2
	39	23	34	38	03	S	♏	08	20	18	21	59	06	S	2

The Names of the Fixed Stars.

Arcturus
The Bright Star in the Southern *Ballance*
The bright Star in the Northern *Crown*
The bright Star in the Neck of the *Serpent*
The Heart of the *Scorpion Antares*
The Head of *Hercules*
The Head of *Serpentarius*
The following Shoulder of *Serpentarius*
In the Head of the *Dragon*, a Bright Star
The Bright Star in the *Harp*
The middlemoſt of the three in the Head of *Sagittary*
The Bright Star in the *Eagle*
The Hand of *Antinous*
The following Horn of *Capricorn*
The following Shoulder of *Aquarius*
The bright Star in the Tail of *Capricorn*
The Mouth of the Southern *Fiſh Fomalhaut*
In the flying *Horſe*, the bright Star *Scheat*
In the flying *Horſe*, the bright Star *Marchab*
The Head of *Andromeda*
In the flying *Horſe*, *Algenib*

And for the Benefit of the Southern Navigation, I have added
tions of the Four remarkable Southern Stars, commonly cal-

The preceding of the two middlemoſt in the *Croſs*
The Southern Foot of the *Croſs*
The Northermoſt of the Four
The following, or Eaſtermoſt of the Four

Th

Right Aſcenſion.			Declination.			Denomination.		Longitude.			Latitude.			Denomination.	Magnitude.
D.	M.	S.	D.	M.	S.			D.	M.	S.	D.	M.	S.		
210	47	42	20	38	44	N	♎	20	23	52	30	57	00	N	1
218	57	37	14	52	52	S	♏	11	16	40	00	22	51	N	2
230	46	40	27	39	08	N		08	25	56	44	21	17	N	2
232	41	37	07	18	12	N		18	13	22	25	31	56	N	2
243	10	40	25	47	22	S	♐	05	56	04	04	34	26	S	1
255	32	58	14	43	16	N		12	20	03	37	19	15	N	3
260	32	35	12	46	49	N		18	35	32	35	53	16	N	2
262	29	02	04	42	23	N		21	00	44	27	58	00	N	3
267	35	29	51	32	16	N		24	12	48	74	58	20	N	2
276	54	03	38	33	02	N	♑	11	27	18	61	45	31	N	1
282	04	37	22	06	12	S		11	10	42	00	54	38	N	4
294	19	56	08	10	01	N		27	53	24	29	19	11	N	1
299	17	08	01	36	44	S	♒	01	05	30	18	45	35	N	3
301	24	21	15	37	01	S	♒	00	13	57	04	27	37	N	3
327	55	42	01	38	12	S		29	32	16	10	40	38	N	3
322	58	46	27	19	10	S		19	43	14	02	32	19	S	3
340	34	51	31	03	20	N	♒	29	58	59	21	04	54	S	1
342	37	09	26	35	44	N	♓	25	32	13	31	08	06	N	2
342	46	03	13	43	40			19	39	13	19	24	37	N	2
358	33	20	27	34	12	N	♈	10	28	53	25	41	01	N	2
359	47	00	13	39	02	N	♈	05	10	50	12	35	12	N	2

the Longitudes, Latitudes, Right Aſcenſions and Declina-
led the *Croſiers* ; deduced from Dr. *Halley*'s Obſervations.

180	21	30	57	10	37	S	♍	01	56	40	50	18	00	S	3
183	05	20	61	31	16			08	07	40	52	45	00	S	2
184	10	20	55	30	10			02	57	40	47	41	00	S	2
188	10	48	58	05	57			07	53	40	48	29	00	S	2

The large Table of Declination, the Solar Tables, together with the Solar Theory, alſo an Account of the inequality of the Solar Days, and ſome other uſeful Matters that were deſign'd to follow next, will be found towards the End of the firſt Volume.

A *TABLE* of the Latitudes of the Principal *Sea-Ports*, *Headlands*, *Iſlands*, and eminent Towns in the moſt frequented Parts of the World; with their Longitudes from the Meridian of *LONDON*, deduced from the lateſt and beſt *Obſervations* and *Journals*.

Places Names.	Latit.			Longit.		
	D	M		D	M	
Acapulco in Mexico	17	30	North Latitude.	106	10	W
Abbeville in France	50	07		1	57	E
Aberdeen in Scotland	57	24		1	37	W
Agde in Spain	43	19		3	33	E
Agra, the Moguls Court	28	30		83	20	E
Aix in Provence	43	30		5	50	E
Alby	43	55		2	23	E
Alenſon	48	29		0	03	E
Alexandria in Eygypt	31	07		33	00	E
Alexandria in Syria	36	32		36	25	E
Aleppo in Syria	37	20		36	20	E
Algiers	36	40		3	05	E
Amiens in France	49	54		2	20	E
Amoy in the Eaſt Indies	24	35		116	55	E
Amſterdam in Holland	52	21		4	51	W
Ancone in Italy	43	31		14	30	E
Angers in France	47	27		0	29	W
Anout in Jutland	56	42		10	58	E
Antibes in France	43	34		6	43	E
Antwerp in Flanders	51	10		4	10	E
Antiocha in Syria	36	40		35	58	E
Arras in France	50	18		2	50	E
Arles in France	43	40		4	46	E
Aguatulco, Weſt Coaſt of America	15	27		101	03	W

Places Names.	Latit.			Longit.		
	D	M		D	M	
Ariea, on the Weſt Coaſt of America	18	12	S	74	10	W
Archangle Bar	64	30	N	40	30	E
Athens	38	05	N	34	29	E
Queen Ann's Foreland	63	32	N	67	30	W
Avignon in France	43	51	N	4	35	E
Aurillac	44	55	N	2	32	E
Auxerre	47	46	N	3	35	E
Babylon	34	30	N	48	30	E
Ballaſore in the Bay of Bengall	21	20	N	86	40	E
Baldivia, on the Weſt-Coaſt of America	39	35	S	81	20	W
Bantam, in the Iſle of Java	5	47	N	104	54	E
Bay Bonaventura, Weſt Coaſt of America	3	28	N	80	06	W
Bay of Breſt, in Newfoundland	52	10	N	55	00	W
Bay Todos Scantos Brazile	13	00	S	40	36	W
Bay of Bulls	47	40	N	54	45	W
Baigarers, in Point Iſland	66	30	N	17	12	W
Barbadoes	13	30	N	58	10	W
Barcelona in Spain	41	26	N	2	26	W
Bayone	43	29	N	1	30	W
Batavia	6	05	S	105	46	E
Belfaſt in Ireland	54	36	N	6	50	W
Beachy Head in England	50	48	N	0	25	E
Bencola in the Iſle of Sumatra	3	50	S	101	13	E
Bengal in India	22	27	N	91	49	E
Bay of Trinity	48	27	N	54	54	W
Beeronberg, or John Main's Iſland	71	45	N	4	30	E
Berwic in England	55	50	N	1	39	W
Bilboa in Spain	43	30	N	3	00	W
Beauvais, in France	49	26	N	2	10	E
Berlin in Pruſſia	52	33	N	13	33	E
Beſanzon	47	20	N	6	08	E
Beſiers	43	20	N	3	17	E
Bergen in Norway	61	00	N	7	30	E
Beeſora, by the River Euphrates	30	00	N	50	00	E
Boulogne in Picardy	50	42	N	1	46	E

Boulogne

Places Names.	Latit.			Longit.		
	D	M		D	M	
Boulogne in Italy	44	30	N	11	42	E
Bourdeaux in France	44	50	N	00	24	W
Bononia in Italy	44	30	N	11	50	E
Boſton in New England	42	25	N	70	37	W
Bornholm in Denmark	55	27	N	14	53	E
Bourges in France	47	05	N	02	29	E
Bareſlav in Sileſia	51	03	N	17	13	E
Breſt in France	48	23	N	04	25	W
Bridge Town in Barbadoes	12	58	N			
Bruges in Flanders	51	11	N	03	12	E
Bruſſels in Flanders	50	51	N	04	20	E
Brill in Holland	51	56	N	04	10	W
Burlings of Lisbon	39	39	N	10	25	W
Buenos Aires, on the E. Coaſt of America	36	10	N	58	00	W
Cape Bona in the Streights	37	08	N	10	35	E
Cape Bayador, on the Coaſt of Barbary	26	12	N	15	30	W
Cape Blanco	20	05	N	17	45	W
Cape de Baſſus	4	00	N	44	50	E
Cape Bona Eſperance	34	15	S	17	00	E
Cape Cautin on the Coaſt of Barbary	32	16	N	09	00	W
Cape Corientes in New Spain	19	40	N	110	30	W
Cape Churchill	69	50	N	95	15	W
Cape Charles on the Coaſt of Virginia	37	14	N	74	15	W
C. Charles on Coaſt of Hudſons Streights	65	45	N	69	30	W
Cape Collone in the Streights	39	10	N	17	16	E
Cape Coquibaca, in South America	12	50	N	7[illegible]	40	E
Cape St. Angelo in the Streights	3[illegible]	41	N	23	20	E
Cape del Agujo, in Peru	6	38	S	83	50	W
Cape St. Auguſtine in Brazil	8	05	S	35	38	W
Cape Cod	42	10	N	67	45	W
Cape Blanco, near River Camarono	46	50	S	72	05	W
Cape Correntes, on Eaſt Coaſt of Africa	23	52	S	35	45	E
Cape Comerin in India	8	00	N	78	15	E
Cape de Geer	30	04	N	10	10	W

Places Names.	Latit.			Longit.		
	D	M		D	M	
Cape Clear in Ireland	51	10	N	10	30	W
Cape de tres Forcas, in the Streights	35	34	N	01	25	W
Cape Formoſa, in India	4	35	N	05	55	E
Cape Gallo, in the Iſland Ceylan	6	07	N	80	00	E
Cape de Gat in the Streights	37	03	N	01	33	W
Cape Guardaſui, by the Red Sea	11	50	N	51	30	E
Cape La Hogue, in France	49	46	N	02	06	W
Cape Finiſter	43	06	N	10	00	W
Cape Lagulas, near the Cape of Good Hope	34	50	S	18	15	E
Cape Lopez, on the Weſt Coaſt of Africa	01	05	S	10	10	E
Cape St. Luca, on Weſt Coaſt of America	23	25	N	111	56	W
Cape de Lampo, on the Coaſt of China	28	30	N	121	10	E
Cape Martin, *in the Streights*	39	00	N	00	30	E
Cape Matapan, *in the Streights*	36	35	N	22	20	E
Cape Martelo, *South Point of Negropont*	38	00	N	25	45	E
Cape Monte *Scanto, in the Streights*	40	05	N	26	05	E
Cape M*iſorato, in the Streights*	32	43	N	16	23	E
Cape St. Mary, *in the Streights*	39	56	N	18	31	E
Cape Negro, on the Weſt Coaſt of Africa	16	08	S	13	03	E
Cape M*uca, in the Gulph of Perſia*	26	56	N	57	10	E
Cape Negraes, in the Bay of Bengal	16	23	N	93	55	E
Cape Palmeras, in the Bay of Bengal	20	55	N	87	10	E
Cape Ortegal, in the Bay of Biſcay	44	02	N	07	45	W
Cape Palos, in the Streights	37	58	N	00	10	W
Cape Spartel, in the mouth of Streights	35	46	N	05	55	W
Cape Spartivento, in the Streights	38	00	N	16	11	E
Cape St. Sebaſtian, on E. Coaſt of Africa	21	40	S	35	43	E
Cape M*eſurada, on the Coaſt of Guinea*	5	50	N	10	20	W
Cape Palmas, on the Coaſt of Guinea	4	03	N	07	20	W
Cape Olreada, on the W. Coaſt of Africa	23	41	N	16	40	W
Cape St. Sebaſtian, W. Coaſt of America	42	40	N	129	55	W
Cape Ruzato, in the Streights	33	15	N	21	55	E
Cape St Thomas, in the Streights	24	40	N	15	06	E
Cape Three Points, on the Coaſt of Guinea	4	13	N	02	10	W

Cape Ra-

Places Names.	Latit.			Longit.		
	D	M		D	M	
Cape Raſalgat, in India	22	27	N	60	45	E
Cape Trafalgar, on Weſt Coaſt of Africa	36	04	N	5	56	W
Cape Verde, at the Iſle of Goree	14	43	N	17	10	W
Cape St. *Vincent*	37	06	N	9	32	W
Cape Britain, in Newfoundland	46	20	N	58	15	W
Cape Bonaviſta	49	11	N	52	03	W
Cape Farewell	60	00	N	50	25	W
Cape Horn	57	58	S	79	55	W
Cape Hateras, in Carolina	35	10	N	74	25	W
Cape Henry, in Virginia	37	00	N	74	30	W
Cape Hinloper, in Maryland	39	04	N	73	25	W
Cape Florida	24	48	N	81	55	W
Cape Heneritta Maria, *C. Hudſons-Bay*	55	05	N	83	00	W
North Cape, on the Eaſt Coaſt America	02	05	N	49	55	W
North Cape	71	25	N	22	00	E
Cape Jones, Hudſons-Bay Coaſt	55	03	N	78	15	W
Cape Frio, in Brazille	23	20	S	42	56	W
Cape Race, in Newfoundland	46	30	N	53	50	W
Cape Roy, in Newfoundland	47	58	N	58	00	W
Cape Roque, in Brazille	05	00	S	35	52	W
Cape Sable, in Nova Scotia	43	45	N	63	25	W
Cape St. *John, in Newfoundland*	50	09	N	52	35	W
Cape St. Maries, *in Nova Scotia*	46	00	N	55	30	W
Cape Victory, in the Streights of Magellan	52	15	S	83	20	W
Cape Walſingham	63	00	N	77	40	W
Cape Southampton	61	57	N	88	30	W
Cadiz, in Spain	36	20	N	6	06	W
Calais, in France	50	57	N	1	55	W
Cambray, in France	50	10	N	3	19	E
Cambodia, in India	10	30	N	104	20	E
Canea, in Crete	35	29	N	24	07	E
Candea	35	18	N	25	10	E
Campeche, in the Weſt-Indies	19	20	N	93	05	W
Catenoſe	65	43	N	35	14	E

Places Names	Latit. D	Latit. M		Longit. D	Longit. M	
Cayanne in the Weſt-Indies	04	56	N	51	25	W
Grand Cairo, in Egypt	30	04	N	34	20	E
Carthagena	10	50	N	75	50	W
Canton, in China	23	30	N	113	00	E
New Callabar, on the Coaſt of Guinea	04	10	N	8	20	E
Old-Callabar	04	15	N	9	27	E
Calecut, in India	11	17	N	74	54	E
Caſkets, in the Channel of England	49	50	N	2	32	W
Cheuſan, in China	30	00	N	121	35	E
Cochin, in the Eaſt-Indies	10	02	N	75	45	E
Charles Town, upon Aſhly River	32	40	N	78	50	W
Chriſtiana, in Norway	59	10	N	9	45	E
Conimbra, in Portugal	40	30	N	9	40	W
Coningsberg, in Pruſſia	54	43	N	20	35	E
Conception-Bay	47	53	N	54	20	W
Conſtantinople	41	07	N	31	45	E
Copenhagen, in Denmark	55	40	N	12	30	E
Cracow, in Poland	50	10	N	19	35	E
Cuſcow in Peru	12	25	S	73	40	W
Dantzic, in Poland	54	22	N	19	05	E
Darby, in England	52	57	N	0	5½	W
Darien, in the Weſt-Indies	08	30	N	78	45	W
St. *David's-Head, in England*	51	55	N	5	22	W
Delphos	38	50	N			
Derwinda, in Courland, upon the Baltic	57	20	N	22	00	E
Dew Point, in India	15	50	N	81	50	E
Diep, in France	49	56	N	1	06	E
Dover, in England	51	08	N	0	30	
Dronton, in Norway	64	00	N	10	40	E
Dublin, in Ireland	53	12	N	6	55	W
Dundee, in Scotland	56	30	N	2	36	W
Dunkirk, in Flanders	51	01	N	2	20	E
Dunnoſe, in the Iſle of Wight	50	38	N	1	23	W
Durazzo, in Dalmatia	41	58	N	20	20	E

Places Names	Latit.			Longit.		
	D	M		D	M	
Edenburgh in Scotland	55	57		02	56	W
Elſinour, in the Baltic Sea	56	00		12	32	E
Embrun, in France	44	35		06	25	E
Emden, on the Coaſt of Holland	53	05		07	40	E
Epheſus	37	54		27	[illegible]	E
Eſcondide, in the Gulph of Mexico	28	20		99	05	W
Fair Foreland, in Greenland	79	18		10	50	E
Fair Foreland Iſland	65	10		29	45	W
Farra Head, in Scotland	58	23		05	05	W
Ferrara, in Italy	44	54		11	51	E
La Flech, in France	47	42		00	03	W
Fez	33	10		05	55	W
The Fly, on the Coaſt of Holland	53	18		05	35	E
Florence	43	41	North Latitude.	11	50	E
Fort St. George, in India	13	08		81	10	E
Frankfort, on the Main	50	04		08	50	E
Gallipoli, in Italy	40	08		18	07	E
Gallipoli, in Anatolia	40	30		27	58	E
Galloway, in Ireland	53	10		10	03	W
Geneva	46	22		06	35	E
Genoa, in the Streights	44	27		09	50	E
Glaſcow, in Scotland	55	53		04	05	W
Ghent, in Flanders	51	01		03	50	E
Gibralter, mouth of the Streights	36	11		05	13	W
Gottenberg, in Sweedland	57	33		12	25	E
Goa, in India	15	30		73	50	E
Goes, in Zeland	51	39		04	05	E
Grenoble	45	16		06	35	E
Greenwich, at the Royal Obſervatory	51	28½		00	05	E
Guatimala, Weſt Coaſt of America	14	25		96	00	W
Guernſey, in the Britiſh Channel	49	36		02	40	W
Hacluits Headland, in Greenland	79	55		12	00	E
Halliford, in Iſland	64	30		27	15	W
Hamburg, in the Baltic Sea	53	41		10	35	W

Places Names	Latit.			Longit.		
	D	M		D	M	
Hartland Point, in England	51	06		04	35	W
Havre de Grace, in France	49	30		00	12	E
Heidelbourg, in Germany	49	20		09	05	E
Helies Sound, in Greenland	79	15		12	50	E
Heligbland, in Greenland	54	28		08	35	E
Hoaignam, in China	33	35		119	05	E
Holy Head, in England	53	23		04	40	W
Horn Sound, in Greenland	77	23		14	20	E
St. *John's Harbour, in Newfoundland*	47	28		53	50	W
Bahama Iſlands.						
Atkins Key	22	00	North Latitude.	74	15	W
Atwoods Key	23	09		73	30	W
Abrolebo, North-point	21	40		68	30	W
Abacco, South-point	25	50		77	25	W
Andras, North point	25	15		79	00	W
Bahama Iſland	26	30		78	20	W
Bahama Bank, North-point	28	22		78	20	W
Bermudas	32	30		63	45	W
Cancas Bank, North-point	21	50		71	00	W
Cat-Iſland	24	30		76	12	W
Crooked Island, North-point	23	03		74	00	W
Exuma	23	30		76	05	W
French Keys	22	40		73	35	W
Harbour Island	25	45		76	35	W
Hogſtyes	21	15		73	40	W
Hyneago	20	47		72	15	W
Illatbera, South-point	25	30		76	12	W
North-point	24	35		76	55	W
Mayaguana	22	38		72	30	W
Merapervouz	21	57		74	40	W
Providence	25	00		77	20	W
Platwrack	20	20		68	10	W

Rum-

Places Names.	Latit. D	Latit. M	North Latitude.	Longit. D	Longit. M	Weſt Longitude
Rum Key	23	47		75	08	
Watling-Iſland	24	07		74	50	
Turks Iſland	21	30		69	80	
Cape de Verd Iſlands.						
St *Antonio*	17	20		24	50	
Bonaviſta	16	05		22	35	
Bravo	14	55		24	35	
Fuego	15	00		24	05	
Jago	15	10		23	30	
St. *Lucia*	17	20		24	00	
May	15	20		22	35	
St. *Nicholas*	17	12		23	30	
Sal	16	45		22	30	
St. *Vincent*	17	10		24	2c	
Canary-Iſlands.						
annarie	27	40		16	10	
erro	27	45		18	50	
ortaventura	27	50		14	50	
omero	27	50		17	50	
ancerota	28	20		13	30	
adera, Weſt end	32	20		17	30	
alma	28	20		18	30	
orto Sancto	32	45		16	05	
eneriff	27	50		17	05	
Caribbe Iſlands.						
nquilla	18	20		62	10	
ntegoa	17	30		60	40	
arbadoes	13	30		58	10	
rieque	17	55		63	30	

Places Names.	Latit. D	Latit. M		Longit. D	Longit. M	
Berbuda	17	58	North Latitude.	60	40	Weſt Longitude
St. *Bartholomew*	18	15		61	45	
Cape St *Antonio*	21	30		86	00	
Cape Cruz, Hiſpaniola	19	20		78	20	
St. *Chriſtophers*	17	55		61	30	
St. *Cruz*	18	00		63	25	
Curaſco	11	32		68	20	
St. *Domingo, in Hiſpaniola*	18	20		69	55	
Dominico	15	30		60	35	
Eaſt End of Cuba	20	05		74	00	
Granada	12	05		60	35	
Guardalupa	16	00		61	10	
Havana	22	40		83	00	
St. *Lucia*	13	58		60	15	
Marigalante	16	01		60	00	
Martinico	14	44		60	50	
St. *Martins*	18	15		62	10	
Monſerat.	17	05		60	45	
Nevis	17	20		61	10	
Porto Rico, St. *Johns*	18	30		64	30	
Port Royal, in Jamaica	17	40		76	00	
Trinadada	10	05		60	18	
Tobago	11	20		59	40	
St. *Vincent*	13	25		60	20	
Iſlands in the Eaſt Indies.						
Abdeleur	12	27	N	52	35	Eaſt Longitude.
De Alonſo	05	35	N	138	32	
Achar Baniau	12	50	N	69	30	
Almircant Iſles, the Eaſtermoſt.	03	42	S	52	20	
Amboina	03	55	S	156	20	
Amſterdam	38	50	S	71	30	
Amſterdam	25	55	N	130	00	

Places Names.	Latit. D	Latit. M		Longit. D	Longit. M
Andama, North End	13	00	N	91	30
Bantam, in Javes	05	37	S	105	11
Batavia	05	47	S	106	27
Banca, South End	03	00	S	105	30
Banda Iſles	04	30	S	127	30
St. *Bartalamo*	08	55	N	147	05
Baly	08	05	S	114	05
Babelmandel, in the Mouth of the Red Sea	12	25	N	45	45
Baſſas de Chagos	06	05	S	71	00
Baſſas de Barchas	04	30	S	50	00
Baſſas de Patron	04	35	S	46	10
Baſſas de Indea	21	30	S	40	30
Baſſas de Padua	12	35	N	71	05
St. *Brandon*	16	24	S	64	45
Borneo, North point	07	20	N	111	30
South point	04	14	S	112	52
Borneo	04	20	S	109	50
Biblan	10	57	N	133	30
Buro	03	38	S	124	00
Cape D' Ambra, South End	11	51	S	52	05
Comero	11	05	S	43	05
Cebes North End	02	02	N	121	30
South End	05	00	S	117	27
P. *Condor*	08	45	N	105	55
Cheuſan	30	00	N	121	35
Ainam Weſt point	19	05	N	107	10
Eaſt point	19	30	N	110	00
Almizant Iſles, the Weſtermoſt	02	30	S	58	10
De Caſa	06	10	N	136	45
Carimon	05	55	S	109	65
De Curia Muria	18	35		55	35
Dina	40	55	S	21	05
Das Reys Magos	25	10	N	122	55
Deſerta	25	30	N	143	05

Eaſt Longitude

Places Names	Latit. D	Latit. M		Longit. D	Longit. M
Duvs Colunas	25	00	N	144	20
Digo Reges	19	35	S	64	15
Digo Gratioſa	08	20	S	69	35
Engano	05	30	S	100	55
Good Fortune	01	28	S	97	20
Formoſa South point	21	30	N	119	10
North point	25	06	N	120	05
St. Franciſco	05	28	S	58	05
De Gallega	09	02	S	55	25
De Gratia	05	35	S	65	35
Java, Eaſt end	06	20	S	113	37
Japan, S. E. point	34	30	N	129	35
S. W. *point*	35	20	N	141	50
St. Juan de Nova	16	40	S	41	20
It des Keeſſe	08	50	N	128	05
Joanna	12	10	S	44	32
Floris, N E. *end*	07	59	S	122	05
S. W. *end*	09	02	S	119	38
Ladrone Iſles — *Guerga*	20	02	N	139	35
Pagau	18	55	N	139	05
Gregau	17	55	N	139	05
Mabau	16	30	N	139	00
La Guana	13	30	N	138	50
Lucon, South point	12	50	N	120	50
North point	18	55	N	118	40
Languin	27	55	N	119	05
Los Hormanos	19	02	N	155	40
La Martienes	05	50	N	139	05
Los M*artieres*	11	05	N	133	22
Maca	03	55	S	114	05
Matalotes	08	40	N	129	30
Madura	06	25	S	112	00
Madagaſcar, South end of St. *Sebaſtian*	25	32	S	47	15
Mayetta	12	29	S	45	25

Eaſt Longitude

Places Names	Latit. D	Latit. M		Longit. D	Longit. M	East Longitude
Muldivies, North end	07	45	N	72	0	
South end	00	30	N	75	25	
Mulique	09	0	N	72	00	
Mintan	00	28	S	96	50	
Mindano, Weſt point	06	50	N	118	35	
Mobilla	12	10	S	43	42	
Mauritias	20	0[illegible]	S	55	40	
La Malha	10	00	S	62	2[illegible]	
Moſcarenhas	20	5	S	54	06	
Mombaſa	03	35	S	38	25	
Menſca	08	05	S	38	55	
Marſveen	41	25	S	21	02	
Mazira	20	35	N	58	5[illegible]	
Nicober	07	35	N	92	55	
Naſſau Iſland	02	41	S	98	32	
Nuas	01	0	N	96	10	
St. Paulo	37	45	S	72	26	
Pemba	05	25	S	38	50	
Princes Iſland	06	30	S	104	02	
Quabella	03	49	S	50	30	
Romeros de Caſtellamas	28	05	S	67	00	
Sumatra, N. W. *End*	05	28	N	93	55	
S. E.	05	42	S	104	34	
Selam	08	20	S	102	13	
Streights of Sunday	05	52	S	104	18	
Savedra	12	02	N	133	02	
De Saltadores	03	55	N	151	25	
Summing	32	30	N	120	50	
Tagima	06	30	N	118	45	
Tanixima	29	55	N	130	00	
Fimer, North End	08	30	S	125	28	
South End	10	40	S	122	25	
Verkins Iſland	02	30	N	94	45	
Uno le Cura	21	30	N	149	42	

Tas.

Places Names.	Latit.			Longit.		
	D	M		D	M	
Ras de Dego Rays	00	00		72	05	E
Ras de Amber	00	00		50	30	E
Zocatra	12	28	N	54	20	E
Zanziber	06	55	S	38	52	E
Southern Iſlands.						
Annabona	01	05	S	07	30	E
Aſcenſion	07	40	S	14	50	W
St. Hellena	00	00	S	06	30	W
Fardinando a Poo	02	48	N	09	58	E
St. Matthews	01	40	S	07	50	W
Princeps	01	35	N	09	03	E
St. Thomas	00	00		08	00	E
Iſlands within the Streights						
Alboran	36	00		02	00	W
Cape Paſſaro, in Scicilly	36	55		15	00	E
Cape St. John, Weſt end of Canada	35	20		24	07	E
Cape Solomon, the Eaſt end of Canada	35	25		27	10	E
Corfu	39	40		20	05	E
Capraia	43	06	North Latitude	10	40	E
Corſica, North End	43	02		09	50	E
South End	41	30		09	06	E
Chefalonica	38	30		21	30	E
Cyprus Weſt End	35	21		33	00	E
Eaſt End	35	35		35	55	E
Formentaria	38	44		02	01	E
Gallita	37	40		08	50	E
Gorgona	43	44		10	38	E
Lemnos	39	58		26	05	E
Libboa	42	45		11	05	E
Majorca City	39	35		02	35	E

Malta

Places Names.	Latit.			Longit.		
	D	M		D	M	
Malta	36	15	N	14	30	East Longitude
Maritimo	38	12	N	12	55	
Meſſina in Sicilly	38	21	N	16	00	
Mendon Morea	36	50	N	20	42	
Port Mahon, Minorca	39	30	N	04	20	
Rodes City	36	42	N	28	40	
Sardina South End	38	50	N	08	50	
Scio	38	20	N	24	04	
Syracuſa, in Siclly	37	04	N	15	20	
Yuica	39	05	N	02	00	
Zant	37	45	N	21	10	
Weſtern Iſlands						
Corvo	40	00	N	31	55	W
Flores	39	32	N	31	52	W
Fyal	39	18	N	30	05	W
St. George	39	17	N	29	00	W
St. Marces	37	00	N	24	55	W
St. Michael	37	50	N	24	52	W
Pico	38	45	N	29	36	W
Tarcena	39	58	N	27	00	W
Admiralty Iſland, Nova Zembla	75	05	N	52	50	E
Baſley Iſle, in St. George's Channel	52	46	N	04	58	W
Bell Iſle, the Bay of Biſcay	47	20	N	03	10	W
Bell Iſland, Newfoundland	52	05	N	54	10	W
Buttons Iſle, Hudſons Streights	60	05	N	69	50	W
Cherry or Bear Iſland	74	35	N	18	05	E
Croſs Iſland, White Sea	66	31	N	36	33	E
Dew Iſland in France	46	40	N	02	15	W
Fero Iſles	62	06	N	05	00	W
Iſland Juan Ferdinando, in the South Sea	33	20	S	86	48	W
Iſland Ferdinando Loronho	03	58	S	32	30	W
Iſland Gallopegas	00	00		91	00	W

Green-

Places Names	Latit.			Longit.		
	D	M		D	M	
Greens Iſland, in Iſeland	66	51	N	24	45	W
Hope Iſland	76	22	N	22	00	E
Long Iſland, in North America	40	50	N	70	40	W
North Point, Lewins Iſland	58	20	N	07	15	W
Manſfield Iſle, in Hudſon's Bay	62	20	N	75	30	W
North part of Orkney	50	13	N	03	30	W
Nottingham Iſle	63	38	N	74	47	W
Reſolution Iſle, Hudſons Streights	61	55	N	64	50	W
Sky Iſlands in Scotland	57	50	N	05	21	W
Sable Iſland	44	30	N	60	65	W
Jeruſalem	31	40	N	37	00	E
Ingolſtat, in Germany	48	40	N	11	42	E
Inſpruck, in Germany	47	15	N	11	50	E
St. Katherines, on the E. Coaſt of America	28	00	N	47	50	W
Kalmar, in Denmark	56	40	N	16	40	E
St. Kilda, in Scotland	58	02	N	10	05	W
Kelduyn, in Finland	69	32	N	30	15	E
Legborn, in Florence	43	18	N	11	00	E
Lepant in Greece	38	20	N	21	55	E
Lands End of England	50	06	N	06	00	W
Liege, in Flanders	50	40	N	06	05	E
Leipſick, in Germany	51	19	N	13	20	E
Lints, in Auſtria	48	16	N	15	05	E
Liſle, in Flanders	50	40	N	03	05	E
Lions, in France	45	45	N	05	05	E
Lisbon, in Portugal	38	50	N	10	25	W
Lizard, in England	50	00	N	05	14	W
Liverpool, in England	53	20	N	02	58	W
Limrick, in Ireland	52	22	N	09	48	W
Lima, in Peru	12	30	S	78	30	W
Larwick, on the Coaſt of Norway	58	54	N	09	20	E
London	51	32	N	00	00	
London Derry, in Ireland	54	55	N	08	00	W
Liampo, in India	30	30	N	120	00	E
Lundy Iſle	51	20	N	04	40	W

Places Names.	Latit. D	Latit. M		Longit. D	Longit. M	
Lubeck, Norway	54	25	N	11	05	E
Leſou, Norway	57	05	N	10	40	E
Lees Foreland	78	05	N	23	25	E
Loefort, S. W. point	67	47	N	09	35	E
La Sarena, Weſt Coaſt of America	24	40	S	76	32	W
Macao, in China	22	13	N	116	00	E
Madagaſcar, Bay of Terra de Gada	19	29	S	44	35	E
Madrid, in Spain	40	10	N	03	10	W
Malacca, in India	23	32	N	104	05	E
Marſeiles, in France	43	20	N	05	27	E
Mardou, in Norway	58	25	N	09	05	E
Maſterland, in Norway	57	58	N	11	45	E
Magellan Eaſt Entrance	51	58	S	75	30	W
Marchants Foreland	63	20	N	17	05	W
St. *Maries, in Sicilly*	49	57	N	06	45	W
Morlaix, in France	48	37	N	03	50	W
Mozambique, in Coaſt of India	15	05	S	40	00	W
Moſcow	55	34	N	38	45	E
St *Maloes, in France*	48	38	N	02	05	W
Mantua, in Italy	45	11	N	11	10	E
Mentz, in Germany	49	14	N	06	40	E
Mexico, in Peru	20	06	N	102	05	W
Munchen, in Bavaria	48	28	N	11	50	E
Montpelier	43	36	N	03	50	E
Milan, in Italy	45	20	N	09	00	E
Modena	44	30	N	11	17	E
Naze of Norway	57	50	N	07	22	E
Narva, in Livonia	58	39	N	28	14	E
Nanquim, in India	32	55	N	129	30	E
Naples	41	05	N	15	40	E
Nantz, in France	47	14	N	01	36	W
Namur, in Flanders	50	25	N	05	05	E
Nangaſack Japan	32	53	N	127	50	E
Nancy, in Lorrain	48	39	N	06	50	E

Places Names.	Latit. D	Latit. M		Longit. D	Longit. M	
Narbon, in the Streights	43	15	N	02	20	E
Narſinga, India	18	05	N	83	30	E
New Caſtle	54	58	N	01	30	W
New York	41	00	N	72	05	W
Northbergen, in Norway	60	16	N	05	40	E
Nice, in Provence	43	38	N	07	20	E
Ningpo, or Leampo in China	29	58	N	120	20	E
Noremberg	49	29	N	12	20	E
North Foreland	51	28	N	01	10	E
Nicaragua Entrance, Eaſt America	11	15	N	85	05	W
Olinda Brazils, or Pernambuck	07	48	S	34	55	W
Orleans in France	47	04	N	02	00	E
Oſtend in Flanders	51	11	N	02	56	E
Oxford, in England	51	44	N	01	10	W
Ozala, Japan	35	05	N	133	05	E
Padua, in Italy	45	31	N	11	20	E
Paris, in France	48	50	N	02	20	E
Pall, in France	43	30	N	00	25	W
Panama, in the Weſt part of America	08	56	N	82	18	W
Placientia, in Newfoundland	47	57	N	55	00	W
Parma, in Italy	44	45	N	10	53	E
Pekin, in China	39	55	N	117	50	E
Portobello, in America	9	55	N	80	15	W
Perpinian, in France	42	41	N	02	59	E
Piſa, in Italy	43	42	N	10	26	E
Poieters, in France	46	35	N	00	05	E
Paudickers, in India	11	54	N	80	20	E
Port a Port, in Portugal	41	18	N	09	20	E
Portland, in England	50	30	N	02	48	E
Port Stevens, Weſt Coaſt of America	46	50	S	82	36	W
Port Segura, in the Eaſt Coaſt of America	16	22	S	40	58	W
Point lookout	76	40	N	16	25	E
Petersburgh	59	24	N	29	50	E
Prague, in Bohemia	50	40	N	14	35	E

Qua-

Places Names.	Latit.			Longit.		
	D	M		D	M	
Quaqua, on the Coast of Guinea	4	16	N	04	04	W
Quebeck	47			69	55	W
River Angola	-	00	N			
River Albany	52	32	N	83	07	W
River de Angria	0	54	N	10	50	F
River Amazones	0	00	N	50	00	W
River Comarones	3	58	N	10	50	E
River Congo, Mouth	5	45	S	14	00	E
River Gambia, Mouth	13	30	N	16	20	W
River Grand	31	55	S	51	40	W
River St. *Franciſco, E. Coaſt of America*	10	55	S	37	46	W
River Julian, Eaſt Coaſt of America	48	40	S	74	32	W
River Formoſa	7	10		05	05	E
River Oronoque	8	15		59	05	W
River Senegal, in Guinea	15	29		16	30	E
River Sierra Leone	8	49		13	01	W
River Vulto	5	45		00	55	E
Ruperts River	51	16		78	55	W
Ratisbon, in Germany	48	59		12	20	E
Reggia in Italy	42	15	North Latitude	13	50	E
Revel, in Livonia	59	00		26	45	E
Riga	56	50		24	50	E
Rhodes	36	42		33	05	E
Rochel	46	10		1	14	W
Rome	41	51		13	05	E
Roſtock	54	10		12	50	E
Roterdam	51	55		04	21	E
Rock of Lisbon	46	13		01	30	W
Rhee Iſland, in France	46	13		01	30	W
Roſes in France	42	10		03	20	E
Salle on the Coaſt of Barbary	33	43		06	30	W
Salamanca, in Spain	41	12		03	55	W
Siam, in India	14	18		100	50	E

Places Names.	Latit.			Longit.		
	D	M		D	M	
Siam Entrance	13	10	N	101	01	E
St. *Salvadore, Brazil*	12	47	S			
Scanderoon	36	00	N	36	00	E
Seuta	35	55	N	05	47	E
Scaw in Denmark	57	26	N	10	10	E
Sevil in Spain	37	36	N	06	25	W
Smyrna in Ionia	38	28	N	27	20	E
Sparta	37	10	N			
Salonica, in Greece	40	41	N	23	13	E
Spurn	53	35	N	00	30	E
Spaahan, in Perſia	36	14	N	65	05	E
Shetland point in Scotland	59	52	N	01	30	W
Slime Head, in Ireland	53	20	N	11	15	W
Shark Point, Hudſons Streights	64	27	N	74	20	W
Stockholm, in Sweedland	59	20	N	18	25	E
Stralſound	54	45	N	13	20	E
Straſbourg	48	35	N	07	55	E
Sluce Coaſt, in Flanders	51	19	N	03	50	E
Start Point	50	09	N	02	45	W
Stockton, in England	54	33	N	01	25	W
Spirito Sanctos, Eaſt Coaſt of America	19	58	S	42	00	W
Sound Royal	66	22	N	15	15	W
Stadland in Norway	62	10	N	04	38	E
Surinam, Weſt Indies	6	00	N	56	55	W
Surrat, India	21	08	N	73	25	E
Surroy Iſle	71	05	N	18	40	E
Sweetnoſe, in Lapland	68	08	N	34	42	E
Syracuſa in Scicilly	37	04	N	15	20	E
Tangier	35	55	N	06	12	E
Ti deus, Sumatra	00	36	N	99	25	E
Tetuan	35	28	N	05	52	W
Texel, in Holland	53	10	N	04	59	E
Thebes	38	22	N			

Places Names.	Latit.			Longit.		
	D	M		D	M	
Toledo, in Spain	39	46	North Latitude.	03	25	W
Tobaun, in France	43	06		05	50	E
Townley, in England	53	44		00	02	W
Tripoly, in Barbary	32	54		12	00	E
Tripoly in Syria	34	40		35	58	E
Toulouſe, in France	43	37		01	30	E
Tours, in France	47	23		00	40	E
Troy, in Champaigne	48	15		04	10	E
Tredagh, in Ireland	52	40				
Tromſound, in Norway	70	18		16	00	E
Tubing, in Germany	48	34		09	20	E
Triſtian de Cunha, E. Coaſt of America	37	05		15	38	W
Tunis, in Barbary	36	58		09	10	E
Turin, in Italy	44	50		07	35	E
Valparaiſo, Weſt Coaſt of America	32	50		77	30	W
Valentia, in Spain	39	30		00	50	E
Valſterborn, in Denmark	55	20		12	55	E
Venice	45	18		12	40	E
Viſapatam, Coaſt of Cormandel	17	40		83	47	E
Vienna, in Germany	48	22		17	20	E
Upſal, in Sweedland	59	00		18	05	E
Uraniburg	55	54		12	50	E
Utric, in Holland	52	05		05	05	E
Uſhant, in France	48	30		05	05	W
Warſaw, in Poland	52	14		21	40	E
Waterford, in Ireland	52	07		07	52	W
Wexford, in Ireland	52	13		07	16	W
Weſtmania Iſles	63	55		25	30	W
White Haven, in England	54	25		03	15	W
Whales Head	77	18		21	30	E
Widdaw, Coaſt of Guinea	06	35		02	10	E
Wirtenbourg, in Saxony	51	53		13	05	E
Wisby, in Gotland	57	37		18	50	E

Places Names	Latit. D	Latit. M		Longit. D	Longit. M	
Wybourg	61	05	N	30	30	E
Whales Back	63	44	N	17	05	W
Wolfenbuttle, Germany	52	11	N	11	05	E
Wologda, Moſcovy	59	19	N			
Woſlay, in Moſcovy	61	15	N			
Yarmouth, in England	52	45	N	01	40	E
Tereſlaw, in Moſcovy	57	44	N			
York, in England	54	00	N	00	55	

A Table of *Logarithms*

For Numbers increaſing in their Natural Order, from an Unit to 10, 000.

With a *Table* of Artificial *Sines*, *Tangents*, and *Secants*, the *Radius* 10.000000; and to every Degree and Minute of the *Quadrant*.

Num.	Logar.	Num.	Logar.	Num	Logar.
1	0.0000000	34	1.5314789	67	1.8260748
2	0,3010300	35	1.5440680	68	1.8325089
3	0.4771213	36	1,5563025	69	1.8388491
4	0.6020600	37	1.5682017	70	1 8450980
5	0.6989700	38	1.5797836	71	1.8512583
6	0.7781513	39	1.5910646	72	1.8573325
7	0.8450980	40	1.6020600	73	1.8633229
8	0.9030900	41	1 6127839	74	1,8692317
9	0.9542425	42	1.6232493	75	1.8750613
10	1,0000000	43	1.6334685	76	1.8808136
11	1.0413927	44	1.6434527	77	1.8864907
12	1.0791812	45	1.6532125	78	1.8920946
13	1.1139434	46	1.6627578	79	1.8976271
14	1.1461280	47	1.6720979	80	1.9030900
15	1.1760913	48	1.6812412	81	1,9084850
16	1.2041200	49	1.6901961	82	1,9138139
17	1.2304489	50	1.6989700	83	1.9190781
18	1.2552725	51	1.7075702	84	1.9242793
19	1.2787536	52	1.7160033	85	1,9294189
20	1.3010300	53	1.7242759	86	1.9344985
21	1.3222193	54	1.7323938	87	1.9395193
22	1.3424227	55	1.7403627	88	1.9444827
23	1.3617278	56	1.7481880	89	1.9493900
24	1 3802112	57	1.7558749	90	1.9542425
25	1.3979400	58	1.7634280	91	1.9590414
26	1.4149733	59	1.7708520	92	1.9637878
27	1.4313638	60	1.7781513	93	1.9684829
28	1.4471580	61	1,7853298	94	1.9731279
29	1.4623980	62	1.7923917	95	1.9777236
30	1.4771213	63	1.7993405	96	1.9822712
31	1,4913617	64	1 8061800	97	1.9867717
32	1.5051500	65	1.8129134	98	1.9912261
33	1,5185139	66	1.8195439	99	1.9956352

A TABLE of Logarithms.

Num	0	1	2	3	4
100	2.0000000	2.0004341	2.0008677	2.0013009	2.0017337
101	2.0043214	2.0047512	2.0051805	2.0056094	2.0060380
102	2.0086002	2.0090257	2.0094509	2.0098756	2.0103000
103	2.0128372	2.0132587	2.0136797	2.0141003	2.0145205
104	2.0170333	2.0174507	2.0178677	2.0182843	2.0187005
105	2.0211893	2.0216027	2.0220157	2.0224284	2.0228406
106	2.0253059	2.0257154	2.0261245	2.0265333	2.0269416
107	2.0293838	2.0297895	2.0301948	2.0305997	2.0310043
108	2.0334238	2.0338257	2.0342273	2.0346285	2.0350293
109	2.0374265	2.0378248	2.0382226	2.0386202	2.0390173
110	2.0413927	2.0417873	2.0421816	2.0425755	2.0429691
111	2.0453230	2.0457141	2.0461048	2.0464952	2.0468852
112	2.0492180	2.0496056	2.0499929	2.0503798	2.0507663
113	2.0530784	2.0534626	2.0538464	2.0542299	2.0546131
114	2.0569049	2.0572856	2.0576661	2.0580462	2.0584260
115	2.0606978	2.0610753	2.0614525	2.0618293	2.0622058
116	2.0644580	2.0648322	2.0652061	2.0655797	2.0659530
117	2.0681859	2.0685569	2.0689276	2.0692980	2.0696681
118	2.0718820	2.0722499	2.0726175	2.0729847	2.0733517
119	2.0755470	2.0759118	2.0762763	2.0766404	2.0770043
120	2.0791812	2.0795430	2.0799045	2.0802656	2.0806265
121	2.0827854	2.0831441	2.0835026	2.0838608	2.0842187
122	2.0863598	2.0867157	2.0870712	2.0874265	2.0877814
123	2.0899051	2.0902581	2.0906107	2.0909631	2.0913152
124	2.0934217	2.0937718	2.0941216	2.0944711	2.0948204
125	2.0969100	2.0972573	2.0976043	2.0979511	2.0982975
126	2.1003705	2.1007151	2.1010594	2.1014034	2.1017471
127	2.1038037	2.1041455	2.1044871	2.1048284	2.1051694
128	2.1072100	2.1075491	2.1078880	2.1082267	2.1085650
129	2.1105897	2.1109262	2.1112625	2.1115985	2.1119343
130	2.1139434	2.1142773	2.1146110	2.1149444	2.1152776
131	2.1172713	2.1176027	2.1179338	2.1182647	2.1185954
132	2.1205739	2.1209028	2.1212315	2.1215598	2.1218880
133	2.1238516	2.1241781	2.1245042	2.1248301	2.1251558
134	2.1271048	2.1274288	2.1277525	2.1280760	2.1283993
135	2.1303338	2.1306553	2.1309769	2.1312978	2.1316187
136	2.1335389	2.1338581	2.1341771	2.1344959	2.1348144
137	2.1367206	2.1370375	2.1373541	2.1376705	2.1379867
138	2.1398791	2.1401937	2.1405080	2.1408222	2.1411361
139	2.1430148	2.1433271	2.1436392	2.1439511	2.1442628
140	2.1461280	2.1464381	2.1467480	2.1470577	2.1473671
141	2.1492191	2.1495270	2.1498347	2.1501422	2.1504494
142	2.1522883	2.1525941	2.1528996	2.1532049	2.1535100
143	2.1553360	2.1556396	2.1559430	2.1562462	2.1565492
144	2.1583625	2.1586640	2.1589653	2.1592663	2.1595672
145	2.1613680	2.1616674	2.1619666	2.1622656	2.1625644
146	2.1643529	2.1646502	2.1649474	2.1652443	2.1655411
147	2.1673173	2.1676127	2.1679078	2.1682027	2.1684975
148	2.1702617	2.1705551	2.1708482	2.1711412	2.1714339
149	2.1731863	2.1734776	2.1737680	2.1740598	2.1743506

Num.	5	6	7	8	9
100	2.0021661	2.0025980	2.0030295	2.0034605	2.0038912
101	2.0064660	2.0068937	2.0073209	2.0077478	2.0081742
102	2.0107239	2.0111474	2.0115704	2.0119931	2.0124154
103	2.0149403	2.0153598	2.0157787	2.0161978	2.0166155
104	2.0191163	2.0195317	2.0199467	2.0203613	2.0207755
105	2.0232525	2.0236639	2.0240750	2.0244857	2.0248960
106	2.0273496	2.0277572	2.0281644	2.0285713	2.0289777
107	2.0314084	2.0318123	2.0322157	2 0326188	2.0330214
108	2.0354297	2.0358298	2.0362295	2.0366289	2.0370279
109	2.0394141	2.0398106	2.0402066	2.0406023	2.0409977
110	2.0433623	2.0437551	2.0441476	2.0445398	2.0449315
111	2.0472749	2.0476642	2:0480532	2.0484418	2.0488301
112	2.0511525	2.0515384	2.0519239	2.0523091	2.0526939
113	2.0549959	2.0553783	2.0557605	2.0561423	2.0565237
114	2.0588055	2.0591846	2.0595634	2.0599419	2.0603200
115	2.0625820	2.0629578	2.0633334	2.0637086	2.0640834
116	2.0663259	2.0666986	2.0670709	2.0674428	2.0678145
117	2.0700379	2.0704073	2.0707765	2.0711453	2.0715138
118	2.0737184	2.0740847	2.0744507	2.0748164	2.0751819
119	2.0773679	2.0777312	2.0780942	2.0784568	2.0788192
120	2.0809870	2.0813473	2.0817073	2.0820669	2.0824263
121	2.0845763	2.0849336	2.0852906	2.0856473	2.0860037
122	2.0881361	2.0884905	2.0888446	2.0891984	2.0895519
123	2.0916670	2.0920185	2.0923697	2.0927206	2.0930713
124	2.0951694	2.0955180	2.0958665	2.0962146	2.0965624
125	2.0986437	2.0989896	2.0993353	2.0996806	2.1000257
126	2.1020905	2.1024337	2.1027766	2.1031193	2.1034616
127	2.1055102	2.1058507	2.1061909	2.1065309	2.1068705
128	2.1089031	2.1092410	2.1095785	2.1099159	2.1102529
129	2 1122698	2.1126050	2.1129400	2.1132747	2.1136092
130	2 1156105	2.1159432	2.1162756	2.1166077	2.1169396
131	2.1189258	2.1192559	2.1195858	2.1199154	2.1202448
132	2.1222159	2.1225435	2.1228709	2.1231981	2.1235250
133	2.1254813	2.1258065	2.1261314	2.1264561	2.1267806
134	2.1287223	2.1290451	2.1293676	2.1296899	2.1300119
135	2.1319393	2.1322597	2.1325798	2.1328998	2.1332195
136	2.1351327	2.1354507	2.1357685	2.1360861	2.1364034
137	2.1383027	2.1386184	2.1389339	2.1392492	2.1395643
138	2.1414498	2.1417632	2.1420765	2.1423895	2.1427022
139	2.1445742	2.1448854	2.1451964	2.1455072	2.1458177
140	2.1476763	2.1479853	2.1482941	2.1486027	2.1489110
141	2.1507564	2.1510633	2.1513699	2.1516762	2.1519824
142	2.1538149	2.1541195	2.1544240	2.1547282	2.1550322
143	2.1568519	2.1571544	2.1574568	2.1577589	2.1580608
144	2.1598678	2.1601683	2.1604685	2.1607686	2.1610684
145	2 1628630	2.1631614	2.1634596	2.1637575	2.1640553
146	2.1558376	2.1661340	2.1664301	2.1667261	2.1670218
147	2.1687920	2.1690864	2.1693805	2.1696744	2.1699682
148	2.1717264	2.1720188	2.1723110	2.1726029	2.1728947
149	2.1746512	2.1749316	2.1752218	2 1755118	2.1758016

A TABLE of Logarithms.

Num	0	1	2	3	4
150	2.1760913	2.1763807	2.1766699	2.1769590	2.1772478
151	2.1789769	2.1792645	2.1795518	2.1798389	2.1801259
152	2.1818436	2.1821292	2.1824146	2.1826999	2.1829850
153	2.1846914	2.1849752	2.1852588	2.1855422	2.1858254
154	2.1875207	2.1878026	2.1880844	2.1883659	2.1886473
155	2.1903317	2.1906118	2.1908917	2.1911714	2.1914510
156	2.1931246	2.1934029	2.1936810	2.1939590	2.1942367
157	2.1958997	2.1961762	2.1964525	2.1967287	2.1970047
158	2.1986571	2.1989319	2.1992065	2.1994809	2.1997552
159	2.2013971	2.2016702	2.2019431	2.2022158	2.2024883
160	2.2041200	2.2043913	2.2046625	2.2049335	2.2052044
161	2.2068259	2.2070955	2.2073650	2.2076344	2.2079035
162	2.2095150	2.2097830	2.2100508	2.2103185	2.2105860
163	2.2121876	2.2124540	2.2127202	2.2129862	2.2132521
164	2.2148438	2.2151086	2.2153732	2.2156376	2.2159018
165	2.2174839	2.2177471	2.2180100	2.2182729	2.2185355
166	2.2201081	2.2203696	2.2206310	2.2208922	2.2211533
167	2.2227165	2.2229764	2.2232363	2.2234959	2.2237555
168	2.2253093	2.2255677	2.2258260	2.2260841	2.2263421
169	2.2278867	2.2281436	2.2284004	2.2286570	2.2289134
170	2.2304489	2.2307043	2.2309596	2.2312146	2.2314696
171	2.2329961	2.2332500	2.2335038	2.2337574	2.2340108
172	2.2355284	2.2357809	2.2360331	2.2362853	2.2365373
173	2.2380461	2.2382971	2.2385479	2.2387986	2.2390491
174	2.2405492	2.2407988	2.2410482	2.2412974	2.2415465
175	2.2430380	2.2432861	2.2435341	2.2437819	2.2440296
176	2.2455127	2.2457594	2.2460059	2.2462523	2.2464986
177	2.2479733	2.2482186	2.2484637	2.2487087	2.2489536
178	2.2504200	2.2506639	2.2509077	2.2511513	2.2513949
179	2.2528530	2.2530956	2.2533380	2.2535803	2.2538224
180	2.2552725	2.2555137	2.2557548	2.2559957	2.2562365
181	2.2576786	2.2579185	2.2581582	2.2583978	2.2586373
182	2.2600714	2.2603099	2.2605484	2.2607867	2.2610248
183	2.2624511	2.2626883	2.2629255	2.2631625	2.2633993
184	2.2648178	2.2650538	2.2652896	2.2655253	2.2657609
185	2.2671717	2.2674064	2.2676410	2.2678754	2.2681097
186	2.2695129	2.2697464	2.2699797	2.2702129	2.2704459
187	2.2718416	2.2720738	2.2723058	2.2725378	2.2727696
188	2.2741578	2.2743888	2.2746196	2.2748503	2.2750809
189	2.2764618	2.2766915	2.2769211	2.2771506	2.2773800
190	2.2787536	2.2789821	2.2792105	2.2794388	2.2796669
191	2.2810334	2.2812607	2.2814879	2.2817150	2.2819419
192	2.2833012	2.2835274	2.2837534	2.2839793	2.2842051
193	2.2855573	2.2857823	2.2860071	2.2862319	2.2864565
194	2.2878017	2.2880255	2.2882492	2.2884728	2.2886963
195	2.2900346	2.2902573	2.2904798	2.2907022	2.2909246
196	2.2922561	2.2924776	2.2926990	2.2929203	2.2931415
197	2.2944662	2.2946866	2.2949069	2.2951271	2.2953471
198	2.2966652	2.2968845	2.2971037	2.2973227	2.2975417
199	2.2988531	2.2990713	2.2992893	2.2995073	2.2997252

Num.	5	6	7	8	9
150	2.1775365	2.1778250	2.1781133	2.1784013	2.1786892
151	2.1804126	2.1806992	2.1809856	2.1812718	2.1815578
152	2.1832698	2.1835545	2.1838390	2.1841234	2.1844075
153	2.1861084	2.1863912	2.1866739	2.1869563	2.1872386
154	2.1889285	2.1892095	2.1894903	2.1897710	2.1900514
155	2.1917304	2.1920096	2.1922886	2.1925675	2.1928461
156	2.1945143	2.1947918	2.1950690	2.1953461	2.1956229
157	2.1972806	2.1975562	2.1978317	2.1981070	2.1983821
158	2.2000293	2.2003032	2.2005769	2.2008505	2.2011239
159	2.2027607	2.2030329	2.2033049	2.2035768	2.2038485
160	2.2054750	2.2057455	2.2060159	2.2062860	2.2065560
161	2.2081725	2.2084414	2.2087100	2.2089785	2.2092468
162	2.2108534	2.2111205	2.2113876	2.2116544	2.2119211
163	2.2135178	2.2137833	2.2140487	2.2143139	2.2145790
164	2.2161659	2.2164298	2.2166936	2.2169572	2.2172207
165	2.2187980	2.2190603	2.2193225	2.2195845	2.2198464
166	2.2214142	2.2216750	2.2219356	2.2221960	2.2224563
167	2.2240148	2.2242740	2.2245331	2.2247920	2.2250507
163	2.2265999	2.2268576	2.2271151	2.2273724	2.2276296
169	2.2291697	2.2294258	2.2296818	2.2299377	2.2301934
170	2.2317244	2.2319790	2.2322335	2.2324879	2.2327421
171	2.2342541	2.2345173	2.2347703	2.2350232	2.2352759
172	2.2367891	2.2370408	2.2372923	2.2375437	2.2377950
173	2.2392995	2.2395497	2.2397998	2.2400498	2.2402996
174	2.2417954	2.2420442	2.2422929	2.2425414	2.2427898
175	2.2442771	2.2445245	2.2447718	2.2450189	2.2452658
176	2.2467447	2.2469907	2.2472365	2.2474823	2.2477278
177	2.2491984	2.2494430	2.2496874	2.2499318	2.2501759
178	2.2516382	2.2518815	2.2521246	2.2523675	2.2526103
179	2.2540645	2.2543063	2.2545481	2.2547897	2.2550312
180	2.2564772	2.2567177	2.2569582	2.2571984	2.2574386
181	2.2588766	2.2591158	2.2593549	2.2595939	2.2598327
182	2.2612629	2.2615008	2.2617385	2.2619762	2.2622137
183	2.2636361	2.2638727	2.2641092	2.2643455	2.2645817
184	2.2659964	2.2662317	2.2664669	2.2667020	2.2669369
185	2.2683439	2.2685780	2.2688119	2.2690457	2.2692794
186	2.2706788	2.2709116	2.2711443	2.2713769	2.2716093
187	2.2730013	2.2732328	2.2734643	2.2736956	2.2739268
188	2.2753114	2.2755417	2.2757719	2.2760020	2.2762320
189	2.2776092	2.2778383	2.2780673	2.2782962	2.2785250
190	2.2798950	2.2801229	2.2803507	2.2805784	2.2808059
191	2.2821688	2.2823955	2.2826221	2.2828486	2.2830750
192	2.2844307	2.2846563	2.2848817	2.2851070	2.2853322
193	2.2866810	2.2869054	2.2871296	2.2873538	2.2875778
194	2.2889196	2.2891428	2.2893660	2.2895890	2.2898118
195	2.2911468	2.2913689	2.2915908	2.2918127	2.2920344
196	2.2933626	2.2935835	2.2938044	2.2940251	2.2942457
197	2.2955671	2.2957869	2.2960067	2.2962263	2.2964458
198	2.2977605	2.2979792	2.2981979	2.2984164	2.2986348
199	2.2999429	2.3001605	2.3003781	2.3005955	2.3008128

A TABLE of Logarithms.

Num	0	1	2	3	4
200	2.3010300	2.3012471	2.3014641	2.3016809	2.3018977
201	2.3031961	2.3034121	2.3036280	2.3038438	2.3040595
202	2.3053514	2,3055663	2.3057812	2.3059959	2.3062105
203	2.3074960	2.3077099	2.3079237	2.3081374	2.3083509
204	2.3096302	2.3098430	2.3100557	2.3102684	2.3104809
205	2.3117539	2.3119657	2.3121774	2.3123889	2.3126004
206	2.3138672	2.3140780	2.3142887	2.3144992	2.3147097
207	2.3159703	2.3161801	2.3163898	2.3165993	2.3168083
208	2.3180633	2.3182721	2.3184807	2.3186893	2.3188977
209	2.3201463	2.3203540	2.3205617	2.3207692	2.3209767
210	2.3222193	2,3224261	2.3226327	2.3228393	2.3230457
211	2.3242825	2.3244882	2.3246939	2.3248995	2.3251050
212	2.3263359	2.3265407	2.3267454	2.3269500	2.3271545
213	2.3283796	2.3285834	2.3287872	2.3289909	2.3291944
214	2,3304138	2.3306167	2.3308195	2.3310222	2.3312248
215	2.3324385	2.3326404	2.3328423	2.3330440	2.3332457
216	2.3344538	2.3346548	2.3348557	2.3350565	2.3352573
217	2.3364597	2.3366598	2.3368598	2.3370597	2.3372595
218	2.3384565	2.3386557	2.3388547	2.3390537	2.3392526
219	2.3404441	2.3406424	2.3408405	2.3410386	2.3412366
220	2.3424227	2.3426200	2.3428173	2.3430145	2.3432116
221	2.3443923	2.3445887	2.3447851	2.3449814	2,3451776
222	2.3463530	2.3465486	2.3467441	2.3469395	2.3471348
223	2.3483049	2.3484996	2.3486942	2.3488887	2.3490832
224	2.3502480	2.3504419	2.3506356	2.3508293	2,3510229
225	2.3521825	2.3523755	2.3525684	2.3527612	2,3529539
226	2.3541084	2.3543006	2.3544926	2.3546846	2.3548764
227	2.3560259	2.3562171	2.3564083	2.3565994	2.3567905
228	2.3579348	2.3581253	2.3583156	2.3585059	2.3586961
229	2.3598355	2.3600251	2.3602146	2.3604041	2.3605934
230	2.3617278	2.3619166	2.3621053	2.3622939	2.3624825
231	2.3636120	2.3638000	2.3639878	2.3641756	2.3643633
232	2.3654880	2.3656751	2.3658622	2.3660492	2.3662361
233	2.3673559	2.3675423	2.3677285	2.3679147	2.3681008
234	2.3692159	2.3694014	2.3695869	2.3697723	2.3699576
235	2.3710679	2.3712526	2.3714373	2.3716219	2.3718065
236	2.3729120	2.3730960	2.3732799	2.3734637	2.3736475
237	2.3747483	2.3749316	2.3751147	2.3752977	2.3754807
238	2.3765770	2.3767594	2.3769418	2.3771240	2.3773062
239	2.3783979	2.3785796	2.3787612	2.3789427	2.3791241
240	2,3802112	2.3803922	2.3805730	2.3807538	2.3809345
241	2.3820170	2.3821972	2.3823773	2.3825573	2.3827373
242	2.3838154	2.3839948	2.3841741	2.3843534	2.3845326
243	2.3856063	2.3857850	2.3859636	2.3861421	2.3863206
244	2.3873898	2.3875678	2.3877457	2.3879235	2.3881012
245	2.3891661	2.3893433	2.3895205	2.3896975	2.3898746
246	2.3909351	2.3911116	2.3912880	2.3914644	2.3916407
247	2.3926970	2.3928727	2.3930485	2.3932241	2.3933997
248	2.3944517	2.3946268	2.3948018	2.3949767	2.3951516
249	2.3961993	2.3963737	2.3965480	2.3967223	2.3968964

Num.	5	6	7	8	9
200	2.3021144	2.3023309	2.3025474	2.3027637	2.3029799
201	2.3042751	2.3044905	2.3047059	2.3049212	2.3051363
202	2.3064250	2.3066394	2.3068537	2.3070680	2.3072820
203	2.3085644	2.3087778	2.3089910	2.3092042	2.3094172
204	2.3106923	2.3109056	2.3111178	2.3113300	2.3115420
205	2.3128118	2.3130231	2.3132343	2.3134454	2.3136563
206	2.3149201	2.3151303	2.3153405	2.3155505	2.3157605
207	2.3170181	2.3172273	2.3174365	2.3176455	2.3178545
208	2.3191061	2.3193143	2.3195224	2.3197305	2.3199384
209	2.3211840	2.3213913	2.3215984	2.3218055	2.3220124
210	2.3232521	2.3234584	2.3236645	2.3238706	2.3240766
211	2.3253104	2.3255157	2.3257209	2.3259260	2.3261310
212	2.3273589	2.3275633	2.3277675	2.3279716	2.3281757
213	2.3293979	2.3296012	2.3298045	2.3300077	2.3302108
214	2.3314273	2.3316297	2.3318320	2.3320343	2.3322364
215	2.3334473	2.3336488	2.3338501	2.3340514	2.3342526
216	2.3354579	2.3356585	2.3358589	2.3360593	2.3362596
217	2.3374593	2.3376589	2.3378584	2.3380579	2.3382572
218	2.3394514	2.3396502	2.3398488	2.3300473	2.3402458
219	2.3414345	2.3416323	2.3418301	2.3420277	2.3422252
220	2.3434086	2.3436055	2.3438023	2.3439991	2.3441957
221	2.3453737	2.3455698	2.3457657	2.3459615	2.3461573
222	2.3473300	2.3475252	2.3477202	2.3479152	2.3481101
223	2.3492775	2.3494718	2.3496660	2.3498601	2.3500541
224	2.3512163	2.3514098	2.3516031	2.3517963	2.3519895
225	2.3531465	2.3533391	2.3535316	2.3537239	2.3539162
226	2.3550682	2.3552599	2.3554515	2.3556431	2.3558345
227	2.3569814	2.3571723	2.3577630	2.3575537	2.3577443
228	2.3588862	2.3590762	2.3593662	2.3594560	2.3596458
229	2.3607827	2.3609719	2.3611610	2.3613500	2.3615390
230	2.3626709	2.3628593	2.3630476	2.3632358	2.3634239
231	2.3645510	2.3647386	2.3649260	2.3651134	2.3653007
232	2.3664230	2.3666097	2.3667964	2.3669830	2.3671695
233	2.3682869	2.3684728	2.3686587	2.3688445	2.3690302
234	2.3701428	2.3703280	2.3705131	2.3706981	2.3708830
235	2.3719909	2.3721753	2.3723596	2.3725438	2.3727279
236	2.3738311	2.3740147	2.3741983	2.3743817	2.3745651
237	2.3756636	2.3758464	2.3760292	2.3762118	2.3763944
238	2.3774884	2.3776704	2.3778524	2.3780343	2.3782161
239	2.3793055	2.3794868	2.3796680	2.3798492	2.3800302
240	2.3811151	2.3812956	2.3814761	2.3816565	2.3818368
241	2.3829171	2.3830969	2.3832766	2.3834563	2.3836359
242	2.3847117	2.3848908	2.3850698	2.3852487	2.3854275
243	2.3864990	2.3866773	2.3868555	2.3870337	2.3872118
244	2.3882789	2.3884565	2.3886340	2.3888114	2.3889888
245	2.3900515	2.3902284	2.3904052	2.3905819	2.3907585
246	2.3918169	2.3919931	2.3921691	2.3923452	2.3925211
247	2.3935752	2.3937506	2.3939260	2.3941013	2.3942765
248	2.3953264	2.3955011	2.3956758	2.3958504	2.3960249
249	2.3970705	2.3972446	2.3974185	2.3975924	2.3977663

A TABLE of Logarithms.

Num	0	1	2	3	4
250	2.3979400	2.3981137	2.3982873	2.3984608	2.3986343
251	2.3996737	2.3998467	2.4000196	2.4001925	2.4003653
252	2.4014005	2,4015728	2.4017451	2.4019173	2.4020894
253	2.4031205	2.4032921	2.4034637	2.4036352	2.4038066
254	2.4048337	2.4050047	2.4051755	2.4053464	2.4055171
255	2.4065402	2.4067105	2.4068807	2.4070508	2.4072209
256	2.4082400	2.4084096	2.4085791	2.4087486	2.4089180
257	2.4099331	2.4101021	2.4102710	2.4104398	2.4106085
258	2.4116197	2.4117880	2.4119562	2.4121244	2.4122925
259	2.4132998	2.4134674	2.4136350	2.4138025	2.4139700
260	2.4149733	2,4151404	2.4153073	2.4154742	2.4156410
261	2.4166405	2.4168069	2.4169732	2.4171394	2.4173056
262	2.4183013	2.4184670	2.4186327	2.4187983	2.4189638
263	2.4199557	2.4201208	2.4202859	2.4204509	2.4206158
264	2,4216039	2.4217684	2.4219328	2.4220972	2.4222615
265	2.4232459	2.4234097	2.4235735	2.4237372	2.4239009
266	2.4248816	2.4250449	2.4252081	2.4253712	2.4255342
267	2.4265113	2.4266739	2.4268365	2.4269990	2.4271614
268	2.4281348	2.4282968	2.4284588	2.4286207	2.4287825
269	2.4297523	2.4299137	2.4300751	2.4302364	2.4303976
270	2.4313638	2.4315246	2.4316853	2.4318460	2.4320067
271	2.4329693	2.4331295	2.4332897	2.4334498	2.4336098
272	2.4345689	2.4347285	2.4348881	2.4350476	2.4352071
273	2.4361626	2.4363217	2.4364807	2.4366396	2.4367985
274	2.4377506	2.4379090	2.4380675	2.4382258	2.4383841
275	2.4393327	2.4394906	2.4396484	2.4398062	2.4399639
276	2.4409091	2.4410664	2.4412237	2.4413809	2.4415380
277	2.4424798	2.4426365	2.4427932	2.4429499	2.4431065
278	2.4440448	2.4442010	2.4443571	2.4445132	2.4446692
279	2.4456042	2.4457598	2.4459154	2.4460709	2.4462264
280	2.4471580	2.4473131	2.4474681	2.4476231	2.4477780
281	2.4487063	2.4488608	2.4490153	2.4491697	2.4493241
282	2.4502491	2.4504031	2.4505570	2.4507109	2.4508647
283	2.4517864	2.4519399	2.4520932	2.4522466	2.4523998
284	2.4533183	2.4534712	2.4536241	2.4537769	2.4539296
285	2.4548449	2.4549972	2.4551495	2.4553018	2.4554540
286	2.4563660	2.4565179	2.4566696	2.4568213	2.4569730
287	2.4578819	2.4580332	2.4581844	2.4583356	2.4584868
288	2.4593925	2.4595433	2.4596940	2.4598446	2.4599953
289	2.4608978	2.4610481	2.4611983	2.4613484	2.4614985
290	2,4623980	2.4625477	2.4626974	2.4628470	2.4629966
291	2.4638930	2.4640422	2.4641914	2.4643405	2.4644895
292	2.4653829	2.4655316	2.4656802	2.4658288	2.4659774
293	2.4668676	2.4670158	2.4671640	2.4673121	2.4674601
294	2.4683473	2.4684950	2.4686427	2.4687903	2.4689378
295	2.4698220	2.4699692	2.4701164	2.4702634	2.4704105
296	2.4712917	2.4714384	2 4715851	2.4717317	2.4718782
297	2 4727564	2 4729027	2.4730488	2.4731949	2.4733410
298	2.4742163	2.4743620	2.4745076	2.4746533	2 4747988
299	2.4756712	2.4758164	2.4759616	2.4761067	2.4762518

Num.	5	6	7	8	9
250	2.3988077	2.3989811	2.3991543	2.3993275	2.3995007
251	2.4005380	2.4007106	2.4008832	2.4010557	2.4012282
252	2.4022614	2.4024333	2.4026052	2.4027771	2.4029488
253	2.4039780	2.4041492	2.4043205	2.4044916	2.4046627
254	2.4056878	2.4058584	2.4060289	2.4061994	2.4063698
255	2.4073909	2.4075608	2.4077307	2.4079005	2.4080703
256	2.4090874	2.4092567	2.4094259	2.4095950	2.4097641
257	2.4107772	2.4109459	2.4111144	2.4112829	2.4114513
258	2.4124605	2.4126285	2.4127964	2.4129643	2.4131320
259	2.4141374	2.4143047	2.4144719	2.4146391	2.4148063
260	2.4158077	2.4159744	2.4161410	2.4163076	2.4164741
261	2.4174717	2.4176377	2.4178037	2.4179696	2.4181355
262	2.4191293	2.4192947	2.4194601	2.4196254	2.4197906
263	2.4207806	2.4209454	2.4211101	2.4212748	2.4214394
264	2.4224257	2.4225898	2.4227539	2.4229180	2.4230820
265	2.4240645	2.4242281	2.4243916	2.4245550	2.4247183
266	2.4256972	2.4258601	2.4260230	2.4261858	2.4263486
267	2.4273238	2.4274861	2.4276484	2.4278106	2.4279727
268	2.4289443	2.4291060	2.4292677	2.4294293	2.4295908
269	2.4305588	2.4307199	2.4308809	2.4310419	2.4312029
270	2.4321673	2.4323278	2.4324883	2.4326487	2.4328090
271	2.4337698	2.4339298	2.4340896	2.4342495	2.4344092
272	2.4353665	2.4355259	2.4356851	2.4358444	2.4360035
273	2.4369573	2.4371161	2.4372748	2.4374334	2.4375920
274	2.4385423	2.4387005	2.4388587	2.4390167	2.4391747
275	2.4401216	2.4402792	2.4404368	2.4405943	2.4407517
276	2.4416951	2.4418522	2.4420092	2.4421664	2.4423230
277	2.4432630	2.4434195	2.4435759	2.4437322	2.4438885
278	2.4448252	2.4449811	2.4451370	2.4452928	2.4454485
279	2.4463818	2.4465372	2.4466925	2.4468477	2.4470029
280	2.4479329	2.4480877	2.4482424	2.4483971	2.4485517
281	2.4494784	2.4496327	2.4497868	2.4499410	2.4500951
282	2.4510185	2.4511722	2.4513258	2.4514794	2.4516329
283	2.4525531	2.4527062	2.4528593	2.4530124	2.4531654
284	2.4540823	2.4542349	2.4543875	2.4545400	2.4546924
285	2.4556061	2.4557582	2.4559102	2.4560622	2.4562142
286	2.4571246	2.4572762	2.4574277	2.4575791	2.4577305
287	2.4586378	2.4587889	2.4589399	2.4590908	2.4592417
288	2.4601458	2.4602963	2.4604468	2.4605972	2.4607475
289	2.4616486	2.4617986	2.4619485	2.4620984	2.4622482
290	2.4631461	2.4632956	2.4634450	2.4635944	2.4637437
291	2.4646386	2.4647875	2.4649364	2.4650853	2.4652341
292	2.4661259	2.4662743	2.4664227	2.4665711	2.4667194
293	2.4676081	2.4677561	2.4679039	2.4680518	2.4681996
294	2.4690853	2.4692327	2.4693801	2.4695275	2.4696748
295	2.4705575	2.4707044	2.4708513	2.4709982	2.4711450
296	2.4720247	2.4721711	2.4723175	2.4724639	2.4726102
297	2.4734870	2.4736329	2.4737788	2.4739247	2.4740705
298	2.4749443	2.4750898	2.4752352	2.4753806	2.4755259
299	2.4763968	2.4765418	2.4766867	2.4768316	2.4769765

A TABLE of Logarithms.

Num	0	1	2	3	4
300	2.4771213	2.4772660	2.4774017	2.4775553	2.4776999
301	2.4785665	2.4787108	2.4788550	2.4789991	2.4791432
302	2.4800069	2.4801507	2.4802945	2.4804381	2.4805818
303	2.4814426	2.4815859	2.4817292	2.4818724	2.4820156
304	2.4828736	2.4830164	2.4831592	2.4833019	2.4834446
305	2.4842998	2.4844422	2.4845845	2.4847268	2.4848690
306	2.4857214	2.4858633	2.4860052	2.4861470	2.4862888
307	2.4871384	2.4872798	2.4874212	2.4875626	2.4877039
308	2.4885507	2.4886917	2.4888326	2.4889735	2.4891144
309	2.4899585	2.4900990	2.4902395	2.4903799	2.4905203
310	2.4913617	2.4915018	2.4916418	2.4917818	2.4919217
311	2.4927604	2.4929000	2.4930396	2.4931791	2.4933186
312	2.4941546	2.4942938	2.4944329	2.4945720	2.4947110
313	2.4955443	2.4956831	2.4958218	2.4959604	2.4960990
314	2.4969296	2.4970679	2.4972062	2.4973444	2.4974825
315	2.4983106	2.4984484	2.4985862	2.4987240	2.4988617
316	2.4996871	2.4998245	2.4999619	2.5000992	2.5002365
317	2.5010593	2.5011962	2.5013332	2.5014701	2.5016069
318	2.5024271	2.5025637	2.5027002	2.5028366	2.5029731
319	2.5037907	2.5039268	2.5040629	2.5041989	2.5043349
320	2.5051500	2.5052857	2.5054213	2.5055569	2.5056925
321	2.5065050	2.5066403	2.5067755	2.5069107	2.5070459
322	2.5078559	2.5079907	2.5081255	2.5082603	2.5083950
323	2.5092025	2.5093370	2.5094714	2.5096057	2.5097400
324	2.5105450	2.5106790	2.5108130	2.5109469	2.5110808
325	2.5118834	2.5120170	2.5121505	2.5122841	2.5124175
326	2.5132176	2.5133508	2.5134840	2.5136171	2.5137501
327	2.5145478	2.5146805	2.5148133	2.5149460	2.5150787
328	2.5158738	2.5160062	2.5161386	2.5162709	2.5164031
329	2.5171959	2.5173279	2.5174598	2.5175917	2.5177236
330	2.5185139	2.5186455	2.5187771	2.5189086	2.5190400
331	2.5198280	2.5199592	2.5200903	2.5202214	2.5203525
332	2.5211381	2.5212689	2.5213996	2.5215303	2.5216610
333	2.5224442	2.5225746	2.5227050	2.5228353	2.5229656
334	2.5237465	2.5238765	2.5240064	2.5241364	2.5242663
335	2.5250448	2.5251744	2.5253040	2.5254336	2.5255631
336	2.5263393	2.5264685	2.5265977	2.5267269	2.5268560
337	2.5276299	2.5277588	2.5278875	2.5280163	2.5281451
338	2.5289167	2.5290452	2.5291736	2.5293020	2.5294304
339	2.5301997	2.5303278	2.5304558	2.5305839	2.5307118
340	2.5314789	2.5316066	2.5317343	2.5318619	2.5319896
341	2.5327544	2.5328817	2.5330090	2.5331363	2.5332635
342	2.5340261	2.5341531	2.5342800	2.5344069	2.5345338
343	2.5352941	2.5354207	2.5355473	2.5356738	2.5358003
344	2.5365584	2.5366847	2.5368109	2.5369370	2.5370631
345	2.5378191	2.5379450	2.5380708	2.5381965	2.5383223
346	2.5390761	2.5392016	2.5393271	2.5394525	2.5395779
347	2.5403295	2.5404546	2.5405797	2.5407048	2.5408298
348	2.5415792	2.5417040	2.5418288	2.5419535	2.5420781
349	2.5428254	2.5429498	2.5430742	2.5431986	2.5433229

Num.	5	6	7	8	9
300	2.4778445	2.4779890	2.4781334	2.4782778	2.4784222
301	2.4792873	2.4794313	2.4795753	2.4797192	2.4798631
302	2.4807254	2.4808689	2.4810124	2.4811559	2.4812993
303	2.4821587	2.4823018	2.4824448	2,4825878	2.4827307
304	2.4835873	2.4837299	2.4838725	2.4840150	2.4841574
305	2.4850112	2.4851533	2.4852954	2.4854375	2.4855795
306	2.4864305	2.4865721	2.4867138	2.4868554	2.4869969
307	2.4878451	2.4879863	2.4881275	2.4882686	2 4884097
308	2.4892552	2.4893959	2.4895366	2.4896773	2.4898179
309	2.4906607	2.4908009	2.4909412	2.4910814	2.4912216
310	2.4920616	2.4922015	2.4923413	2.4924810	2.4926207
311	2.4934581	2.4935974	2.4937368	2.4938761	2 4940154
312	2.4948500	2.4949890	2.4951279	2.4952667	2.4954056
313	2.4962375	2.4963761	2.4965145	2.4966529	2.4967913
314	2.4976206	2.4977587	2.4978967	2.4980347	2.4981727
315	2.4989994	2.4991370	2.4992746	2.4994121	2.4995496
316	2.5003737	2.5005109	2.5006481	2.5007852	2.5009222
317	2.5017437	2.5018805	2.5020172	2.5021539	2.5022905
318	2.5031094	2.5032458	2.5033821	2.5035183	2.5036545
319	2.5044709	2.5046068	2.5047426	2.5048785	2.5050142
320	2.5058280	2.5059635	2.5060990	2.5062344	2.5063697
321	2.5071810	2.5073160	2.5074511	2.5075860	2,5077210
322	2.5085297	2.5086644	2.5087990	2.5089335	2.5090680
323	2.5098743	2.5100085	2.5101427	2.5102768	2.5104109
324	2.5112147	2.5113485	2.5114823	2.5116160	2.5117497
325	2.5125510	2.5126844	2.5128178	2.5129511	2.5130844
326	2.5138832	2.5140162	2.5141491	2.5142820	2.5144149
327	2.5152113	2.5153439	2.5154764	2.5156089	2.5157414
328	2.5165354	2.5166676	2.5167997	2.5169318	2.5170639
329	2.5178554	2.5179872	2.5181189	2.5182506	2.5183823
330	2.5191715	2.5193028	2.5194342	2.5195655	2.5196968
331	2.5204835	2.5206145	2.5207455	2.5208764	2.5210073
332	2.5217916	2.5219222	2.5220528	2.5221833	2.5223138
333	2.5230958	2.5232260	2.5233562	2.5234863	2.5236164
334	2.5243961	2.5245259	2.5246557	2.5247854	2.5249151
335	2.5256925	2.5258219	2.5259513	2.5260807	2.5262100
336	2.5269851	2.5271141	2.5272431	2.5273721	2.5275010
337	2.5282738	2.5284024	2.5285311	2.5286596	2.5287882
338	2.5295587	2.5296869	2.5298152	2.5299434	2.5300716
339	2.5308398	2.5309677	2.5310955	2.5312234	2.5313512
340	2.5321171	2.5322446	2.5323721	2.5324996	2.5326270
341	2.5333907	2.5335179	2.5336450	2.5337721	2.5338991
342	2.5346606	2.5347874	2.5349141	2.5350408	2 5351675
343	2.5359267	2.5360532	2.5361795	2.5363059	2.5364322
344	2.5371892	2.5373153	2.5374413	2.5375673	2.5376932
345	2.5384481	2.5385737	2.5386994	2.5388250	2.5389506
346	2.5397032	2.5398286	2.5399538	2.5400791	2.5402043
347	2.5409548	2.5410798	2.5412047	2.5413296	2.5414544
348	2.5422028	2.5423274	2.5424519	2.5425765	2.5427010
349	2.5434472	2.5435714	2.5436956	2.5438198	2.5439439

A TABLE of Logarithms.

Num	0	1	2	3	4
350	2.5440680	2.5441921	2.5443161	2.5444401	2.5445641
351	2.5453071	2.5454308	2.5455545	2.5456781	2.5458018
352	2.5465427	2.5466660	2,5467894	2.5469126	2.5470359
353	2.5477747	2.5478977	2.5480207	2.5481436	2.5482665
354	2.5490033	2.5491259	2.5492486	2.5493712	2.5494937
355	2.5502284	2.5503507	2.5504730	2.5505952	2.5507174
356	2.5514500	2.5515720	2.5516939	2.5518158	2.5519377
357	2.5526682	2.5527899	2.5529115	2.5530330	2.5531545
358	2.5538830	2.5540043	2.5541256	2.5542468	2.5543680
359	2.5550944	2.5552154	2.5553363	2.5554572	2.5555781
360	2.5563025	2.5564231	2.5565437	2.5566643	2.5567848
361	2.5575072	2.5576275	2.5577477	2.5578680	2.5579881
362	2.5587086	2.5588285	2.5589484	2.5590683	2.5591882
363	2.5599066	2.5600262	2,5601458	2.5602654	2.5603849
364	2.5611014	2.5612207	2.5613399	2.5614592	2.5615784
365	2.5622929	2.5624118	2.5625308	2.5626497	2.5627685
366	2.5634811	2.5635997	2.5637183	2.5638369	2.5639555
367	2.5646661	2.5647844	2.5649027	2.5650209	2.5651392
368	2.5658478	2.5659658	2.5660838	2.5662017	2.5663196
369	2.5670264	2.5671440	2.5672617	2.5673793	2.5674969
370	2.5682017	2.5683191	2.5684364	2.5685537	2.5686710
371	2.5693739	2.5694910	2.5696080	2.5697249	2.5698419
372	2.5705429	2.5706597	2.5707764	2.5708930	2.5710097
373	2.5717088	2.5718252	2.5719416	2.5720580	2.5721743
374	2.5728716	2.5729877	2.5731038	2.5732198	2.5733358
375	2.5740313	2.5741471	2.5742628	2.5743786	2.5744943
376	2.5751878	2.5753033	2.5754188	2.5755342	2.5756496
377	2.5763414	2.5764565	2.5765717	2.5766868	2.5768019
378	2.5774918	2.5776067	2.5777215	2.5778363	2.5779511
379	2.5786392	2.5787538	2.5788683	2.5789828	2.5790973
380	2.5797836	2.5798979	2.5800121	2.5801263	2.5802405
381	2.5809250	2.5810389	2.5811529	2.5812668	2.5813807
382	2.5820634	2.5821770	2.5822907	2.5824043	2.5825179
383	2.5831988	2.5833122	2.5834255	2.5835388	2.5836521
384	2.5843312	2.5844443	2.5845574	2.5846704	2.5847834
385	2.5854607	2.5855735	2.5856863	2.5857990	2.5859117
386	2.5865873	2.5866998	2.5868123	2.5869247	2.5870371
387	2.5877110	2.5878232	2.5879353	2.5880475	2.5881596
388	2.5888317	2.5889436	2.5890555	2.5891674	2.5892792
389	2.5899496	2.5900612	2.5901728	2.5902844	2.5903959
390	2.5910646	2.5911763	2.5912873	2.5913986	2.5915098
391	2.5921768	2,5922878	2.5923988	2.5925098	2.5926208
392	2.5932861	2,5933968	2.5935076	2.5936183	2.5937290
393	2,5943926	2.5945030	2.5946135	2.5947239	2.5948344
394	2,5954962	2.5956064	2.5957166	2.5958268	2.5959369
395	2.5965971	2.5967070	2.5968169	2.5969268	2.5970367
396	2.5976952	2.5978048	2.5979145	2.5980241	2.5981336
397	2.5987905	2.5988999	2.5990092	2.5991186	2.5992279
398	2.5998831	2.5999922	2.6001013	2,6002103	2.6003193
399	2.6009729	2.6010817	2.6011905	2.6012993	2.6014081

Num.	5	6	7	8	9
350	2.5446880	2.5448119	2.5449358	2.5450596	2.5451834
351	2.5459253	2.5460489	2.5461724	2.5462958	2.5464193
352	2.5471591	2.5472823	2.5474055	2.5475286	2.5476517
353	2.5483894	2.5485123	2.5486351	2.5487578	2.5488806
354	2.5496162	2.5497387	2.5498612	2.5499836	2.5501060
355	2.5508396	2.5509618	2.5510839	2.5512059	2.5513280
356	2.5520595	2.5521813	2.5523031	2.5524248	2.5525465
357	2.5532760	2.5533975	2.5535189	2.5536403	2.5537617
358	2.5544892	2.5546103	2.5547314	2.5548524	2.5549735
359	2.5556989	2.5558197	2.5559404	2.5560612	2.5561818
360	2.5569053	2.5570257	2.5571461	2.5572665	2.5573869
361	2.5581083	2.5582284	2.5583485	2.5584686	2.5585886
362	2.5593080	2.5594278	2.5595476	2.5596673	2.5597870
363	2.5605044	2.5606239	2.5607433	2.5608627	2.5609821
364	2.5616975	2.5618167	2.5619358	2.5620548	2.5621739
365	2.5628874	2.5630062	2.5631250	2.5632437	2.5633624
365	2.5640740	2.5641925	2.5643109	2.5644293	2.5645477
367	2.5652573	2.5653755	2.5654936	2.5656117	2.5657298
363	2.5664375	2.5665553	2.5666731	2.5667909	2.5669087
369	2.5676144	2.5677320	2.5678494	2.5679669	2.5680843
370	2.5687882	2.5689054	2.5690226	2.5691397	2.5692568
371	2.5699588	2.5700757	2.5701926	2.5703094	2.5704262
372	2.5711263	2.5712429	2.5713594	2.5714759	2.5715924
373	2.5722906	2.5724069	2.5725231	2.5726393	2.5727555
374	2.5734518	2.5735678	2.5736837	2.5737996	2.5739154
375	2.5746099	2.5747256	2.5748412	2.5749568	2.5750723
376	2.5757650	2.5758803	2.5759956	2.5761109	2.5762251
377	2.5769170	2.5770320	2.5771470	2.5772620	2.5773769
378	2.5780659	2.5781806	2.5782953	2.5784100	2.5785246
379	2.5792118	2.5893262	2.5794406	2.5795550	2.5796693
380	2.5803547	2.5804688	2.5805829	2.5806969	2.5808110
381	2.5814945	2.5816084	2.5817222	2.5818359	2.5819497
382	2.5826314	2.5827450	2.5828585	2.5829719	2.5830854
383	2.5837654	2.5838786	2.5839918	2.5841050	2.5842181
384	2.5848963	2.5850093	2.5851222	2.5852351	2.5853479
385	2.5860244	2.5861370	2.5862496	2.5863622	2.5864748
386	2.5871495	2.5872618	2.5873742	2.5874865	2.5875987
387	2.5882717	2.5883838	2.5884958	2.5886078	2.5887198
388	2.5893910	2.5895028	2.5896145	2.5897263	2.5898379
389	2.5905075	2.5906189	2.5907304	2.5908418	2.5909532
390	2.5916210	2.5917322	2.5918434	2.5919546	2.5920657
391	2.5927318	2.5928427	2.5929536	2.5930644	2.5931753
392	2.5938397	2.5939503	2.5940609	2.5941715	2.5942820
393	2.5949447	2.5950551	2.5951654	2.5952757	2.5953860
394	2.5960470	2.5961571	2.5962671	2.5963771	2.5964871
395	2.5971465	2.5972563	2.5973661	2.5974758	2.5975855
396	2.5982432	2.5983527	2.5984622	2.5985717	2.5986811
397	2.5993371	2.5994464	2.5995556	2.5996648	2.5997739
398	2.6004283	2.6005373	2.6006462	2.6007551	2.6008640
399	2.6015168	2.6016255	2.6017341	2.6018428	2.6019514

A TABLE of Logarithms.

Num.	0	1	2	3	4
400	2.6020600	2.6021685	2.6022771	2.6023856	2.6024941
401	2.6031444	2.6032527	2.6033609	2.6034692	2.6035774
402	2.6042261	2.6043341	2.6044421	2.6045500	2.6046580
403	2.6053050	2.6054128	2.6055205	2.6056282	2.6057358
404	2.6063814	2.6064889	2.6065963	2.6067037	2.6068111
405	2.6074550	2.6075622	2.6076694	2.6077766	2.6078837
406	2.6085260	2.6086330	2.6087399	2.6088468	2.6089537
407	2.6095944	2.6097011	2.6098078	2.6099144	2.6100210
408	2.6106602	2.6107666	2.6108730	2.6109794	2.6100857
409	2.6117233	2.6118295	2.6119356	2.6120417	2.612147
410	2.6127839	2.6128898	2.6129957	2.6131015	2.6132074
411	2.6138418	2.6139475	2.614 531	2.6141587	2.6142643
412	2.6148972	2.6150026	2.6151080	2.6152133	2.6153187
413	2.6159501	2.6160552	2.6161603	2.6162654	2.6163705
414	2.6170003	2.6171052	2.6172101	2.6173149	2.6174197
415	2.6180481	2.6181527	2.6182573	2.6183619	2.6184665
416	2.6190933	2.6191977	2.6193021	2.6194064	2.6195107
417	2.6201360	2.6202402	2.6203443	2.6204484	2.6205524
418	2.6211763	2.6212802	2.6213840	2.6214879	2.6215917
419	2.6222140	2.6223177	2.6224213	2.6225249	2.6226284
420	2.6232493	2.6233527	2.6234560	2.6235594	2.6236627
421	2.6242821	2.6243852	2.6244884	2.6245915	2.6246945
422	2.6253125	2.6254154	2.6255182	2.6256211	2.6257239
423	2.6263404	2.6264430	2.6265457	2.6266483	2.6267509
424	2.6273659	2.6274683	2.6275707	2.6276730	2.6277754
425	2.6283889	2.6284911	2.6285933	2.6286954	2.6287975
426	2.6294096	2.6295115	2.6296134	2.6297153	2.6298172
427	2.6304279	2.6305296	2.6306312	2.6307329	2.6308345
428	2.6314438	2.6315452	2.6316467	2.6317481	2.6318495
429	2.6324573	2.6325585	2.6326597	2.6327609	2.6328620
430	2.6334685	2.6335694	2.6336704	2.6337713	2.6338723
431	2.6344773	2.6345780	2.6346788	2.6347795	2.6348801
432	2.6354837	2.6355843	2.6356848	2.6357852	2.6358857
433	2.6364879	2.6365882	2.6366884	2.6367887	2.6368889
434	2.6374897	2.6375898	2.6376898	2.6377898	2.6378898
435	2.6384893	2.6385891	2.6386889	2.6387887	2.6388884
436	2.6394865	2.6395861	2.6396857	2.6397852	2.6398847
437	2.6404814	2.6405808	2.6406802	2.6407795	2.6408788
438	2.6414741	2.6415733	2.6416724	2.6417715	2.6418705
439	2.6424645	2.6425634	2.6426623	2.6427612	2.6428601
440	2.6434527	2.6435514	2.6436500	2.6437487	2.6438473
441	2.6444386	2.6445371	2.6446355	2.6447339	2.6448323
442	2.6454223	2.6455205	2.6456187	2.6457169	2.6458151
443	2.6464037	2.6465018	2.6465998	2.6466977	2.6467957
444	2.6473830	2.6474808	2.6475786	2.6476763	2.6477741
445	2.6483600	2.6484576	2.6485552	2.6486527	2.6487502
446	2.6493349	2.6494322	2.6495296	2.6496269	2.6497241
447	2.6503075	2.6504047	2.6505018	2.6505989	2.6506960
448	2.6512780	2.6513749	2.6514719	2.6515687	2.6516656
449	2.6522463	2.6523431	2.6524397	2.6525364	2.6526331

Num.	5	6	7	8	9
400	2.6026025	2.6027109	2.6028193	2.6029277	2.6030361
401	2.6036855	2.6037937	2.6039018	2.6040099	2.6041180
402	2.6047659	2.6048738	2.6049816	2.6050895	2.6051973
403	2.6058435	2.6059512	2.6060587	2.6061663	2.6062739
404	2.6069185	2.6070259	2.6071332	2.6072405	2.6073478
405	2.6079909	2.6080979	2.6082050	2.6083120	2.6084191
406	2.6090605	2.6091674	2.6092742	2.6093809	2.6094877
407	2.6101276	2.6102342	2.6103407	2.6104472	2.6105537
408	2.6111921	2.6112984	2.6114046	2.6115109	2.6116171
409	2.6122539	2.6123599	2.6124660	2.6125720	2.6126779
410	2.6133132	2.6134189	2.6135247	2.6136304	2.6137361
411	2.6143698	2.6144754	2.6145809	2.6146863	2.6147918
412	2.6154240	2.6155292	2.6156345	2.6157397	2.6158449
413	2.6164755	2.6165805	2.6166855	2.6167905	2.6168954
414	2.6175245	2.6176293	2.6177340	2.6178387	2.6179434
415	2.6185710	2.6186755	2.6187800	2.6188845	2.6189889
416	2.6196150	2.6197193	2.6198235	2.6199277	2.6200319
417	2.6206565	2.6207605	2.6208645	2.6209684	2.6210724
418	2.6216955	2.6217992	2.6219030	2.6220067	2.6221104
419	2.6227320	2.6228355	2.6229390	2.6230424	2.6231459
420	2.6237660	2.6238693	2.6239725	2.6240757	2.6241789
421	2.6247976	2.6249006	2.6250036	2.6251066	2.6252095
422	2.6258267	2.6259295	2.6260322	2.6261350	2.6262377
423	2.6268534	2.6269560	2.6270585	2.6271610	2.6272634
424	2.6278777	2.6279800	2.6280823	2.6281845	2.6282867
425	2.6288996	2.6290016	2.6291037	2.6292057	2.6293076
426	2.6299190	2.6300209	2.6301226	2.6302244	2.6303262
427	2.6309361	2.6310377	2.6311393	2.6312408	2.6313423
428	2.6319508	2.6320522	2.6321535	2.6322548	2.6323560
429	2.6329632	2.6330643	2.6331654	2.6332664	2.6333674
430	2.6339732	2.6340740	2.6341749	2.6342757	2.6343765
431	2.6349808	2.6350814	2.6351820	2.6352826	2.6353832
432	2.6359861	2.6360865	2.6361869	2.6362872	2.6363876
433	2.6369891	2.6370893	2.6371894	2.6372895	2.6373897
434	2.6379898	2.6380897	2.6381896	2.6382895	2.6383894
435	2.6389882	2.6390879	2.6391876	2.6392872	2.6393869
436	2.6399842	2.6400837	2.6401832	2.6402826	2.6403820
437	2.6409781	2.6410773	2.6411765	2.6412758	2.6413749
438	2.6419696	2.6420686	2.6421676	2.6422666	2.6423656
439	2.6429589	2.6430577	2.6431565	2.6432552	2.6433540
440	2.6439459	2.6440445	2.6441431	2.6442416	2.6443401
441	2.6449307	2.6450291	2.6451274	2.6452257	2.6453240
442	2.6459133	2.6460114	2.6461095	2.6462076	2.6463057
443	2.6468936	2.6469915	2.6470894	2.6471873	2.6472851
444	2.6478718	2.6479695	2.6480671	2.6481648	2.6482624
445	2.6488477	2.6489452	2.6490426	2.6491401	2.6492375
446	2.6498215	2.6499187	2.6500160	2.6501132	2.6502104
447	2.6507930	2.6508901	2.6509871	2.6510841	2.6511811
448	2.6517624	2.6518593	2.6519561	2.6520528	2.6521496
449	2.6527297	2.6528263	2.6529229	2.6530195	2.6531160

A TABLE of Logarithms.

Num	0	1	2	3	4
450	2.6532125	2.6533090	2.6534055	2.6535019	2.6535984
451	2.6541765	2.6542728	2.6543691	2.6544653	2.6545616
452	2.6551384	2.6552345	2.6553306	2.6554266	2.6555226
453	2.6560982	2.6561941	2,6562899	2.6563857	2.6564815
454	2.6570559	2.6571515	2.6572471	2.6573427	2.6574383
455	2.6580114	2.6581068	2.6582023	2.6582977	2.6583930
456	2.6589648	2.6590601	2.6591553	2.6592505	2.6593456
457	2.6599162	2.6600112	2.6601062	2.6602012	2.6602962
458	2.6608655	2.6609603	2.6610551	2.6611499	2.6612440
459	2.6618127	2.6619073	2.6620019	2.6620964	2.6621910
460	2.6627578	2,6628522	2.6629466	2.6630410	2.6631353
461	2.6637009	2.6637951	2.6638893	2.6639835	2.6640776
462	2.6646420	2.6647360	2.6648299	2.6649239	2.6650178
463	2.6655819	2.6656748	2.6657686	2.6658623	2.6659560
464	2,6665180	2.6666116	2.6667051	2.6667987	2.6668922
465	2.6674530	2.6675463	2.6676397	2.6677331	2.6678264
466	2.6683859	2.6684791	2.6685723	2.6686654	2.6687585
467	2.6693169	2.6694099	2.6695028	2.6695958	2.6696887
468	2.6702459	2.6703386	2.6704314	2.6705242	2.6706169
469	2.6711728	2.6712654	2.6713580	2.6714506	2.6715431
470	2.6720979	2.6721903	2.6722826	2.6723750	2.6724673
471	2.6730209	2.6731131	2.6732053	2.6732974	2,6733896
472	2.6739420	2.6740340	2.6741260	2.6742179	2.6743099
473	2.6748611	2.6749529	2.6750447	2.6751365	2.6752283
474	2.6757783	2.6758700	2.6759615	2.6760531	2.6761417
475	2.6766936	2.6767850	2.6768764	2.6769678	2.6770592
476	2.6776070	2.6776982	2.6777894	2.6778806	2.6779718
477	2.6785184	2.6786094	2.6787004	2.6787914	2.6788824
478	2.6794279	2.6795187	2.6796096	2.6797004	2.6797912
479	2.6803355	2.6804262	2.6805168	2.6806074	2.6806980
480	2.6812412	2.6813317	2.6814222	2.6815126	2.6816030
481	2.6821451	2.6822354	2.6823256	2.6824159	2.6825061
482	2.6830470	2.6831371	2.6832272	2.6833173	2.6834073
483	2.6839471	2.6840370	2.6841269	2.6842168	2.6843066
484	2.6848454	2.6849351	2.6850248	2.6851145	2.6852041
485	2.6857417	2.6858313	2.6859208	2.6860103	2.6860998
486	2.6866363	2.6867256	2.6868150	2.6869043	2.6869936
487	2,6875290	2.6876181	2.6877073	2.6877964	2.6878855
488	2.6884198	2.6885088	2.6885978	2.6886867	2.6887757
489	2.6893089	2.6893977	2.6894864	2.6895752	2.6896640
490	2.6901961	2.6902847	2.6903733	2.6904619	2.6905505
491	2.6910815	2.6911699	2.6912584	2.6913468	2.6914352
492	2.6919651	2.6920534	2.6921416	2.6922298	2.6923180
493	2.6928469	2.6929350	2.6930231	2.6931111	2.6931991
494	2.6937269	2.6938149	2.6939027	2.6939906	2.6940785
495	2.6946052	2.6946929	2.6947806	2.6948683	2.6949560
496	2.6954817	2.6955692	2.6956568	2.6957443	2.6958318
497	2.6963564	2.6964438	2.6965311	2.6966185	2.6967058
498	2.6972293	2.6973165	2.6974037	2.6974909	2.6975780
499	2.6981005	2.6981876	2.6982746	2.6983616	2.6984485

Num.	5	6	7	8	9
450	2.6536948	2.6537912	2.6538876	2.6539839	2.6540802
451	2.6546578	2.6547539	2.6548501	2.6549462	2.6550423
452	2.6556186	2.6557145	2.6558105	2.6559064	2.6560023
453	2.6565773	2.6566730	2.6567688	2.6568645	2.6569602
454	2.6575339	2.6576294	2.6577250	2.6578205	2.6579159
455	2.6584884	2.6585837	2.6586790	2.6587743	2.6588696
456	2.6594408	2.6595359	2.6596310	2.6597261	2.6598212
457	2.6603911	2.6604860	2.6605809	2.6606758	2.6607706
458	2.6613393	2.6614340	2.6615287	2.6616234	2.6617181
459	2.6622855	2.6623800	2.6524745	2.6625690	2.6626634
460	2.6632296	2.6633239	2.6634182	2.6635125	2.6636067
461	2.6641717	2.6642658	2.6643599	2.6644539	2.6645480
462	2.6651117	2.6652056	2.6652995	2.6653933	2.6654872
463	2.6660497	2.6661434	2.6662371	2.6663307	2.6664244
464	2.6669857	2.6670792	2.6671727	2.6672661	2.6673595
465	2.6679197	2.6680130	2.6681062	2.6681995	2.6682927
466	2.6688516	2.6689447	2.6590378	2.6691308	2.6692239
467	2.6697816	2.6698745	2.6699674	2.6700602	2.6701530
468	2.6707096	2.6708023	2.6708950	2.6709876	2.6710802
469	2.6716356	2.6717281	2.6718206	2.6719130	2.6720054
470	2.6725596	2.6726519	2.6727442	2.6728365	2.6729287
471	2.6734817	2.6735738	2.6736659	2.6737579	2.6738500
472	2.6744018	2.6744937	2.6745856	2.6746775	2.6747693
473	2.6753200	2.6754117	2.6755034	2.6755951	2.6756867
474	2.6762362	2.6763277	2.6764192	2.6765107	2.0766022
475	2.6771505	2.6772418	2.6773332	2.6774244	2.6775157
476	2.6780629	2.6781540	2.6782452	2.6783362	2.6784273
477	2.6789734	2.6790643	2.6791552	2.6792461	2.6793370
478	2.6798819	2.6799727	2.6800634	2.6801541	2.6802448
479	2.6807886	2.6808792	2.6809697	2.6810602	2.6811507
480	2.6816934	2.6817838	2.6818741	2.6819645	2.6820548
481	2.6825963	2.6826865	2.6827765	2.6828668	2.6829569
482	2.6834973	2.6835873	2.6836773	2.6837673	2.6838572
483	2.6843965	2.6844863	2.6845761	2.6846659	2.6847556
484	2.6852938	2.6853834	2.6854730	2.6855626	2.6856522
485	2.6861892	2.6862787	2.6863681	2.6854575	2.6865469
486	2.6870828	2.6871721	2.6872613	2.6873506	2.6874393
487	2.6879746	2.6880637	2.6881528	2.6882418	2.6883308
488	2.6888646	2.6889535	2.6890423	2.6891312	2.6892200
489	2.6897527	2.6898414	2.6899301	2.6900188	2.6901074
490	2.6906390	2.6907275	2.6908161	2.6909046	2.6909930
491	2.6915235	2.6916119	2.6917002	2.6917885	2.6918768
492	2.6924062	2.6924944	2.6925826	2.6926707	2.6927588
493	2.6932872	2.6933752	2.6934631	2.6935511	2.6936390
494	2.6941663	2.6942541	2.6943419	2.6944297	2.6945174
495	2.6950437	2.6951313	2.6952189	2.6953065	2.6953941
496	2.6959193	2.6960067	2.6960942	2.6961816	2.6962690
497	2.6967931	2.6968804	2.6969676	2.6970549	2.6971421
498	2.6976652	2.6977523	2.6978394	2.6979264	2.6980135
499	2.6985355	2.6986224	2.6987093	2.6987963	2.6988831

A TABLE of Logarithms.

Num	0	1	2	3	4
500	2.6989700	2.6990569	2.6991437	2.6992305	2.6993173
501	2.6998377	2.6999244	2.7000111	2.7000977	2.7001843
502	2.7007037	2.7007902	2.7008767	2.7009632	2.7010496
503	2.7015680	2.7016543	2.7017406	2.7018269	2.7019132
504	2.7024305	2.7025167	2.7026028	2.7026890	2.7027751
505	2.7032914	2.7033774	2.7034633	2.7035493	2.7036352
506	2.7041505	2.7042363	2.7043221	2.7044079	2.7044937
507	2.7050080	2.7050936	2.7051792	2.7052649	2.7053505
508	2.7058637	2.7059492	2.7060347	2.7061201	2.7062055
509	2.7067178	2.7068031	2.7068884	2.7069737	2.7070589
510	2.7075702	2.7076553	2.7077405	2.7078256	2.7079107
511	2.7084209	2.7085059	2.7085908	2.7086758	2.7087607
512	2.7092700	2.7093548	2.7094396	2.7095244	2.7096091
513	2.7101174	2.7102020	2.7102866	2.7103713	2.7104559
514	2.7109631	2.7110476	2.7111321	2.7112165	2.7113010
515	2.7118072	2.7118915	2.7119759	2.7120601	2.7121444
516	2.7126497	2.7127339	2.7128180	2.7129021	2.7129862
517	2.7134905	2.7135745	2.7136585	2.7137425	2.7138264
518	2.7143298	2.7144136	2.7144974	2.7145812	2.7146650
519	2.7151674	2.7152510	2.7153347	2.7154183	2.7155019
520	2.7160033	2.7160869	2.7161703	2.7162538	2.7163373
521	2.7168377	2.7169211	2.7170044	2.7170877	2.7171710
522	2.7176705	2.7177537	2.7178369	2.7179200	2.7180032
523	2.7185017	2.7185847	2.7186677	2.7187507	2.7188337
524	2.7193313	2.7194142	2.7194970	2.7195799	2.7196627
525	2.7201593	2.7202420	2.7203247	2.7204074	2.7204901
526	2.7209857	2.7210683	2.7211508	2.7212334	2.7213159
527	2.7218106	2.7218930	2.7219754	2.7220578	2.7221401
528	2.7226339	2.7227162	2.7227984	2.7228806	2.7229628
529	2.7234557	2.7235378	2.7236198	2.7237019	2.7237839
530	2.7242759	2.7243578	2.7244397	2.7245216	2.7246035
531	2.7250945	2.7251763	2.7252581	2.7253398	2.7254216
532	2.7259116	2.7259933	2.7260749	2.7261565	2.7262380
533	2.7267272	2.7268087	2.7268901	2.7269716	2.7270530
534	2.7275413	2.7276226	2.7277039	2.7277852	2.7278664
535	2.7283538	2.7284350	2.7285161	2.7285972	2.7286784
536	2.7291648	2.7292458	2.7293268	2.7294078	2.7294888
537	2.7299743	2.7300552	2.7301360	2.7302168	2.7302977
538	2.7307823	2.7308630	2.7309437	2.7310244	2.7311051
539	2.7315888	2.7316693	2.7317499	2.7318304	2.7319109
540	2.7323938	2.7324742	2.7325546	2.7326350	2.7327153
541	2.7331973	2.7332775	2.7333578	2.7334380	2.7335183
542	2.7339993	2.7340794	2.7341595	2.7342396	2.7343197
543	2.7347998	2.7348798	2.7349598	2.7350397	2.7351196
544	2.7355989	2.7356787	2.7357585	2.7358383	2.7359181
545	2.7363965	2.7364762	2.7365558	2.7366355	2.7367151
546	2.7371926	2.7372722	2.7373517	2.7374312	2.7375107
547	2.7379873	2.7380667	2.7381461	2.7382254	2.7383048
548	2.7387806	2.7388598	2.7389390	2.7390182	2.7390974
549	2.7395723	2.7396514	2.7397305	2.7398096	2.7398887

Num.	5	6	7	8	9
500	2.6994041	2.6994908	2.6995776	2.6996643	2.6997510
501	2.7002709	2.7003575	2.7004441	2.7005307	2.7006173
502	2.7011361	2.7012225	2.7013089	2.7013953	2.7014816
503	2.7019995	2.7020857	2.7021719	2.7022582	2.7023444
504	2.7028612	2.7029472	2.7030333	2.7031193	2.7032054
505	2.7037212	2.7038071	2.7038929	2.7039788	2.7040647
506	2.7045794	2.7046652	2.7047509	2.7048366	2.7049223
507	2.7054360	2.7055216	2.7056072	2 7056927	2 7057782
508	2.7062910	2.7063764	2.7064617	2.7065471	2.7066325
509	2.7071442	2.7072294	2:7073146	2.7073998	2.7074850
510	2.7079957	2.7080808	2.7081659	2.7082509	2.7083359
511	2.7088456	2.7089305	2.7090154	2.7091003	2.7091851
512	2.7096939	2.7097786	2.7098633	2.7099480	2.7100327
513	2.7105404	2.7106250	2.7107096	2.7107941	2.7108786
514	2.7113854	2.7114698	2.7115542	2.7116385	2.7117229
515	2.7122287	2.7123129	2.7123971	2.7124813	2.7125655
516	2.7130703	2.7131544	2.7132385	2.7133225	2.7134065
517	2.7139104	2.7139943	2.7140782	2.7141620	2.7142459
518	2.7147488	2.7148325	2.7149162	2.7150000	2.7150837
519	2.7155856	2.7156691	2.7157527	2.7158363	2.6159198
520	2.7164207	2.7165042	2.7165876	2.7166710	2.7167544
521	2.7172543	2.7173376	2.7174208	2.7175041	2.7175873
522	2.7180863	2.7181694	2.7182525	2.7183356	2.7184186
523	2.7189167	2.7189996	2.7190826	2.7191655	2.7192484
524	2.7197455	2.7198283	2.7199111	2.7199938	2.7200766
525	2.7205727	2.7206554	2.7207380	2.7208206	2.7209032
526	2.7213984	2.7214809	2.7215633	2.7216458	2.7217282
527	2.7222225	2.7223048	2.7223871	2.7224694	2.7225517
528	2.7230450	2.7231272	2.7232093	2.7232914	2.7233736
529	2.7238660	2.7239480	2.7240300	2.7241120	2.7241939
530	2.7246854	2.7247672	2.7248491	2.7249309	2.7250127
531	2.7255033	2.7255850	2.7256667	2.7257483	2.7258300
532	2.7263196	2.7264012	2.7264827	2.7265642	2.7266457
533	2.7271344	2.7272158	2.7272972	2.7273786	2.7274599
534	2.7279477	2.7280290	2.7281102	2.7281914	2.7282726
535	2.7287595	2.7288406	2.7289216	2.7290027	2.7290838
536	2.7295697	2.7296507	2.7297316	2.7298125	2.7298934
537	2.7303785	2.7304593	2.7305400	2.7306208	2.7307015
538	2.7311857	2.7312663	2.7313470	2.7314276	2.7315082
539	2.7319914	2.7320719	2.7321524	2.7322329	2.7323133
540	2.7327957	2.7328760	2.7329564	2.7330367	2.7331170
541	2.7335985	2.7336787	2.7337588	2.7338390	2.7339192
542	2.7343997	2.7344798	2.7345598	2.7346398	2 7347198
543	2.7351995	2.7352794	2.7353593	2.7354392	2.7355191
544	2.7359979	2.7360776	2.7361574	2.7362371	2.7363168
545	2.7367948	2.7368744	2.7369540	2.7370335	2.7371131
546	2.7375902	2.7376696	2.7377491	2.7378285	2.7379079
547	2.7383841	2.7384634	2.7385427	2.7386220	2.7387013
548	2.7391766	2.7392558	2.7393350	2.7394141	2.7394932
549	2.7399677	2.7400467	2.7401257	2.7402047	2.7402837

A TABLE of Logarithms.

Num.	0	1	2	3	4
600	2.7781512	2.7782236	2.7782960	2.7783683	2.7784407
601	2.7788745	2.7789467	2.7790190	2.7790912	2.7791634
602	2.7795965	2.7796686	2.7797407	2.7798129	2.7798850
603	2.7803173	2.7803893	2.7804613	2.7805333	2.7806053
604	2.7810369	2.7811088	2.7811807	2.7812526	2.7813245
605	2.7817554	2.7818272	2.7818989	2.7819707	2.7820424
606	2.7824726	2.7825443	2.7826159	2.7826876	2.7827592
607	2.7831887	2.7832602	2.7833318	2.7834033	2.7834748
608	2.7839036	2.7839750	2.7840464	2.7841178	2.7841892
609	2.7846173	2.7846886	2.7847599	2.7848312	2.7849024
610	2.7853298	2.7854010	2.7854722	2.7855434	2.7856145
611	2.7860412	2.7861123	2.7861833	2.7862544	2.7863254
612	2.7867514	2.7868224	2.7868933	2.7869643	2.7870352
613	2.7874605	2.7875313	2.7876021	2.7876730	2.7877439
614	2.7881684	2.7882391	2.7883098	2.7883805	2.7884512
615	2.7888751	2.7889457	2.7890163	2.7890869	2.7891575
616	2.7895807	2.7896512	2.7897217	2.7897922	2.7898625
617	2.7902852	2.7903555	2.7904259	2.7904963	2.7905665
618	2.7909885	2.7910587	2.7911290	2.7911992	2.7912695
619	2.7916906	2.7917608	2.7918309	2.7919011	2.7919712
620	2.7923917	2.7924617	2.7925318	2.7926018	2.7926718
621	2.7930916	2.7931615	2.7932314	2.7933013	2.7933712
622	2.7937904	2.7938602	2.7939300	2.7939998	2.7940696
623	2.7944880	2.7945577	2.7946274	2.7946971	2.7947668
624	2.7951846	2.7952542	2.7953238	2.7953933	2.7954629
625	2.7958800	2.7959495	2.7960190	2.7960884	2.7961579
626	2.7965743	2.7966437	2.7967131	2.7967824	2.7968517
627	2.7972675	2.7973368	2.7974060	2.7974753	2.7975445
628	2.7979596	2.7980288	2.7980979	2.7981671	2.7982362
629	2.7986506	2.7987197	2.7987887	2.7988577	2.7989267
630	2.7993405	2.7994095	2.7994784	2.7995473	2.7996162
631	2.8000294	2.8000982	2.8001670	2.8002358	2.8003046
632	2.8007171	2.8007858	2.8008545	2.8009232	2.8009919
633	2.8014037	2.8014723	2.8015409	2.8016095	2.8016781
634	2.8020893	2.8021577	2.8022262	2.8022947	2.8023632
635	2.8027737	2.8028421	2.8029105	2.8029789	2.8030472
636	2.8034571	2.8035254	2.8035937	2.8036619	2.8037302
637	2.8041394	2.8042076	2.8042758	2.8043439	2.8044121
638	2.8048207	2.8048887	2.8049568	2.8050248	2.8050929
639	2.8055009	2.8055688	2.8056368	2.8057047	2.8057725
640	2.8061800	2.8062478	2.8063157	2.8063835	2.8064513
641	2.8068580	2.8069258	2.8069935	2.8070612	2.8071290
642	2.8075350	2.8076027	2.8076703	2.8077379	2.8078055
643	2.8082110	2.8082785	2.8083460	2.8084135	2.8084811
644	2.8088859	2.8089533	2.8090207	2.8090881	2.8091555
645	2.8095597	2.8096270	2.8096944	2.8097617	2.8098290
646	2.8102325	2.8102997	2.8103670	2.8104342	2.8105013
647	2.8109043	2.8109714	2.8110385	2.8111056	2.8111727
648	2.8115750	2.8116420	2.8117090	2.8117760	2.8118430
640	2.8122447	2.8123116	2.8123785	2.8124454	2.8125123

Num.	5	6	7	8	9
600	2.7785130	2.7785853	2.7786576	2.7787299	2.7788022
601	2.7792356	2.7783078	2.7793800	2.7794522	2.7795243
602	2.7799570	2.7800291	2.7801012	2.7801732	2.7802453
603	2.7806773	2.7807492	2.7808212	2.7808931	2.7809650
604	2.7813963	2.7814681	2.7815400	2.7816118	2.7816836
605	2.7821141	2.7821859	2.7822576	2.7823293	2.7824010
606	2.7828308	2.7829024	2.7829740	2.7830456	2.7831171
607	2.7835463	2.7836178	2.7836892	2.7837607	2.7838321
608	2.7842606	2.7843319	2.7844033	2.7844746	2.7845460
609	2.7849737	2.7850450	2.7851162	2.7851874	2.7852586
610	2.7856857	2.7857568	2.7858279	2.7858990	2.7859701
611	2.7863965	2.7864675	2.7865385	2.7866095	2.7866804
612	2.7871061	2.7871770	2.7872479	2.7873188	2.7873896
613	2.7878146	2.7878853	2.7879561	2.7880269	2.7880976
614	2.7885219	2.7885926	2.7886632	2.7887339	2.7888045
615	2.7892281	2.7892986	2.7893691	2.7894397	2.7895102
616	2.7899331	2.7900035	2.7900739	2.7901444	2.7902148
617	2.7906370	2.7907073	2.7907776	2.7908479	2.7909182
618	2.7913397	2.7914099	2.7914801	2.7915503	2.7916205
619	2.7920413	2.7921114	2.7921815	2.7922516	2.7923216
620	2.7927418	2.2928118	2.7928817	2.7929517	2.7930217
621	2.7934411	2.7935110	2.7935809	2.7936507	2.7937206
622	2.7941394	2.7942091	2.7942789	2.7943486	2.7944183
623	2.7948365	2.7949061	2.7949757	2.7950454	2.7951150
624	2.7955324	2.7956020	2.7956715	2.7957410	2.7958105
625	2.7962273	2.7962967	2.7963662	2.7964356	2.7965049
626	2.7969211	2.7969904	2.7970597	2.7971290	2.7971983
627	2.7976137	2.7976829	2.7977521	2.7978213	2.7978905
628	2.7983053	2.7983744	2.7984435	2.7985125	2.7985816
629	2.7989957	2.7990647	2.7991337	2.7992027	2.7992716
630	2.7996851	2.7997540	2.7998228	2.7998917	2.7999605
631	2.8003733	2.8004421	2.8905109	2.8005796	2.8006484
632	2.8010605	2.8011292	2.8011978	2.8012665	2.8013351
633	2.8017466	2.8018152	2.8018837	2.8019522	2.8020207
634	2.8024316	2.8025001	2.8025685	2.8026369	2.8027053
635	2.8031155	2.8031839	2.8032522	2.8033205	2.8033888
636	2.8037984	2.8038666	2.8039348	2.8040030	2.8040712
637	2.8044802	2.8045483	2.8046164	2.8046845	2.8047526
638	2.8051609	2.8052289	2.8052969	2.8053649	2.8054329
639	2.8058405	2.8059085	2.8059763	2.8060442	2.8061121
640	2.8065191	2.8065869	2.8066547	2.8067225	2.8067903
641	2.8071967	2.8072644	2.8073320	2.8073997	2.8074674
642	2.8078731	2.8079407	2.8080083	2.8080759	2.8081434
643	2.8085485	2.8086160	2.8086835	2.8087510	2.8088184
644	2.8092229	2.8092903	2.8093577	2.8094250	2.8094924
645	2.8098962	2.8099635	2.8100308	2.8100980	2.8101653
646	2.8105685	2.8106357	2.8107029	2.8107700	2.8108371
647	2.8112398	2.8113068	2.8113739	2.8114409	2.8115080
648	2.8119100	2.8119769	2.8120439	2.8121108	2.8121778
649	2.8125792	2.8126460	2.8127129	2.8127797	2.8128465

A TABLE of Logarithms.

Num.	0	1	2	3	4
650	2.8129134	2.8129802	2.8130470	2.8131138	2.8131805
651	2.8135810	2.8136477	2.8137144	2.8137811	2.8138478
652	2.8142476	2.8543142	2.8143808	2.8144474	2.8145140
653	2.8149132	2.8149797	2.8150462	2.8151127	2.8151791
654	2.8155777	2.8156441	2.8157105	2.8157769	2.8158433
655	2.8162413	2.8163076	2.8163739	2.8164402	2.8165064
656	2.8169038	2.8169700	2.8170362	2.8171024	2.8171686
657	2.8175654	2.8176315	2.8176976	2.8177636	2.8178297
658	2.8182259	2.8182919	2.8183579	2.8184239	2.8184898
659	2.8188854	2.8189513	2.8190172	2.8190831	2.8191489
660	2.8195439	2.8196097	2.8196755	2.8197413	2.8198071
661	2.8202015	2.8202672	2.8203328	2.8203985	2.8204642
662	2.8208580	2.8209236	2.8209892	2.8210548	2.8211203
663	2.8215135	2.8215790	2.8216445	2.8217100	2.8217755
664	2.8221681	2.8222335	2.8222989	2.8223643	2.8224296
665	2.8228216	2.8228869	2.8229522	2.8230175	2.8230828
666	2.8234742	2.8235394	2.8236046	2.8236698	2.8237350
667	2.8241258	2.8241909	2.8242560	2.8243211	2.8243862
668	2.8247765	2.8248415	2.8249065	2.8249715	2.8250364
669	2.8254261	2.8254910	2.8255559	2.8256208	2.8256857
670	2.8260748	2.8261396	2.8262044	2.8262692	2.8263340
671	2.8267225	2.8267872	2.8268519	2.3269166	2.8269813
672	2.8273693	2.8274339	2.8274985	2.8275631	2.8276277
673	2.8280151	2.8280796	2.8281441	2.8282086	2.8282731
674	2.8286599	2.8287243	2.8287887	2.8288532	2.8289176
675	2.8293038	2.8293681	2.8294324	2.8294967	2.8295611
676	2.8299467	2.8300109	2.8300752	2.8301394	2.8302036
677	2.8305887	2.8306528	2.8307169	2.8307811	2.8308452
678	2.8312297	2.8312937	2.8313578	2.8314218	2.8314858
679	2.8318698	2.8319337	2.8319977	2.8320616	2.8321255
680	2.8325089	2.8325728	2.8326366	2.8327005	2.8327643
681	2.8331471	2.8332109	2.8332746	2.8333384	2.8334021
682	2.8337844	2.8338480	2.8339117	2.8339754	2.8340390
683	2.8344207	2.8344843	2.8345479	2.8346114	2.8346750
684	2.8350561	2.8351196	2.8351831	2.8352465	2.8353100
685	2.8356906	2.8357540	2.8358174	2.8358807	2.8359441
686	2.8363241	2.8363874	2.8364507	2.8365140	2.8365773
687	2.8369567	2.8370199	2.8370832	2.8371403	2.8372095
688	2.8375884	2.8376516	2.8377147	2.8377778	2.8378409
689	2.8382192	2.8382822	2.8383453	2.8384083	2.8384713
690	2.8388491	2.8389120	2.8389750	2.8390379	2.8391008
691	2.8394780	2.8395409	2.8396037	2.8396666	2.8397294
692	2.8401061	2.8401688	2.8402316	2.8402943	2.8403571
693	2.8407332	2.8407959	2.8408586	2.8409212	2.8409838
694	2.8413595	2.8414220	2.8414846	2.8415472	2.8416097
695	2.8419848	2.8420473	2.8421098	2.8421722	2.8422347
696	2.8426092	2.8426716	2.8427340	2.8427964	2.8428588
697	2.8422328	2.8432952	2.8433574	2.8434197	2.8434819
698	2.8438554	2.8439176	2.8439798	2.8440420	2.8441042
699	2.8444772	2.8445393	2.8446014	2.8446635	2.8447256

Num	5	6	7	8	9
650	2.8132473	2.8133141	2.8133808	2.8134475	2.8135143
651	2.8139144	2.8139811	2.8140477	2.8141144	2.8141810
652	2.8145805	2.8146471	2.8147136	2.8147801	2.8148467
653	2.8152456	2.8153120	2.8153785	2.8154449	2.8155113
654	2.8159096	2.8159760	2.8160423	2.8161087	2.8161750
655	2.8165727	2.8166389	2.8167052	2.8167714	2.8168376
656	2.8172347	2.8173009	2.8173670	2.8174331	2.8174993
657	2.8178958	2.8179618	2.8180278	2.8180939	2.8181599
658	2.8185558	2.8186217	2.8186877	2.8187536	2.8188195
659	2.8192148	2,8192806	2.8193465	2.8194123	2.8194781
660	2.8198728	2.8199386	2.8200043	2.8200700	2.8201357
661	2-8205298	2.8205955	2.8206611	2.8207268	2.8207924
662	2.8211859	2.8212514	2.8213170	2,8213825	2.8214480
663	2.8218409	2.8219064	2.8219718	2.8220372	2.8221027
664	2.8224950	2.8225603	2.8226257	2,8226910	2.8227563
665	2.8231481	2.8232133	2.8232786	2.8233438	2,8234090
666	2.8238002	2.8238653	2.8239305	2.8239956	2.8240607
667	2.8244513	2.8245163	2.8245814	2.8246464	2.8247114
668	2.8251014	2.8251664	2.8252313	2.8252963	2.8253612
669	2.8257506	2 8258154	2.8258803	2.8259451	2.8260100
670	2.8263988	2.8264635	2.8265283	2.8265931	2,8266578
671	2.8270460	2.8271107	2.8271753	2.8272400	2.8273046
672	2.8276923	2.8277569	2.8278214	2.8278860	2.8279505
673	2.8283376	2.8284021	2.8284665	2.8285310	2.8285955
674	2.8289820	2.8290463	2.8291107	2.8291751	2.8292394
675	2.8296254	2.8296896	2.8297539	2.8298182	2.8298824
676	2.8302678	2.8303320	2.8303962	2.8304604	2.8305245
677	2.8309093	2.8309734	2.8310375	2-8311016	2-8311656
678	2-8315499	2,8316139	2.8316778	2.8317418	2.8318058
679	2.8321895	2.8322534	2.8323173	2-8323812	2.8324450
680	2.8328281	2.8328919	2.8329558	2.8330195	2.8330833
681	2.8334659	2.8335296	2.8335933	2.8336570	2.8337207
682	2.8341027	2.8341663	2.8342299	2.8342935	2.8343571
683	2.8347385	2.8348021	2.8348656	2.8349291	2.8349926
684	2.8353735	2.8354369	2.8355003	2.8355638	2.8356272
685	2.8360075	2.8360708	2.8361341	2.8361975	2.8362608
686	2.8366405	2.8367038	2.8367670	2.8368303	2.8368935
687	2,8372727	2.8373359	2.8373990	2.8374622	2.8375253
688	2.8379039	2.8379670	2-8380301	2.8380931	2.8381562
689	2.8385343	2.8385973	2.8386602	2.8387232	2-8387861
690	2.8391637	2.8392266	2.8392895	2.8393523	2.8394152
691	2.8397922	2.8398550	2.8399178	2.8399806	2.8400433
692	2.8404198	2.8404825	2.8405452	2.8406079	2.8406706
693	2.8410465	2.8411091	2.8411717	2.8412343	2.8412969
694	2.8416723	2.8417348	2.8417973	2.8418598	2.8419223
695	2.8422971	2.8423596	2.8424220	2.8424844	2.8425468
696	2.8429211	2,8429835	2.8430458	2.8431081	2.8431705
697	2.8435442	2.8436065	2.8436687	2-8437310	2.8437932
698	2.8441664	2.8442286	2.8442907	2.8443529	2.8444150
699	2.8447877	2.8448498	2.8449119	2.8449739	2.8450360

Num.	0	1	2	
650	2.8129134	2.8129802	2.8130470	2
651	2.8135810	2.8136477	2.8137144	
652	2.8142476	2.8543142	2.814380	
653	2.8149132	2.8149797	2.8150	
654	2.8155777	2.8156441	2.815	
655	2.8162413	2.8163076	2.8	
656	2.8169038	2.8169700	2.	
657	2.8175654	2.8176315	2	
658	2.8182259	2.8182919		
659	2.8188854	2.8189513		
660	2.8195439	2.81960		
661	2.8202015	2.82026		
662	2.8208580	2.820		
663	2.8215135	2.82		
664	2.8221681	2.8		
665	2.8228216	2.		
666	2.8234742			
667	2.8241258			
668	2.82477			
669	2.82542			
670	2.8260			
671	2.826			
672	2.82			
673	2.8			
674	2			
675				
676				
677				
678				
67				
68				

					4
				2	2.8453461
					2.8459658
				27	2.8465845
				406	2.8472024
				7577	2.8478193
				3739	2.8484355
				89892	2.8490577
				496037	2.8496651
				.8502172	2.8502786
				2.8508300	2.8508912
				2.8514418	2.8515030
				2.8520528	2.8521139
			0	2.8526629	2.8527239
			113	2.8532722	2.8533331
			198	2.8538807	2.8539414
			1275	2.8544882	2.8545489
			0343	2.8550950	2.8551556
			56403	2.8557008	2.8557614
			562454	2.8563059	2.8563663
			.8568497	2.8569101	2.8569704
			2.8574531	2.8575134	2.8575737
			2.8580557	2.8581159	2.8581761
			2.8586575	2.8587176	2.8587777
		4	2.8592584	2.8593185	2.8593785
		85	2.8598585	2.8599185	2.8599784
		979	2.8604578	2.8605177	2.8605776
		9964	2.8610562	2.8611160	2.8611758
		15941	2.8616539	2.8617136	2.8617733
		621910	2.8622507	2.8623103	2.8623699
		2.8627871	2.8628467	2.8629062	2.8629658
		2.8633823	2.8634418	2.8635013	2.8635608
		2.8639768	2.8640362	2.8640956	2.8641552
	11	2.8645704	2.8646297	2.8646890	2.8647483
	040	2.8651632	2.8652225	2.8652817	2.8653409
	6961	2.8657552	2.8658144	2.8658735	2.8659327
	62873	2.8663464	7.8664055	2.8664646	2.8665236
	668778	2.8669368	2.8669958	2.8670548	2.8671138
	2.8674675	2.8675264	2.8675853	2.8676442	2.8677031
	2.8680564	2.8681152	2.8681740	2.8682329	2.8682917
	2.8686444	2.8687032	2.8687620	2.8688207	2.8688794
	2.8692317	2.8692904	2.8693491	2.8694077	2.8694654
1	2.8698182	2.8698768	2.8699354	2.8699940	2.8700520
42	2.8704039	2.8704624	2.8705210	2.8705794	2.8706380
743	2.8709888	2.8710473	2.8711057	2.8711641	2.8712226
744	2.8715729	2.8716313	2.8716897	2.8717480	2.8718064
745	2.8721563	2.8722146	2.8722728	2.8723311	2.8723894
746	2.8727388	2.8727970	2.8728552	2.8729134	2.8729716
747	2.8733206	2.8733787	2.8734369	2.8734950	2.8735531
748	2.8739016	2.8739597	2.8740177	2.8740757	2.8741338
749	2.8744818	2.8745398	2.8745978	2.8746557	2.8747137

	5	6	7	8	9
	2.8454081	2.8454701	2.8455321	2.8455941	2.8456561
	8460277	2.8460896	2.8461515	2.8462134	2.8462752
	66463	2.8467081	2.8467700	2.8468318	2.8468935
	641	2.8473258	2.8473876	2.8474493	2.8475110
	0	2.8479426	2.8480043	2.8480659	2.8481275
		2.8485586	2.8486201	2.8486817	2.8487432
		2.8491736	2.8492351	2.8492965	2.8493580
		497878	2.8498492	2.8499106	2.8499719
		4011	2.8504624	2.8505237	2.8505850
		36	2.8510748	2.8511360	2.8511972
			2.8516863	2.8517474	2.8518085
			2.8522970	2.8523580	2.8524190
			2.8529068	2.8529677	2.8530286
		6	2.8535157	2.8535765	2.8536374
		0	2.8541238	2.8541845	2.8542453
		703	2.8547310	2.8547917	2.8548524
		2768	2.8553374	2.8553980	2.8554586
		58824	2.8559429	2.8560035	2.8560640
		564872	2.8565476	2.8566081	2.8566685
		8570912	2.8571515	2.8572118	2.8572722
		2.8576943	2.8577545	2.8578148	2.8578750
		2.8582965	2.8583567	2.8584169	2.8584770
	9	2.8588980	2.8589581	2.8590181	2.8590782
	85	2.8594986	2.8595586	2.8596186	2.8596786
	0384	2.8600983	2.8601583	2.8602182	2.8602781
	06374	2.8606973	2.8607571	2 8608170	2.8608768
	.8612356	2.8612954	2.8613552	2.8614149	2.8614747
	2.8618330	2.8618927	2.8619524	2.8620120	2 8620717
	2,8624296	2,8624892	2.8625488	2.8626084	2.8626680
	2.8630253	2.8630848	2.8631443	2.8632039	2.8632634
0	2.8636202	2.8636797	2.8637391	2.8637985	2.8638580
731	2.8642143	2.8642737	2.8643331	2.8643924	2.8644517
732	2.8648076	2.8648669	2.8649262	2.8649855	2.8650447
733	2.8654001	2.8654593	2.8655185	2.8655777	2.8656369
734	2.8659918	2.8660509	2.8661100	2.8661691	2.8662289
735	2.8665827	2.8666417	2.8667008	2.8667598	2.8668188
736	2.8671728	2.8672317	2.8672907	2.8673496	2.8674086
737	2.8677620	2.8678209	2.8678798	2.8679387	2.8679975
738	2.8683505	2.8684093	2.8684681	2.8685269	2.8685857
739	2.8689382	2.8689969	2.8690556	2.8691143	2.8691730
740	2.8695251	2.8695837	2.8696423	2.8697010	2.8697596
741	2.8701117	2.8701697	2.8702283	2.8702868	2.8703454
742	2.8706965	2.8707549	2.8708134	2.8708719	2.8709304
743	2.8712810	2.8713394	2.8713978	2 8714562	2.8715146
744	2.8718647	2.8719230	2.8719814	2.8720397	2.8720980
745	2.8724476	2.8725059	2.8725641	2.8726224	2.8726806
746	2.8730298	2.8730880	2.8731462	2.8732043	2.8732625
747	2.8736112	2,8736693	2.8737274	2 8737855	2.8738435
748	2.8741918	2.8742498	2.8743078	2.8743658	2.8744238
749	2.8747716	2.8748296	2.8748875	2 8749454	2.8750031

A TABLE of Logarithms.

Num.	0	1	2	3	4
700	2.8450980	2.8451601	2.8452221	2.8452841	2.8453461
701	2.8457180	2.8457800	2.8458419	2.8459038	2.8459658
702	2.8463371	2.8463990	2.8464608	2.8465227	2.8465845
703	2.8469553	2.8470171	2.8470789	2.8471406	2.8472024
704	2.8475727	2.8476343	2.8476960	2.8477577	2.8478193
705	2.8481891	2.8482507	2.8483123	2.8483739	2.8484355
706	2.8488047	2.8488662	2.8489277	2.8489892	2.8490507
707	2.8494194	2.8494808	2.8495423	2.8496037	2.8496651
708	2.8500333	2.8500946	2.8501559	2.8502172	2.8502786
709	2.8506462	2.8507075	2.8507687	2.8508300	2.8508912
710	2.8512583	2.8513195	2.8513807	2.8514418	2.8515030
711	2.8518696	2.8519307	2.8519917	2.8520528	2.8521139
712	2.8524800	2.8525410	2.8526020	2.8526629	2.8527239
713	2.8530895	2.8531504	2.8532113	2.8532722	2.8533331
714	2.8536982	2.8537590	2.8538198	2.8538807	2.8539414
715	2.8543060	2.8543668	2.8544275	2.8544882	2.8545489
716	2.8549130	2.8549737	2.8550343	2.8550950	2.8551556
717	2.8555192	2.8555797	2.8556403	2.8557008	2.8557614
718	2.8561244	2.8561849	2.8562454	2.8563059	2.8563663
719	2.8567289	2.8567893	2.8568497	2.8569101	2.8569704
720	2.8573325	2.8573928	2.8574531	2.8575134	2.8575737
721	2.8579353	2.8579955	2.8580557	2.8581159	2.8581761
722	2.8585372	2.8585973	2.8586575	2.8587176	2.8587777
723	2.8591383	2.8591984	2.8592584	2.8593185	2.8593785
724	2.8597386	2.8597985	2.8598585	2.8599185	2.8599784
725	2.8603380	2.8603979	2.8604578	2.8605177	2.8605776
726	2.8609366	2.8609964	2.8610562	2.8611160	2.8611758
727	2.8615344	2.8615941	2.8616539	2.8617136	2.8617733
728	2.8621314	2.8621910	2.8622507	2.8623103	2.8623699
729	2.8627275	2.8627871	2.8628467	2.8629062	2.8629658
730	2.8633229	2.8633823	2.8634418	2.8635013	2.8635608
731	2.8639174	2.8639768	2.8640362	2.8640956	2.8641550
732	2.8645111	2.8645704	2.8646297	2.8646890	2.8647483
733	2.8651040	2.8651632	2.8652225	2.8652817	2.8653409
734	2.8656961	2.8657552	2.8658144	2.8658735	2.8659327
735	2.8662873	2.8663464	2.8664055	2.8664646	2.8665236
736	2.8668778	2.8669368	2.8669958	2.8670548	2.8671138
737	2.8674675	2.8675264	2.8675853	2.8676442	2.8677031
738	2.8680564	2.8681152	2.8681740	2.8682329	2.8682917
739	2.8686444	2.8687032	2.8687620	2.8688207	2.8688794
740	2.8692317	2.8692904	2.8693491	2.8694077	2.8694664
741	2.8698182	2.8698768	2.8699354	2.8699940	2.8700526
742	2.8704039	2.8704624	2.8705210	2.8705794	2.8706380
743	2.8709888	2.8710473	2.8711057	2.8711641	2.8712226
744	2.8715729	2.8716313	2.8716897	2.8717480	2.8718064
745	2.8721563	2.8722146	2.8722728	2.8723311	2.8723894
746	2.8727388	2.8727970	2.8728552	2.8729134	2.8729716
747	2.8733206	2.8733787	2.8734369	2.8734950	2.8735531
748	2.8739016	2.8739597	2.8740177	2.8740757	2.8741338
749	2.8744818	2.8745398	2.8745978	2.8746557	2.8747137

Num.	5	6	7	8	9
700	2.8454081	2.8454701	2.8455321	2.8455941	2.8456561
701	2.8460277	2.8460896	2.8461515	2.8462134	2.8462752
702	2.8466463	2.8467081	2.8467700	2.8468318	2.8468935
703	2.8472641	2.8473258	2.8473876	2.8474493	2.8475110
704	2.8478810	2.8479426	2.8480043	2.8480659	2.8481275
705	2.8484970	2.8485586	2.8486201	2.8486817	2.8487432
706	2.8491122	2.8491736	2.8492351	2.8492965	2.8493580
707	2.8497264	2.8497878	2.8498492	2.8499106	2.8499719
708	2.8503399	2.8504011	2.8504624	2.8505237	2.8505850
709	2.8509524	2.8510136	2.8510748	2.8511360	2.8511972
710	2.8515641	2.8516252	2.8516863	2.8517474	2.8518085
711	2.8521749	2.8522359	2.8522970	2.8523580	2.8524190
712	2.8527849	2.8528458	2.8529068	2.8529677	2.8530286
713	2.8533940	2.8534548	2.8535157	2.8535765	2.8536374
714	2.8540022	2.8540630	2.8541238	2.8541845	2.8542453
715	2.8546096	2.8546703	2.8547310	2.8547917	2.8548524
716	2.8552162	2.8552768	2.8553374	2.8553980	2.8554586
717	2.8558219	2.8558824	2.8559429	2.8560035	2.8560640
718	2.8564268	2.8564872	2.8565476	2.8566081	2.8566685
719	2.8570308	2.8570912	2.8571515	2.8572118	2.8572722
720	2.8576340	2.8576943	2.8577545	2.8578148	2.8578750
721	2.8582363	2.8582965	2.8583567	2.8584169	2.8584770
722	2.8588379	2.8588980	2.8589581	2.8590181	2.8590782
723	2.8594385	2.8594986	2.8595586	2.8596186	2.8596786
724	2.8600384	2.8600983	2.8601583	2.8602182	2.8602781
725	2.8606374	2.8606973	2.8607571	2.8608170	2.8608768
726	2.8612356	2.8612954	2.8613552	2.8614149	2.8614747
727	2.8618330	2.8618927	2.8619524	2.8620120	2.8620717
728	2.8624296	2.8624892	2.8625488	2.8626084	2.8626680
729	2.8630253	2.8630848	2.8631443	2.8632039	2.8632634
730	2.8636202	2.8636797	2.8637391	2.8637985	2.8638580
731	2.8642143	2.8642737	2.8643331	2.8643924	2.8644517
732	2.8648076	2.8648669	2.8649262	2.8649855	2.8650447
733	2.8654001	2.8654593	2.8655185	2.8655777	2.8656369
734	2.8659918	2.8660509	2.8661100	2.8661691	2.8662289
735	2.8665827	2.8666417	2.8667008	2.8667598	2.8668188
736	2.8671728	2.8672317	2.8672907	2.8673496	2.8674085
737	2.8677620	2.8678209	2.8678798	2.8679387	2.8679975
738	2.8683505	2.8684093	2.8684681	2.8685269	2.8685857
739	2.8689382	2.8689969	2.8690556	2.8691143	2.8691730
740	2.8695251	2.8695837	2.8696423	2.8697010	2.8697596
741	2.8701112	2.8701697	2.8702283	2.8702868	2.8703454
742	2.8706965	2.8707549	2.8708134	2.8708719	2.8709304
743	2.8712810	2.8713394	2.8713978	2.8714562	2.8715146
744	2.8718647	2.8719230	2.8719814	2.8720397	2.8720980
745	2.8724476	2.8725059	2.8725641	2.8726224	2.8726806
746	2.8730298	2.8730880	2.8731462	2.8732043	2.8732625
747	2.8736112	2.8736693	2.8737274	2.8737855	2.8738435
748	2.8741918	2.8742498	2.8743078	2.8743658	2.8744238
749	2.8747716	2.8748296	2.8748875	2.8749154	2.8750031

Num.	0	1	2	3	4
750	2.8750613	2.8751192	2.8751771	2.8752349	2.8752928
751	2.8756399	2.8756978	2.8757556	2.8758134	2.8758712
752	2.8762178	2.8762756	2.8763333	2.8763911	2.8764488
753	2.8767950	2.8768526	2.8769103	2.8769680	2.8770256
754	2.8773713	2.8774289	2.8774865	2.8775441	2.8776017
755	2.8779470	2.8780045	2.8780620	2.8781195	2.8781770
756	2.8795218	2.8785792	2.8786367	2.8786941	2.8787515
757	2.8790959	2.8791532	2.8792106	2.8792680	2.8793253
758	2.8796692	2.8797265	2.8797838	2.8798411	2.8798983
759	2.8802418	2.8802990	2.8803562	2.8804134	2.8804706
760	2.8808136	2.8808707	2.8809279	2.8809850	2.8810421
761	2.8813847	2.8814417	2.8814988	2.8815558	2.8816129
762	2.8819550	2.8820120	2.8820689	2.8821259	2.8821829
763	2.8825245	2.8825815	2.8826384	2.8826953	2.8827522
764	2.8830934	2.8831502	2.8832070	2.8832639	2.8833207
765	2.8836614	2.8837182	2.8837750	2.8838317	2.8838885
766	2.8842288	2.8842855	2.8843421	2.8843988	2.8844555
767	2.8847954	2.8848520	2.8849086	2.8849652	2.8850218
768	2.8853612	2.8854178	2.8854743	2.8855308	2.8855874
769	2.8859263	2.8859828	2.8860393	2.8860957	2.8861522
770	2.8864907	2.8865471	2.8866035	2.8866599	2.8867163
771	2.8870544	2.8871107	2.8871670	2.8872233	2.8872796
772	2.8876173	2.8876736	2.8877298	2.8877860	2.8878423
773	2.8881795	2.8882357	2.8882918	2.8883480	2.8884042
774	2.8887410	2.8887971	2.8888532	2.8889093	2.8889653
775	2.8893017	2.8893577	2.8894138	2.8894698	2.8895258
776	2.8898617	2.8899177	2.8899736	2.8900296	2.8900855
777	2.8904210	2.8904769	2.8905328	2.8905887	2.8906445
778	2.8909796	2.8910354	2.8910912	2.8911470	2.8912028
779	2.8915375	2.8915932	2.8916489	2.8917047	2.8917604
780	2.8920946	2.8921503	2.8922059	2.8922616	2.8923173
781	2.8926510	2.8927066	2.8927622	2.8928178	2.8928734
782	2.8932068	2.8932623	2.8933178	2.8933733	2.8934288
783	2.8937618	2.8938172	2.8938727	2.8939281	2.8939836
784	2.8943161	2.8943715	2.8944268	2.8944822	2.8945376
785	2.8948697	2.8949250	2.8949803	2.8950356	2.8950909
786	2.8954225	2.8954778	2.8955330	2.8955883	2.8956435
787	2.8959748	2.8960299	2.8960851	2.8961403	2.8961954
788	2.8965262	2.8965813	2.8966364	2.8966915	2.8967466
789	2.8970770	2.8971320	2.8971871	2.8972421	2.8972971
790	2.8976271	2.8976821	2.8977370	2.8977920	2.8978469
791	2.8981765	2.8982314	2.8982863	2.8983412	2.8983960
792	2.8987252	2.8987800	2.8988348	2.8988897	2.8989445
793	2.8992732	2.8993279	2.8993827	2.8994375	2.8994922
794	2.8998205	2.8998752	2.8999299	2.8999845	2.9000392
795	2.9003671	2.9004218	2.9004764	2.9005310	2.9005856
796	2.9009131	2.9009676	2.9010222	2.9010767	2.9011313
797	2.9014583	2.9015128	2.9015673	2.9016218	2.9016762
798	2.9020029	2.9020573	2.9021117	2.9021661	2.9022205
799	2.9025468	2.9026011	2.9026555	2.9027098	2.9027641

Num.	5	6	7	8	9
750	2.8753507	2.8754086	2.8754664	2.8755243	2.8755821
751	2.8759290	2.8759868	2.8760446	2.8761023	2.8761601
752	2.8765065	2.8765642	2.8766219	2.8766796	2.8767373
753	2.8770833	2.8771409	2.8771985	2.8772561	2.8773137
754	2.8776592	2.8777168	2.8777743	2.8778319	2.8778894
755	2.8782345	2.8782919	2.8783494	2.8784069	2.8784643
756	2.8788089	2.8788663	2.8789237	2.8789811	2.8790385
757	2.8793826	2.8794400	2.8794973	2.8795546	2.8796119
758	2.8799556	2.8800128	2.8800701	2.8801273	2.8801846
759	2.8805278	2.8805850	2.8806421	2.8806993	2.8807564
760	2.8810992	2.8811563	2.8812134	2.8812705	2.8813276
761	2.8816699	2.8817269	2.8817840	2.8818410	2.8818980
762	2.8822398	2.8822968	2.8823537	2.8824107	2.8824676
763	2.8828090	2.8828659	2.8829228	2.8829797	2.8830365
764	2.8833775	2.8834343	2.8834911	2.8835479	2.8836047
765	2.8839452	2.8840019	2.8840586	2.8841154	2.8841721
766	2.8845122	2.8845688	2.8846255	2.8846821	2.8847387
767	2.8850784	2.8851350	2.8851915	2.8852481	2.8853047
768	2.8856439	2.8857004	2.8857569	2.8858134	2.8858699
769	2.8862085	2.8862651	2.8863215	2.8863779	2.8864343
770	2.8867726	2.8868290	2.8868854	2.8869417	2.8869980
771	2.8873359	2.8873922	2.8874485	2.8875048	2.8875610
772	2.8878985	2.8879547	2.8880109	2.8880671	2.8881233
773	2.8884603	2.8885165	2.8885726	2.8886287	2.8886848
774	2.8890214	2.8890775	2.8891336	2.8891896	2.8892457
775	2.8895818	2.8896378	2.8896938	2.8897498	2.8898058
776	2.8901415	2.8901974	2.8902533	2.8903092	2.8903651
777	2.8907004	2.8907563	2.8908121	2.8908679	2.8909238
778	2.8912586	2.8913144	2.8913702	2.8914259	2.8914817
779	2.8918161	2.8918718	2.8919275	2.8919832	2.8920389
780	2.8923729	2.8924285	2.8924842	2.8925398	2.8925954
781	2.8929290	2.8929846	2.8930401	2.8930957	2.8931512
782	2.8934843	2.8935398	2.8935953	2.8936508	2.8937063
783	2.8940390	2.8940944	2.8941498	2.8942053	2.8942607
784	2.8945929	2.8946483	2.8947037	2.8947590	2.8948143
785	2.8951462	2.8952015	2.8952568	2.8953120	2.8953673
786	2.8956987	2.8957539	2.8958092	2.8958644	2.8959195
787	2.8962506	2.8963057	2.8963608	2.8964160	2.8964711
788	2.8968017	2.8968568	2.8969118	2.8969669	2.8970220
789	2.8973521	2.8974071	2.8974621	2.8975171	2.8975721
790	2.8979019	2.8979568	2.8980117	2.8980657	2.8981216
791	2.8984509	2.8985058	2.8985606	2.8986155	2.8986703
792	2.8989993	2.8990541	2.8991089	2.8991636	2.8992184
793	2.8995469	2.8996017	2.8996564	2.8997111	2.8997658
794	2.9000939	2.9001486	2.9002032	2.9002579	2.9003125
795	2.9006402	2.9006948	2.9007494	2.9008039	2.9008585
796	2.9011858	2.9012403	2.9012948	2.9013493	2.9014038
797	2.9017307	2.9017851	2.9018396	2.9018940	2.9019485
798	2.9022749	2.9023293	2.9023837	2.9024381	2.9024924
799	2.9028185	2.9028728	2.9029271	2.9029814	2.9030357

A TABLE of Logarithms.

Num	0	1	2	3	4
800	2.9030900	2.9031443	2.9031985	2.9032528	2.9033071
801	2.9036325	2.9036867	2.9037409	2.9037951	2.9038493
802	2.9041744	2.9042285	2.9042827	2.9043368	2.9043909
803	2.9047155	2.9047696	2.9048237	2.9048778	2.9049318
804	2.9052560	2.9053101	2.9053641	2.9054181	2.9054721
805	2.9057959	2.9058498	2.9059038	2.9059577	2.9060116
806	2.9063350	2.9063889	2.9064428	2.9064967	2.9065505
807	2.9068735	2.9069273	2.9069812	2.9070350	2.9070887
808	2.9074114	2.9074651	2.9075188	2.9075726	2.9076262
809	2.9079485	2.9080022	2.9080559	2.9081095	2.9081632
810	2.9084850	2.9085386	2.9085922	2.9086458	2.9086994
811	2.9090209	2.9090744	2.9091279	2.9091815	2.9092350
812	2.9095560	2.9096095	2.9096630	2.9097165	2.9097699
813	2.9100905	2.9101440	2.9101974	2.9102508	2.9103042
814	2.9106244	2.9106778	2.9107311	2.9107844	2.9108378
815	2.9111576	2.9112109	2.9112642	2.9113174	2.9113707
816	2.9116902	2.9117434	2.9117966	2.9118498	2.9119030
817	2.9122221	2.9122752	2.9123284	2.9123815	2.9124346
818	2.9127533	2.9128064	2.9128595	2.9129126	2.9129656
819	2.9132839	2.9133369	2.9133899	2.9134430	2.9134960
820	2.9138139	2.9138668	2.9139198	2.9139727	2.9140257
821	2.9143432	2.9143961	2.9144489	2.9145018	2.9145547
822	2.9148718	2.9149246	2.9149775	2.9150303	2.9150831
823	2.9153998	2.9154526	2.9155054	2.9155581	2.9156109
824	2.9159272	2.9159799	2.9160326	2.9160853	2.9161380
825	2.9164539	2.9165066	2.9165592	2.9166118	2.9166645
826	2.9169800	2.9170326	2.9170852	2.9171378	2.9171903
827	2.9175055	2.9175580	2.9176105	2.9176630	2.9177155
828	2.9180303	2.9180828	2.9181352	2.9181877	2.9182401
829	2.9185545	2.9186067	2.9186593	2.9187117	2.9187640
830	2.9190781	2.9191304	2.9191827	2.9192350	2.9192873
831	2.9196010	2.9196533	2.9197055	2.9197578	2.9198100
832	2.9201233	2.9201755	2.9202277	2.9202799	2.9203321
833	2.9206450	2.9206971	2.9207493	2.9208014	2.9208535
834	2.9211661	2.9212181	2.9212702	2.9213222	2.9213743
835	2.9216865	2.9217385	2.9217965	2.9218425	2.9218945
836	2.9222063	2.9222582	2.9223102	2.9223621	2.9224140
837	2.9227255	2.9227773	2.9228292	2.9228811	2.9229330
838	2.9232440	2.9232958	2.9233477	2.9233995	2.9234513
839	2.9237620	2.9238137	2.9238655	2.9239172	2.9239690
840	2.9242793	2.9243310	2.9243827	2.9244344	2.9244860
841	2.9247960	2.9248476	2.9248993	2.9249509	2.9250025
842	2.9253121	2.9253637	2.9254152	2.9254668	2.9255184
843	2.9258276	2.9258791	2.9259306	2.9259821	2.9260336
844	2.9263424	2.9263939	2.9264453	2.9264968	2.9265482
845	2.9268567	2.9269081	2.9269595	2.9270109	2.9270622
846	2.9273704	2.9174217	2.9274730	2.9275243	2.9275757
847	2.9278834	2.9279347	2.9279859	2.9280372	2.9280885
848	2.9283959	2.9284471	2.9284983	2.9285495	2.9286007
849	2.9289077	2.9289588	2.9290100	2.9290611	2.9291123

Num.	5	6	7	8	9
800	2.9033613	2.9034156	2.9034698	2.9035241	2.9035783
801	2.9039035	2.9039577	2.9040119	2.9040661	2.9041202
802	2.9044450	2.9044992	2.9045533	2.9046074	2.9046615
803	2.9049859	2.9050399	2.9050940	2.9051480	2.9052020
804	2.9055260	2.9055800	2.9056340	2.9056880	2.9057419
805	2.9060655	2.9061195	2.9061734	2.9062273	2.9062812
806	2.9066044	2.9066582	2.9067121	2.9067659	2.9068197
807	2.9071425	2.9071963	2.9072501	2.9073038	2.9073576
808	2.9076800	2.9077337	2.9077874	2.9078411	2.9078948
809	2.9082169	2.9082705	2.9083241	2.9083778	2.9084314
810	2.9087530	2.9088066	2.9088602	2.9089137	2.9089673
811	2.9092885	2.9093420	2.9093955	2.9094490	2.9095025
812	2.9098234	2.9098768	2.9099302	2.9099837	2.9100371
813	2.9103576	2.9104109	2.9104643	2.9105177	2.9105710
814	2.9108911	2.9109444	2.9109977	2.9110510	2.9111043
815	2.9114240	2.9114772	2.9115305	2.9115837	2.9116369
816	2.9119562	2.9120094	2.9120626	2.9121157	2.9121689
817	2.9124878	2.9125409	2.9125940	2.9126471	2.9127002
818	2.9130187	2.9130717	2.9131248	2.9131778	2.9132309
819	2.9135490	2.9136019	2.9136549	2.9137079	2.9137609
820	2.9140786	2.9141315	2.9141844	2.9142373	2.9142903
821	2.9146076	2.9146604	2.9147133	2.9147661	2.9148190
822	2.9151359	2.9151887	2.9152415	2.9152943	2.9153471
823	2.9156636	2.9157163	2.9157691	2.9158218	2.9158745
824	2.9161907	2.9162433	2.9162960	2.9163487	2.9164013
825	2.9167171	2.9167697	2.9168223	2.9168749	2.9169275
826	2.9172429	2.9172954	2.9173479	2.9174005	2.9174530
827	2.9177680	2.9178205	2.9178730	2.9179254	2.9179779
828	2.9182925	2.9183449	2.9183973	2.9184497	2.9185021
829	2.9188164	2.9188687	2.9189211	2.9189734	2.9190258
830	2.9193396	2.9193919	2.9194442	2.9194965	2.9195488
831	2.9198623	2.9199145	2.9199667	2.9200189	2.9200711
832	2.9203842	2.9204364	2.9204886	2.9205407	2.9205929
833	2.9209056	2.9209577	2.9210098	2.9210619	2.9211140
834	2.9214263	2.9214784	2.9215304	2.9215824	2.9216345
835	2.9219465	2.9219984	2.9220504	2.9221024	2.9221543
836	2.9224659	2.9225179	2.9225698	2.9226217	2.9226736
837	2.9229848	2.9230367	2.9230885	2.9231404	2.9231922
838	2.9235031	2.9235549	2.9236066	2.9236584	2.9237102
839	2.9240207	2.9240724	2.9241242	2.9241759	2.9242276
840	2.9245377	2.9245894	2.9246410	2.9246927	2.9247444
841	2.9250541	2.9251057	2.9251573	2.9252089	2.9252605
842	2.9255699	2.9256215	2.9256730	2.9257245	2.9257761
843	2.9260851	2.9261366	2.9261880	2.9262395	2.9262910
844	2.9265997	2.9266511	2.9267025	2.9267539	2.9268053
845	2.9271136	2.9271650	2.9272163	2.9272677	2.9273190
846	2.9276270	2.9276783	2.9277296	2.9277808	2.9278321
847	2.9281397	2.9281909	2.9282422	2.9282934	2.9283446
848	2.9286518	2.9287030	2.9287542	2.9288054	2.9288565
849	2.9291634	2.9292145	2.9292656	2.9293167	2.9293678

A TABLE of Logarithms.

Num	0	1	2	3	4
850	2.9294189	2.9294700	2.9295211	2.9295722	2.9296233
851	2.9299296	2.9299806	2.9300316	2.9300826	2.9301336
852	2.9304396	2.9304906	2.9305415	2.9305925	2.9306434
853	2.9309490	2.9309999	2.9310508	2.9311017	2.9311526
854	2.9314579	2.9315087	2.9315596	2.9316104	2.9316612
855	2.9319661	2.9320169	2.9320677	2.9321185	2.9321692
856	2.9324738	2.9325245	2.9325752	2.9326259	2.9326767
857	2.9329808	2.9330315	2.9330822	2.9331328	2.9331835
858	2.9334873	2.9335379	2.9335885	2.9336391	.9336897
859	2.9339932	2.9340437	2.9340943	2.9341448	2.9341953
860	2.9344985	2.9345489	2.9345994	2.9346499	2.9347004
861	2.9350032	2.9350536	2.9351040	2.9351544	2.9352049
862	2.9355073	2.9355576	2.9356080	2.9356584	2.9357087
863	2.9360108	2.9360611	2.9361114	2.9361617	2.9362120
864	2.9365137	2.9365640	2.9366143	2.9366645	2.9367145
865	2.9370161	2.9370663	2.9371165	2.9371667	2.9372169
866	2.9375179	2.9375680	2.9376182	2.9376683	2.9377184
867	2.9380191	2.9380692	2.9381193	2.9381693	2.9382194
868	2.9385197	2.9385698	2.9386198	2.9386698	2.9387193
869	2.9390198	2.9390697	2.9391197	2.9391697	2.9392196
870	2.9395193	2.9395692	2.9396191	2.9396690	2.9397189
871	2.9400182	2.9400680	2.9401179	2.9401677	2.9402176
872	2.9405165	2.9405663	2.9406161	2.9406659	2.9407157
873	2.9410142	2.9410640	2.9411137	2.9411635	2.9412132
874	2.9415114	2.9415611	2.9416108	2.9416605	2.9417101
875	2.9420081	2.9420577	2.9421073	2.9421569	2.9422065
876	2.9425041	2.9425537	2.9426032	2.9426528	2.9427024
877	2.9429996	2.9430491	2.9430986	2.9431481	2.9431976
878	2.9434945	2.9435440	2.9435934	2.9436429	2.9436923
879	2.9439889	2.9440383	2.9440877	2.9441371	2.9441865
880	2.9444827	2.9445320	2.9445814	2.9446307	2.9446800
881	2.9449759	2.9450252	2.9450745	2.9451238	2.9451730
882	2.9454686	2.9455178	2.9455671	2.9456163	2.9456655
883	2.9459607	2.9460099	2.9460591	2.9461082	2.9461574
884	2.9464523	2.9465014	2.9465505	2.9465996	2.9466487
885	2.9469433	2.9469923	2.9470414	2.9470905	2.9471395
886	2.9474337	2.9474827	2.9475317	2.9475807	2.9476297
887	2.9479236	2.9479726	2.9480215	2.9480705	2.9481194
888	2.9484130	2.9484619	2.9485108	2.9485597	2.9486085
889	2.9489018	2.9489506	2.9489995	2.9490483	2.9490971
890	2.9493900	2.9494388	2.9494876	2.9495364	2.9495852
891	2.9498777	2.9499264	2.9499752	2.9500239	2.9500726
892	2.9503649	2.9504135	2.9504622	2.9505109	2.9505596
893	2.9508515	2.9509001	2.9509487	2.9509973	2.9510459
894	2.9513375	2.9513861	2.9514347	2.9514832	2.9515318
895	2.9518230	2.9518716	2.9519201	2.9519686	2.9520171
896	2.9523080	2.9523565	2.9524049	2.9524534	2.9525018
897	2.9527924	2.9528409	2.9528893	2.9529377	2.9529861
898	2.9532763	2.9533247	2.9533731	2.9534214	2.9534697
899	2.9537597	2.9538080	2.9538563	2.9539046	2.9539529

Nm.	5	6	7	8	9
850	2.9296743	2.9297254	2.9297764	2.9298275	2.9298785
851	2.9301847	2.9302357	2.9302866	2.9303376	2.9303886
852	2.9306944	2.9307453	2.9307963	2.9308472	2.9308981
853	2.9312035	2.9312544	2.9313053	2.9313562	2.9314070
854	2.9317121	2.9317629	2.9318137	2.9318645	2.9319153
855	2.9322200	2.9322708	2.9323215	2.9323723	2.9324230
856	2.9327274	2.9327781	2.9328288	2.9328795	2.9329301
857	2.9332341	2.9332848	2.9333354	2.9333860	2.9334367
858	2.9337403	2.9337909	2.9338415	2.9338920	2.9339426
859	2.9342459	2.9342964	2.9343469	2.9343974	2.9344479
860	2.9347509	2.9348013	2.9348518	2.9349023	2.9349527
861	2.9352553	2.9353057	2.9353561	2.9354065	2.9354569
862	2.9357591	2.9358095	2.9358598	2.9359101	2.9359605
863	2.9362623	2.9363126	2.9363629	2.9364132	2.9364635
864	2.9367650	2.9368152	2.9368655	2.9369157	2.9369659
865	2.9372671	2.9373172	2.9373674	2.9374176	2.9374677
866	2.9377686	2.9378187	2.9378688	2.9379189	2.9379690
867	2.9382695	2.9383195	2.9383696	2.9384196	2.9384697
868	2.9387698	2.9388198	2.9388698	2.9389198	2.9389698
869	2.9392696	2.9393195	2.9393695	2.9394194	2.9394693
870	2.9397688	2.9398187	2.9398685	2.9399184	2.9395683
871	2.9402674	2.9403172	2.9403670	2.9404169	2.9404667
872	2.9407654	2.9408152	2.9408650	2.9409147	2.9409645
873	2.9412629	2.9413126	2.9413623	2.9414120	2.9414617
874	2.9417598	2.9418095	2.9418591	2.9419088	2.9419584
875	2.9422562	2.9423058	2.9423553	2.9424049	2.9424545
876	2.9427519	2.9428015	2.9428510	2.9429005	2.9429501
877	2.9432471	2.9432966	2.9433461	2.9433956	2.9434450
878	2.9437418	2.9437912	2.9438406	2.9438900	2.9439395
879	2.9442358	2.9442852	2.9443346	2.9443840	2.9444333
880	2.9447294	2.9447787	2.9448280	2.9448773	2.9449266
881	2.9452223	2.9452716	2.9453208	2.9453701	2.9454193
882	2.9457147	2.9457639	2.9458131	2.9458623	2.9459115
883	2.9462066	2.9462557	2.9463049	2.9463540	2.9464031
884	2.9466978	2.9467469	2.9467960	2.9468451	2.9468942
885	2.9471886	2.9472376	2.9472866	2.9473357	2.9473847
886	2.9476787	2.9477277	2.9477767	2.9478257	2.9478747
887	2.9481684	2.9482173	2.9482662	2.9483151	2.9483641
888	2.9486574	2.9487063	2.9487552	2.9488040	2.9488529
889	2.9491460	2.9491948	2.9492436	2.9492924	2.9493412
890	2.9496339	2.9496827	2.9497315	2.9497802	2.9498290
891	2.9501213	2.9501701	2.9502188	2.9502675	2.9503162
892	2.9506082	2.9506569	2.9507055	2.9507542	2.9508028
893	2.9510946	2.9511432	2.9511918	2.9512404	2.9512889
894	2.9515803	2.9516289	2.9516774	2.9517260	2.9517745
895	2.9520656	2.9521141	2.9521626	2.9522111	2.9522595
896	2.9525503	2.9525987	2.9526472	2.9526956	2.9527440
897	2.9530345	2.9530828	2.9531312	2.9531796	2.9532280
898	2.9535181	2.9535664	2.9536147	2.9536631	2.9537114
899	2.9540012	2.9540494	2.9540977	2.9541460	2.9541942

A TABLE of Logarithms.

Num	0	1	2	3	4
900	2.9542425	2.9542908	2.9543390	2.9543873	2.9544355
901	2.9547248	2.9547730	2.9548212	2.9548694	2.9549176
902	2.9552065	2.9552547	2.9553028	2.9553510	2.9553991
903	2.9556878	2.9557358	2.9557839	2.9558320	2.9558801
904	2.9561684	2.9562165	2.9562645	2.9563125	2.9563605
905	2.9566486	2.9566966	2.9567445	2.9567925	2.9568405
906	2.9471282	2.9571761	2.9572241	2.9572720	2.9573199
907	2.9576073	2.9576552	2.9577030	2.9577509	2.9577988
908	2.9580858	2.9581337	2.9581815	2.9582293	2.9582771
909	2.9585639	2.9586117	2.9586594	2.9587072	2.9587549
910	2.9590414	2.9590891	2.9591368	2.9591845	2.9592322
911	2.9595184	2.9595660	2.9596137	2.9596614	2.9597090
912	2.9599948	2.9600425	2.9600901	2.9601377	2.9601853
913	2.9604708	2.9605183	2.9605659	2.9606135	2.9606610
914	2.9609462	2.9609937	2.9610412	2.9610887	2.9611362
915	2.9614211	2.9614686	2.9615160	2.9615635	2.9616109
916	2.9618955	2.9619429	2.9619903	2.9620377	2.9520851
917	2.9623693	2.9624167	2.9624640	2.9625114	2.9625587
918	2.9628427	2.9628900	2.9629373	2.9629846	2.9630319
919	2.9633155	2.9633628	2.9634100	2.9634573	2.9635045
920	2.9637878	2.9638350	2.9638822	2.9639294	2.9639766
921	2.9642596	2.9643068	2.9643539	2.9644011	2.9644482
922	2.9647309	2.9647780	2.9648251	2.9548722	2.9649193
923	2.9652017	2.9652488	2.9652958	2.9653428	2.9653899
924	2.9656720	2.9657190	2.9657660	2.9658130	2.9658599
925	2.9661417	2.9661887	2.9662356	2.9662826	2.9663295
926	2.9666110	2.9666579	2.9667048	2.9667517	2.9667985
927	2.9670797	2.9671266	2.9671734	2.9672203	2.9672671
928	2.9675480	2.9675948	2.9676416	2.9676884	2.9677351
929	2.9680157	2.9680625	2.9681092	2.9681559	2.9682027
930	2.9684829	2.9685296	2.9685763	2.9686230	2.9686697
931	2.9689497	2.9689963	2.9690430	2.9690896	2.9691362
932	2.9694159	2.9694625	2.9695091	2.9695557	2.9696023
933	2.9698816	2.9699282	2.9699747	2.9700213	2.9700678
934	2.9703469	2.9703934	2.9704399	2.9704863	2.9705328
935	2.9708116	2.9708581	2.9709045	2.9709509	2.9709974
936	2.9712758	2.9713222	2.9713686	2.9714150	2.9714614
937	2.9717396	2.9717859	2.9718323	2.9718786	2.9719249
938	2.9722028	2.9722491	2.9722954	2.9723417	2.9723880
939	2.9726656	2.9727118	2.9727581	2.9728043	2.9728506
940	2.9731279	2.9731741	2.9732202	2.9732664	2.9733126
941	2.9735896	2.9736358	2.9736819	2.9737281	2.9737742
942	2.9740509	2.9740970	2.9741431	2.9741892	2.9742353
943	2.9745117	2.9745577	2.9746038	2.9746498	2.9746959
944	2.9749720	2.9750180	2.9750640	2.9751100	2.9751560
945	2.9754318	2.9754778	2.9755237	2.9755697	2.9756156
946	2.9758911	2.9759370	2.9759829	2.9760288	2.9760747
947	2.9763500	2.9763958	2.9764417	2.9764875	2.9765334
948	2.9768083	2.9768541	2.9769000	2.9769458	2.9769915
949	2.9772662	2.9773120	2.9773577	2.9774035	2.9774492

Nm.	5	6	7	8	9
900	2.9544837	2.9545319	2.9545802	2.9546284	2.9546766
901	2.9549657	2.9550139	2.9550621	2.9551102	2.9551584
902	2.9554472	2.9554953	2.9555434	2.9555916	2.9556397
903	2.9559282	2.9559762	2.9560243	2.9560723	2.9561204
904	2.9564086	2.9564566	2.9565046	2.9565526	2.9566006
905	2.9568885	2.9569364	2.9569844	2.9570323	2.9570803
906	2.9573678	2.9574157	2.9574636	2.9575115	2.9575594
907	2.9578466	2.9578945	2.9579423	2.9579902	2.9580380
908	2.9583249	2.9583727	2.9584205	2.9584683	2.9585161
909	2.9588027	2.9588505	2.9588932	2.9589459	2.9589937
910	2.9592800	2.9593276	2.9593755	2.9594230	2.9594707
911	2.9597567	2.9598043	2.9598520	2.9598996	2.9599472
912	2.9602329	2.9602805	2.9603281	2.9603756	2.9604232
913	2.9607086	2.9607561	2.9608036	2.9608512	2.9608987
914	2.9611837	2.9612312	2.9512787	2.9613262	2.9613736
915	2.9616583	2.9617058	2.9617532	2.9618006	2.9618481
916	2.9621325	2.9621799	2.9622272	2.9622746	2.9623220
917	2.9626061	2.9626534	2.9627007	2.9627481	2.9627954
918	2.9630792	2.9631264	2.9631737	2.9632210	2.9632583
919	2.9635517	2.9635990	2.9636462	2.9636934	2.9637406
920	2.9640238	2.9640710	2.9641181	2.9641653	2.9642125
921	2.9644953	2.9645425	2.9645896	2.9646367	2.9646838
922	2.9549664	2.9650135	2.9650605	2.9651076	2.9651547
923	2.9654369	2.9654839	2.9655309	2.9655780	2.9656250
924	2.9659069	2.9659539	2.9660009	2.9660478	2.9660948
925	2.9663764	2.9664233	2.9564703	2.9665172	2.9665641
926	2.9668454	2.9668923	2.9669392	2.9669860	2.9670329
927	2.9673139	2.9673607	2.9674076	2.9674544	2.9675012
928	2.9677819	2.9678287	2.9678754	2.9679222	2.9679690
929	2.9682494	2.9682961	2.9683428	2.9683895	2.9684362
930	2.9687164	2.9687630	2.9688097	2.9688564	2.9689030
931	2.9691829	2.9692295	2.9692761	2.9693227	2.9693693
932	2.9696488	2.9696954	2.9697420	2.9697885	2.9698351
933	2.9701143	2.9701608	2.9702074	2.9702539	2.9703004
934	2.9705793	2.9706258	2.9706722	2.9707187	2.9707652
935	2.9710438	2.9710902	2.9711366	2.9711830	2.9712294
936	2.9715078	2.9715542	2.9716005	2.9716469	2.9716932
937	2.9719713	2.9720176	2.9720639	2.9721102	2.9721565
938	2.9724343	2.9724805	2.9725268	2.9725731	2.9726193
939	2.9728968	2.9729430	2.9729892	2.9730354	2.9730816
940	2.9733588	2.9734050	2.9734511	2.9734973	2.9735435
941	2.9738203	2.9738664	2.9739126	2.9739587	2.9740048
942	2.9742814	2.9743274	2.9743735	2.9744196	2.9744656
943	2.9747419	2.9747879	2.9748340	2.9748800	2.9749260
944	2.9752020	2.9752479	2.9752939	2.9753399	2.9753858
945	2.9756615	2.9757075	2.9757534	2.9757993	2.9758452
946	2.9761206	2.9761665	2.9762124	2.9762582	2.9763041
947	2.9765792	2.9766251	2.9766709	2.9767167	2.9767625
948	2.9770373	2.9770831	2.9771289	2.9771747	2.9772204
949	2.9774950	2.9775407	2.9775864	2.9776322	2.9776779

A TABLE of Logarithms.

Num	0	1	2	3	4
950	2.9777236	2.9777693	2.9778150	2.9778607	2.9779064
951	2.9781805	2.9782262	2.9782718	2.9783175	2.9783631
952	2.9786369	2.9786826	2.9787282	2.9787738	2.9788194
953	2.9790929	2.9791385	2.9791840	2.9792296	2.9792751
954	2.9795484	2.9795939	2.9796394	2.9796849	2.9797304
955	2.9800034	2.9800488	2.9800943	2.9801398	2.9801852
956	2.9804579	2.9805033	2.9805487	2.9805942	2.9806396
957	2.9809119	2.9809573	2.9810027	2.9810481	2.9810934
958	2.9813655	2.9814108	2.9814562	2.9815015	2.9815468
959	2.9818186	2.9818639	2.9819092	2.9819544	2.9819997
960	2.9822712	2.9823165	2.9823617	2.9824069	2.9824522
961	2.9827234	2.9827686	2.9828138	2.9828589	2.9829041
962	2.9831751	2.9832202	2.9832654	2.9833105	2.9833556
963	2.9836263	2.9836714	2.9837165	2.9837616	2.9838066
964	2.9840770	2.9841221	2.9841671	2.9842122	2.9842572
965	2.9845273	2.9845723	2.9846173	2.9846623	2.9847073
966	2.9849771	2.9850221	2.9850670	2.9851120	2.9851569
967	2.9854265	2.9854714	2.9855163	2.9855612	2.9856061
968	2.9858754	2.9859202	2.9859651	2.9860099	2.9860548
969	2.9863238	2.9863686	2.9864134	2.9864582	2.9865030
970	2.9867717	2.9868165	2.9868613	2.9869060	2.9869508
971	2.9872192	2.9872640	2.9873087	2.9873534	2.9873981
972	2.9876663	2.9877109	2.9877556	2.9878003	2.9878450
973	2.9881128	2.9881575	2.9882021	2.9882467	2.9882913
974	2.9885590	2.9886035	2.9886481	2.9886927	2.9887373
975	2.9890046	2.9890492	2.9890937	2.9891382	2.9891828
976	2.9894498	2.9894943	2.9895388	2.9895833	2.9896278
977	2.9898946	2.9899390	2.9899835	2.9900279	2.9900723
978	2.9903389	2.9903833	2.9904277	2.9904721	2.9905164
979	2.9907829	2.9908271	2.9908714	2.9909158	2.9909601
980	2.9912261	2.9912704	2.9913147	2.9913590	2.9914033
981	2.9916690	2.9917133	2.9917575	2.9918018	2.9918461
982	2.9921115	2.9921557	2.9921999	2.9922441	2.9922884
983	2.9925535	2.9925977	2.9926419	2.9926860	2.9927302
984	2.9929951	2.9930392	2.9930834	2.9931275	2.9931716
985	2.9934362	2.9934803	2.9935244	2.9935685	2.9936126
986	2.9938769	2.9939210	2.9939650	2.9940090	2.9940531
987	2.9943171	2.9943611	2.9944051	2.9944491	2.9944931
988	2.9947569	2.9948009	2.9948448	2.9948888	2.9949327
989	2.9951963	2.9952402	2.9952841	2.9953280	2.9953719
990	2.9956352	2.9956791	2.9957229	2.9957668	2.9958106
991	2.9960736	2.9961175	2.9961613	2.9962051	2.9962489
992	2.9965117	2.9965554	2.9965992	2.9966430	2.9966868
993	2.9969492	2.9969930	2.9970367	2.9970804	2.9971242
994	2.9973864	2.9974301	2.9974738	2.9975174	2.9975611
995	2.9978231	2.9978667	2.9979104	2.9979540	2.9979976
996	2.9982593	2.9983029	2.9983465	2.9983901	2.9984337
997	2.9986952	2.9987387	2.9987823	2.9988258	2.9988694
998	2.9991305	2.9991740	2.9992176	2.9992611	2.9993046
999	2.9995655	2.9996090	2.9996524	2.9996959	2.9997393

Num.	5	6	7	8	9
950	2.9779521	2.9779978	2.9780435	2.9780892	2.9781348
951	2.9784088	2.9784544	2.9785001	2.9785457	2.9785913
952	2.9788650	2.9789106	2.9789562	2.9790017	2.9790473
953	2.9793207	2.9793662	2.9794118	2.9794573	2.9795028
954	2.9797759	2.9798214	2.9798669	2.9799124	2.9799579
955	2.9802307	2.9802761	2.9803216	2.9803670	2.9804125
956	2.9806850	2.9807304	2.9807758	2.9808212	2.9808666
957	2.9811388	2.9811841	2.9812295	2.9812748	2.9813202
958	2.9815921	2.9816374	2.9816827	2.9817280	2.9817733
959	2.9820450	2.9820902	2.9821355	2.9821807	2.9822260
960	2.9824974	2.9825426	2.9825878	2.9826330	2.9826782
961	2.9829493	2.9829945	2.9830396	2.9830848	2.9831299
962	2.9834007	2.9834459	2.9834910	2.9835361	2.9835812
963	2.9838517	2.9838968	2.9839419	2.9839869	2.9840320
964	2.9843022	2.9843473	2.9843923	2.9844373	2.9844823
965	2.9847523	2.9847973	2.9848422	2.9848872	2.9849322
966	2.9852019	2.9852468	2.9852917	2.9853366	2.9853816
967	2.9856510	2.9856959	2.9857407	2.9857856	2.9858305
968	2.9860996	2.9861445	2.9861893	2.9862341	2.9862790
969	2.9865478	2.9865926	2.9866374	2.9866822	2.9867270
970	2.9869955	2.9870403	2.9870850	2.9871298	2.9871745
971	2.9874428	2.9874875	2.9875322	2.9875769	2.9876216
972	2.9878896	2.9879343	2.9879789	2.9880236	2.9880682
973	2.9883360	2.9883806	2.9884252	2.9884698	2.9885144
974	2.9887818	2.9888264	2.9888710	2.9889155	2.9889601
975	2.9892273	2.9892718	2.9893163	2.9893608	2.9894053
976	2.9896722	2.9897167	2.9897612	2.9898057	2.9898501
977	2.9901168	2.9901612	2.9902056	2.9902500	2.9902944
978	2.9905608	2.9906052	2.9906496	2.9906940	2.9907383
979	2.9910044	2.9910488	2.9910931	2.9911374	2.9911818
980	2.9914476	2.9914919	2.9915362	2.9915805	2.9916247
981	2.9918903	2.9919345	2.9919788	2.9920230	2.9920673
982	2.9923326	2.9923768	2.9924210	2.9924651	2.9925093
983	2.9927744	2.9928185	2.9928627	2.9929068	2.9929510
984	2.9932157	2.9932598	2.9933039	2.9933480	2.9933921
985	2.9936566	2.9937007	2.9937448	2.9937888	2.9938329
986	2.9940971	2.9941411	2.9941851	2.9942291	2.9942731
987	2.9945371	2.9945811	2.9946251	2.9946690	2.9947130
988	2.9949767	2.9950206	2.9950645	2.9951085	2.9951524
989	2.9954158	2.9954597	2.9955036	2.9955474	2.9955913
990	2.9958545	2.9958983	2.9959422	2.9959860	2.9960298
991	2.9962927	2.9963365	2.9963803	2.9964241	2.9964679
992	2.9967305	2.9967743	2.9968180	2.9968618	2.9969055
993	2.9971679	2.9972116	2.9972553	2.9972990	2.0973427
994	2.9976048	2.9976485	2.9976921	2.9977358	2.9977794
995	2.9980413	2.9980849	2.9981285	2.9981721	2.9982157
996	2.9984773	2.9985209	2.9985645	2.9986080	2.9986516
997	2.9989129	2.9989564	2.9990000	2.9990435	2.9990870
998	2.9993481	2.9993916	2.9994350	2.9994785	2.9995220
999	2.9997828	2.9998262	2.9998697	2.9999131	2.9999566

A Table of Artificial Sines,

0 Degrees.

M	Sine		Tang.		Secant.		
0	0.0000000	10.0000000	0.0000000	Infinite.	10.0000000	Infinite.	60
1	6.4637261	9.9999999	6.4637261	13.5362739	10.0000000	13.5362739	59
2	6.7647561	9.9999999	6.7647562	13.2352438	10.0000001	13.2352439	58
3	6.9408473	9.9999998	6.9408475	13.0591525	10.0000002	13.0591527	57
4	7.0657860	9.9999997	7.0657863	12.9342137	10.0000003	12.9342140	56
5	7.1626960	9.9999995	7.1626964	12.8373036	10.0000005	12.8373040	55
6	7.2418771	9.9999993	7.2418778	12.7581222	10.0000007	12.7581229	54
7	7.3088239	9.9999991	7.3088248	12.6911752	10.0000009	12.6911761	53
8	7.3668157	9.9999988	7.3668169	12.6331831	10.0000012	12.6331843	52
9	7.4179681	9.9999985	7.4179696	12.5820304	10.0000015	12.5820319	51
10	7.4637255	9.9999982	7.4637273	12.5362727	10.0000018	12.5362745	50
11	7.5051181	9.9999978	7.5051203	12.4948797	10.0000022	12.4948819	49
12	7.5429065	9.9999974	7.5429091	12.4570909	10.0000026	12.4570935	48
13	7.5776684	9.9999969	7.5776715	12.4223285	10.0000031	12.4223316	47
14	7.6098530	9.9999964	7.6098566	12.3901434	10.0000036	12.3901470	46
15	7.639816	9.9999959	7.6398201	12.3601799	10.0000041	12.3601840	45
16	7.6678445	9.9999953	7.6678492	12.3321508	10.0000047	12.3321555	44
17	7.6941733	9.9999947	7.6941786	12,3058214	10.0000053	12.3058267	43
18	7.7189966	9.9999940	7.7190026	12.2809974	10.0000060	12.2810034	42
19	7.7424775	9.9999934	7.742484	12.2575159	10.0000066	12.2575225	41
20	7.7647537	9.9999927	7.7647610	12.2352390	10.0000073	12.2352463	40
21	7.785942	9.9999919	7.7859508	12.2140492	10.0000081	12.2140573	39
22	7.8061458	9.9999911	7.8061547	12.1938453	10.0000089	12.1938542	38
23	7.8254507	9.9999903	7.8254604	12.1745396	10.0000097	12.1745493	37
24	7.8439338	9.9999894	7.8439444	12.1560556	10.0000106	12.1560662	36
25	7.8616623	9.9999885	7,8616738	12.1383262	10.0000115	12.1383377	35
26	7.8786953	9.9999876	7.8787077	12.1212923	10.0000124	12.1213047	34
27	7.8950854	9.9999866	7.8950988	12.1049012	10.0000134	12.1049146	33
28	7.9108793	9.9999856	7.9108938	12.0891062	10.0000144	12.0891207	32
29	7.9261190	9.9999845	7.9261344	12.0738656	10.0000155	12.0738810	31
30	7.9408419	9.9999835	7.9408584	12.0591416	10.0000165	12.0591581	30
		Sine.		Tang.		Secant.	

89 Degrees.

0 Degrees.

M	Sine.		Tang.		Secant.		
30	7.9408419	9.9999835	7.9408584	12.0591416	10.0000165	12.0591581	30
31	7.9550819	9.9999823	7.9550996	12.0449064	10.0000177	12.0449181	29
32	7.9688698	9.9999812	7.9688886	12.0311114	10.0000188	12.0311302	28
33	7.9822334	9.9999800	7.9822534	12.0177466	10.0000200	12.0177666	27
34	7.9951980	9.9999788	7.9952192	12.0047808	10.0000212	12.0048020	26
35	8.0077867	9.9999775	8.0078092	11.9921908	10.0000225	11.9922133	25
36	8.0200207	9.9999762	8.0200445	11.9799555	10.0000238	11.9799793	24
37	8.0319195	9.9999748	8.0319446	11.9680554	10.0000252	11.9680805	23
38	8.0435009	9.9999735	8.0435274	11.9564726	10.0000265	11.9564991	22
39	8.0547814	9.9999721	8.0548094	11.9451906	10.0000275	11.9452186	21
40	8.0657763	9.9999706	8.0658057	11.9341943	10.0000294	11.9342237	20
41	8.0764997	9.9999691	8.0765306	11.9234694	10.0000309	11.9235003	19
42	8.0869646	9.9999676	8.0869970	11.9130030	10.0000324	11.9130354	18
43	8.0971832	9.9999660	8.0972172	11.9027828	10.0000340	11.9028168	17
44	8.1071669	9.9999644	8.1072025	11.8927975	10.0000356	11.8928331	16
45	8.1169262	9.9999628	8.1169634	11.8830366	10.0000372	11.8830738	15
46	8.1264710	9.9999611	8.1265099	11.8734901	10.0000389	11.8735290	14
47	8.1358104	9.9999594	8.1358510	11.8641490	10.0000406	11.8641896	13
48	8.1449532	9.9999577	8.1449956	11.8550044	10.0000423	11.8550468	12
49	8.1539075	9.9999559	8.1539516	11.8460484	10.0000441	11.8460925	11
50	8.1626808	9.9999541	8.1627267	11.8372733	10.0000459	11.8373192	10
51	8.1712804	9.9999522	8.1713282	11.8286718	10.0000478	11.8287196	9
52	8.1797129	9.9999503	8.17976[illegible]6	11.8202374	10.0000497	11.8202871	8
53	8.1879848	9.9999484	8.1880364	11.8119636	10.0000516	11.8120152	7
54	8.1961020	9.9999464	8.1961556	11.8038444	10.0000536	11.8038980	6
55	8.2040703	9.9999444	8.2041259	11.7958741	10.0000556	11.7959297	5
56	8.2118949	9.9999424	8.2119526	11.7880474	10.0000576	11.7881051	4
57	8.2195811	9.9999403	8.2196408	11.7803592	10.0000597	11.7804189	3
58	8.2271335	9.9999382	8.2271953	11.7728047	10.0000618	11.7728665	2
59	8.2345568	9.9999360	8.2346208	11.7653792	10.0000640	11.7654432	1
60	8.2418553	9.9999338	8.2419215	11.7580785	10.0000662	11.7581447	0
		Sine.		Tang.		Secant.	M

89 Degrees.

1 *Degree.*

M	Sine.		Tang.		Secant.		
0	8.2418553	9.9999338	8.2419215	11.7580785	10.0000066	11.7581447	60
1	8.2490332	9.9999316	8.2491015	11.7508985	10.0000684	11.7509668	59
2	8.2560942	9.9999294	8.2561649	11.7438351	10.0000706	11.7439057	58
3	8.2630424	9.9999271	8.2631153	11.7368847	10.0000729	11.7369576	57
4	8.2698810	9.9999247	8.2699563	11.7300437	10.0000753	11.7301190	56
5	8.2766136	9.9999224	8.2766912	11.7233088	10.0000776	11.7233864	55
6	8.2832434	9.9999200	8.2833234	11.7166766	10.0000800	11.7107566	54
7	8.2897734	9.9999175	8.2898559	11.7101441	10.0000825	11.7102266	53
8	8.2962067	9.9999150	8.2962917	11.7037083	10.0000850	11.7037933	52
9	8.3025460	9.9999125	8.3026335	11.6973665	10.0000875	11.6974540	51
10	8.3087941	9.9999100	8.3088842	11.6911158	10.0000900	11.6912059	50
11	8.3149536	9.9999074	8.3150462	11.6849538	10.0000926	11.6850464	49
12	8.3210269	9.9999047	8.3211221	11.6788779	10.0000953	11.6789731	48
13	8.3270163	9.9999021	8.3271143	11.6728857	10.0000979	11.6729837	47
14	8.3329243	9.9998994	8.3330249	11.6669751	10.0001006	11.6670757	46
15	8.3387529	9.9998966	8.3388563	11.6611437	10.0001034	11.6612471	45
16	8.3445043	9.9998939	8.3446105	11.6553895	10.0001061	10.6554957	44
17	8.3501805	9.9998911	8.3502895	11.6497105	10.0001089	10.6498195	43
18	8.3557835	9.9998882	8.3558953	11.6441047	10.0001118	10.6442165	42
19	8.3613150	9.9998853	8.3614297	11.6385703	10.0001147	10.6386850	41
20	8.3667765	9.9998824	8.3668945	11.6331055	10.0001176	10.6332235	40
21	8.3721710	9.9998794	8.3722915	11.6277085	10.0001206	11.6278290	39
22	8.3774988	9.9998764	8.3776223	11.6223777	10.0001236	11.6225012	38
23	8.3827620	9.9998734	8.3828886	11.6171114	10.0001266	11.6172380	37
24	8.3879622	9.9998703	8.3880918	11.6119082	10.0001297	11.6120378	36
25	8.3931008	9.9998672	8.3932336	11.6067664	10.0001328	11.6068992	35
26	8.3981793	9.9998341	8.3983152	11.6016848	10.0001359	11.6018207	34
27	8.4031990	9.9998609	8.4033381	11.5966619	10.0001391	11.5968010	33
28	8.4081614	9.9998577	8.4083037	11.5916963	10.0001423	11.5918386	32
29	8.4130676	9.9998544	8.4132132	11.5867868	10.0001456	11.5869324	31
30	8.4179190	9.9998512	8.4180679	11.5819321	10.0001488	11.5820810	30
		Sine.		Tang.		Secant.	M

88 *Degrees.*

1 *Degrees.*

M	Sine.		Tang		Secant,		
30	8.4179190	9.9998512	8.4180679	11,5819321	10.0001488	11.5820810	30
31	8.4227168	9.9998478	8.4228690	11,5771310	10.0001522	11.5772832	29
32	8.4274621	9.9998445	8.4276170	11.5723824	10.0001555	11.5725379	28
33	8.4321561	9.9998411	8.4323150	11.5676850	10.0001589	11.5678439	27
34	8.4367999	9.9998376	8.4369622	11.5630378	10.0001624	11.5632001	26
35	8.4413944	9.9998342	8.4415603	11.5584397	10.0001658	11.5586058	25
36	8.4459409	9.9998306	8.4461103	11.5538897	10.0001694	11.5540591	24
37	8.4504402	9.9998271	8.4506131	11.5493869	10.0001729	11.5495598	23
38	8.4548934	9.9998235	8.4550699	11.5449301	10,0001765	11.5451060	22
39	8.4593013	9.9998199	8.4594814	11.5405186	10.0001801	11.5406987	21
40	8.4636649	9.9998162	8.4638486	11.5361514	10.0001838	11.5363351	20
41	8.4679850	9.9998125	8.4681725	11.5318275	10.0001875	11.5320150	19
42	8.4722626	9,9998088	8.4724538	11.5275462	10.0001912	11.5277374	18
43	8.4764984	9.9998050	8.4766933	11.5233067	10.0001950	11.5235016	17
44	8.4806932	9.9998012	8.4808920	11.5191080	10.0001988	11.5193068	16
45	8.4848479	9.9997974	8.4850505	11.5149495	10.0002026	10.5151521	15
46	8.4889632	9,9997935	8.4891696	11.5108304	10.0002065	11.5110368	14
47	8.4930398	9.9997896	8.4932502	11.5067498	10.0002104	11.5069602	13
48	8.4970784	9.9997856	8.4972928	11.5027072	10.0002144	11.5029216	12
49	8.5010798	9.9997817	8.5012982	11.4987018	10.0002183	11.4989202	11
50	8.5050447	9.9997776	8.5052671	11.4947329	10.0002224	11.4949553	10
51	8.5089736	9.9997736	8.5092001	11,4907999	10.0002264	11.4910264	9
52	8.5128673	9.9997695	8.5130978	11,4869022	10.0002305	11.4871327	8
53	8.5167264	9.9997653	8.5169610	11.4830390	10.0002347	11.4832736	7
54	8.5205514	9.9997612	8.5207902	11.4792098	10.0002388	11.4794486	6
55	8,5243430	9.9997570	8.5245860	11.4754140	10.0002430	11.4756570	5
56	8,5281017	9.9997527	8.5283490	11.4716510	10.0002473	11.4718983	4
57	8.5318281	9.9997484	8.5320797	11.4679203	10.0002516	11.4681719	3
58	8 5355228	9.0997441	8.5357787	11.4642213	10.0002559	11.4644772	2
59	8.5391863	9.9997398	8.5394466	11.4605534	10.0002602	11.4608137	1
60	8.5428192	9.9997354	8.5430838	11.4569162	10.0002646	11.4571808	0
		Sine.		Tang.		Secant.	M

88 *Degrees.*

A Table of Artificial Sines.

2 Degrees.

M	Sine.	[illegible]	Tang.	[illegible]	Secant.	[illegible]	
0	8.5428192	9.9997354	8.5430838	11.4569162	10.0002646	11.4571808	60
1	8.5464218	9.9997309	8.5466909	11.4533091	10.0002691	11.4535782	59
2	8.5499948	9.9997265	8.5502683	11.4497317	10.0002735	11.4500052	58
3	8.5535380	9.9997220	8.5538160	11.4461834	10.0002780	11.4464614	57
4	8.5570536	9.9997174	8.5573362	11.4426638	10.0002826	11.4429464	56
5	8.5605404	9.9997128	8.5608276	11.4391724	10.0002872	11.4394596	55
6	8.5639994	9.9997082	8.5642912	11.4357088	10.0002918	11.4360006	54
7	8.5674310	9.9997036	8.5677275	11.4322725	10.0002964	11.4325690	53
8	8.5708357	9.9996989	8.5711368	11.4288632	10.0003011	11.4291643	52
9	8.5742139	9.9996942	8.5745197	11.4254803	10.0003058	11.4257861	51
10	8.5775660	9.9996894	8.5778766	11.4221234	10.0003106	11.4224340	50
11	8.5808923	9.9996846	8.5812077	11.4187923	10.0003154	11.4191077	49
12	8.5841933	9.9996798	8.5845136	11.4154864	10.0003202	11.4158067	48
13	8.5874694	9.9996749	8.5877945	11.4122055	10.0003251	11.4125306	47
14	8.5907209	9.9996700	8.5910509	11.4089491	10.0003300	11.4092791	46
15	8.5939483	9.9996651	8.5942832	11.4057168	10.0003350	11.4060517	45
16	8.5971517	9.9996601	8.5974917	11.4025083	10.0003399	11.4028483	44
17	8.6003317	9.9996550	8.6006767	11.3993233	10.0003450	11.3996683	43
18	8.6034886	9.9996500	8.6038386	11.3961614	10.0003500	11.3965114	42
19	8.6066226	9.9996445	8.6069777	11.3930223	10.0003551	11.3933774	41
20	8.6097341	9.9996398	8.6100943	11.3899057	10.0003602	11.3902659	40
21	8.6128235	9.9996346	8.6131889	11.3868111	10.0003654	11.3871765	39
22	8.6158910	9.9996294	8.6162616	11.3837384	10.0003706	11.3841090	38
23	8.6189369	9.9996242	8.6193127	11.3806873	10.0003758	11.3810631	37
24	8.6219616	9.9996189	8.6223427	11.3776573	10.0003811	11.3780384	36
25	8.6249659	9.9996136	8.6253518	11.3746482	10.0003864	11.3750341	35
26	8.6279484	9.9996082	8.6283402	11.3716598	10.0003918	11.3720516	34
27	8.6309111	9.9996028	8.6313083	11.3686917	10.0003972	11.3690889	33
28	8.6338537	9.9995974	8.6342563	11.3657437	10.0004026	11.3661463	32
29	8.6367764	9.9995919	8.6371845	11.3628155	10.0004081	11.3632236	31
30	8.6396796	9.9995865	8.6400931	11.3599069	10.0004135	11.3603204	30
[illegible]	[illegible]	Sine.	[illegible]	Tang.	[illegible]	Secant.	M

87 Degrees.

2 Degrees.

M	Sine.		Tang		Secant		
30	8.6396796	9.9995865	8.6400931	11.3599069	10.0004135	11.3603204	30
31	8.6425634	9.9995809	8.6429825	11.3570175	10.0004191	11.3574366	29
32	8.6454282	9.9995753	8,6458528	11.3541472	10.0004247	11.3545718	28
33	8.6482742	9.9995697	8.6487044	11.3512956	10.0004303	11.3517258	27
34	8.6511016	9,9995641	8.6515375	11.3484625	10.0004359	11.3488984	26
35	8.6539107	9.9995584	8.6543522	11.3456478	10.0004416	11.3460893	25
36	8.6567017	9.9995527	8.6571490	11.3428510	10.0004473	11.3432983	24
37	8.6594748	9.9995469	8.6599279	11.3400721	10.0004531	11.3405252	23
38	8.6622303	9.9995411	8.6626891	11.3373109	10.0004589	11.3377697	22
39	8.6649684	9.9995353	8.6654331	11.3345669	10.0004647	11.3350316	21
40	8.6676893	9.9995295	8,6681598	11.3318402	10.0004705	11.3323107	20
41	8.6703932	9,9995236	8.6708697	11.3291303	10 0004764	11.3296068	19
42	8.6730804	9.9995176	8.6735628	11.3264372	10.0004824	11.3269196	18
43	8.6757510	9.9995116	8.6762393	11.3237607	10.0004884	11.3242490	17
44	8.6784052	9.9995056	8.6788996	11.3211004	10.0004944	11.3215948	16
45	8.6810433	9 9994996	8.6815437	11.3184563	10.0005004	11.3189567	15
46	8.6836654	9.9994935	8 6841719	11.3158281	10.0005065	11.3163346	14
47	8.6862718	9.9994874	8.6867844	11.3132156	10 0005126	11.3137282	13
48	8.6888625	9.9994812	8.6893813	11.3106187	10.0005188	11.3111375	12
49	8.6914379	9.9994750	8.6919629	11.3080371	10.0005250	11.3085621	11
50	8.6939980	9.9994688	8.6945292	11.3054708	10.0005312	11.3060020	10
51	8.6965431	9.9994625	8.6970806	11.3029194	10.0005375	11.3034569	9
52	8.6990734	9.9994562	8.6996172	11.3003828	10.0005438	11.3009266	8
53	8.7015889	9.9994498	8.7021390	11.2978610	10.0005502	11.2984111	7
54	8.7040899	9.9994435	8.7046465	11.2953535	10.0005565	11.2959101	6
55	8.7065766	9.9994370	8.7071395	11.2928605	10.0005630	11.2934234	5
56	8.7090490	9.9994306	8.7096185	11.2903815	10.0005694	11.2909510	4
57	8.7115075	9.9994241	8.7120834	11.2879166	10.0005759	11.2884925	3
58	8.7139520	9.9994176	8.7145345	11.2854655	1.00005824	11.2860480	2
59	8,7163829	9.9994110	8.7169719	11.2830281	10.0005890	11.2836171	1
60	8.7188002	9.9994044	8.7193958	11.2806042	10.0005956	11.2811998	0
		Sine.		Tang.		Secant.	M

87 Degrees.

A Table of Artificial Sines,

3 *Degree.*

M	Sine.		Tang.		Secant		
0	8.7188002	9.9994044	8.7193958	11.2806042	10.0005950	11.2811998	60
1	8.7212040	9.9993978	8.7218063	11.2781937	10.0006022	11.2787960	59
2	8.7235946	9.9993911	8.7242035	11.2757965	10.0006089	11.2764054	58
3	8.7259721	9.9993844	8.7265877	11.2734123	10.0006156	11.2740279	57
4	8.7283366	9.9993776	8.7289589	11.2710411	10.0006224	11.2716634	56
5	8.730688	9.9993708	8.7313174	11.2686826	10.0006292	11.2693118	55
6	8.733027	9.9993640	8.7336631	11.2663369	10.0006360	11.2669728	54
7	8.735353	9.9993572	8.7359964	11.2640036	10.0006428	11.2646465	53
8	8.737667	9.9993503	8.7383172	11.2616828	10.0006497	11.2623325	52
9	8.7399691	9.9993433	8.7406258	11.2593742	10.0006567	11.2600309	51
10	8.7422586	9.9993364	8.7429222	11.2570778	10.0006630	11.2577414	50
11	8.7445360	9.9993293	8.7452067	11.2547933	10.0006707	11.2554640	49
12	8.746801	9.9993223	8.7474792	11.2525208	10.0006777	11.2531985	48
13	8.749055	9.9993152	8.7497400	11.2502600	10.0006848	11.2509447	47
14	8.751297	9.9993081	8.7519892	11.2480108	10.0006919	11.2487027	46
15	8.7535278	9.9993009	8.7542269	11,2457731	10.0006991	11.2464722	45
16	8.7557469	9.9992938	8 7564531	11.2435469	10.0007062	11.2442531	44
17	8.7579546	9.9992865	8.7586681	11.2413319	10.0007135	11.2420454	43
18	8,760151	9.9992793	8.7608719	11.2391281	10.0007207	11.2398488	42
19	8.7623366	9.9992720	8.7630647	11.2369353	10.0007280	11.2376634	41
20	8.7645111	9.9992646	8.7652465	11.2347535	10.0007354	11.2354889	40
21	8.7666747	9.9992572	8.7674175	11.2325825	10.0007428	11.2333253	39
22	8.7688275	9.9992498	8.7695777	11.2304223	10.0007502	11.2311725	38
23	8.7709697	9.9992424	8.7717274	11.2282726	10.0007576	11.2290303	37
24	8.773101	9.9992349	8.7738665	11.2261335	10.0007651	11.2268986	36
25	8.7752226	9.9992274	8.7759952	11.2240048	10.0007726	11.2247774	35
26	8 777333	9.9992198	8.7781136	11.2218864	10.0007802	11.2226666	34
27	8 779434	9.9992122	8.7802218	11.2197782	10.0007878	11.2205660	33
28	8.7815244	9.9992046	8.7823199	11.2176801	10.0007954	11.2184756	32
29	8.7836048	9.9991969	8.7844079	11.2155921	00.0008031	11.2163952	31
30	8.785675	9.9991892	8.7864861	11.2135139	00.0008108	11.2143247	30
		Sine.		Tang.		Secant.	M

86 *Degrees.*

3 *Degrees.*

M	Sine.		Tang.		Secant.		
30	8.7856753	9.9991892	8.7864861	11.2135139	10.0008108	11.2143247	30
31	8.7877359	9.9991815	8.7885544	11.2114456	10.0008185	11.2122641	29
32	8.7897867	9.9991737	8.7906130	11.2093870	10.0008263	11.2102133	28
33	8.7918278	9.9991659	8.7926620	11.2073380	10.0008341	11.2081722	27
34	8.7938594	9.9991580	8.7947014	11.2052986	10.0008420	11.2061406	26
35	8.7958814	9.9991501	8.7967313	11.2032687	10.0008499	11.2041186	25
36	8.7978941	9.9991422	8.7987519	11.2012481	10.0008578	11.2021059	24
37	8.7998974	9.9991342	8.8007632	11.1992368	10.0008658	11,2001026	23
38	8.8018915	9.9991262	8.8027653	11.1972347	10.0008738	11.1961085	22
39	8.8038764	9.9991182	8.8047583	11.1952417	10.0008818	11.1961236	21
40	8.8058523	9.9991101	8.8067422	11.1932578	10.0008899	11.1941477	20
41	8.8078192	9.9991020	8.8087172	11.1912828	10.0008980	11.1921808	19
42	8.8097772	9.9990939	8.8106834	11.1893166	10.0009062	11.1902228	18
43	8.8117264	9.9990856	8.8126407	11.1873593	10.0009144	11.1882736	17
44	8.8136668	9.9990774	8.8145894	11.1854106	10.0009226	11.1863332	16
45	8.8155985	9.9990691	8.8165294	11.1834706	10.0009309	11.1844015	15
46	8.8175217	9.9990608	8.8184608	11.1815392	10.0009392	11.1824783	14
47	8.8194363	9.9990525	8.8203838	11.1796162	10.0009475	11.1805637	13
48	8.8213425	9.9990441	8.8222984	11.1777016	10.0009559	11.1786575	12
49	8.8232404	9.9990357	8.8242046	11.1757954	10.0009643	11.1767596	11
50	8.8251299	9.9990273	8.8261026	11.1738974	10.0009727	11.1748701	10
51	8.8270112	9.9990188	8.8279924	11.1720076	10.0009812	11.1729888	9
52	8.8288844	9.9990103	8.8298741	11.1701259	10.0009897	11.1711156	8
53	8.8307495	9.9990017	8.8317478	11.1682522	10.0009983	11.1692505	7
54	8.8326066	9.9989931	8.8336134	11.1663866	10.0010069	11.1673934	6
55	8.8344557	9.9989845	8,8354712	11.1645288	10.0010155	11.1655443	5
56	8.8362969	9.9989758	8.8373211	11.1626789	10.0010242	11.1637031	4
57	8.8381304	9.9989671	8.8391633	11.1608367	10.0010329	11.1618696	3
58	8.8399561	9.9989584	8.8409977	11.1590023	10.0010416	11.1600439	2
59	8.8417741	9.9989496	8.8428245	11.1571755	10.0010504	11.1582259	1
60	8.8435845	9.9989408	8.8446437	11.1553563	10.0010592	11.1564155	0
		Sine.		Tang.		Secant.	M

86 *Degrees.*

A Table of Artificial Sines,

4 Degrees.

M	Sine.		Tang.		Secant.		
0	8.8435845	9.9989408	8.8446437	11.1553563	10.0010592	11.1564155	60
1	8.8453874	9.9989315	8.8464554	11.1535446	10.0010685	11.1546126	59
2	8.8471827	9.9989230	8.8482597	11.1517403	10.0010770	11.1528173	58
3	8.8489707	9.9989141	8.8500566	11.1499434	10.0010859	11.1510293	57
4	8.8507512	9.9989052	8.8518461	11.1481535	10.0010948	11.1492488	56
5	8.8525245	9.9988952	8.8536283	11.1463717	10.0011038	11.1474755	55
6	8.8542905	9.998887	8.8554034	11.1445966	10.0011129	11.1457095	54
7	8.8560493	9.9988780	8.8571713	11.1428287	10.0011220	11.1439507	53
8	8.8578010	9.9988689	8.8589321	11.1410679	10.0011311	11.1421990	52
9	8.8595457	9.9988596	8.8606859	11.1393141	10.0011402	11.1404543	51
10	8.861283	9.9988506	8.8624327	11.1375673	10.0011494	11.1387167	50
11	8.8630135	9.998841	8.8641725	11.1358275	10.0011586	11.1369861	49
12	8.8647379	9.9988321	8.8659055	11.1340945	10.0011679	11.1352621	48
13	8.8664545	9.9988228	8.8676317	11.1323683	10.0011772	11.1335455	47
14	8.8681646	9.9988135	8.8693511	11.1306489	10.0011865	11.1318354	46
15	8.8698680	9.9988041	8.8710638	11.1289362	10.0011959	11.1301320	45
16	8.8715646	9.9987947	8.8727699	11.1272301	10.0012053	11.1284354	44
17	8.8732946	9.9987853	8.8744694	11.1255306	10.0012147	11.1267454	43
18	8.8749381	9.9987758	8.8761623	11.1238377	10.0012242	11.1250619	42
19	8.8766150	9.9987663	8.8778487	11.1221513	10.0012337	11.1233850	41
20	8.8782854	9.9987567	8.8795286	11.1204714	10.0012433	11.1217146	40
21	8.8799493	9.9987471	8.8812022	11.1187978	10.0012529	11.1200507	39
22	8.8816069	9.9987375	8.8828694	11.1171306	10.0012625	11.1183931	38
23	8.8832581	9.9987278	8.8845303	11.1154697	10.0012722	11.1167419	37
24	8.8849031	9.9987181	8.8861850	11.1138150	10.0012819	11.1150969	36
25	8.8865418	9.9987084	8.8878334	11.1121666	10.0012916	11.1134582	35
26	8.8881743	9.9986986	8.8894757	11.1105243	10.0013014	11.1118257	34
27	8.8898007	9.9986888	8.8911119	11.1088888	10.0013112	11.1101993	33
28	8.8914209	9.9986790	8.8927420	11.1072580	10.0013210	11.1085791	32
29	8.8930351	9.9986691	8.8943660	11.1056340	10.0013309	11.1069649	31
30	8.8946433	9.9986591	8.8959842	11.1040158	10.0013409	11.1053567	30
		Sine.		Tang.		Secant.	M

85 Degrees.

4 Degrees.

M	Sine.		Tang.		Secant.		
30	8.8946433	9.9986591	8.8959842	11.1040158	10.0013409	11.1053567	30
31	8.8962455	9.9986492	8.8975963	11.1024037	10.0013508	11.1037545	29
32	8.8978418	9.9986392	8.8992026	11.1007974	10.0013608	11.1021582	28
33	8.8994322	9.9986292	8.9008030	11.0991970	10.0013708	11.1005678	27
34	8.9010168	9.9986191	8.9023977	11.0976023	10.0013809	11.0989832	26
35	8.9025955	9.9986090	8.9039866	11.0960134	10.0013910	11.0974045	25
36	8.9041685	9.9985988	8.9055697	11.0944303	10.0014012	11.0958315	24
37	8.9057358	9.9985886	8.9071472	11.0928528	10.0014114	11.0942642	23
38	8.9072975	9.9985784	8.9087190	11.0912810	10.0014216	11.0927025	22
39	8.9088535	9.9985682	8.9102853	11.0897147	10.0014318	11.0911465	21
40	8.9104039	9.9985579	8.9118460	11.0881540	10.0014421	11.0895961	20
41	8.9119487	9.9985475	8.9134012	11.0865988	10.0014525	11.0880513	19
42	8.9134881	9.9985372	8.9149509	11.0850491	10.0014628	11.0865119	18
43	8.9150219	9.9985268	8.9164952	11.0835048	10.0014732	11.0849781	17
44	8.9165504	9.9985163	8.9180340	11.0819660	10.0014837	11.0834496	16
45	8.9180734	9.9985058	8.9195675	11.0804325	10.0014942	11.0819266	15
46	8.9195911	9.9984953	8.9210957	11.0789043	10.0015047	11.0804089	14
47	8.9211034	9.9984848	8.9226186	11.0773814	10.0015152	11.0788966	13
48	8.9226105	9.9984742	8.9241363	11.0758637	10.0015258	11.0773895	12
49	8.9241123	9.9984636	8.9256487	11.0743513	10.0015364	11.0758877	11
50	8.9256089	9.9984529	8.9271560	11.0728440	10.0015471	11.0743911	10
51	8.9271003	9.9984422	8.9286581	11.0713419	10.0015578	11.0728997	9
52	8.9285866	9.9984315	8.9301552	11.0698448	10.0015685	11.0714134	8
53	8.9300678	9.9984207	8.9316471	11.0683529	10.0015793	11.0699322	7
54	8.9315439	9.9984099	8.9331340	11.0668660	10.0015901	11.0684561	6
55	8.9330150	9.9983990	8.9346160	11.0653840	10.0016010	11.0669850	5
56	8.9344811	9.9983881	8.9360929	11.0639071	10.0016119	11.0655189	4
57	8.9359422	9.9983772	8.9375650	11.0624350	10.0016228	11.0642078	3
58	8.9373983	9.9983663	8.9390321	11.0609679	10.0016337	11.0626017	2
59	8.9388496	9.9983553	8.9404944	11.0595056	10.0016447	11.0611504	1
60	8.9402960	9.9983442	8.9419518	11.0580482	10.0016558	11.0597040	0
		Sine.		Tang.		Secant.	M

85 Degrees.

5 *Degrees.*

M	Sine.		Tang.		Secant.		
0	8.9402960	9.9983442	8.9419518	11.0580482	10.0016558	11.0597040	60
1	8.9417376	9.9983332	8.9434044	11.0565956	10.0016668	11.0582624	59
2	8.9431743	9.9983220	8.9448523	11.0551477	10.0016780	11.0568257	58
3	8.9446063	9.9983109	8.9462954	11.0537046	10.0016891	11.0553937	57
4	8.9460335	9.9982997	8.9477338	11.0522662	10.0017003	11.0539665	56
5	8.9474561	9.9982885	8.9491676	11.0508324	10.0017115	11.0525439	55
6	8.9488739	9.9982772	8.9505967	11.0494033	10.0017228	11.0511261	54
7	8.9502871	9.9982660	8.9520211	11.0479789	10.0017340	11.0497129	53
8	8.9516957	9.9982546	8.9534410	11.0465590	10.0017454	11.0483043	52
9	8.9530996	9.9982433	8.9548564	11.0451436	10.0017567	11.0469004	51
10	8.9544991	9.9982318	8.9562672	11.0437328	10.0017682	11.0455009	50
11	8.9558940	9.9982204	8.9576735	11.0423265	10.0017796	11.0441060	49
12	8.9572843	9.9982089	8.9590754	11.0409246	10.0017911	11.0427157	48
13	8.9586703	9.9981974	8.9604728	11.0395272	10.0018026	11.0413297	47
14	8.9600517	9.9981859	8.9618655	11.0381341	10.0018141	11.0399483	46
15	8.9614288	9.9981743	8.9632545	11.0367457	10.0018255	11.0385712	45
16	8.9628014	9.9981626	8.9646388	11.0353612	10.0018374	11.0371986	44
17	8.9641697	9.9981510	8.9660188	11.0339812	10.0018490	11.0358303	43
18	8.9655337	9.9981393	8.9673944	11.0326056	10.0018607	11.0344663	42
19	8.9668934	9.9981275	8.9687658	11.0312342	10.0018725	11.0331066	41
20	8.9682487	9.9981158	8.9701330	11.0298670	10.0018842	11.0317513	40
21	8.9695999	9.9981040	8.9714959	11.0285041	10.0018960	11.0304001	39
22	8.9709468	9.9980921	8.9728547	11.0271453	10.0019079	11.0290532	38
23	8,9722895	9.9980802	8.9742092	11.0257908	10.0019198	11.0277105	37
24	8.9736280	9.9980683	8.9755597	11,0244403	10.0019317	11.0263720	36
25	8.9749624	9 9980563	8.9769060	11.0230940	10.0019437	11.0250376	35
26	8.9762926	9.9980443	8.9782483	11.0217517	10.0019557	11.0237074	34
27	8.9776188	9.9980323	8.9795865	11.0204135	10.0019677	11.0223812	33
28	8.9789408	9.9980202	8.9809206	11.0190794	10.0019798	11.0210592	32
29	8.9802589	9.9980081	8.9822507	11.0177493	10.0019919	11.0197411	31
30	8.9815729	9.9979960	8.9835769	11.0164231	10.0020040	11.0184271	30
		Sine.		Tang.		Secant.	M

84 *Degrees.*

5 *Degrees.*

M	Sine.		Tang		Secant,		
30	8.9815729	9.9979960	8.9835765	11.0164231	10.0020040	11.0184271	30
31	8.9828829	9.9979838	8.9848991	11.0151009	10.0020162	11.0171171	29
32	8.9841889	9.9979716	8.9862173	11.0137827	10.0020284	11.0158111	28
33	8.9854910	9.9979593	8.9875317	11.0124683	10.0020407	11.0145090	27
34	8.9867891	9.9979470	8.9888421	11.0111579	10.0020530	11.0132109	26
35	8.9880834	9.9979347	8.9901487	11.0098513	10.0020653	11.0119166	25
36	8.9893737	9.9979223	8.9914514	11.0085486	10.0020777	11.0106263	24
37	8.9906602	9.9979099	8.9927503	11.0072497	10.0020901	11.0093398	23
38	8.9919429	9.9978975	8.9940454	11.0059546	10.0021025	11.0080571	22
39	8.9932217	9.9978850	8.9953367	11.0046633	10.0021150	11.0067783	21
40	8.9944968	9.9978725	8.9966243	11.0033757	10.0021275	11.0055032	20
41	8.9957681	9.9978599	8.9979081	11.0020919	10.0021411	11.0042315	19
42	8.9970356	9.9978473	8.9991883	11.0008117	10.0021527	11.0029644	18
43	8.9982994	9.9978347	9.0004647	10.9995353	10.0021653	11.0017006	17
44	8.9995595	9.9978220	9.0017375	10.9982625	10.0021780	11.0004405	16
45	9.0008160	9.9978093	9.0030066	10.9969934	10.0021907	10.9991840	15
46	9.0020687	9.9977966	9.0042721	10.9957279	10.0022034	10.9979313	14
47	9.0033179	9.9977838	9.0055340	10.9944660	10.0022162	10.9966821	13
48	9.0045634	9.9977710	9.0067924	10.9932076	10.0022290	10.9954366	12
49	9.0058053	9.9977582	9.0080471	10.9919529	10.0022418	10.9941947	11
50	9.0070436	9.9977453	9.0092984	10.9907016	10.0022547	10.9929564	10
51	9.0082784	9.9977323	9.0105461	10.9894539	10.0022677	10.9917216	9
52	9.0095096	9.9977194	9.0117903	10.9882097	10.0022806	10.9904904	8
53	9.0107374	9.9977064	9.0130310	10.9869690	10.0022936	10.9892626	7
54	9.0119616	9.9976933	9.0142682	10.9857318	10.0023067	10.9880384	6
55	9.0131823	9.9976803	9.0155021	10.9844979	10.0023197	10.9868177	5
56	9.0143996	9.9976672	9.0167325	10.9832675	10.0023328	10.9856004	4
57	9.0156135	9.9976540	9.0179594	10.9820406	10.0023460	10.9843865	3
58	9.0168239	9.0976408	9.0191831	10.9808169	10.0023592	10.9831761	2
59	9.0180309	9.9976276	9.0204033	10.9795967	10.0023724	10.9819691	1
60	9.0192346	9.9976143	9.0216202	10.9783798	10.0023857	10.9807654	0
		Sine.		Tang.		Secant.	M

84 *Degrees.*

A Table of Artificial Sines,

6 Degrees.

M	Sine.		Tang.		Secant.		
0	9.0192346	9.9976143	9.0216202	10.9783798	10.0023857	10.9807654	60
1	9.0204348	9.9976011	9.0228338	10.9771662	10.0023989	10.9795652	59
2	9.0216318	9.9975877	9.0240441	10.9759559	10.0024123	10.9783682	58
3	9.0228254	9.9975743	9.0252510	10.9747490	10.0024257	10.9771746	57
4	9.0240157	9.9975609	9.0264548	10.9735452	10.0024391	10.9759843	56
5	9.0252027	9.9975475	9.0276552	10.9723448	10.0024525	10.9747973	55
6	9.0263864	9.9975340	9.0288524	10.9711476	10.0024660	10.9736136	54
7	9.0275669	9.9975205	9.0300464	10.9699536	10.0024795	10.9724331	53
8	9.0287442	9.9975069	9.0312373	10.9687627	10.0024931	10.9712558	52
9	9.0299182	9.9974933	9.0324249	10.9675751	10.0025067	10.9700818	51
10	9.0310890	9.9974797	9.0336093	10.9663907	10.0025203	10.9689110	50
11	9.0322567	9.9974660	9.0347906	10.9652094	10.0025340	10.9677433	49
12	9.0334212	9.9974523	9.0359688	10.9640312	10.0025477	10.9665788	48
13	9.0345825	9.9974386	9.0371439	10.9628561	10.0025614	10.9654175	47
14	9.0357407	9.9974248	9.0383159	10.9616841	10.0025752	10.9642593	46
15	9.0368958	9.9974110	9.0394848	10.9605152	10.0025890	10.9631042	45
16	9.0380477	9.9973971	9.0406506	10.9593494	10.0026029	10.9619523	44
17	9.0391966	9.9973833	9.0418134	10.9581866	10.0026167	10.9608034	43
18	9.0403424	9.9973693	9.0429731	10.9570269	10.0026307	10.9596576	42
19	9.0414852	9.9973554	9.0441299	10.9558701	10.0026446	10.9585148	41
20	9.0426249	9.9973414	9.0352836	10.9547164	10.0026586	10.9573751	40
21	9.0437617	9.9973273	9.0464343	10.9535657	10.0026727	10.9562383	39
22	9.0448954	9.9973132	9.0475821	10.9524179	10.0026868	10.9551046	38
23	9.0460261	9.9972991	9.0487270	10.9512730	10.0027009	10.9539739	37
24	9.0471538	9.9972850	9.0498689	10.9501311	10.0027150	10.9528462	36
25	9.0482786	9.9972708	9.0510078	10.9489922	10.0027292	10.9517214	35
26	9.0494005	9.9972566	9.0521439	10.9478561	10.0027434	10.9505995	34
27	9.0505194	9.9972423	9.0532771	10.9467229	10.0027577	10.9494806	33
28	9.0516354	9.9972280	9.0544074	10.9455926	10.0027720	10.9483646	32
29	9.0527485	9.9972137	9.0555349	10.9444651	10.0027863	10.9472515	31
30	9.0538588	9.9971993	9.0566595	10.9433405	10.0028007	10.9461412	30
		Sine.		Tang.		Secant.	M

83 Degrees.

6 Degrees.

M	Sine.		Tang.		Secant.		
30	9.0538588	9.9971993	90.566595	10.9433405	10.0028007	10 9461412	30
31	9.0549661	9.9971849	9.0577813	10.9422187	10.0028151	10.9450339	29
32	9.0560706	9.9971704	9.0589002	10.9410998	10.0028296	10.9439294	28
33	9.0571723	9.9971559	9.0600164	10.9399836	10.0028441	10.9428277	27
34	9.0582711	9,9971414	9.0611297	10.9388703	10.0028586	10 9417289	26
35	9.0593672	9.9971268	9.0622403	10.9377597	10.0028732	10.9406328	25
36	9.0604604	9.9971122	9.0633482	10.9366518	10.0028878	10.9395396	24
37	9.0615509	9.9970976	9.0644533	10.9355467	10.0029024	10.9384491	23
38	9.0626386	9.9970829	9·0655556	10.9344444	10.0029171	10.9373614	22
39	9.0637235	9.9970682	9.0666553	10.9333447	10.0029318	10 9362765	21
40	9.0648057	9.9970535	9.0677522	10.9322478	10.0029465	10.9351943	20
41	9.0658852	9,9970387	9.0688465	10.9311535	10.0029613	10.9341148	19
42	9.0669619	9.9970239	9.0699381	10.9300619	10.0029762	10.9330381	18
43	9.0680360	9.9970090	9.0710270	10.9289730	10.0029910	10.9319640	17
44	9.0691074	9.9969941	9.0721133	10.9278867	10.0030059	10.9308926	16
45	9.0701761	9.9969792	9.0731969	10.9268031	10.0030208	10.9298239	15
46	9.0712421	9.9969642	9.0742779	10.9257221	10.0030358	10.9287579	14
47	9.0723055	9.9969492	9.0753563	10.9246437	10 0030508	10.9276945	13
48	9.0733663	9.9969342	9.0764321	10.9235679	10.0030658	10.9266337	12
49	9.0744244	9.9969191	9.0775053	10.9224947	10.0030809	10.9255756	11
50	9.0754799	9.9969040	9.0785760	10.9214240	10.0030960	10.9245201	10
51	9.0765329	9.9968888	9.0796441	10.9203559	10.0031112	10.9234671	9
52	9.0775832	9.9968736	9.0807096	10.9192904	10.0031264	10.9224168	8
53	9.0786310	9.9968584	9.0817725	10.9182274	10.0031416	10.9213690	7
54	9.0796762	9.9968431	9.0828331	10.9171669	10.0031569	10.9203238	6
55	9.0807189	9.9968278	9.0838911	10.9161089	10.0031722	10.9192811	5
56	9.0817590	9.9968125	9.0849466	10.9150534	10.0031875	10.9182410	4
57	9.0827966	9.9967971	9.0859996	10.9140004	10.0032029	10.9172034	3
58	9.0838317	9.9967817	9.0870501	10.9129499	10.0032183	10.9161683	2
59	9,0848643	9.9967662	9.0880981	10.9119019	10.0032338	10.9151357	1
60	9.0858945	9.9967507	9.0891438	10.9108562	10.0032493	10.9141055	0
		Sine.		Tang.		Secant.	M

83 *Degrees.*

A Table of Artificial Sines,

7 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.0858945	9.9967507	9.0991438	10.9108562	10.0032493	10.9141055	60
1	9.0869221	9.9967352	9.0901869	10.9098131	10.0032648	10.9130779	59
2	9.0879473	9.9967196	9.0912277	10.9087723	10.0032804	10.9120527	58
3	9.0889700	9.9967040	9.0922660	10.9077340	10.0032960	10.9110300	57
4	9.0899903	9.9966884	9.0933020	10.9066980	10.0033116	10.9100097	56
5	9.0910082	9.9966727	9.0943355	10.9056645	10.0033273	10.9089918	55
6	9.0920237	9.9966570	9.0953667	10.9046333	10.0033430	10.9079763	54
7	9.0930367	9.9966412	9.0963955	10.9036045	10.0033588	10.9069633	53
8	9.0940474	9.9966254	9.0974219	10.9025781	10.0033746	10.9059526	52
9	9.0950556	9.9966096	9.0984460	10.9015540	10.0033904	10.9049444	51
10	9.0960615	9.9965937	9.0994678	10.9005322	10.0034063	10.9039385	50
11	9 0970651	9.9965778	9.1004872	10.8995128	10.0034222	10.9029349	49
12	9.0980662	9.9965619	9.1015044	10.8984956	10.0034381	10.9019338	48
13	9.0990651	9.9965459	9.1025192	10.8974808	10.0034541	10.9009349	47
14	9.1000616	9.9965299	9.1035317	10.8964683	10.0034701	10.8999384	46
15	9.1010558	9.9965138	9.1045420	10.8954580	10.0034862	10.8989442	45
16	9.1020477	9.9964977	9.1055500	10.8944500	10.0035023	10.8979523	44
17	9.1030373	9.9964816	9.1065557	10.8934443	10.0035184	10.8969627	43
18	9.1040246	9.9964655	9.1075591	10.8924409	10.0035345	10.8959754	42
19	9.1050096	9.9964493	9.1085604	10.8914396	10.0035507	10.8949904	41
20	9.1059924	9.9964330	9.1095594	10.8904406	10.0035670	10.8940076	40
21	9.1069729	9.9964167	9.1105562	10.8894438	10.0035833	10.8930271	39
22	9.1079512	9.9964004	9.1115508	10.8884492	10.0035996	10.8920488	38
23	9,1089272	9.9963841	9.1125431	10,8874569	10.0036159	10.8910728	37
24	9.1099010	9.9963677	9.1135333	10.8864667	10.0036323	10.8900990	36
25	9.1108726	9.9963513	9.1145213	10.8854787	10.0036487	10.8891274	35
26	9.1118420	9.9963348	9.1155072	10.8844928	10.0036652	10.8881580	34
27	9.1128092	9.9963183	9.1164909	10.8835091	10.0036817	10.8871908	33
28	9.1137742	9.9963018	9.1174724	10.8825276	10.0036982	10.8862258	32
29	9.1147370	9.9962852	9.1184518	10.8815482	10.0037148	10.8852630	31
30	9.1156977	9.9962686	9.1194291	10.8805709	10.0037314	10.8843023	30
		Sine.		Tang.		Secant.	M

82 *Degrees.*

7 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.1156977	9.9962680	9.1194291	10.8805709	10.0037314	10.8843023	30
31	9.1160562	9.9962519	9.1204043	10.8795957	10.0037481	10.8833438	29
32	9.1176125	9.9962352	9.1213773	10.8786227	10.0037648	10.8823875	28
33	9.1185667	9.9962185	9.1223482	10.8776518	10.0037815	10.8814333	27
34	9.1195188	9.9962017	9.1233171	10.8766829	10.0037983	10.8804812	26
35	9.1204688	9.9961849	9.1242839	10.8757161	10.0038151	10.8795312	25
36	9.1214167	9.9961681	9.1252486	10.8747514	10.0038319	10.8785833	24
37	9.1223624	9.9961512	9.1262112	10.8737888	10.0038488	10.8776376	23
38	9.1233061	9.9961343	9.1271718	10.8728282	10.0038657	10.8766939	22
39	9.1242477	9.9961174	9.1281303	10.8718697	10.0038826	10.8757523	21
40	9.1251872	9.9961004	9.1290868	10.8709132	10.0038996	10.8748128	20
41	9.1261246	9.9960834	9.1300413	10.8699587	10.0039166	10.8738754	19
42	9.1270600	9.9960663	9.1309937	10.8690063	10.0039337	10.8729400	18
43	9.1279934	9.9960492	9.1319442	10.8680558	10.0039508	10.8720066	17
44	9.1289247	9.9960321	9.1328926	10.8671074	10.0039679	10.8710753	16
45	9.1298539	9.9960149	9.1338391	10.8661609	10.0039851	10.8701461	15
46	9.1307812	9.9959977	9.1347835	10.8652165	10.0040023	10.8692188	14
47	9.1317064	9.9959804	9.1357260	10.8642740	10.0040196	10.8682936	13
48	9.1326297	9.9959631	9.1366665	10.8633335	10.0040369	10.8673703	12
49	9.1335509	9.995945	9.1376051	10.8623949	10.0040542	10.8664491	11
50	9.1344702	9.9959284	9.1385417	10.8614583	10.0040716	10.8655298	10
51	9.1353875	9.9959111	9.1394764	10.8605236	10.0040889	10.8646125	9
52	9.1363028	9.9958936	9.1404092	10.8595908	10.0041064	10.8636972	8
53	9.1372161	9.9958761	9.1413400	10.8586600	10.0041239	10.8627839	7
54	9.1381275	9.9958586	9.1422689	10.8577311	10.0041414	10.8618725	6
55	9.1390370	9.9958411	9.1431959	10.8568041	10.0041589	10.8609630	5
56	9.1399445	9.9958235	9.1441210	10.8558790	10.0041765	10.8600555	4
57	9.1408501	9.9958059	9.1450442	10.8549558	10.0041941	10.8591499	3
58	9.1417537	9.995788	9.1459655	10.8540345	10.0042118	10.8582463	2
59	9.14 6555	9.9957705	9.1468850	10.8531150	10.0042295	10.8573445	1
60	9.1435553	9.9957528	9.1478025	10.8521975	10.0042472	10.8564447	0
		Sine.		Tang.		Secant.	M

82 *Degrees.*

8 Degrees.

	Sine.		Tang.		Secant.	
0	9.1435553	9.9957528	9.1478025	10.8521975	10.0042472	10.8564447
1	9.1444532	9.9957350	9.1487182	10.8512818	10.0042650	10.8555468
2	9.1453493	9.9957172	9.1496321	10.8503679	10.0042828	10.8546507
3	9.1462435	9.9956993	9.1505441	10.8494559	10.0043007	10.8537565
4	9.1471358	9.9956815	9.1514543	10.8485457	10.0043185	10.8528642
5	9.1480262	9.9956635	9.1523627	10.8476373	10.0043365	10.8519738
6	9.1489148	9.9956456	9.1532692	10.8467308	10.0043544	10.8510852
7	9.1498015	9.9956276	9.1541739	10.8458261	10.0043724	10.8501985
8	9.1506864	9.9956095	9.1550769	10.8449231	10.0043905	10.8493136
9	9.1515694	9.9955915	9.1559780	10.8440220	10.0044085	10.8484306
10	9.1524507	9.9955734	9.1568773	10.8431227	10.0044266	10.8475493
11	9.1533301	9.9955552	9.1577748	10.8422252	10.0044448	10.8466699
12	9.1542076	9.9955370	9.1586706	10.8413294	10.0044630	10.8457924
13	9.1550834	9.9955188	9.1595646	10.8404354	10.0044812	10.8449166
14	9.1559574	9.9955005	9.1604569	10.8395431	10.0044995	10.8440426
15	9.1568290	9.9954822	9.1613473	10.8386527	10.0045178	10.8431710
16	9.1577000	9.9954639	9.1622361	10.8377639	10.0045361	10.8423000
17	9.1585686	9.9954455	9.1631231	10.8368769	10.0045545	10.8414314
18	9.1594354	9.9954271	9.1640083	10.8359917	10.0045729	10.8405646
19	9.1603005	9.9954087	9.1648919	10.8351081	10.0045913	10.8396995
20	9.1611639	9.9953902	9.1657737	10.8342263	10.0046098	10.8388361
21	9.1620254	9.9953717	9.1666538	10.8333462	10.0046283	10.8379746
22	9.1628853	9.9953531	9.1675322	10.8324678	10.0046469	10.8371147
23	9.1637434	9.9953345	9.1684089	10.8315911	10.0046655	10.8362566
24	9.1645998	9.9953159	9.1692839	10.8307161	10.0046841	10.8354002
25	9.1654544	9.9952972	9.1701572	10.8298428	10.0047028	10.8345456
26	9.1663074	9.9952785	9.1710289	10.8289711	10.0047215	10.8336926
27	9.1671586	9.9952597	9.1718989	10.8281011	10.0047403	10.8328414
28	9.1680081	9.9952409	9.1727672	10.8272328	10.0047591	10.8319919
29	9.1688559	9.9952221	9.1736338	10.8263662	10.0047779	10.8311441
30	9.1697021	9.9952033	9.1744988	10.8255012	10.0047967	10.8302979
		Sine.		Tang.		Secant.

81 Degrees.

8 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.1697021	9.9952033	9.1744988	10.8255012	10.0047967	10,8302979	30
31	9.1705465	9.9951844	9.1753622	10.8246378	10.0048156	10.8294535	29
32	9.1713893	9.9951654	9.1762239	10.8237761	10.0048346	10.8286107	28
33	9.1722305	9.9951464	9.1770840	10.8229160	10.0048536	10.8277695	27
34	9.1730699	9.9951274	9.1779425	10 8220575	10.0048726	10.8269301	26
35	9.1739077	9.9951084	9.1787993	10.8212007	10.0048916	10.8260923	25
36	9.1747439	9.9950893	9.1796546	10.8203454	10.0049107	10.8252561	24
37	9.1755784	9.9950702	9.1805082	10.8194918	10.0049298	10.8244216	23
38	9.1764112	9.9950510	9.1813602	10.8186398	10.0049490	10.8235888	22
39	9.1772 25	9.9950318	9.1822106	10.8177894	10.0049682	10.8227575	21
40	9.1780721	9.9950126	9.1830595	10.8169405	10.0049874	10.8219279	20
41	9.1789001	9.9949933	9.1839068	10.8160932	10.0050067	10.8210999	19
42	9.1797265	9.9949740	9.1847525	10.8152475	10.0050260	10.8202735	18
43	9.1805512	9.9949546	9.1855966	10.8144034	10.0050454	10.8194488	17
44	9.1813744	9.9949352	9.1864392	10.8135608	10.0050648	10.8186256	16
45	9.1821960	9 9949158	9.1872802	10.8127198	10.0050842	10.8178040	15
46	9.1830160	9.9948964	9.1881196	10.8118804	10.0051036	10.8169840	14
47	9.1838344	9.9948769	9.1889575	10.8110425	10.0051231	10.8161656	13
48	9.1846512	9,9948573	9.1897939	10.8102061	10.0051427	10.8153488	12
49	9.1854665	9.9948377	9.1906287	10.8093713	10.0051625	10.8145335	11
50	9.1862802	9.9948181	9.1914621	10.808 379	10.0051819	10.8137198	10
51	9.1870923	9.9947985	9.1922939	10.8077061	10.0052015	10.8129077	9
52	9.1879029	9.9947788	9.1931241	10.8068759	10.0052212	10.8120971	8
53	9.1887120	9.9947591	9.1939529	10.8060471	10.0052409	10.8112880	7
54	9.1895195	9.9947393	9.1947802	10.8052198	10.0052607	10.8104805	6
55	9,1903254	9.99471 5	9.1956059	10.8043941	10.0052805	10.8096746	5
56	9.1911299	9.9946997	9.1964302	10.8035698	10.0053003	10.3088701	4
57	9.1919328	9.9946798	9.1972530	10.8027470	10.0053202	10.8080672	3
58	9.1927342	9.9946599	9.1980743	10.8019257	10.0053401	10.8072658	2
59	9 1935341	9.9946399	9.1988941	10.8011059	10.0053601	10.8064659	1
60	9.1943324	9.9946199	9.1997125	10.8002875	10.0053801	10.8056676	0
		Sine.		Tang.		Secant.	*M*

81 *Degrees.*

A Table of Artificial Sines,

9 *Degrees.*

M	Sine.		Tang.		Secant		
0	9.1943324	9.9946199	9.1997125	10.8002875	10.0053801	10.8056676	60
1	9.1951293	9.9945999	9.2005294	10.7994706	10.0054001	10.8048707	59
2	9.1959247	9.9945798	9.2013449	10.7986551	10.0054202	10.8040753	58
3	9.1967186	9.9945597	9.2021588	10.7978412	10.0054403	10.8032814	57
4	9.1975111	9.9945396	9.2029714	10.7970286	10.0054604	10.8024890	56
5	9.1983019	9.9945194	9.2037825	10.7962175	10.0054806	10.8016981	55
6	9.1990913	9.9944992	9.2045922	10.7954078	10.0055008	10.8009087	54
7	9.1998793	9.9944789	9.2054004	10.7945996	10.0055211	10.8001207	53
8	9.2006658	9.9944587	9.2062072	10.7937928	10.0055413	10.7993342	52
9	9.2014509	9.9944383	9.2070126	10.7929874	10.0055617	10.7985491	51
10	9.202234	9.9944180	9.2078165	10.7921835	10.0055820	10.7977655	50
11	9.2030167	9.9943975	9.2086191	10.7913809	10.0056025	10.7969833	49
12	9.2037974	9.9943771	9.2094203	10.7905797	10.0056229	10.7962026	44
13	9.2045766	9.9943566	9.2102200	10.7897800	10.0056434	10.7954234	47
14	9.2053545	9.9943361	9.2110184	10.7889816	10.0056639	10.7946455	46
15	9.2061309	9.9943156	9.2118153	10.7881847	10.0056844	10.7938691	45
16	9.2069055	9.9942950	9 2126109	10.7873891	10.0057050	10.7930941	44
17	9.2076795	9.9942743	9.2134051	10.7865949	10.0057257	10.7923205	43
18	9.2084516	9.9942537	9.2141980	10.7858020	10.0057463	10.7915484	42
19	9.2092224	9.9942330	9.2149894	10.7850106	10.0057670	10.7907776	41
20	9.2099917	9.9942122	9.2157795	10.7842205	10.0057878	10.7900083	40
21	9.2107597	9.9941914	9.2165683	10.7834317	10.0058086	10.7892403	39
22	9.2115263	9.9941706	9.2173556	10.7826444	10.0058294	10.7884737	38
23	9.2122914	9.9941498	9.2181417	10.7818583	10.0058502	10.7877086	37
24	9.2130552	9.9941289	9.2189264	10.7810736	10.0058711	10.7869448	36
25	9.2138176	9.9941079	9.2197097	10.7702903	10.0058921	10.7861824	35
26	9.2145787	9.9940870	9.2204917	10.7795083	10.0059130	10.7854213	34
27	9.2153384	9.9940659	9.2212724	10.7787276	10.0059341	10.7846616	33
28	9.2160967	9.9940449	9.2220518	10.7779482	10.0059551	10.7839033	32
29	9.2168536	9.9940238	9.2228298	10.7771702	10.0059762	10.7831464	31
30	9.2176092	9.9940027	9.2236065	10.7763935	10.0059973	10.7823908	30
		Sine.		Tang.		Secant.	M

80 *Degrees.*

9 *Degrees.*

M	Sine.		Tang		Secant,	-	
30	9.2176092	9.9940027	9.2236065	0.7763935	10 0059973	10.7823908	30
31	9.2183635	9.9939815	9.2243819	10,7756181	10.0060185	10.7816365	29
32	9.2191164	9.9939603	9.2251561	10.7748439	10.0060397	10.7808836	28
33	9.2198680	9.9939391	9.2259289	10.7740711	10,0060609	10.7801320	27
34	9.2206182	9.9939178	9.2267004	10.7732996	10.0060822	10.7793818	26
35	9.2213671	9.9938965	9.2274706	10.7725294	10.0061035	10.7786325	25
36	9.2221147	9 9938752	9.2282395	20.7717605	10.0061248	10.7778853	24
37	9.2228609	9.9938538	9.2290071	10.7709929	10.0061462	10.7771391	23
38	9.2236059	9.9938324	9.2297735	10.7702265	10.0061676	10.7763941	22
39	9.2243495	9 9938109	9.2305386	10.7694614	10.0061891	10.7756505	21
40	9.2250918	9.9937894	9.2313024	10.7686976	10.0062106	10.7749082	20
41	9.2258328	9.9937679	9.2320650	10.7679350	10.0062321	10.7741672	19
42	9.2265725	9.9937463	9.2328262	10.7671728	10.0062537	20.7734275	18
43	9.2273110	9.9937247	9.2335863	10.7664137	10.0062753	10.7726890	17
44	9.2280481	9.9937030	9.2343451	10.7656549	10.0062970	10.7719519	16
45	9.2287839	9.9936813	9.2351026	10.7648974	10 0063187	10.7712161	15
46	9.2295185	9,9936596	9.2358589	10.7641411	10.0063404	10.7704815	14
47	9.2302518	9.9936378	9.2366139	10.7633861	10.0063622	10.7697482	13
48	9.2309838	9.9936160	9.2373676	10.7626322	10-0063840	10.7690162	12
49	9.2317145	9.9935942	9.2381203	10.7618797	10.0064058	10.7682855	11
50	9.2324440	9.9935723	9.2388717	10.7611283	10.0064277	10.7675560	10
51	9.2331722	9.9935504	9.2396218	10,7603782	10.0064496	10.7668278	9
52	9.2338992	9.9935285	9.2403708	10,7596292	10.0064715	10.7661008	8
53	9.2346249	9.9935065	9.2411185	10.7588815	10.0064935	10.7653751	7
54	9.2353494	9.9934844	9.2418650	10.7581350	10.0065156	10.7646506	6
55	9,2360726	9.9934624	9.2426103	10.7573897	10.0065376	10.7639274	5
56	9,2367946	9.9934403	9.2433543	10.7566457	10.0065597	10.7632054	4
57	9 2375153	9.9934181	9,2440972	10.7559028	10.0065819	10.7624847	3
58	9.2382349	9.0933959	9.2448389	10.7551611	10.0066041	10.7617651	2
59	9.2389532	9.9933737	9.2455794	10.7544206	10.0066263	10.7610468	1
60	9.2396702	9.9933515	9.2463188	10.7536812	10.0066485	10.7603298	0
		Sine.		Tang.		Secant.	*M*

80 *Degrees.*

A Table of Artificial Sines,

10 *Degrees.*

M	Sine		Tang.		Secant.		
0	9.2396702	9.9933515	9.2463188	10.7536812	10.0066485	10.7603298	60
1	9.2403861	9.9933292	9.2470569	10.7529431	10.0066708	10.7596139	59
2	9.2411007	9.9933068	9.2477939	10.7522061	10.0066932	10.7588993	58
3	9.2418141	9.9932845	9.2485297	10.7514703	10.0067155	10.7581859	57
4	9.2425264	9.9932621	9.2492643	10.7507357	10.0067379	10.7574736	56
5	9.2432374	9.9932396	9.2499978	10.7500022	10.0067604	10.7567626	55
6	9.2439472	9.9932171	9.2507301	10.7492699	10.0067829	10.7560528	54
7	9.2446558	9.9931946	9.2514612	0.7485388	10.0068054	10.7553442	53
8	9.2453632	9.9931720	9.2521912	10.7478088	10.0068280	10.7546368	52
9	9.2460695	9.9931494	9.2529200	10.7470800	10.0068506	10.7539305	51
10	9.2467746	9.9931268	9.2536477	10.7463523	10.0068732	10.7532254	50
11	9.2474784	9.9931041	9.2543743	10.7456257	10.0068959	10.7525216	49
12	9.2481811	9.9930814	9.2550997	10.7449003	10.0069186	10.7518189	48
13	9.2488827	9.9930587	9.2558240	10.7441760	10.0069413	10.7511173	47
14	9.2495830	9.9930359	9.2565472	10.7434528	10.0069641	10.7504170	46
15	9.2502822	9.9930131	9.2572692	10.7427308	10.0069869	10.7497178	45
16	9.2509803	9.9929902	9.2579901	10.7420099	10.0070098	10.7490197	44
17	9.2516772	9.9929673	9.2587099	10.7412901	10.0070327	10.7483228	43
18	9.2523729	9.9929444	9.2594285	10.7405715	10.0070556	10.7476271	42
19	9.2530675	9.9929214	9.2601461	10.7398539	10.0070786	10.7469325	41
20	9.2537609	9.9928984	9.2608625	10.7391375	10.0071016	10.7462391	40
21	9.2544532	9.9928753	9.2615779	10.7384221	10.0071247	10.7455468	39
22	9.2551444	9.9928522	9.2622921	10.7377079	10.0071478	10.7448556	38
23	9.2558344	9.9928291	9.2630053	10.7369947	10.0071709	10.7441656	37
24	9.2565233	9.9928059	9.2637173	10.7362827	10.0071941	10.7434767	36
25	9.2572110	9.9927827	9.2644283	10.7355717	10.0072173	10.7427890	35
26	9.2578977	9.9927595	9.2651382	10.7348618	10.0072405	10.7421023	34
27	9.2585832	9.9927362	9.2658470	10.7341530	10.0072638	10.1414168	33
28	9.2592676	9.9927129	9.2665547	10.7334453	10.0072871	10.7407324	32
29	9.2599509	9.9926895	9.2672613	10.7327387	10.0073105	10.7400491	31
30	9.2606330	9.9926664	9.2679669	10.7320331	10.0073335	10.7393670	30
		Sine.		Tang.		Secant.	M

79 *Degrees.*

10 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.2606330	9.9926661	9.2679669	10.7320331	10.0073339	10.7393670	30
31	9.2613141	9.9926427	9.2686714	10.7313286	10.0073573	10.7386859	29
32	9.2519941	9.9926192	9.2693749	10.7306251	10.0073808	10.7380059	28
33	9.2626729	9.9925957	9.2700772	10.7299228	10.0074043	10.7373271	27
34	9.2633507	9.9925722	9.2707786	10.7292214	10.0074278	10.7366493	26
35	9.2640274	9.9925486	9.2714788	10.7285212	10.0074514	10.7359726	25
36	9.2647030	9.9925250	9.2721780	10.7278220	10.0074750	10.7352970	24
37	9.2653775	9.9925013	9.2728762	10.7271238	10.0074987	10.7346225	23
38	9.2660509	9.9924770	9.2735733	10.7264267	10.0075224	10.7339491	22
39	9.2667232	9.9924539	9.2742694	10.7257306	10.0075461	10.7332768	21
40	9.2673945	9.9924301	9.2749644	10.7250356	10.0075699	10.7326055	20
41	9.2680647	9.9924063	9.2756584	10.7243416	10.0075937	10.7319353	19
42	9.2687338	9.9923824	9.2763514	10.7236486	10.0076176	10.7312662	18
43	9.2694019	9.9923585	9.2770434	10.7229566	10.0076415	10.7305981	17
44	9.2700689	9.9923346	9.2777343	10.7222657	10.0076654	10.7299311	16
45	9.2707348	9.9923106	9.2784242	10.7215758	10.0076894	10.7292652	15
46	9.2713997	9.9922866	9.2791131	10.7208869	10.0077134	10.7286003	14
47	9.2720635	9.9922626	9.2798009	10.7201991	10.0077374	10.7279365	13
48	9.2727263	9.9922385	9.2804878	10.7195122	10.0077615	10.7272737	12
49	9.2733880	9.9922144	9.2811736	10.7188264	10.0077856	10.7266120	11
50	9.2740487	9.9921902	9.2818585	10.7181415	10.0078098	10.7259513	10
51	9.2747083	9.9921660	9.2825423	10.7174577	10.0078340	10.7252917	9
52	9.2753669	9.9921418	9.2832251	10.7167749	10.0078582	10.7246331	8
53	9.2760245	9.9921175	9.2839070	10.7160930	10.0078825	10.7239755	7
54	9.2766811	9.9920932	9.2845878	10.7154122	10.0079068	10.7233189	6
55	9.2773366	9.9920689	9.2852677	10.7147323	10.0079311	10.7226634	5
56	9.2779911	9.9920445	9.2859466	10.7140534	10.0079555	10.7220089	4
57	9.2786445	9.9920201	9.2866245	10.7133755	10.0079799	10.7213555	3
58	9.2792970	9.9919956	9.2873014	10.7126986	10.0080044	10.7207030	2
59	9.2799484	9.9919711	9.2879773	10.7120227	10.0080289	10.7200516	1
60	9.2805988	9.9919466	9.2886523	10.7113477	10.0080534	10.7194012	0
		Sine.		Tang.		Secant.	M

79 *Degrees.*

11 Degrees.

M	Sine.		Tang.		Secant.		
0	9.2805988	9.9919466	9.2886523	10.7113477	10.0080534	10.7194012	60
1	9.2812483	9.9919220	9.2893263	10.7106737	10.0080780	10.7187517	59
2	9.2818967	9.9918974	9.2899993	10.7100007	10.0081026	10.7181033	58
3	9.2825441	9.9918727	9.2906713	10.7093287	10.0081273	10.7174559	57
4	9.2831905	9.9918480	9.2913424	10.7086576	10.0081520	10.7168095	56
5	9.2838359	9.9918231	9.2920126	10.7079874	10.0081767	10.7161641	55
6	9.2844803	9.9917986	9.2926817	10.7073183	10.0082014	10.7155197	54
7	9.2851237	9.9917737	9.2933500	10.7066500	10.0082263	10.7148763	53
8	9.2857661	9.9917489	9.2940172	10.7059828	10.0082511	10.7142339	52
9	9.2864076	9.9917240	9.2946836	10.7053164	10.0082760	10.7135924	51
10	9.2870480	9.9916991	9.2953489	10.7046511	10.0083009	10.7129520	50
11	9.2876875	9.9916741	9.2960134	10.7039866	10.0083259	10.7123125	49
12	9.2883260	9.9916492	9.2966769	10.7033231	10.0083508	10.7116740	48
13	9.2869636	9.9916241	9.2973395	10.7026605	10.0083755	10.7110364	47
14	9.2896001	9.9915990	9.2980011	10.7019989	10.0084010	10.7103999	46
15	9.2902357	9.9915739	9.2986618	10.7013382	10.0084261	10.7097643	45
16	9.2908704	9.9915488	9.2993216	10.7006784	10.0084512	10.7091296	44
17	9.2915040	9.9915236	9.2999804	10,7000196	10.0084764	10.7084960	43
18	9.2921367	9.9914984	9.3006383	10.6993617	10.0085016	10.7078633	42
19	9.2927685	9.9914731	9.3012954	10.6987046	10.0085267	10.7072315	41
20	9.2933993	9.9914478	9.3019514	10.6980486	10.0085522	10.7066007	40
21	9.2940251	9.9914225	9.3026066	10.6973934	10.0085775	10.7059709	39
22	9.2946580	9.9913971	9.3032609	10.6967391	10.0086029	10.7053420	38
23	9.2952855	9.9913717	9.3039143	10.6960857	10.0086283	10.7047141	37
24	9.2959125	9.9913462	9.3045667	10.6954333	10.0086538	10.7040871	36
25	9.2965390	9.9913207	9.3052183	10.6947817	10.0086793	10.7034610	35
26	9.2971641	9.9912952	9.3058689	10.6941311	10.0087048	10.7028359	34
27	9.2977883	9.9912696	9.3065187	10.6934813	10.0087304	10.7022117	33
28	9.2984116	9.9912440	9.3071675	10.6928325	10.0087560	10.7015884	32
29	9.2990339	9.9912184	9.3078155	10.6921845	10.0087816	10.7009661	31
30	9.2996553	9.9911927	9.3084626	10.6915374	10.0088073	10.7003447	30
		Sine.		Tang.		Secant.	M

78 Degrees.

11 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.2996553	9.9911927	9.3084626	10.6915374	10.0088073	10.7003447	30
31	9.3002758	9.9911670	9.3091088	10.6908912	10.0088330	10.6997242	29
32	9.3008953	9.9911412	9.3097541	10.6902459	10.0088585	10.6991047	28
33	9.3015140	9.9911154	9.3103985	10.6896015	10.0088846	10.6984860	27
34	9.3021317	9.9910896	9.3110421	10.6889579	10.0089104	10.6978683	26
35	9.3027485	9.9910637	9.3116848	10.6883152	10.0089363	10.6972515	25
36	9.3033644	9.9910378	9.3123266	10.6876734	10.0089622	10.6966356	24
37	9.3039794	9.9910119	9.3129675	10.6870325	10.0089881	10.6960206	23
38	9.3045934	9.9909859	9.3136076	10.6863924	10.0090141	10.6954066	22
39	9.3052066	9.9909598	9.3142468	10.6857532	10.0090402	10.6947934	21
40	9.3058189	9.9909338	9.3148851	10.6851149	10.0090662	10.6941811	20
41	9.3064303	9.9909077	9.3155226	10.6844774	10.0090923	10.6935697	19
42	9.3070407	9.9908815	9.3161592	10.6838408	10.0091185	10.6929593	18
43	9.3076503	9.9908553	9.3167950	10.6832050	10.0091447	10.6923497	17
44	9.3082590	9.9908291	9.3174299	10.6825701	10.0091709	10.6917410	16
45	9.3088668	9.9908029	9.3180640	10.6819360	10.0091971	10.6911332	15
46	9.3094737	9.9907766	9.3186972	10.6813028	10.0092234	10.6905263	14
47	9.3100798	9.9907502	9.3193295	10.6806705	10.0092498	10.6899202	13
48	9.3106849	9.9907239	9.3199611	10.6800389	10.0092761	10.6893151	12
49	9.3112892	9.9906974	9.3205918	10.6794082	10.0093026	10.6887108	11
50	9.3118926	9.9906710	9.3212216	10.6787784	10.0093290	10.6881074	10
51	9.3124951	9.9906445	9.3218506	10.6781494	10.0093555	10.6875049	9
52	9.3130968	9.9906180	9.3224788	10.6775212	10.0093820	10.6869032	8
53	9.3136976	9.9905914	9.3231061	10.6768939	10.0094086	10.6863024	7
54	9.3142975	9.9905648	9.3237327	10.6762673	10.0094352	10.6857025	6
55	9.3148965	9.9905382	9.3243584	10.6756416	10.0094618	10.6851035	5
56	9.3154947	9.9905115	9.3249832	10.6750168	10.0094885	10.6845053	4
57	9.3160921	9.9904848	9.3256073	10.6743927	10.0095152	10.6839079	3
58	9.3166885	9.9904580	9.3262305	10.6737695	10.0095420	10.6833115	2
59	9.3172841	9.9904312	9.3268529	10.6731471	10.0095688	10.6827159	1
60	9.3178789	9.9904044	9.3274745	10.6725255	10.0095956	10.6821211	0
		Sine.		Tang.		Secant.	M

78 *Degrees.*

A Table of Artificial Sines,

12 Degrees.

	Sine.		Tang.		Secant		
	9.3178789	9.9904044	9.3274745	10.6725255	10.0095956	10.6821211	6
1	9.3184728	9.9903776	9.3280953	10.6719047	10.0096225	10.6815272	59
2	9.3190659	9.9903500	9.3287153	10.6712847	10.0096494	10.6809341	5
3	9.3196581	9.9903237	9.3293345	10.6706655	10.0096763	10.6803419	57
4	9.3202495	9.9902967	9.3299528	10.6700472	10.0097033	10.6797505	5
5	9.3208406	9.9902693	9.3305704	10.6694296	10.0097303	10.6791600	5
6	9.3214297	9.9602426	9 3311872	10.6688128	10.0097574	10.6785703	54
7	9.3220186	9.9902155	9.3318031	10.6681969	10.0097845	10.6779814	53
8	9.3226066	9.9901883	9.3324183	10.6675817	10.0098117	10.6773934	52
9	9.3231938	9.9901612	9.3330327	10.6669673	10.0098388	10.6768062	51
10	9.3237802	9.9901339	9.3336463	10.6663537	10.0098661	10.6762198	50
11	9.3243657	9.9901067	9.3342591	10.6657409	10.0098933	10.6756343	49
12	9.3249505	9.9900794	9.3348711	10.6651289	10.0099206	10.6750495	4
13	9.3255344	9.9900521	9.3354823	10.6645177	10.0099479	10.6744656	47
14	9.3261174	9.9900247	9.3360927	10.6639073	10.0099753	10.6738826	4
15	9.3266997	9.9899973	9.3367024	10.6632976	10.0100027	10.6733003	45
16	9.3272811	9.9899698	9.3373113	10.6626887	10.0100302	10.6727189	44
17	9,3278617	9.9899423	9,3379194	10.6620806	10.0100577	10.6721383	43
18	9.3284416	9.9899148	9.3385267	10.6614733	10.0100852	10.6715584	42
19	9.3290206	9.9898873	9.3391333	10.6608667	10.0101127	10.6709794	41
20	9.3295988	9.9898597	9.3397391	10.6602609	10.0101403	10 6704012	4
21	9.3301761	9.9898320	9.3403441	10.6596559	10.0101680	10.6698239	3
22	9.3307527	9.9898043	9.3409484	10.6590516	10.0101957	10 6692473	3
23	9.3313285	9.9897766	9.3415519	10.6584481	10.0102234	10.6686715	37
24	9.3319035	9.9897489	9,3421546	10.6578454	10.0102511	10.6680965	3
25	9.3324777	9.9897211	9.3427566	10.6572434	10.0102789	10.6675223	3
26	9.3330511	9.9896932	9.3433578	10.6566422	10.0103068	10.6669489	36
27	9.3336237	9.9896654	9.3439583	10.6560417	10.0103346	10.6663763	
28	9.3341955	9.9896374	9.3445580	10.6554420	10.0103626	10.6658045	3
29	9.3347665	9.9896095	9.3451570	10.6548430	10.0103909	10.6652335	31
30	9.3353368	9.9895815	9.3457552	10.6542448	10.0104189	10.6646632	
		Sine.		Tang.		Secant.	

77 Degrees.

12 Degrees.

M	Sine.		Tang.		Secant.		
30	9.3353368	9.9895815	9.3457552	10.6542448	10.0104185	10.6646632	30
31	9.3359062	9.9895535	9.3463527	10.6536473	10.0104465	10.6640938	29
32	9.3364749	9.9895254	9.3469494	10.6530506	10.0104746	10.6635251	28
33	9.3370428	9.9894973	9.3475454	10.6524546	10.0105027	10.6629572	27
34	9.3376099	9.9894692	9.3481407	10.6518593	10.0105308	10.6623901	26
35	9.3381762	9.9894410	9.3487352	10.6512648	10.0105590	10.6618238	25
36	9.3387418	9.9894128	9.3493290	10.6506710	10.0105872	10.6612582	24
37	9.3393065	9.9893845	9.3499220	10.6500780	10.0106155	10.6606935	23
38	9.3398700	9.9893562	9.3505143	10.6494857	10.0106438	10.6601294	22
39	9.3404338	9.9893279	9.3511059	10.6488941	10.0106721	10.6595662	21
40	9.3409963	9.9892995	9.3516968	10.6483032	10.0107005	10.6590037	20
41	9.3415580	9.9892711	9.3522869	10.6477131	10.0107289	10.6584420	19
42	9.3421190	9.9892427	9.3528763	10.6471237	10.0107573	10.6578810	18
43	9.3426792	9.9892142	9.3534650	10.6465350	10.0107858	10.6573208	17
44	9.3432386	9.9891856	9.3540530	10.6459470	10.0108144	10.6567614	16
45	9.3437973	9.9891571	9.3546402	10.6453598	10.0108429	10.6562027	15
46	9.3443552	9.9891284	9.3552267	10.6447733	10.0108716	10.6556448	14
47	9.3449124	9.9890998	9.3558126	10.6441874	10.0109002	10.6550876	13
48	9.3454688	9.9890711	9.3563977	10.6436023	10.0109289	10.6545312	12
49	9.3460245	9.9890424	9.3569821	10.6430179	10.0109576	10.6539755	11
50	9.3465794	9.9890137	9.3575658	10.6424342	10.0109863	10.6534206	10
51	9.3471330	9.9889849	9.3581487	10.6418513	10.0110151	10.6528664	9
52	9.3476870	9.9889560	9.3587310	10.6412690	10.0110440	10.6523130	8
53	9.3482397	9.9889271	9.3593126	10.6406874	10.0110729	10.6517603	7
54	9.3487917	9.9888982	9.3598935	10.6401065	10.0111018	10.6512083	6
55	9.3493429	9.9888693	9.3604736	10.6395264	10.0111307	10.6506571	5
56	9.3498934	9.9888403	9.3610531	10.6389469	10.0111597	10.6501066	4
57	9.3504432	9.9888113	9.3616319	10.6383681	10.0111887	10.6495568	3
58	9.3509922	9.9887822	9.3622100	10.6377900	10.0112178	10.6490078	2
59	9.3515405	9.9887531	9.3627874	10.6372126	10.0112469	10.6484595	1
60	9.3520880	9.9887239	9.3633641	10.6366359	10.0112761	10.6479120	0
		Sine.		Tang.		Secant.	M

77 Degrees.

A Table of Artificial Sines,

13 *Degrees.*

	Sine.		Tang.		Secant.		
0	9.3520880	9.9887239	9.3633641	10.6366359	10.0112761	10.6479120	60
1	9.3526349	9.9886947	9.3639401	10.6360599	10.0113053	10.6473651	59
2	9.3531810	9.9886655	9.3645155	10.6354845	10.0113345	10.6468190	58
3	9.3537264	9.9886363	9.3650901	10.6349099	10.0113637	10.6462736	57
4	9.3542710	9.9886070	9.3656641	10.6343359	10.0113930	10.6457290	56
5	9.3548150	9.9885776	9.3662374	10.6337626	10.0114224	10.6451850	55
6	9.3553582	9.9885482	9.3668100	10.6331900	10.0114518	10.6446418	54
7	9.3559007	9.9885188	9.3673819	10.6326181	10.0114812	10.6440993	53
8	9.3564426	9.9884894	9.3679532	10.6320468	10.0115106	10.6435574	52
9	9.3569836	9.9884599	9.3685238	10.6314762	10.0115401	10.6430164	51
10	9.3575240	9.9884303	9.3690937	10.6309063	10.0115697	10.6424760	50
11	9.3580637	9.9884008	9.3696629	10.6303371	10.0115992	10.6419363	49
12	9.3586027	9.9883712	9.3702315	10.6297685	10.0116288	10.6413973	48
13	9.3591409	9.9883415	9.3707994	10.6292006	10.0116585	10.6408591	47
14	9.3596785	9.9883118	9.3713667	10.6286333	10.0116882	10.6403215	46
15	9.3602154	9.9882821	9.3719333	10.6280667	10.0117179	10.6397846	45
16	9.3607515	9.9882523	9.3724992	10.6275008	10.0117477	10.6392485	44
17	9.3612870	9.9882225	9.3730645	10.6269355	10.0117775	10.6387130	43
18	9.3618217	9.9881927	9.3736291	10.6263709	10.0118073	10.6381783	42
19	9.3623558	9.9881628	9.3741930	10.6258070	10.0118372	10.6376442	41
20	9.3628892	9.9881329	9.3747563	10.6252437	10.0118671	10.6371108	40
21	9.3634219	9.9881029	9.3753190	10.6246810	10.0118971	10.6365781	39
22	9.3639539	9.9880729	9.3758810	10.6241190	10.0119271	10.6360461	38
23	9.3644852	9.9880429	9.3764423	10.6235577	10.0119571	10.6355148	37
24	9.3650158	9.9880128	9.3770030	10.6229970	10.0119872	10.6349842	36
25	9.3655458	9.9879827	9.3775631	10.6224369	10.0120173	10.6344542	35
26	9.3660750	9.9879525	9.3781225	10.6218775	10.0120475	10.6339250	34
27	9.3666036	9.9879223	9.3786813	10.6213187	10.0120777	10.6333964	33
28	9.3671315	9.9878921	9.3792394	10.6207606	10.0121079	10.6328685	32
29	9.3676587	9.9878618	9.3797969	10.6202031	10.0121382	10.6323413	31
30	9.3681853	9.9878315	9.3803537	10.6196463	10.0121685	10.6318147	30
		Sine.		Tang.		Secant.	M

76 *Degrees.*

13 Degrees.

M	Sine.		Tang.		Secant.		
30	9.3681853	9.9878315	9.3803537	10.6196463	10.0121685	10.6318147	30
31	9.3687111	9.9878012	9.3809100	10.6190900	10.0121988	10.6312889	29
32	9.3692363	9.9877708	9.3814655	10.6185345	10.0122292	10.6307637	28
33	9.3697608	9.9877404	9.3820205	10.6179795	10.0122596	10.6302392	27
34	9.3702847	9.9877099	9.3825748	10.6174252	10.0122901	10.6297153	26
35	9.3708079	9 9876794	9.3831285	10.6168715	10.0123206	10.6291921	25
36	9.3713304	9.9876488	9.3836816	10.6163184	10'0123512	10.6286696	24
37	9.3718523	9.9876183	9.3842340	10.6157660	10.0123817	10,6281477	23
38	9.3723735	9.9875876	9.3847858	10.6152142	10.0124124	10.6276265	22
39	9.3728940	9.9875570	9.3853370	10.6146630	10.0124430	10.6271060	21
40	9.3734139	9.9875263	9.3858876	10.6141124	10.0124737	10.6265861	20
41	9.3739331	9.9874955	9.3864376	16.6135624	10.0125045	10.6260669	19
42	9.3744517	9 9874645	9.3869869	10.6130131	10.0125352	10.6255483	18
43	9.3749696	9.9874339	9.3875356	10.6124644	10.0125661	10.6250304	17
44	9.3754868	9.9874031	9.3880837	10.6119163	10.0125969	10.6245132	16
45	9.3760034	9 9873722	9.3886312	10.6113688	10.0126278	10.6239966	15
46	9.3765194	9.9873413	9.3891781	10.6108219	10.0126587	10.6234806	14
47	9.3770347	9.9873103	9.3897244	10.6102756	10.0126897	10.6229653	13
48	9.3775493	9.9872793	9.3902700	10.6097300	10.0127207	10.6224507	12
49	9.3780633	9.9872482	9.3908151	10.6091849	10.0127518	10.6219367	11
50	9.3785767	9.9872171	9.3913595	10.6086405	10.0127829	10.6214233	10
51	9.3790894	9 9871860	9.3919034	19.6080966	10.0128140	10.6209106	9
52	9.3796015	9.9871549	9.3924466	10.6075534	10.0128451	10.6203985	8
53	9.3801129	9.9871236	9.3929893	10.6070107	10.0128764	10.6198871	7
54	9.3806237	9.9870924	9.3935313	10.6064687	10 0129076	10.6193763	6
55	9.3811339	9.9870611	9,3940727	10.6059273	10.0129389	10.6188661	5
56	9.3816434	9.9870298	9.3946136	10.6053864	10.0129702	10.6183566	
57	9.3821523	9 9869984	9.3951538	10.6048462	10.0130016	10.6178477	3
58	9.3826605	9.9869670	9.3956935	10.6043065	10.0130330	10.6173395	2
59	9.3831682	9.9869356	9.3962326	10.6037674	10.0130644	10.6168318	1
60	9.3836752	9.9869041	9'3967711	10'6032289	10.0130959	10.6163248	0
		Sine.		Tang.		Secant.	M

76 Degrees.

14 Degrees.

	Sine.		Tang.		Secant.		
0	9.3836752	9.9869041	9.3967711	10.6032289	10.0130959	10.6163248	60
1	9.3841815	9.9868726	9.3973089	10.6026911	10.0131274	10.6158185	59
2	9.3846873	9.9868410	9.3978463	10.6021537	10.0131590	10.6153127	58
3	9.3851924	9.9868094	9.3983830	10.6016170	10.0131906	10.6148076	57
4	9.3856969	9.9867778	9.3989191	10.6010809	10.0132222	10.6143031	56
5	9.3862008	9.9867461	9.3994547	10.6005453	10.0132539	10.6137992	55
6	9.3867040	9.9867144	9.3999896	10.6000104	10.0132856	10.6132960	54
7	9.3872067	9.9866827	9.4005240	10.5994760	10.0133173	10.6127933	53
8	9.3877087	9.9866509	9.4010578	10.5989422	10.0133491	10.6122913	52
9	9.3882101	9.9866191	9.4015910	10.5984090	10.0133809	10.6117899	51
10	9.3887109	9.9865873	9.4021237	10.5978763	10.0134127	10.6112891	50
11	9.3892111	9.9865553	9.4026558	10.5973442	10.0134447	10.6107889	49
12	9.3897106	9.9865234	9.4031873	10.5968127	10.0134766	10.6102894	48
13	9.3902096	9.9864913	9.4037182	10.5962818	10.0135087	10.6097904	47
14	9.3907079	9.9864593	9.4042486	10.5957514	10.0135407	10.6092921	46
15	9.3912057	9.9864273	9.4047784	10.5952216	10.0135727	10.6087943	45
16	9.3917028	9.9863952	9.4053076	10.5946924	10.0136048	10.6082972	44
17	9.3921993	9.9863630	9.4058363	10.5941637	10.0136370	10.6078007	43
18	9.3926952	9.9863308	9.4063644	10.5936356	10.0136692	10.6073048	42
19	9.3931905	9.9862986	9.4068919	10.5931081	10.0137014	10.6068095	41
20	9.3936852	9.9862663	9.4074189	10.5925811	10.0137337	10.6063148	40
21	9.3941794	9.9862340	9.4079453	10.5920547	10.0137660	10.6058206	39
22	9.3946729	9.9862017	9.4084712	10.5915288	10.0137983	10.6053271	38
23	9.3951658	9.9861693	9.4089965	10.5910035	10.0138307	10.6048342	37
24	9.3956581	9.9861369	9.4095212	10.5904788	10.0138631	10.6043419	36
25	9.3961499	9.9861045	9.4100454	10.5899546	10.0138955	10.6038501	35
26	9.3966410	9.9860720	9.4105690	10.5894310	10.0139280	10.6033590	34
27	9.3971315	9.9860394	9.4110921	10.5889079	10.0139606	10.6028685	33
28	9.3976215	9.9860069	9.4116146	10.5883854	10.0139931	10.6023785	32
29	9.3981109	9.9859742	9.4121366	10.5878634	10.0140258	10.6018891	31
30	9.3985996	9.9859416	9.4126581	10.5873419	10.0140584	10.6014004	30
		Sine.		Tang.		Secant.	M

75 Degrees.

14 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.3985996	9.9859416	9.4126581	10.5873419	10.0140584	10.6014004	30
31	9.3990878	9.9859089	9.4131789	10.5868211	10.0140911	10.6009122	29
32	9.3995754	9.9858762	9.4136993	10.5863007	10.0141238	10.6004246	28
33	9.4000625	9.9858434	9.4142191	10.5857809	10.0141566	10.5999375	27
34	9.4005489	9.9858106	9.4147383	10.5852617	10.0141894	10.5994511	26
35	9.4010348	9.9857777	9.4152570	10.5847430	10.0142223	10.5989652	25
36	9.4015201	9.9857449	9.4157752	10.5842248	10.0142551	10.5984799	24
37	9.4020048	9.9857119	9.4162928	10.5837072	10.0142881	10.5979952	23
38	9.4024889	9.9856790	9.4168099	10.5831901	10.0143210	10.5975111	22
39	9.4029724	9.9856460	9.4173265	10.5826735	10.0143540	10.5970276	21
40	9.4034554	9.9856129	9.4178425	10.5821575	10.0143871	10.5965446	20
41	9.4039378	9.9855798	9.4183580	10.5816420	10.0144202	10.5960622	19
42	9.4044196	9.9855467	9.4188729	10.5811271	10.0144533	10.5955804	18
43	9.4049009	9.9855135	9.4193874	10.5806126	10.0144865	10.5950991	17
44	9.4053816	9.9854803	9.4199013	10.5800987	10.0145197	10.5946184	16
45	9.4058617	9.9854471	9.4204146	10.5795854	10.0145529	10.5941383	15
46	9.4063413	9.9854138	9.4209275	10.5790725	10.0145862	10.5936587	14
47	9.4068203	9.9853805	9.4214398	10.5785602	10.0146195	10.5931797	13
48	9.4072987	9.9853471	9.4219515	10.5780485	10.0146529	10.5927013	12
49	9.4077766	9.9853138	9.4224628	10.5775372	10.0146862	10.5922234	11
50	9.4082539	9.9852803	9.4229735	10.5770265	10.0147197	10.5917461	10
51	9.4087306	9.9852468	9.4234838	10.5765162	10.0147532	10.5912694	9
52	9.4092068	9.9852133	9.4239935	10.5760065	10.0147867	10.5907932	8
53	9.4096824	9.9851798	9.4245026	10.5754974	10.0148202	10.5903176	7
54	9.4101575	9.9851462	9.4250113	10.5749887	10.0148538	10.5898425	6
55	9.4106320	9.9851125	9.4255194	10.5744806	10.0148875	10.5893680	5
56	9.4111059	9.9850789	9.4260271	10.5739729	10.0149211	10.5888941	4
57	9.4115793	9.9850452	9.4265342	10.5734658	10.0149548	10.5884207	3
58	9.4120522	9.9850114	9.4270408	10.5729592	10.0149886	10.5879478	2
59	9.4125245	9.9849776	9.4275469	10.5724531	10.0150224	10.5874755	1
60	9.4129962	9.9849438	9.4280525	10.5719475	10.0150562	10.5870038	0
		Sine.		Tang.		Secant.	M

75 *Degrees.*

A Table of Artificial Sines,

15 Degrees.

	Sine.		Tang.		Secant.		
[illegible]	9.4129962	9.9849438	9.4280525	10.5719475	10.0150562	10.5870038	6
1	9.4134674	9.9849099	9.42855[illegible]	10.5714425	10.0150901	10.5865326	59
2	9.4139381	9.9848760	9.4290621	10.5709379	10.0151240	10.5860619	58
3	9.414408	9.9848420	9.429566	10.5704339	10.0151580	10.5855918	57
4	9.4148778	9.9848081	9.430069	0.5699303	10.0151919	10.5851222	56
5	9.4153468	9.9847740	9.430572	0.5694273	10.0152260	10.5846532	55
6	9.4158152	9.9847400	9.4310755	10.5689247	10.0152600	10.5841848	54
7	9.4162832	9.9847059	9.431577	10 5684227	10.0152941	10.5837168	53
8	9.4167506	9.9846717	9.4320789	10.5679211	10.0153283	10.5832494	52
9	9.4172174	9.9846375	9.4325799	0.5674201	10.0153625	10.5827826	51
10	9.4176837	9.9846033	9.433080	0.5669196	10.0153967	10.5823163	50
11	9.4181495	9.9845690	9.4335805	10.5664195	10.0154310	10.5818505	49
12	9.4186148	9.9845347	9.4340800	10.5659200	10.0154653	10.5813852	48
13	9.4190795	9.9845004	9.4345791	10.5654209	10.0154996	10.5809205	47
14	9.4195436	9.9844660	9.4350776	10.5649224	10.0155340	10.5804564	46
15	9.4200073	9.9844316	9.4355757	10.5644243	10.0155684	10.5799927	45
16	9.4204704	9.9843971	9.4360733	10.5639267	10.0156029	10.5795296	44
17	9.4209330	9.9843626	9.4365704	10.5634296	10.0156374	10.5790670	43
18	9.4213950	9.9843281	9.4370670	10.5629330	10.0156719	10.5786050	42
19	9.4218566	9.9842935	9.4375631	10.5624369	10.0157065	10.5781434	41
20	9.4223176	9.9842589	9.4380587	10.5619413	10.0157411	10.5776824	40
21	9.4227780	9.9842242	9.4385538	10.5614462	10.0157758	10.5772220	39
22	9.4232380	9.9841895	9.4390485	10.5609515	10.0158105	10.5767620	38
23	9,4236974	9.9841548	9.4395426	10.5604574	10.0158452	10.5763026	37
24	9.4241563	9.9841200	9.4400363	10.5599637	10.0158800	10.5758437	36
25	9.4246147	9.9840852	9.4405295	10.5594705	10.0159148	10.5753853	35
26	9.4250726	9.9840503	9.4410222	10.5589778	10.0159497	10.5749274	34
27	9.4255299	9.9840154	9.4415145	10.5584855	10.0159846	10.5744701	33
28	9.4259867	9.9839805	9.4420062	10.5579938	10.0160195	10.5740133	32
29	9.4264430	9.9839455	9.4424975	10.5575025	10.0160545	10.5735570	31
30	9.4268988	9.9839105	9.4429883	10.5570117	10.0160895	10.5731012	30
		Sine.		Tang.		Secant.	M

74 Degrees.

15 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.4268988	9.983910	9.4429883	10.5570117	10.0160895	10.5731012	30
31	9.427354	9.9838755	9.4434786	10.5565214	10.016124	10.572645	29
32	9.4278089	9.9838404	9.4439685	10.5560315	10.0161596	10.572 9 1	28
33	9.4282631	9.9838052	9.4444579	10.5555421	10.0161948	10.5717 09	27
34	9.4287169	9.9837701	9.4449468	10.5550532	10.0162299	10.57 23	26
35	9.4291701	9.9837348	9.4454352	10.5545648	10.016265	10.570	25
36	9.4296228	9.9836996	9.4459232	10.5540768	10.01633 4	10.57 772	24
37	9.430075	9.9836643	9.4464107	10.5535893	10.0163357	10.5699 5	23
38	9.4305267	9.9836290	9.4468978	10.5531022	10.01637	10.5 733	22
39	9.4309779	9.9835936	9.4473843	10.5526157	10.0164004	10.569 21	21
40	9.4314286	9.9835582	9.4478704	10.5521296	10.0164418	10.5 857 4	20
41	9.4318788	9.9835227	9.4483561	10.5516439	10.0164773	10.568 212	19
42	9.4323285	9.9834872	9.4488413	10.5511587	10.0165128	10.56767	18
43	9.4327777	9.9834517	9.4493260	10.5506740	10.0165483	10.5672223	17
44	9.433226	9.9834161	9.4498102	10.5501898	10.0165839	10.5667736	16
45	9.433674	9.9833805	9.4502940	10.5497060	10.0166195	10.5663254	15
46	9.4341223	9.9833449	9.4507774	10.5492226	10.0166551	10.5658777	14
47	9.4345694	9.9833092	9.4512602	10.5487398	10.0166908	10.5654306	13
48	9.4350161	9.9832735	9.4517427	10.5482573	10.0167265	10.5649839	12
49	9.4354623	9.9832377	9.4522246	10.5477754	10.0167623	10.5645377	11
50	9.435908	9.9832019	9.4527061	10.5472939	10.0167981	10.5640920	10
51	9.4363532	9.9831661	9.4531872	10.5468128	10.0168339	10.5636468	9
52	9.436798	9.9831302	9.4536678	10.5463322	10.0168698	10.5632020	8
53	9.4372422	9.9830942	9.4541479	10.5458521	10.0169058	10.5627578	7
54	9.4376859	9.9830583	9.4546276	10.5453724	10.0169417	10.5623141	6
55	9.4381292	9.9830223	9.4551069	10.5448931	10.0169777	10.5618708	5
56	9.4385719	9.9829862	9.4555857	10.5444143	10.0170138	10.5614281	4
57	9.4390142	9.9829501	9.4560641	10.5439359	10.0170499	10.5609858	3
58	9.4394560	9.9829140	9.4565420	10.5434580	10.0170860	10.5605440	2
59	9.4398973	9.9828778	9.4570194	10.5429806	10.0171222	10.5601027	1
60	9.440338	9.9828416	9.4574964	10.5425036	10.0171584	10.5596619	0
		Sine.		Tang.		Secant.	*M*

74 *Degrees.*

A Table of Artificial Sines,

16 *Degrees.*

	Sine.		Tang.		Secant.		
0	9.4403381	9.9828416	9.4574964	10.5425036	10.0171584	10.5596619	60
1	9.4407784	9.9828054	9.4579730	10.5420270	10.0171946	10.5592216	59
2	9.4412182	9.9827691	9.4584491	10.5415509	10.0172309	10.5587818	58
3	9.4416576	9.9827328	9.4589248	10.5410752	10.0172672	10.5583424	57
4	9.4420965	9.9826964	9.4594001	10.5405999	10.0173036	10.5579035	56
5	9.4425349	9.9826600	9.4598749	10.5401251	10.0173400	10.5574651	55
6	9.4429728	9.9826236	9.4603492	10.5396508	10.0173764	10.5570272	54
7	9.4434103	9.9825871	9.4608232	10.5391768	10.0174129	10.5565897	53
8	9.443847	9.9825506	9.4612967	10.5387033	10.0174494	10.5561528	52
9	9.4442837	9.9825140	9.461769	10.5382303	10.017486	10.5557163	51
10	9.4447197	9.9824774	9.4622423	10.5377577	10.0175226	10.5552803	50
11	9.4451553	9.9824408	9.4627145	10.5372855	10.0175592	10.5548447	49
12	9.4455904	9.9824041	9.4631863	10.5368137	10.0175959	10.5544096	48
13	9.4460250	9.9823674	9.4636576	10.5363424	10.0176326	10.5539750	47
14	9.4464591	9.9823306	9.4641285	10.5358715	10.0176694	10.5535409	46
15	9.4468927	9.9822938	9.4645990	10.5354010	10.0177062	10.5531073	45
16	9.4473259	9.9822569	9.4650690	10.5349310	10.0177431	10.5526741	44
17	9.4477586	9.9822201	9.4655386	10.5344614	10.0177799	10.5522414	43
18	9.4481909	9.9821831	9.4660078	10.5339922	10.0178169	10.5518091	42
19	9.4486227	9.9821462	9.4664765	10.5335235	10.0178538	10.5513773	41
20	9.4490540	9.9821092	9.4669448	10.5330552	10.0178908	10.5509460	40
21	9.4494849	9.9820721	9.4674127	10.5325873	10.0179279	10.5505151	39
22	9.4499153	9.9820351	9.4678802	10.5321198	10.0179649	10.5500847	38
23	9.4503452	9.9819979	9.4683473	10.5316527	10.0180021	10.5496548	37
24	9.4507747	9.9819608	9.4688139	10.5311861	10.0180392	10.5492253	36
25	9.4512037	9.9819236	9.4692801	10.5307199	10.0180764	10.5487963	35
26	9.4516322	9.9818863	9.4697459	10.5302541	10.0181137	10.5483678	34
27	9.4520603	9.9818490	9.4702112	10.5297888	10.0181510	10.5479397	33
28	9.4524879	9.9818117	9.4706762	10.5293238	10.0181883	10.5475121	32
29	9.4529151	9.9817744	9.4711407	10.5288593	10.0182256	10.5470849	31
30	9.4533418	9.9817370	9.4716048	10.5283952	10.0182630	10.5466582	30
		Sine.		Tang.		Secant.	M

73 *Degrees.*

16 Degrees.

M	Sine.		Tang.		Secant.		
30	9.4533415	9.9817370	9.4716048	10.5283952	10.0182630	10.5466582	30
31	9.4537681	9.9816995	9.4720685	10.5279315	10.0183005	10.5462319	29
32	9.4541935	9.9816620	9.4725318	10.5274682	10.0183380	10.5458061	28
33	9.4546192	9.9816245	9.4729947	10.5270053	10.0183755	10.5453808	27
34	9.4550441	9.9815870	9.4734572	10.5265428	10.0184130	10.5449559	26
35	9.4554686	9.9815494	9.4739192	10.5260808	10.0184506	10.5445314	25
36	9.4558926	9.9815117	9.4743808	10.5256192	10.0184883	10.5441074	24
37	9.4563161	9.9814740	9.4748421	10.5251579	10.0185260	10.5436839	23
38	9.4567392	9.9814363	9.4753029	10.5246971	10.0185637	10.5432608	22
39	9.4571618	9.9813986	9.4757633	10.5242367	10.0186014	10.5428382	21
40	9.4575840	9.9813608	9.4762233	10.5237767	10.0186392	10.5424160	20
41	9.4580058	9.9813229	9.4766829	10.5233171	10.0186771	10.5419942	19
42	9.4584271	9.9812850	9.4771421	10.5228579	10.0187150	10.5415729	18
43	9.4588480	9.9812471	9.4776009	10.5223991	10.0187529	10.5411520	17
44	9.4592684	9.9812091	9.4780592	10.5219408	10.0187909	10.5407316	16
45	9.4596884	9.9811711	9.4785172	10.5214828	10.0188289	10.5403116	15
46	9.4601075	9.9811331	9.4789748	10.5210252	10.0188669	10.5398921	14
47	9.4605270	9.9810950	9.4794319	10.5205681	10.0189050	10.5394730	13
48	9.4609456	9.9810569	9.4798887	10.5201113	10.0189431	10.5390544	12
49	9.4613638	9.9810187	9.4803451	10.5196549	10.0189813	10.5386362	11
50	9.4617816	9.9809805	9.4808011	10.5191989	10.0190195	10.5382184	10
51	9.4621989	9.9809423	9.4812566	10.5187434	10.0190577	10.5378011	9
52	9.4626158	9.9809040	9.4817118	10.5182882	10.0190960	10.5373842	8
53	9.4630323	9.9808657	9.4821666	10.5178334	10.0191343	10.5369677	7
54	9.4634483	9.9808273	9.4826210	10.5173790	10.0191727	10.5365517	6
55	9.4638639	9.9807889	9.4830750	10.5169250	10.0192111	10.5361361	5
56	9.4642790	9.9807505	9.4835286	10.5164714	10.0192495	10.5357210	4
57	9.4646938	9.9807120	9.4839818	10.5160182	10.0192880	10.5353062	3
58	9.4651081	9.9806735	9.4844346	10.5155654	10.0193265	10.5348919	2
59	9.4655219	9.9806349	9.4848870	10.5151130	10.0193651	10.5344781	1
60	9.4659353	9.9805963	9.4853390	10.5146610	10.0194037	10.5340647	0
		Sine.		Tang.		Secant.	M

73 Degrees.

A Table of Artificial Sines.

17 Degrees.

M	Sine		Tang.		Secant.		
0	9.4659353	9.9805969	9.4853390	10.5146610	10.0194037	10.5340647	60
1	9.4663483	9.9805577	9.4857907	10.5142093	10.0194423	10.5336517	59
2	9.4667609	9.9805190	9.4862415	10.5137581	10.0194810	10.5332391	58
3	9.4671730	9.9804803	9.4866928	10.5133072	10.0195197	10.5328270	57
4	9.4675848	9.9804415	9.4871433	10.5128567	10.0195585	10.5324152	56
5	9.4679960	9.9804027	9.4875933	10.5124067	10.0195973	10.5320040	55
6	9.4684069	9.9803639	9.4880430	10.5119570	10.0196361	10.5315931	54
7	9.4688173	9.9803250	9.4884924	10.5115076	10.0196750	10.5311827	53
8	9.4692273	9.9802860	9.4889413	10.5110587	10.0197140	10.5307727	52
9	9.4696369	9.9802471	9.4893898	10.5106102	10.0197529	10.5303631	51
10	9.4700461	9.9802081	9.4898380	10.5101620	10.0197919	10.5299539	50
11	9.4704548	9.9801690	9.4902858	10.5097142	10.0198310	10.5295452	49
12	9.4708631	9.9801295	9.4907332	10.5092668	10.0198701	10.5291369	48
13	9.4712710	9.9800908	9.4911802	10.5088198	10.0199092	10.5287290	47
14	9.4716785	9.9800516	9.4916269	10.5083731	10.0199484	10.5283215	46
15	9.4720856	9.9800124	9.4920731	10.5079269	10.0199876	10.5279144	45
16	9.4724922	9.9799732	9.4925190	10.5074810	10.0200268	12.5275078	44
17	9.4728985	9.9799339	9.4929646	10,5070354	10.0200661	12.5271015	43
18	9.4733043	9.9798946	9.4934097	10.5065903	10.0201054	12.5266957	42
19	9.4737097	9.9798552	9.4938545	10.5061455	10.0201448	12.5262903	41
20	9.4741146	9.9798158	9 4942988	10.5057012	10.0201842	12.5258854	40
21	9.4745192	9.9797764	9.4947429	10.5052571	10.0202236	10.4254808	39
22	9.4749234	9.9797369	9.4951865	10.5048135	10.0202631	10.5250766	38
23	9.4753271	9.9796973	9.4956298	10.5043702	10.0203027	10.5246729	37
24	9.4757304	9.9796578	9.4960727	10.5039273	10.0203422	10.5242696	36
25	9.4761334	9.9796182	9.4965152	10.5034848	10.0203818	10.5238666	35
26	9.4765359	9.9795785	9.4969574	10.5030426	10.0204215	10.5234641	34
27	9.4769380	9.9795388	9.4973991	10.5026009	10.0204612	10.5230620	33
28	9.4773396	9.9794991	9.4978406	10.5021594	10.0205005	10.5226604	32
29	9.4777409	9.9794593	9.4982816	10.5017184	10.0205407	10.5222591	31
30	9.4781418	9.9794195	9.4987223	10.5012777	10.0205805	10.5218582	30
		Sine.		Tang.		Secant.	M

72 Degrees.

17 Degrees.

M	Sine.		Tang		Secant.		
30	9.4781418	9.9794195	9.4987223	10.5012777	10.0205805	10.5218582	30
31	9.4785423	9.9793796	9.4991626	10.5008374	10.0206204	10.5214577	29
32	9.4789423	9.9793398	9.4996026	10.5003974	10.0206602	10.5210577	28
33	9.4793420	9.9792998	9.5000422	10.4999578	10.0207002	10.5206580	27
34	9.4797412	9.9792598	9.5004814	10.4995186	10.0207402	10.5202588	26
35	9.4801401	9.9792198	9.5009203	10.4990797	10.0207802	10.5198599	25
36	9.4805385	9.9791798	9.5013588	10.4986412	10.0208202	10.5194615	24
37	9.4809366	9.9791397	9.5017969	10.4982031	10.0208603	10.5190634	23
38	9.4813342	9.9790996	9.5022347	10.4977653	10.0209004	10.5186658	22
39	9.4817315	9.9790594	9.5026721	10.4973279	10.0209406	10.5182685	21
40	9.4821283	9.9790192	9.5031092	10.4968908	10.0209808	10.5178717	20
41	9.4825248	9.9789789	9.5035459	10.4964541	10.0210211	10.5174752	19
42	9.4829208	9.9789386	9.5039822	10.4960178	10.0210614	10.5170792	18
43	9.4833165	9.9788983	9.5044182	10.4955818	10.0211017	10.5166835	17
44	9.4837117	9.9788579	9.5048538	10.4951462	10.0211421	10.5162883	16
45	9.4841066	9.9788175	9.5052891	10.4947109	10.0211825	10.5158934	15
46	9.4845010	9.9787770	9.5057240	10.4942760	10.0212230	10.5154990	14
47	9.4848951	9.9787365	9.5061586	10.4938414	10.0212635	10.5151049	13
48	9.4852888	9.9786960	9.5065928	10.4934072	10.0213040	10.5147112	12
49	9.4856820	9.9786554	9.5070267	10.4929733	10.0213446	10.5143180	11
50	9.4860749	9.9786148	9.5074602	10.4925398	10.0213852	10.5139251	10
51	9.4864674	9.9785741	9.5078933	10.4921067	10.0214259	10.5135326	9
52	9.4868595	9.9785334	9.5083261	10.4916739	10.0214666	10.5131405	8
53	9.4872512	9.9784927	9.5087586	10.4912414	10.0215073	10.5127488	7
54	9.4876426	9.9784519	9.5091907	10.4908093	10.0215481	10.5123574	6
55	9.4880335	9.9784111	9.5096224	10.4903776	10.0215889	10.5119665	5
56	9.4884240	9.9783702	9.5100539	10.4899461	10.0216298	10.5115760	4
57	9.4888142	9.9783293	9.5104849	10.4895151	10.0216707	10.5111858	3
58	9.4892040	9.9782883	9.5109156	10.4890844	10.0217117	10.5107960	2
59	9.4895934	9.9782474	9.5113460	10.4886540	10.0217526	10.5104066	1
60	9.4899824	9.9782063	9.5117760	10.4882240	10.0217937	10.5100176	0
		Sine.		Tang.		Secant.	M

72 Degrees.

18 **Degrees.**

M	Sine.		Tang.		Secant.		
0	9.4899824	9.9782063	9.5117760	10.4882240	10.0217937	10.5100176	60
1	9.4903710	9.9781653	9.5122057	10.4877943	10.0218347	10.5096290	59
2	9.4907592	9.9781241	9.5126351	10.4873649	10.0218759	10.5092408	58
3	9.4911471	9.9780830	9.5130641	10.4869359	10.0219170	10.5088529	57
4	9.4915345	9.9780418	9.5134927	10.4865073	10.0219582	10.5084655	56
5	9.4919210	9.9780006	9.5139210	10.4860790	10.0219994	10.5080784	55
6	9.4923083	9.9779593	9.5143490	10.4856510	10.0220407	10.5076917	54
7	9.4926946	9.9779180	9.5147766	10.4852234	10.0220820	10.5073054	53
8	9.4930806	9.9778766	9.5152039	10.4847961	10.0221234	10.5069194	52
9	9.4934661	9.9778353	9.5156309	10.4843691	10.0221647	10.5065339	51
10	9.4938513	9.9777938	·95160575	10.4839425	10.0222062	10.5061487	50
11	9.4942361	9.9777523	9.5164838	10.4835162	10.0222477	10.5057639	49
12	9.4946205	9.9777108	9.5169097	10.4830903	10.0222892	10.5053795	48
13	9.4950046	9.9776693	9.5173353	10.4826647	10.0223307	10.5049954	47
14	9.4953883	9.9776277	9.5177606	10.4822394	10.0223723	10.5046117	46
15	9.4957716	9.9775860	9.5181855	10.4818145	10.0224140	10.5042284	45
16	9.4961545	9.9775444	9.5186101	10.4813899	10.0224556	10.5038455	44
17	9.4965370	9.9775026	9.5190344	10.4809656	10.0225974	10.5034630	43
18	9.4969192	9.9774609	9.5194583	10.4805417	10.0225391	10.5030808	42
19	9.4973010	9.9774191	9.5198819	10.4801181	10.0225809	10.5026990	41
20	9.4976824	9.9773772	9.5203052	10.4796948	10.0226228	10.5023176	40
21	9.4980635	9.9773354	9.5207282	10.4792718	10.0226646	10.5019365	39
22	9.4984442	9.9772934	9.5211508	10.4788492	10.0227066	10.5015558	38
23	9,4988245	9.9772515	9.5215730	10,4784270	10.0227485	10.5011755	37
24	9.4992045	9.9772095	9.5219950	10.4780050	10.0227905	10.5007955	36
25	9.4995840	9.9771674	9.5224166	10.4775834	10.0228326	10.5004160	35
26	9.4999633	9.9771253	9.5228379	10.4771621	10.0228747	10.5000367	34
27	9.5003421	9.9770832	9.5232589	10.4767411	10.0229168	10.4996579	33
28	9.5007206	9.9770410	9.5236795	10.4763205	10.0229590	10.4992794	32
29	9.5010987	9.9769988	9.5240999	10.4759001	10.0230012	10.4989013	31
30	9.5014764	9.9769566	9.5245199	10.4754801	10.0230434	10.4985236	30
		Sine.		Tang.		Secant.	M

71 **Degrees.**

18 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.5014764	9.9769566	9.5245199	10.4754801	10.0230434	10.4985236	30
31	9.5018538	9.9769143	9.5249395	10.4750605	10.0230857	10.4981462	29
32	9.5022308	9.9768720	9.5253589	10.4746411	10.0231280	10.4977692	28
33	9.5026075	9.9768296	9.5257779	10.4742221	10.0231704	10.4973925	27
34	9.5029838	9.9767872	9.5261966	10.4738034	10.0232128	10.4970162	26
35	9.5033597	9.9767447	9.5266150	10.4733850	10.0232553	10.4966403	25
36	9.5037353	9.9767022	9.5270331	10.4729669	10.0232978	10.4962647	24
37	9.5041105	9.9766597	9.5274508	10.4725492	10.0233403	10.4958895	23
38	9.5044853	9.9766171	9.5278682	10.4721318	10.0233829	10.4955147	22
39	9.5048598	9.9765745	9.5282853	10.4717147	10.0234255	10.4951402	21
0	9.5052339	9.9765318	9.5287021	10.4712979	10.0234682	10.4947661	20
1	9.5056077	9.9764891	9.5291186	10.4708814	10.0235109	10.4943923	19
2	9.5059811	9.9764464	9.5295347	10.4704653	10.0235536	10.4940189	18
3	9.5063542	9.9764036	9.5299505	10.4700495	10.0235964	10.4936458	17
4	9.5067269	9.9763608	9.5303661	10.4696339	10.0236392	10.4932731	16
5	9.5070992	9.9763179	9.5307813	10.4692187	10.0236821	10.4929008	15
6	9.5074712	9.9762750	9.5311961	10.4688039	10.0237250	10.4925288	14
7	9.5078428	9.9762321	9.5316107	10.4683893	10.0237679	10.4921572	13
8	9.5082141	9.9761891	9.5320250	10.4679750	10.0238109	10.4917859	12
9	9.5085850	9.9761461	9.5324389	10.4675611	10.0238539	10.4914150	11
0	9.5089556	9.9761030	9.5328526	10.4671474	10.0238970	10.4910444	10
1	9.5093258	9.9760599	9.5332659	10.4667341	10.0239401	10.4906742	9
2	9.5096956	9.9760167	9.5336789	10.4663211	10.0239833	10.4903044	8
3	9.5100651	9.9759736	9.5340916	10.4659084	10.0240264	10.4899349	7
4	9.5104343	9.9759303	9.5345040	10.465496	10.0240697	10.4895657	6
5	9.5108031	9.9758870	9.5349161	10.4650839	10.0241130	10.4891969	5
6	9.5111716	9.9758437	9.5353278	10.4646722	10.0241563	10.4888284	4
7	9.5115397	9.9758004	9.5357393	10.4642607	10.0241596	10.4884603	3
8	9.5119074	9.9757570	9.5361505	10.4638495	10.0242430	10.4880926	2
9	9.5122749	9.9757135	9.5365613	10.4634387	10.0242865	10.4877251	1
	9.5126419	9.9756701	9.5369719	10.4630281	10.0243295	10.4873581	0
		Sine.		Tang.		Secant.	M

71 *Degrees.*

19 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.5126419	9.9756701	9.5369719	10.4630281	10.0243295	10.4873581	60
1	9.5130086	9.9756265	9.5373821	10.4626179	10.0243735	10.4869914	59
2	9.5133750	9.9755830	9.5377920	10.4622080	10.0244170	10.4866250	58
3	9.5137410	9.9755394	9.5382017	10.4617983	10.0244606	10.4862590	57
4	9.5141067	9.9754957	9.5386110	10.4613890	10.0245043	10.4858933	56
5	9.5144721	9.9754521	9.5390200	10.4609800	10.0245479	10.4855279	55
6	9.5148371	9.9754083	9.5394287	10.4605713	10.0245917	10.4851629	54
7	9.5152017	9.9753646	9.5398371	10.4601629	10.0246354	10.4847983	53
8	9.5155660	9.9753208	9.5402453	10.4597547	10.0246792	10.4844340	52
9	9.5159300	9.9752769	9.5406531	10.4593469	10.0247231	10.4840700	51
10	9.5162936	9.9752330	9.5410606	10.4589394	10.0247670	10.4837064	50
11	9.5166569	9.9751891	9.5414678	10.4585322	10.0248109	10.4833431	49
12	9.5170198	9.9751451	9.5418747	10.4581253	10.0248549	10.4829802	48
13	9.5173824	9.9751011	9.5422813	10.4577187	10.0248989	10.4826176	47
14	9.5177447	9.9750570	9.5426877	10.4573123	10.0249430	10.4822553	46
15	9.5181066	9.9750129	9.5430937	10.4569063	10.0249871	10.4818934	45
16	9.5184682	9.9749688	9.5434994	10.4565006	10.0250312	10.4815318	44
17	9.5188295	9.9749246	9.5439048	10.4560952	10.0250754	10.4811705	43
18	9.5191904	9.9748804	9.5443100	10.4556900	10.0251196	10.4808096	42
19	9.5195510	9.9748361	9.5447148	10.4552852	10.0251639	10.4804490	41
20	9.5199112	9.9747918	9.5451193	10.4548807	10.0252082	10.4800888	40
21	9.5202711	9.9747475	9.5455236	10.4544764	10.0252525	10.4797289	39
22	9.5206307	9.9747031	9.5459276	10.4540724	10.0252969	10.4793693	38
23	9.5209899	9.9746587	9.5463312	10.4536688	10.0253413	10.4790101	37
24	9.5213488	9.9746142	9.5467346	10.4532654	10.0253858	10.4786512	36
25	9.5217074	9.9745697	9.5471377	10.4528623	10.0254303	10.4782926	35
26	9.5220656	9.9745252	9.5475405	10.4524595	10.0254748	10.4779344	34
27	9.5224235	9.9744806	9.5479430	10.4520570	10.0255194	10.4775765	33
28	9.5227811	9.9744359	9.5483452	10.4516548	10.0255641	10.4772189	32
29	9.5231383	9.9743913	9.5487471	10.4512529	10.0256087	10.4768617	31
30	9.5234953	9.9743466	9.5491487	10.4508513	10.0256534	10.4765047	30
		Sine.		Tang.		Secant.	M

70 *Degrees.*

19 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.5234953	9.9743466	9.5491487	10.4508513	10.0256534	10.4765047	30
31	9.5238516	9.9743018	9.5495500	10.4504500	10.0256982	10.4761482	29
32	9.5242081	9.9742570	9.5499511	10.4500489	10.0257430	10.4757919	28
33	9.5245640	9.9742122	9.5503519	10.4496481	10.0257878	10.4754360	27
34	9.5249196	9.9741673	9.5507523	10.4492477	10.0258327	10.4750804	26
35	9.5252749	9.9741224	9.5511525	10.4488475	10.0258776	10.4747251	25
36	9.5256298	9.9740774	9.5515524	10.4484476	10.0259226	10.4743702	24
37	9.5259844	9.9740324	9.5519521	10.4480479	10.0259676	10.4740156	23
38	9.5263387	9.9739873	9.5523514	10.4476486	10.0260127	10.4736613	22
39	9.5266927	9.9739422	9.5527504	10.4472496	10.0260578	10.4733073	21
40	9.5270453	9.9738971	9.5531492	10.4468508	10.0261029	10.4729537	20
41	9.5273997	9.9738519	9.5535477	10.4464523	10.0261481	10.4726003	19
42	9.5277520	9.9738067	9.5539459	10.4460541	10.0261933	10.4722474	18
43	9.5281053	9.9737615	9.5543438	10.4456562	10.0262385	10.4718947	17
44	9.5284577	9.9737162	9.5547415	10.4452585	10.0262838	10.4715423	16
45	9.5288092	9.9736709	9.5551388	10.4448612	10.0263291	10.4711903	15
46	9.5291614	9.9736255	9.5555359	10.4444641	10.0263745	10.4708386	14
47	9.5295128	9.9735801	9.5559327	10.4440673	10.0264199	10.4704872	13
48	9.5298638	9.9735346	9.5563292	10.4436708	10.0264654	10.4701362	12
49	9.5302146	9.9734891	9.5567255	10.4432745	10.0265109	10.4697854	11
50	9.5305650	9.9734435	9.5571214	10.4428786	10.0265565	10.4694350	10
51	9.5309151	9.9733980	9.5575171	10.4424829	10.0266020	10.4690849	9
52	9.5312649	9.9733523	9.5579125	10.4420875	10.0266477	10.4687351	8
53	9.5316143	9.9733067	9.5583077	10.4416923	10.0266933	10.4683857	7
54	9.5319635	9.9732610	9.5587025	10.4412975	10.0267390	10.4680365	6
55	9.5323123	9.9732152	9.5590971	10.4409029	10.0267848	10.4676877	5
56	9.5326608	9.9731694	9.5594914	10.4405086	10.0268306	10.4673392	4
57	9.5330090	9.9731236	9.5598854	10.4401146	10.0268764	10.4669910	3
58	9.5333569	9.9730777	9.5602792	10.4397208	10.0269223	10.4666431	2
59	9.5337044	9.9730318	9.5606727	10.4393273	10.0269682	10.4662956	1
60	9.5340517	9.9729858	9.5610659	10.4389341	10.0270142	10.4659483	0
		Sine.		Tang.		Secant.	M

70 *Degrees.*

20 *Degrees.*

M.	Sine.		Tang.		Secant.		
0	9.5340517	9.9729858	9.5610659	10.4389341	10.0270142	10.4659483	60
1	9.5343986	9.9729398	9.5614588	10.4385412	10.0270602	10.4656014	59
2	9.5347452	9.9728938	9.5618515	10.4381485	10.0271062	10.4652548	58
3	9.5350915	9.9728477	9.5622439	10.4377561	10.0271523	10.4649085	57
4	9.5354375	9.9728016	9.5626360	10.4373640	10.0271984	10.4645625	56
5	9.5357832	9.9727554	9.5630278	10.4369722	10.0272446	10.4642168	55
6	9.5361286	9.9727092	9.5634194	10.4365806	10.0272908	10.4638714	54
7	9.5364737	9.9726629	9.5638107	10.4361893	10.0273371	10.4635263	53
8	9.5368184	9.9726166	9.5642018	10.4357982	10.0273834	10.4631816	52
9	9.5371628	9.9725703	9.5645925	10.4354075	10.0274297	10.4628372	51
10	9.5375070	9.9725239	9.5649831	10.4350169	10.0274761	10.4624930	50
11	9.5378508	9.9724775	9.5653733	10.4346267	10.0275225	10.4621492	49
12	9.5381943	9.9724310	9.5657633	10.4342367	10.0275690	10.4618057	48
13	9.5385375	9.9723845	9.5661530	10.4338470	10.0276155	10.4614625	47
14	9.5388804	9.9723380	9.5665424	10.4334576	10.0276620	10.4611196	46
15	9.5392230	9.9722914	9.5669316	10.4330684	10.0277086	10.4607770	45
16	9.5395653	9.9722448	9.5673205	10.4326795	10.0277552	10.4604347	44
17	9.5399073	9.9721981	9.5677091	10.4322909	10.0278019	10.4600927	43
18	9.5402489	9.9721514	9.5680975	10.4319025	10.0278486	10.4597511	42
19	9.5405903	9.9721047	9.5684856	10.4315144	10.0278953	10.4594097	41
20	9.5409314	9.9720579	9.5688735	10.4311265	10.0279421	10.4590686	40
21	9.5412721	9.9720110	9.5692611	10.4307389	10.0279890	10.4587279	39
22	9.5416126	9.9719642	9.5696484	10.4303516	10.0280358	10.4583874	38
23	9.5419527	9.9719172	9.5700355	10.4299645	10.0280828	10.4580473	37
24	9.5422926	9.9718703	9.5704223	10.4295777	10.0281297	10.4577074	36
25	9.5426321	9.9718233	9.5708088	10.4291912	10.0281767	10.4573679	35
26	9.5429713	9.9717762	9.5711951	10.4288049	10.0282238	10.4570287	34
27	9.5433103	9.9717291	9.5715811	10.4284189	10.0282709	10.4566897	33
28	9.5436489	9.9716820	9.5719669	10.4280331	10.0283180	10.4563511	32
29	9.5439873	9.9716348	9.5723524	10.4276476	10.0283652	10.4560127	31
30	9.5443253	9.9715876	9.5727377	10.4272623	10.0284124	10.4556747	30
		Sine.		Tang.		Secant.	M.

69 *Degrees.*

20 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.5443253	9.9715876	9.5727377	10.4272623	10.0284124	10.4556747	30
31	9.5446630	9.9715404	9.5731227	10.4268773	10.0284596	10.4553370	29
32	9.5450005	9.9714931	9.5735074	10.4264926	10.0285069	10.4549995	28
33	9.5453376	9.9714457	9.5738919	10.4261081	10.0285543	10.4546624	27
34	9.5456745	9.9713984	9.5742761	10.4257239	10.0286016	10.4543255	26
35	9.5460110	9.9713509	9.5746601	10.4253399	10.0286491	10.4539890	25
36	9.5463472	9.9713035	9.5750438	10.4249562	10.0286965	10.4536528	24
37	9.5466832	9.9712560	9.5754272	10.4245728	10.0287440	10.4533168	23
38	9.5470189	9.9712084	9.5758104	10.4241896	10.0287916	10.4529811	22
39	9.5473542	9.9711608	9.5761934	10.4238066	10.0288392	10.4526155	21
40	9.5476893	9.9711132	9.5765761	10.4234239	10.0288868	10.4523107	20
41	9.5480240	9.9710655	9.5769585	10.4230415	10.0289345	10.4519763	19
42	9.5483585	9.9710178	9.5773407	10.4226593	10.0289822	10.4516415	18
43	9.5486927	9.9709701	9.5777226	10.4222774	10.0290299	10.4513073	17
44	9.5490266	9.9709223	9.5781043	10.4218957	10.0290777	10.4509734	16
45	9.5493602	9.9708744	9.5784858	10.4215142	10.0291256	10.4506398	15
46	9.5496935	9.9708265	9.5788669	10.4211331	10.0291735	10.4503065	14
47	9.5500265	9.9707786	9.5792479	10.4207521	10.0292214	10.4499735	13
48	9.5503592	9.9707306	9.5796286	10.4203714	10.0292694	10.4496408	12
49	9.5506916	9.9706826	9.5800090	10.4199910	10.0293174	10.4493084	11
50	9.5510237	9.9706346	9.5803892	10.4196108	10.0293654	10.4489763	10
51	9.5513556	9.9705865	9.5807691	10.4192309	10.0294135	10.4486444	9
52	9.5516871	9.9705383	9.5811488	10.4188512	10.0294617	10.4483129	8
53	9.5520184	9.9704902	9.5815282	10.4184718	10.0295098	10.4479816	7
54	9.5523494	9.9704419	9.5819074	10.4180926	10.0295581	10.4476506	6
55	9.5526801	9.9703937	9.5822864	10.4177136	10.0296063	10.4473199	5
56	9.5530105	9.9703454	9.5826651	10.4173349	10.0296546	10.4469895	4
57	9.5533406	9.9702974	9.5830435	10.4169565	10.0297030	10.4466594	3
58	9.5536704	9.9702486	9.5834217	10.4165783	10.0297514	10.4463296	2
59	9.5539999	9.9702002	9.5837997	10.4162003	10.0297998	10.4460001	1
60	9.5543292	9.9701517	9.5841774	10.4158226	10.0298483	10.4476708	0
		Sine.		Tang.		Secant.	M

69 *Degrees.*

21 *Degrees.*

	Sine.		Tang.		Secant.		
0	9.5543292	9.9701517	9.5841774	10.4158226	10.0298483	10.4456708	60
1	9.5546581	9.9701032	9.5845549	10.4154451	10.0298968	10.4453419	59
2	9.5549868	9.9700547	9.5849321	10.4150679	10.0299453	10.4450132	58
3	9.5553152	9.9700061	9.5853091	10.4146909	10.0299939	10.4446848	57
4	9.5556433	9.9699574	9.5856859	10.4143141	10.0300426	10.4443567	56
5	9.5559711	9.9699087	9.5860624	10.4139376	10.0300913	10.4440289	55
6	9.5562987	9.9698600	9.5864386	10.4135614	10.0301400	10.4437013	54
7	9.5566259	9.9698112	9.5868147	10.4131853	10.0301888	10.4433741	53
8	9.5569529	9.9697624	9.5871904	10.4128096	10.0302376	10.4430471	52
9	9.5572796	9.9697136	9.5875660	10.4124340	10.0302864	10.4427204	51
10	9.5576060	9.9696647	9.5879413	10.4120587	10.0303353	10.4423940	50
11	9.5579321	9.9696158	9.5883163	10.4116837	10.0303842	10.4420679	49
12	9.5582579	9.9695668	9.5886912	10.4113088	10.0304332	10.4417421	48
13	9.5585835	9.9695177	9.5890657	10.4109343	10.0304823	10.4414165	47
14	9.5589088	9.9694687	9.5894401	10.4105599	10.0305313	10.4410912	46
15	9.5592338	9.9694196	9.5898142	10.4101858	10.0305804	10.4407662	45
16	9.5595585	9.9693704	9.5901881	10.4098119	10.0306296	10.4404415	44
17	9.5598829	9.9693212	9.5905617	10.4094383	10.0306788	10.4401171	43
18	9.5602071	9.9692720	9.5909351	10.4090649	10.0307280	10.4397929	42
19	9.5605310	9.9692227	9.5913082	10.4086918	10.0307773	10.4394690	41
20	9.5608546	9.9691734	9.5916812	10.4083188	10.0308266	10.4391454	40
21	9.5611779	9.9691241	9.5920539	10.4079461	10.0308759	10.4388221	39
22	9.5615010	9.9690746	9.5924263	10.4075737	10.0309254	10.4384990	38
23	9,5618237	9.9690252	9.5927985	10,4072015	10.0309748	10.4381763	37
24	9.5621462	9.9689757	9.5931705	10.4068295	10.0310243	10.4378538	36
25	9.5624685	9.9689262	9.5935423	10.4064577	10.0310738	10.4375315	35
26	9.5627904	9.9688766	9.5939138	10.4060862	10.0311234	10.4372096	34
27	9.5631121	9.9688270	9.5942851	10.4057149	10.0311730	10.4368879	33
28	9.5634335	9.9687773	9.5946561	10.4053439	10.0312227	10.4365665	32
29	9.5637546	9.9687276	9.5950269	10.4049731	10.0312724	10.4362454	31
30	9.5640754	9.9686779	9.5953975	10.4046025	10.0313221	10.4359246	30
		Sine.		Tang.		Secant.	M

68 *Degrees.*

21 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.5640754	9.9686779	9.5953975	10.4046025	10.0313221	10.4359246	30
31	9.5643960	9.9686281	9.5957679	10.4042321	10.0313719	10.4356040	29
32	9.5647163	9.9685783	9.5961380	10.4038620	10.0314217	10.4352837	28
33	9.5650363	9.9685284	9.5965079	10.4034921	10.0314716	10.4349637	27
34	9.5653561	9.9684785	9.5968776	10 4031224	10.0315215	10.4346439	26
35	9.5656756	9.9684286	9.5972470	10.4027530	10.0315714	10.4343244	25
36	9.5659948	9.9683786	9.5976162	10.4023838	10.0316214	10.4340052	24
37	9.5663137	9.9683285	9.5979852	10.4020148	10.0316715	10.4336863	23
38	9.5666324	9.9682784	9.5983540	10.4016460	10.0317216	10.4333676	22
39	9.5669508	9.9682283	9.5987225	10.4012775	10.0317717	10.4330492	21
40	9.5672689	9.9681781	9.5990908	10.4009092	10.0318215	10.4327311	20
41	9.5675868	9.9681279	9.5994588	10.4005412	10.0318721	10.4324132	19
42	9.5679044	9.9680777	9.5998267	10.4001733	10.0319223	10.4320956	18
43	9.5682217	9.9680274	9.6001943	10.3998057	10.0319726	10.4317783	17
44	9.5685387	9.9679771	9.6005617	10.3994383	10.0320229	10.4314613	16
45	9.5688555	9 9679267	9.6009289	10.3990711	10.0320733	10.4311445	15
46	9.5691721	9.9678763	9.6012958	10.3987042	10.0321237	10.4308279	14
47	9.5694883	9.9678258	9.6016625	10.3983375	10.0321742	10.4305117	13
48	9.5698043	9.9677753	9.6020290	10.3979710	10.0322247	10.4301957	12
49	9.5701200	9.9677247	9.6023953	10.3976047	10.0322753	10.4298800	11
50	9.5704355	9.9676741	9.6027613	10.3972387	10.0323259	10.4295645	10
51	9.5707506	9.9676235	9.6031271	10.3968729	10.0323765	10.4292494	9
52	9.5710656	9.9675728	9.6034927	10.3965073	10.0324272	10.4289344	8
53	9.5713802	9.9675221	9.6038581	10.3961419	10.0324779	10.4286198	7
54	9.5716946	9.9674713	9.6042233	10.3957767	10.0325287	10.4283054	6
55	9.5720087	9.9674205	9.6045882	10.3954118	10.0325795	10.4279913	5
56	9.5723226	9.9673697	9.6049529	10.3950471	10.0326303	10.4276774	4
57	9.5726362	9.9673188	9 6053174	10.3946826	10.0326812	10.4273638	3
58	9.5729495	9.9672679	9.6056817	10.3943183	10.0327321	10.4270505	2
59	9.5732626	9.9672169	9.6060457	10.3939543	10.0327831	10.4267374	1
60	9.5735754	9.9671659	9 6064096	10.3935904	10.0328341	10.4264246	0
		Sine.		Tang.		Secant.	*M*

68 *Degrees.*

A Table of Artificial Sines,

22 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.5735754	9.9671659	9.6064096	10.3935904	10.0328341	10.4264246	
1	9.5738880	9.9071148	9.6067732	10.3932268	10.0328852	10.4261120	
2	9.5742003	9.9670637	9.6071366	10.3928634	10.0329363	10.4257997	
3	9.5745123	9.9670125	9.6074997	10.3925003	10.0329875	10.4254877	
4	9.5748240	9.9669614	9.6078627	10.3921373	10.0330386	10.4251760	
5	9.5751356	9.9669101	9.6082254	10.3917746	10.0330899	10.4248644	
6	9.5754468	9.9668588	9.6085880	10.3914120	10.0331412	10.4245532	
7	9.5757578	9.9668075	9.6089503	10.3910497	10.0331925	10.4242422	
8	9.5760685	9.9667562	9.6093124	10.3906876	10.0332438	10.4239315	
9	9.5763790	9.9667048	9.6096742	10.3903258	10.0332952	10.4236210	
10	9.5766892	9.9666533	9.6100359	10.3899641	10.0333467	10.4233108	
11	9.5769991	9.9666018	9.6103973	10.3896027	10.0333982	10.4230009	
12	9.5773088	9.9665503	9.6107586	10.3892414	10.0334497	10.4226912	
13	9.5776183	9.9664987	9.6111196	10.3888804	10.0335013	10.4223817	
14	9.5779275	9.9664471	9.6114804	10.3885196	10.0335529	10.4220725	
15	9.5782364	9.9663954	9.6118409	10.3881591	10.0336046	10.4217636	
16	9.5785450	9.9663437	9.6122013	10.3877987	10.0336563	10.4214550	
17	9.5788535	9.9662920	9.6125615	10.3874385	10.0337080	10.4211465	
18	9.5791616	9.9662402	9.6129214	10.3870786	10.0337598	10.4208384	
19	9.5794695	9.9661884	9.6132812	10.3867188	10.0338116	10.4205305	
20	9.5797772	9.9661365	9.6136407	10.3863593	10.0338635	10.4202228	
21	9.5800845	9.9660846	9.6140000	10.3860000	10.0339154	10.4199155	
22	9.5803917	9.9660326	9.6143591	10.3856409	10.0339674	10.4196083	
23	9.5806986	9.9659806	9.6147180	10.3852820	10.0340194	10.4193014	
24	9.5810052	9.9659285	9.6150766	10.3849234	10.0340715	10.4189948	
25	9.5813116	9.9658764	9.6154351	10.3845649	10.0341236	10.4186884	
26	9.5816177	9.9658243	9.6157934	10.3842066	10.0341757	10.4183823	
27	9.5819236	9.9657721	9.6161514	10.3838486	10.0342279	10.4180764	
28	9.5822292	9.9657199	9.6165093	10.3834907	10.0342801	10.4177708	
29	9.5825345	9.9656677	9.6168669	10.3831331	10.0343323	10.4174655	
30	9.5828397	9.9656153	9.6172243	10.3827757	10.0343847	10.4171603	
		Sine.		Tang.		Secant.	M

67 *Degrees.*

22 Degrees.

M	Sine.		Tang.		Secant.		
30	9.5828397	9.9656153	9.6172243	10.3827757	10.0343847	10,4171603	30
31	9.5831445	9.9655630	9.6175815	10.3824185	10.0344370	10.4168555	29
32	9.5834491	9.9655106	9.6179385	10.3820615	10.0344894	10.4165509	28
33	9.5837535	9.9654582	9.6182953	10.3817047	10.0345418	10.4162465	27
34	9.5840576	9.9654057	9.6186519	10 3813481	10.0345943	10.4159424	26
35	9.5843615	9.9653532	9.6190083	10.3809917	10 0346468	10.4156385	25
36	9.5846651	9.9653006	9.6193645	10.3806355	10-0346994	10.4153349	24
37	9.5849684	9.9652480	9.6197205	10.3802795	10.0347520	10.4150315	23
38	9.5852716	9.9651953	9.6200762	10.3799238	10.0348047	10.4147284	22
39	9.5855745	9.9651426	9.6204318	10.3795682	10.0348574	10.4144255	21
40	9.5858771	9.9650899	9.6207872	10.3792128	10.0349101	10.4141229	20
41	9.5861795	9.9650371	9.6211423	10.2788577	10.0349629	10.4138205	19
42	9.5864816	9.9649843	9.6214973	10.3785027	10.0350157	10.4135184	18
43	9.5867835	9.9649314	9.6218520	10.3781480	10.0350686	10.4132165	17
44	9.5870851	9.9648785	9.6222066	10.3777934	10.0351215	10.4129149	16
45	9.5873865	9.9648256	9.6225609	10.3774391	10.0351744	10.4126135	15
46	9.5876876	9.9647726	9.6229150	10.3770850	10.0352274	10.4123124	14
47	9.5879885	9.9647195	9.6232690	10.3767310	10.0352805	10.4120115	13
48	9.5882892	9,9646665	9.6236227	10.3763773	10.0353335	10.4117108	12
49	9.5885896	9.9646133	9.6239763	10.3760237	10.0353867	10.4114104	11
50	9,5888897	9.9645602	9.6243296	10.3756704	10.0354398	10.4111103	10
51	9,5891897	9.9645069	9.6246827	10.3753173	10.0354931	10 4108103	9
52	9.5894893	9.9644537	9.6250356	10.3749644	10.0355463	10.4105107	8
53	9.5897888	9.9644004	9.6253884	10.3746116	10.0355996	10.4102112	7
54	9.5900880	9.9643470	9.6257409	10.3742591	10.0356530	10.4099120	6
55	9.5903869	9.9642937	9.6260932	10.3739068	10.0357063	10.4096131	5
56	9.5906856	9.9642402	9.6264454	10.3735546	10.0357598	10.4093144	4
57	9.5909841	9.9641868	9 6267973	10.3732027	10.0358132	10.4090159	3
58	9.5912823	9.9641332	9.6271491	10.3728509	10.0358668	10.4087177	2
59	9.5915803	9.9640797	9.6275006	10.3724994	10.0359203	10.4084197	1
60	9.5918780	9.9640261	9.6278519	10.3721481	10.0359739	10.4081220	0
		Sine.		Tang.		Secant.	M

67 Degrees.

A Table of Artificial Sines,

23 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.5918780	9.9640261	9.6278519	10.3721481	10.0359739	10.4081220	60
1	9.5921755	9.9639724	9.6282031	10.3717969	10.0360276	10.4078245	59
2	9.5924728	9.9639187	9.6285540	10.3714460	10.0360813	10.4075272	58
3	9.5927698	9.9638650	9.6289048	10.3710952	10.0361350	10.4072302	57
4	9.5930666	9.9638112	9.6292553	10.3707447	10.0361888	10.4069334	56
5	9.5933631	9.9637574	9.6296057	10.3703943	10.0362426	10.4066369	55
6	9.5936594	9.9637036	9.6299558	10.3700442	10.0362964	10.4063406	54
7	9.5939555	9.9636496	9.6303058	10.3696942	10.0363504	10.4060445	53
8	9.5942513	9.9635957	9.6306556	10.3693444	10.0364043	10.4057487	52
9	9.5945469	9.9635417	9.6310052	10.3689948	10.0364583	10.4054531	51
10	9.5948422	9.9634877	9.6313545	10.3686455	10.0365123	10.4051578	50
11	9.5951373	9.9634336	9.6317037	10.3682963	10.0365664	10.4048627	49
12	9.5954322	9.9633795	9.6320527	10.3679473	10.0366205	10.4045678	48
13	9.5957268	9.9633253	9.6324015	10.3675985	10.0366747	10.4042732	47
14	9.5960212	9.9632711	9.6327501	10.3672499	10.0367289	10.4039788	46
15	9.5963154	9.9632168	9.6330985	10.3669015	10.0367832	10.4036846	45
16	9.5966093	9.9631625	9.6334468	10.3665532	10.0368375	10.4033907	44
17	9.5969030	9.9631082	9.6337948	10.3662052	10.0368918	10.4030970	43
18	9.5971965	9.9630538	9.6341426	10.3658574	10.0369462	10.4028035	42
19	9.5974897	9.9629994	9.6344903	10.3655097	10.0370006	10.4025103	41
20	9.5977827	9.9629449	9.6348378	10.3651622	10.0370551	10.4022173	40
21	9.5980754	9.9628904	9.6351850	10.3648150	10.0371096	10.4019246	39
22	9.5983679	9.9628358	9.6355321	10.3644679	10.0371642	10.4016321	38
23	9.5986602	9.9627812	9.6358790	10.3641210	10.0372188	10.4013398	37
24	9.5989523	9.9627266	9.6362257	10.3637743	10.0372734	10.4010477	36
25	9.5992441	9.9626719	9.6365722	10.3634278	10.0373281	10.4007559	35
26	9.5995357	9.9626172	9.6369185	10.3630815	10.0373828	10.4004643	34
27	9.5998270	9.9625624	9.6372646	10.3627354	10.0374376	10.4001730	33
28	9.6001181	9.9625076	9.6376106	10.3623894	10.0374924	10.3998819	32
29	9.6004090	9.9624527	9.6379563	10.3620437	10.0375473	10.3995910	31
30	9.6006997	9.9623978	9.6383019	10.3616981	10.0376022	10.3993003	30
		Sine.		Tang.		Secant.	M

66 *Degrees.*

23 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.6006997	9.9623978	9.6383019	10.3616981	10.0376022	10.3993003	30
31	9.6019901	9.9623428	9.6386473	10.3613527	10.0376572	10.3990099	29
32	9.6012803	9.9622878	9.6389925	10.3610075	10.0377122	10.3987197	28
33	9.6015703	9.9622328	9.6393375	10.3606625	10.0377672	10.3984297	27
34	9.6018600	9,9621777	9.6396823	10.3603177	10.0378223	10.3981400	26
35	9.6021495	9.9621226	9.6400269	10.3599731	10.0378774	10.3978505	25
36	9.6024388	9.9620674	9.6403714	10.3596286	10.0379326	10.3975612	24
37	9.6027278	9.9620122	9.6407156	10.3592844	10.0379878	10.3972722	23
38	9.6030166	9.9619569	9.6410597	10.3589403	10.0380431	10.3969834	22
39	9.6033052	9.9619016	9.6414036	10.3585964	10.0380984	10.3966948	21
40	9.6035936	9.9618463	9.6417473	10.3582527	10.0381537	10.3964064	20
41	9.6038817	9,9617909	9.6420908	10.3579092	10.0382091	10.3961183	19
42	9.6041696	9.9617355	9.6424342	10.3575658	10.0382645	10.3958304	18
43	9.6044573	9.9616800	9.6427773	10.3572227	10.0383200	10.3955427	17
44	9.6047448	9.9616245	9.6431203	10.3568797	10.0383755	10.3952552	16
45	9.6050320	9.9615689	9.6434631	10.3565369	10.0384311	10.3949680	15
46	9.6053190	9.9615133	9.6438057	10.3561943	10.0384867	10.3946810	14
47	9.6056057	9.9614576	9.6441481	10.3558519	10.0385424	10.3943943	13
48	9.6058923	9.9614020	9.6444903	10.3555097	10.0385980	10.3941077	12
49	9.6061786	9.9613462	9.6448324	10.3551676	10.0386538	10.3938214	11
50	9.6064647	9.9612904	9.6451743	10.3548257	10.0387096	10.3935353	10
51	9.6067506	9.9612346	9.6455160	10.3544840	10.0387654	10.3932494	9
52	9.6070362	9.9611787	9.6458575	10.3541425	10.0388213	10.3929638	8
53	9.6073216	9.9611228	9.6461988	10.3538012	10.0388772	10.3926784	7
54	9.6076068	9.9610668	9.6465400	10.3534600	10.0389332	10.3923932	6
55	9.6078918	9.9610108	9.6468810	10.3531190	10.0389892	10.3921082	5
56	9.6081765	9.9609548	9.6472217	10.3527783	10.0390452	10.3918235	4
57	9.6084611	9.9608987	9.6475624	10.3524376	10.0391013	10.3915389	3
58	9.6087454	9.9608426	9.6479028	10.3520972	10.0391574	10.3912546	2
59	9,6090294	9.9607864	9.6482431	10.3517569	10.0392136	10.3909706	1
60	9.6093133	9.9607302	9.6485831	10.3514169	10.0392698	10.3906867	0
		Sine.		Tang.		Secant.	M

66 *Degrees.*

24 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.6093133	9.9607302	9.6485831	10.3514169	10.0392698	10.3906867	60
1	9.6095969	9.9606739	9.6489230	10.3510770	10.0393261	10.3904031	59
2	9.6098803	9.9606170	9.6492628	10.3507372	10.0393824	10.3901197	58
3	9.6101635	9.9605612	9.6496023	10.3503977	10.0394388	10.3898365	57
4	9.6104465	9.9605048	9.6499417	10.3500583	10.0394952	10.3895535	56
5	9.6107293	9.9604484	9.6502809	10.3497191	10.0395516	10.3892707	55
6	9.6110118	9.9603919	9.6506199	10.3493801	10.0396081	10.3889882	54
7	9.6112941	9.9603354	9.6509587	10.3490413	10.0396646	10.3887059	53
8	9.6115762	9.9602788	9.6512974	10.3487026	10.0397212	10.3884238	52
9	9.6118580	9.9602222	9.6516359	10.3483641	10.0397778	10.3881420	51
10	9.6121397	9.9601655	9.6519742	10.3480258	10.0398345	10.3878603	50
11	9.6124211	9.9601088	9.6523123	10.3476877	10.0398912	10.3875789	49
12	9.6127023	9.9600520	9.6526503	10.3473497	10.0399480	10.3872977	48
13	9.6129833	9.9599952	9.6529881	10.3470119	10.0400048	10.3870167	47
14	9.6132641	9.9599384	9.6533257	10.3466743	10.0400616	10.3867359	46
15	9.6135446	9.9598815	9.6536631	10.3463369	10.0401185	10.3864554	45
16	9.6138250	9.9598246	9.6540004	10.3459996	10.0401754	10.3861750	44
17	9.6141051	9.9597676	9.6543375	10.3456625	10.0402324	10.3858949	43
18	9.6143850	9.9597106	9.6546744	10.3453256	10.0402894	10.3856150	42
19	9.6146647	9.9596535	9.6550112	10.3449888	10.0403465	10.3853353	41
20	9.6149441	9.9595964	9.6553477	10.3446523	10.0404036	10.3850559	40
21	9.6152234	9.9595393	9.6556841	10.3443159	10.0404607	10.3847766	39
22	9.6155024	9.9594821	9.6560204	10.3439796	10.0405179	10.3844976	38
23	9.6157812	9.9594248	9.6563564	10.3436436	10.0405752	10.3842188	37
24	9.6160599	9.9593675	9.6566923	10.3433077	10.0406325	10.3839401	36
25	9.6163382	9.9593102	9.6570280	10.3429720	10.0406898	10.3836618	35
26	9.6166164	9.9592528	9.6573636	10.3426364	10.0407472	10.3833839	34
27	9.6168944	9.9591954	9.6576989	10.3423011	10.0408046	10.3831059	33
28	9.6171721	9.9591380	9.6580341	10.3419659	10.0408620	10.3828279	32
29	9.6174496	9.9590805	9.6583692	10.3416308	10.0409195	10.3825504	31
30	9.6177270	9.9590229	9.6587041	10.3412959	10.0409771	10.3822730	30
		Sine.		Tang.		Secant.	M

65 *Degrees.*

24 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.6177270	9.9590229	9.6587041	10.3412959	10.0409771	10.3822730	30
31	9.6180041	9.9589653	9.6590387	10.3409613	10.0410347	10.3819959	29
32	9.6182809	9.9589077	9.6593733	10.3406267	10.0410923	10.3817191	28
33	9.6185576	9.9588500	9.6597076	10.3402924	10.0411500	10.3814424	27
34	9.6188341	9.9587923	9.6600418	10.3399582	10.0412077	10.3811659	26
35	9.6191103	9.9587345	9.6603758	10.3396242	10.0412655	10.3808897	25
36	9.6193864	9.9586767	9.6607097	10.3392903	10.0413233	10.3806136	24
37	9.6196622	9.9586188	9.6610434	10.3389566	10.0413812	10.3803378	23
38	9.6199378	9.9585609	9.6613769	10.3386231	10.0414391	10.3800622	22
39	9.6202132	9.9585030	9.6617103	10.3382897	10.0414970	10.3797868	21
40	9.6204884	9.9584450	9.6620434	10.3379566	10.0415550	10.3795116	20
41	9.6207634	9.9583869	9.6623765	10.3376235	10.0416131	10.3792366	19
42	9.6210382	9.9583288	9.6627093	10.3372907	10.0416712	10.3789618	18
43	9.6213127	9.9582707	9.6630420	10.3369580	10.0417293	10.3786873	17
44	9.6215871	9.9582125	9.6633745	10.3366255	10.0417875	10.3784129	16
45	9.6218612	9.9581543	9.6637069	10.3362931	10.0418457	10.3781388	15
46	9.6221351	9.9580961	9.6640391	10.3359609	10.0419039	10.3778649	14
47	9.6224088	9.9580378	9.6643711	10.3356289	10.0419622	10.3775912	13
48	9.6226824	9.9579794	9.6647030	10.3352970	10.0420206	10.3773176	12
49	9.6229557	9.9579210	9.6650346	10.3349654	10.0420790	10.3770443	11
50	9.6232287	9.9578626	9.6653662	10.3346338	10.0421374	10.3767713	10
51	9.6235016	9.9578041	9.6656975	10.3343025	10.0421959	10.3764984	9
52	9.6237743	9.9577456	9.6660288	10.3339712	10.0422544	10.3762257	8
53	9.6240468	9.9576870	9.6663598	10.3336402	10.0423130	10.3759532	7
54	9.6243190	9.9576284	9.6666907	10.3333093	10.0423716	10.3756810	6
55	9.6245911	9.9575697	9.6670214	10.3329786	10.0424303	10.3754089	5
56	9.6248629	9.9575110	9.6673519	10.3326481	10.0424890	10.3751371	4
57	9.6251346	9.9574522	9.6676823	10.3323177	10.0425478	10.3748654	3
58	9.6254060	9.9573934	9.6680126	10.3319874	10.0426066	10.3745940	2
59	9.6256772	9.9573346	9.6683426	10.3316574	10.0426654	10.3743228	1
60	9.6259483	9.9572757	9.6686725	10.3313275	10.0427243	10.3740517	0
		Sine.		Tang.		Secant.	M

65 *Degrees.*

25 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.6259483	9.9572757	9.6686725	10.3313275	10.0427243	10.3740517	60
1	9.6262191	9.9572168	9.6690023	10.3309977	10.0427832	10.3737809	59
2	9.6264897	9.9571578	9.6693319	10.3306681	10.0428422	10.3735103	58
3	9.6267601	9.9570988	9.6696613	10.3303387	10.0429012	10.3732399	57
4	9.6270303	9.9570397	9.6699906	10.3300094	10.0429603	10.3729697	56
5	9.6273003	9.9569806	9.6703197	10.3296803	10.0430194	10.3726997	55
6	9.6275701	9.9569215	9.6706486	10.3293514	10.0430785	10.3724299	54
7	9.6278397	9.9568623	9.6709774	10.3290226	10.0431377	10.3721603	53
8	9.6281090	9.9568030	9 6713060	10.3286940	10.0431970	10.3718910	52
9	9.6283782	9.9567437	9.6716345	10.3283655	10.0432563	10.3716218	51
10	9.6286472	9.9566844	9.6719628	10.3280372	10 0433150	10.3713528	50
11	9.6289160	9.9566250	9.6722910	10.3277090	10.0433750	10.3710840	49
12	9.6291845	9.9565656	9.6726190	10.3273810	10.0434344	10.3708155	48
13	9.6294529	9.9565051	9.6729468	10.3270532	10.0434939	10.3705471	47
14	9.6297211	9.9564466	9.6732745	10.3267255	10.0435534	10.3702789	46
15	9.6299890	9.9563870	9.6736020	10.3263980	10.0436130	10.3700110	45
16	9.6302568	9.9563274	9.6739294	10 3260706	10.0436726	10.3697432	44
17	9.6305243	9.9562678	9.6742566	10.3257434	10.0437322	10.3694757	43
18	9.6307917	9.9562081	9.6745835	10.3254164	10.0437919	10.3692083	42
19	9.6310589	9.9561483	9.6749105	10.3250895	10.0438517	10.3689411	41
20	9.6313258	9.9560886	9.6752372	10.3247628	10.0439114	10.3686742	40
21	9.6315926	9.9560287	9.6755638	10.3244362	10.0439713	10.3684074	39
22	9.6318591	9.9559689	9.6758903	10.3241097	10.0440311	10.3681409	38
23	9.6321255	9.9559089	9.6762165	10.3237835	10.0440911	10.3678745	37
24	9.6323916	9.9558490	9.6765426	10.3234574	10.0441510	10.3676084	36
25	9.6326576	9.9557890	9.6768686	10.3231314	10.0442110	10.3673424	35
26	9.6329233	9.9557289	9.6771944	10.3228056	10 0442711	10.3670767	34
27	9.6331889	9.9556688	9.6775201	10,3224799	10.0443312	10.3668111	33
28	9.6334545	9.9556087	9.6778456	10.3221544	10.0443913	10.3665455	32
29	9.6337194	9.9555485	9.6781709	10.3218291	10.0444515	10.3662806	31
30	9.6339844	9.9554882	9.6784961	10.3215039	10.0445118	10.3660156	30
		Sine.		Tang.		Secant.	M

64 *Degrees.*

25 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.6339844	9.9554882	9.6784961	10.3215039	10.0445118	10,3660156	30
31	9.6342491	9.9554280	9 6788211	10.3211789	10.0445720	10.3557509	29
32	9.6345137	9.9553676	9.6791460	10.3208540	10.0446324	10.3654863	28
33	9.6347780	9.9553073	9.6794708	10.3205292	10.0446927	10.3652220	27
34	9.6350422	9.9552469	9.6797953	10 3202047	10.0447531	10.3649578	26
35	9.6353062	9.9551864	9.6801198	10.3198802	10.0448136	10.3646938	25
36	9.6355695	9.9551259	9.6804440	10.3195560	10-0448741	10.3644301	24
37	9.6358335	9.9550653	9.6807682	10.3192318	10.0449347	10.3641665	23
38	9.6360969	9.9550047	9.6810921	10.3189079	10.0449953	10.3639031	22
39	9.6363601	9.9549441	9.6814160	10.3185840	10.0450559	10.3636399	21
40	9.6366231	9.9548834	9.6817396	10.3182604	10.0451166	10.3633769	20
41	9.6368859	9.9548227	9.6820632	10.2179368	10.0451773	10.3631141	19
42	9.6371484	9.9547619	9.6823865	10.3176135	10.0452381	10.3628516	18
43	9.6374108	9.9547011	9.6827098	10.3172902	10.0452989	10.3625892	17
44	9.6376731	9.9546402	6.6830328	10.3169672	10.0453598	10.3623269	16
45	9.6379351	9.9545793	6.6833557	10.3166443	10.0454207	10.3620649	15
46	9.6381969	9.9545184	6.6836785	10.3163215	10.0454816	10.3618031	14
47	9.6384585	9.9544574	9.6840011	10.3159989	10.0455426	10.3615415	13
48	9.6387199	9,9543963	9.6843236	10.3156764	10.0456037	10.3612801	12
49	9.6389812	9.9543352	9.6846459	10.3153541	10.0456648	10.3610188	11
50	9,6392422	9.9542741	9.6849681	10.3150319	10.0457259	10.3607578	10
51	9,6395030	9.9542129	9.6852901	10.3147099	10.0457871	10.3604970	9
52	9.6397637	9.9541517	9.6856120	10.3143880	10.0458483	10.3602363	8
53	9.6400241	9.9540904	9.6859338	10.3140662	10.0459096	10.3599759	7
54	9.6402844	9.9540291	9.6862553	10.3137447	10.0459709	10.3597156	6
55	9.6405445	9 9539677	9.6865768	10.3134232	10.0460323	10.3594555	5
56	9.6408044	9.9539063	9.6868981	10.3131019	10.0460937	10.3591956	4
57	9.6410640	9.9538448	9.6872192	10.3127808	10.0461552	10.3589360	3
58	9.6413235	9.9537833	9.6875402	10.3124598	10.0462167	10.3586765	2
59	9.6415828	9.9537218	9.6878611	10.3121389	10.0462782	10.3584172	1
60	9 6418420	9.9536602	9.6881818	10.3118182	10.0463398	10.3581580	0
		Sine.		Tang.		Secant.	M

64 *Degrees.*

A Table of Artificial Sines,

26 *Degrees.*

M	Sine.		Tang.		Secant.	
0	9.6418420	9.9536602	9.6881818	10.3118182	10.0463398	10.3581580
1	9.6421009	9.9535985	9.688502[illegible]	10.3114977	10.0464015	10.3578991
2	9.6423596	9.9535369	9.6888227	10.3111773	10.0464631	10.3576404
3	9.6426182	9.9534751	9.689143[illegible]	10.3108570	10.0465249	10.357381[illegible]
4	9.6428765	9.9534134	9.689463[illegible]	10.3105369	10.0465860	10.3571235
5	9.6431347	9.9533515	9.689783[illegible]	10.3102169	10.0466485	10.3568653
6	9.6433926	9.9532897	9.6901030	10.3098970	10.0467103	10.3566074
7	9.6436504	9.9532278	9.6904226	10.3095774	10.0467722	10.3563496
8	9.6439080	9.9531658	9.690742[illegible]	10.3092578	10.0468342	10.356092[illegible]
9	9.6441654	9.9531038	9.6910616	10.3089384	10.0468962	10.3558346
10	9.6444226	9.9530418	9.6913809	10.3086191	10.046958[illegible]	10.3555774
11	9.6446796	9.9529797	9.6917000	10.3083000	10.0470203	10.3553204
12	9.6449365	9.9529175	9.6920189	10.3079811	10.0470825	10.3550635
13	9.6451931	9.9528553	9.692337[illegible]	10.3076622	10.0471447	10.3548069
14	9.6454496	9.9527931	9.692656[illegible]	10.3073435	10.0472069	10.3545504
15	9.6457[illegible]8	9.9527308	9.6929750	10.3070250	10.0472692	10.354254[illegible]
16	9.6459619	9.952668[illegible]	9.693[illegible]293	10.3067166	10.0473315	10.3540381
17	9.6462178	9.9526061	9.693611[illegible]	10.3063883	10.0473939	10.3537822
18	9.64647[illegible]	9.9525437	9.6939298	10.3060702	10.0474563	10.3535265
19	9.6467[illegible]	9.952481	9.694[illegible]47[illegible]	10.3057522	10.0475187	10.353271[illegible]
20	9.6469844	9.952418[illegible]	9.6945656	10.3054344	10.0475812	10.353[illegible]156
21	9.647[illegible]395	9.9523562	9.6948833	10.3051167	10.0476438	10.3527605
22	9.6474945	9.952293[illegible]	9.6952009	10.304799	10.0477064	10.352505[illegible]
23	9.6477492	9.9522310	9.6955183	10.3044817	10.0477690	10.352250[illegible]
24	9.6480038	9.9521683	9.6958355	10.3041645	10.0478317	10.351996[illegible]
25	9.6482582	9.9521055	9.6961527	10.3038473	10.0478945	10.351741[illegible]
26	9.6485124	9.9520425	9.696469[illegible]	10.3035303	10.0479572	10.3514876
27	9.6487665	9.9519799	9.696678[illegible]	10.303213[illegible]	10.0480201	10.3512335
28	9,6490203	9.9519171	9.697[illegible]	10.3028968	10.0480829	10.350979[illegible]
29	9.6492740	9.9518541	9.6974198	10.3025802	10.0481459	10.350726[illegible]
30	9.6495274	9.9517912	9.6977363	10.3022637	10.0482088	10.3504726
		Sine.		Tang.		Secant.

63 *Degrees.*

26 *Degrees.*

M	Sine.		Tang.		Secant		
30	9.6495274	9.9517912	9.6977363	10.3022637	10.0482088	10,3504726	30
3	9.6497807	9.9517282	9.6980526	10.3019474	10.0482718	10.3502193	29
3	9.6500338	9.9516651	9.6983687	10.3016313	10.0483349	10.3499662	28
3	9.6502868	9.9516020	9.6986847	10.3013153	10.0483980	10.3497132	27
34	9.6505395	9.9515389	9.6990006	10.3009994	10.0484611	10.3494605	26
35	9.6507920	9.9514757	9.6993164	10.3006836	10.0485243	10.3492080	25
36	9.6510444	9.9514124	9.6996320	10.3003680	10.0485876	10.3489556	24
37	9.6512966	9.9513492	9.6999474	10.3000526	10.0486508	10.3487034	23
38	9.6515486	9.9512858	9.7002628	10.2997372	10.0487142	10.3484514	22
39	9.6518004	9.9512224	9.7005780	10.2994220	10.0487776	10.3481996	21
40	9.6520521	9.9511590	9.7008930	10.2991070	10.0488410	10.3479479	20
41	9.6523035	9.9510956	9.7012080	10.2987920	10.0489044	0.3476965	19
42	9.6525548	9.9510320	9.7015227	10.2984773	10.0489680	10.3474452	18
43	9.6528059	9.9509685	9.7018374	10.2981626	10.0490315	10.3471941	17
44	9.6530568	9.9509049	6.7021519	10.2978481	10.0490951	10.3469432	16
45	9.6533075	9.9508412	6.7024663	10.2975337	10.0491588	10.3466925	15
46	9.6535581	9.9507775	6.7027805	10.2972195	10.0492225	10.3464419	14
47	9.6538084	9.9507138	9.7030946	10.2969054	10.0492862	10.3461916	13
48	9.6540586	9,9506500	9.7034086	10.2965914	10.0493500	10.3459414	12
49	9.6543086	9.9505861	9.7037225	10.2962775	10.0494139	10.3456914	11
50	9,6545584	9.9505223	6.7040362	10.2959638	10.0494777	10.3454416	10
51	9,6548081	9.9504583	9.7043497	10.2956503	10.0495417	10.3451919	9
52	9.6550575	9.9503944	9.7046632	10.2953368	10.0496056	10.3449425	8
53	9.6553068	9.9503303	9.7049765	10.2950235	10.0496697	10.3446932	7
54	9.6555559	9.9502663	9.7052897	10.2947103	10.0497337	10.3444441	6
55	9.6558048	9.9502022	9.7056027	10.2943973	10.0497978	10.3441952	5
56	9.6560536	9.9501380	9.7059156	10.2940844	10.0498620	10.3439464	4
57	9.6563021	9.9500738	9.7062284	10.2937716	10.0499262	10.3436979	3
58	9.6565505	9.9500095	9.7065410	10.2934590	10.0499905	10.3434495	2
59	9.6567987	9.9499452	9.7068535	10.2931465	10.0500548	10.3432013	1
60	9.6570468	9.9498809	9.7071659	10.2928341	10.0501191	10.3439532	0
		Sine.		Tang.		Secant.	M

63 *Degrees.*

27 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.6570468	9.9498809	9.7071655	10.292834	10.0501191	10.3429532	
1	9.6572946	9.9498165	9.7074781	10.2925219	10.0501835	10.3427054	
2	9.6575423	9.4497521	9.7077902	10.2922098	10.0502479	10.3424577	
3	9.6577898	9.9496876	9.7081022	10.2918978	10.0503124	10.3422102	
4	9.6580371	9.9496230	9.708414	10.2915859	10.0503770	10.3419629	
5	9.6582842	9.9495585	9.7087258	10.2912742	10.0504415	10.3417158	
6	9.6585312	9.9494938	9.7090374	10.2909626	10.0505062	10.3414688	
7	9.6587780	9.9494292	9.7093488	10.290651	10.0505708	10.3412220	
8	9.6590246	9.9493645	9.7096601	10.2903399	10.0506355	10.3409754	
9	9.6592710	9.9492997	9.7099713	10.2900287	10.0507003	10.3407290	
10	9.6595173	9.9492349	9.7102824	10.2897176	10.0507651	10.3404827	
11	9.6597634	9.9491700	9.7105933	10.2894067	10.050830	10.3402367	
12	9.6600093	9.9491051	9.7109041	10.2890959	10.0508945	10.3399907	
13	9.6602550	9.9490402	9.7112148	10.2887852	10.0509598	10.3397450	
14	9.6605005	9.9489752	9.7115254	10.2884746	10.0510248	10.3394995	
15	9.6607459	9.9489101	9.7118358	10.2881642	10.0510899	10.3392541	
16	9.6609911	9.9488450	9.7121461	10.2878539	10.0511550	10.3390089	
17	9.6612361	9.9487799	9.7124562	10.2875438	10.0512201	10.3387639	
18	9.6614810	9.9487147	9.7127662	10.2872338	10.0512853	10.3385190	
19	9.6617257	9.9486495	9.7130761	10.2869239	10.0513505	10.338274	
20	9.6619702	9.9485842	9.7133859	10.2866141	10.0514158	10.338029	
21	9.6622145	9.9485189	9.7136956	10.2863044	10.0514811	10.3377855	
22	9.6624586	9.9484535	9.7140051	10.2859949	10.0515465	10.3375414	
23	9,6627026	9.9483881	9.7143145	10,2856855	10.0516119	10.337297	
24	9.6629464	9.9483227	9.7146237	10.2853763	10.0516773	10.3370536	
25	9.6631900	9 9482572	9.7149329	10.2850671	10.0517428	10.3368100	
26	9 6634335	9.9481916	9.7152415	10.2847581	10.0518084	10.3365665	
27	9.6636768	9.9481260	9.7155508	10.2844492	10.0518740	10.3363232	
28	9.6639199	9.9480604	9.7158595	10.2841405	10.0519396	10.3360801	
29	9.6641628	9.9479947	9.7161682	10.2838318	10.0520053	10.3358372	
30	9.6644056	9.9479289	9.7164767	10.2835233	10.0520711	10.335594	
		Sine.		Tang.		Secant.	M

62 *Degrees.*

27 *Degrees.*

M	Sine.		Tang		Secant.		
30	9.6644056	9.9479289	9.7164767	10.2835233	10.0520711	10.3355944	30
31	9.6646482	9.9478631	9.7167851	10.2832149	10.0521369	10.3353518	29
32	9.6648906	9 9477973	9.7170933	10.2829067	10.0522027	10.3351094	28
33	9.6651329	9.9477314	9.7174014	10.2825986	10.0522686	10.3348671	27
34	9.6653749	9.9476655	9.7177094	10.2822906	10.0523345	10 3346251	26
35	9.6656168	9.9475995	9.7180173	10.2819827	10.0524005	10.3343832	25
36	9.6658580	9.9475335	9.7183251	10.2816749	10.0524665	10.3341414	24
37	9.6661001	9.9474674	9.7186327	10.2813673	10.0525326	10.3338999	23
38	9.6663415	9.9474013	9.7189402	10 2810598	10.0525987	10.3336585	22
39	9.6665828	9.9473352	9.7192476	10.2807524	10.0526648	10.3334172	21
40	9.6668238	9.9472689	9.7195549	10.2804451	10.0527311	10.3331762	20
41	9.6670647	9,9472027	9.7198620	10.2801380	10.0527973	10.3329353	19
42	9.6673054	9.9471364	9.7201690	10.2798310	10.0528636	10.3326946	18
43	9.6675459	9.9470700	9.7204759	10.2795241	10.0529300	10.3324541	17
44	9.6677863	9.9470036	9.7207827	10.2792173	10.0529964	10.3322137	16
45	9.6680265	9 9469372	9.7210893	10.2789107	10.0530628	10.3319735	15
46	9.6682665	9.9468707	9.7213958	10.2786042	10.0531293	10.3317335	14
47	9.6685064	9.9468042	9 7217022	10.2782978	10 0531958	10.3314936	13
48	9.6687461	9.9467376	9.7220085	10.2779915	10.0532624	10.3312539	12
49	9.6689856	9.9466710	9.7223147	10.2776853	10.0533290	10.3310144	11
50	9.6692250	9.9466043	9.7226207	10.2773793	10.0533957	10.3307750	10
51	9.6694642	9.9465376	9.7229266	10.2770734	10.0534624	10.3305358	9
52	9.6697032	9.9464708	9.7232324	10.2767676	10.0535292	10.3302968	8
53	9.6699420	9.9464040	9.7235381	10.2764619	10.0535960	10.3300580	7
54	9.6701807	9.9463371	9.7238436	10.2761564	10.0536629	10.3298193	6
55	9.6704192	9.9462702	9.7241490	10.2758510	10.0537298	10.3295808	5
56	9.6706576	9.9462032	9.7244543	10.2755457	10.0537968	10.3293424	4
57	9.6708958	9.9461362	9.7247595	10.2752405	10.0538638	10.3291042	3
58	9.6711338	9.9460692	9.7250646	10.2749354	10.0539308	10.3288662	2
59	9,6713716	9.9460021	9.7253695	10.2746305	10.0539979	10.3286284	1
60	9.6716093	9.9459349	9.7256744	10.2743256	10.0540651	10.3283907	0
		Sine.		Tang.		Secant.	M

62 *Degrees.*

A Table of Artificial Sines,

28 *Degrees.*

M	Sine.		Tang.		Secant		
0	9.6716093	9.9459349	9.7256744	10.2743256	10.054065	10.3283907	60
1	9.6718468	9.9458074	9.7259791	10.2740209	10.054134	10.3281534	59
2	9.6720841	9.9458009	9.7262837	10.2737163	10.0541993	10.3279159	58
3	9.6723213	9.9457332	9.7265881	10.2734119	10.0442660	10.3270787	57
4	9.6725583	9.9456659	9.7268925	10.2731075	10.0543341	10.3274417	56
5	9.6727952	9.9455985	9.7271967	10.2728033	10.0544015	10.3272048	55
6	9.6730315	9.9455310	9.7275008	10.2724992	10.0544690	10.3269681	54
7	9.6732684	9.9454636	9.7278048	10.2721952	10.0545364	10.3267316	53
8	9.6735047	9.9453960	9.7281087	10.2718913	10.054604	10.3264953	52
9	9.6737409	9.9453285	9.7284124	10.2715876	10.0546715	10.3262591	51
10	9.6739769	9.9452609	9.7287161	10.2712839	10.0547391	10.3260231	50
11	9.6742128	9.9451932	9.7290196	10.2709804	10.0548068	10.3257872	49
12	9.6744485	9.9451255	9.7293230	10.2706770	10.0548745	10.3255515	48
13	9.6746840	9.9450577	9.7296263	10.2703737	10.0549423	10.3253160	47
14	9.6749194	9.9449895	9.7299295	10.2700705	10.0550101	10.3250806	46
15	9.6751546	9.9449220	9.7302325	10.2697675	10.0550780	10.3248454	45
16	9.6753896	9.9448541	9.7305354	10.2694646	10.0551459	10.3246104	44
17	9.6756245	9.9447862	9.7308383	10.2691617	10.0552138	10.3243755	43
18	9.6758592	9.9447182	9.7311410	10.2688590	10.0552818	10.3241408	42
19	9.6760937	9.9446501	9.7314436	10.2685564	10.0553499	10.3239063	41
20	9.6763281	9.9445821	9.7317460	10.2682540	10.0554179	10.3236719	40
21	9.6765623	9.9445139	9.7320484	10.2679516	10.0554861	10.3234377	39
22	9.6767963	9.9444457	9.7323506	10.2676494	10.0555543	10.3232037	38
23	9.6770302	9.9443775	9.7326527	10.2673473	10.0556225	10.3229698	37
24	9. 772640	9.9443092	9.7329547	10.2670453	10.0556908	10.322736	36
25	9.6774975	9.9442409	9.7332566	10.2667434	10.055759	10.3225025	35
26	9.6777309	9.9441725	9.7335584	10.2664416	10.0558275	10.3222691	34
27	9.6779642	9.9441041	9.7338601	10,2661399	10.0558959	10.3220358	33
28	9.6781972	9.9440356	9.7341616	10.2658384	10.0559644	10.3218028	32
29	9.6784301	9.9439671	9.7344631	10.2655369	10.0560329	10.3215699	31
30	9.6786629	9.9438985	9.7347644	10.2652356	10.0561015	10.3213371	30
		Sine.		Tang.		Secant.	M

61 *Degrees.*

28 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.6786629	9.9438985	9.7347644	10.2652356	10.0561015	10.3213371	30
31	9.6788955	9.9438299	9.7350656	10.2649344	10.0561701	10.3211045	29
32	9.6791279	9.9437612	9.7353667	10.2646333	10.0562388	10.3208721	28
33	9.6793602	9.9436925	9.7356677	10.2643323	10.0563075	10.3206398	27
34	9.6795923	9.9436238	9.7359685	10.2640315	10.0563762	10.3204077	26
35	9.6798243	9.9435549	9.7362693	10.2637307	10.0564451	10.3201757	25
36	9.6800560	9.9434861	9.7365699	10.2634301	10.0565139	10.3199440	24
37	9.6802877	9.9434172	9.7368705	10.2631295	10.0565828	10.3197123	23
38	9.6805191	9.9433482	9.7371709	10.2628291	10.0566518	10.3194809	22
39	9.6807504	9.9432792	9.7374712	10.2625288	10.0567208	10.3192496	21
40	9.6809816	9.9432102	9.7377714	10.2622286	10.0567898	10.3190184	20
41	9.6812126	9.9431411	9.7380715	10.2619285	10.0568589	10.3187874	19
42	9.6814434	9.9430720	9.7383714	10.2616286	10.0569280	10.3185566	18
43	9.6816741	9.9430028	9.7386713	10.2613287	10.0569972	10.3183259	17
44	9.6819046	9.9429335	9.7389710	10.2610290	10.0570665	10.3180954	16
45	9.6821349	9.9428643	9.7392707	10.2607293	10.0571357	10.3178651	15
46	9.6823651	9.9427949	9.7395702	10.2604298	10.0572051	10.3176349	14
47	9.6825952	9.9427255	9.7398696	10.2601304	10.0572745	10.3174048	13
48	9.6828250	9.9426561	9.7401689	10.2598311	10.0573439	10.3171750	12
49	9.6830548	9.9425866	9.7404681	10.2595319	10.0574134	10.3169452	11
50	9.6832843	9.9425171	9.7407672	10.2592328	10.0574829	10.3167157	10
51	9.6835137	9.9424476	9.7410662	10.2589338	10.0575524	10.3164863	9
52	9.6837430	9.9423779	9.7413650	10.2586350	10.0576221	10.3162570	8
53	9.6839720	9.9423083	9.7416638	10.2583362	10.0576917	10.3160280	7
54	9.6842010	9.9422386	9.7419624	10.2580376	10.0577614	10.3157990	6
55	9.6844297	9.9421688	9.7422609	10.2577391	10.0578312	10.3155703	5
56	9.6846583	9.9420990	9.7425594	10.2574406	10.0579010	10.3153417	4
57	9.6848868	9.9420291	9.7428577	10.2571423	10.0579709	10.3151132	3
58	9.6851151	9.9419592	9.7431559	10.2568441	10.0580408	10.3148849	2
59	9.6853432	9.9418893	9.7434540	10.2565460	10.0581107	10.3146568	1
60	9.6855712	9.9418193	9.7437520	10.2562480	10.0581807	10.3144288	0
		Sine.		Tang.		Secant.	M

61 *Degrees.*

29 *Degrees.*

M	Sine.		Tang.		Secant.	
0	9.6855712	9.9418193	9.7437520	10.2562480	10.0581807	10.3144288
1	9.6857991	9.9417492	9.7440499	10.2559501	10.0582508	10.3142009
2	9.6860267	9.9416791	9.7443476	10.2556524	10.0583209	10.3139733
3	9.6862542	9.9416090	9.7446453	10.2553547	10.0583910	10.3137458
4	9.6864816	9.9415388	9.7449428	10.2550572	10.0584612	10.3135184
5	9.6867088	9.9414685	9.7452403	10.2547597	10.0585315	10.3132912
6	9.6869359	9.9413982	9.7455376	10.2544624	10.0586018	10.3130641
7	9.6871628	9.9413279	9.7458349	10.2541651	10.0586721	10.3128372
8	9.6873895	9.9412575	9.7461320	10.2538680	10.0587425	10.3126105
9	9.6876161	9.9411871	9.7464290	10.2535710	10.0588129	10.3123839
10	9.6878425	9.9411166	9.7467259	10.2532741	10.0588834	10.3121575
11	9.6880688	9.9410461	9.7470227	10.2529773	10.0589539	10.3119312
12	9.6882949	9.9409755	9.7473194	10.2526806	10.0590245	10.3117051
13	9.6885209	9.9409048	9.7476160	10.2523840	10.0590952	10.3114791
14	9.6887467	9.9408342	9.7479125	10.2520875	10.0591658	10.3112533
15	9.6889723	9.9407634	9.7482089	10.2517911	10.0592366	10.3110277
16	9.6891978	9.9406927	9.7485052	10.2514948	10.0593073	10.3108022
17	9.6894232	9.9406219	9.7488013	10.2511987	10.0593781	10.3105768
18	9.6896484	9.9405510	9.7490974	10.2509026	10.0594490	10.3103516
19	9.6898734	9.9404801	9.7493934	10.2506066	10.0595195	10.3101266
20	9.6900983	9.9404091	9.7496892	10.2503108	10.0595909	10.3100917
21	9.6903231	9.9403381	9.7499850	10.2500150	10.0596619	10.3096769
22	9.6905476	9.9402670	9.7502806	10.2497194	10.0597330	10.3094524
23	9.6907721	9.9401959	9.7505762	10.2494238	10.0598041	10.3092279
24	9.6909964	9.9401248	9.7508716	10.2491284	10.0598752	10.3090036
25	9.6912205	9.9400535	9.7511669	10.2488331	10.0599465	10.3087795
26	9.6914445	9.9399823	9.7514622	10.2485378	10.0600177	10.3085555
27	9.6916683	9.9399110	9.7517573	10.2482427	10.0600890	10.3083317
28	9.6918919	9.9398396	9.7520523	10.2479477	10.0601604	10.3081081
29	9.6921155	9.9397682	9.7523472	10.2476528	10.0602318	10.3078845
30	9.6923388	9.9396968	9.7526420	10.2473580	10.0603032	10.3076612
		Sine.		Tang.		Secant.

60 *Degrees.*

29 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.6923388	9.9396968	9.7526420	10.2473580	10.0603032	10,3076612	30
31	9.6925620	9.9396253	9.7529368	10.2470632	10.0603747	10.3074380	29
32	9.6927851	9.9395537	9.7532314	10.2467686	10.0604463	10.3072149	28
33	9.6930080	9.9394821	9.7535259	10.2464741	10.0605179	10.3069920	27
34	9.6932308	9.9394105	9.7538203	10 2461797	10.0605895	10.3067692	26
35	9.6934534	9.9393388	9.7541146	10.2458854	10.0606612	10.3065466	25
36	9.6936758	9.9392671	9.7544088	10.2455912	10-0607329	10.3063242	24
37	9.6938981	9.9391953	9.7547029	10.2452971	10.0608047	10.3061019	23
38	9.6941203	9.9391234	9.7549969	10.2450031	10.0608766	10.3058797	22
39	9.6943423	9.9390515	9.7552908	10.2447092	10.0609485	10.3056577	21
40	9.6945642	9.9389796	9.7555846	10.2444154	10.0610204	10.3054358	20
41	9.6947859	9.9389076	9.7558783	10.2441217	10.0610924	10.3052141	19
42	9.6950074	9.9388356	9.7561718	10.2438282	10.0611644	10.3043926	18
43	9.6952288	9.9387635	9.7564653	10.2435347	10.0612365	10.3047712	17
44	9.6954501	9.9386914	9.7567587	10.2432413	10.0613086	10.3045499	16
45	9.6956712	9.9386192	9.7570520	10.2429480	10.0613808	10.3043288	15
46	9.6958922	9.9385470	9.7573452	10.2426548	10.0614530	10.3041078	14
47	9.6961130	9.9384747	9.7576383	10.2423617	10.0615253	10.3038870	13
48	9.6963336	9,9384024	9.7579313	10.2420687	10.0615976	10.3036664	12
49	9.6965541	9.9383300	9.7582242	10.2417758	10.0616700	10.3034459	11
50	9,6967745	9.9382576	9.7585170	10.2414830	10.0617424	10.3032255	10
51	9,6969947	9.9381851	9.7588096	10.2411904	10.0618149	10.3030053	9
52	9.6972148	9.9381126	9.7591022	10.2408978	10.0618874	10.3027852	8
53	9.6974347	9.9380400	9.7593947	10.2406053	10.0619600	10.3025653	7
54	9.6976545	9.9379674	9.7596871	10.2403125	10.0620326	10.3023455	6
55	9.6978741	9.9378947	9.7599794	10.2400206	10.0621053	10.3021259	5
56	9.6980936	9.9378220	9.7602716	10.2397284	10.0621780	10.3019064	4
57	9.6983129	9.9377492	9.7605637	10.2394363	10.0622508	10.3016871	3
58	9.6985321	9.9376764	9.7608557	10.2391443	10.0623236	10.3014679	2
59	9.6987511	9.9376035	9.7611476	10.2388524	10.0623965	10.3012489	1
60	9.6989700	9.9375306	9.7614394	10.2385606	10.0624694	10.3010300	0
		Sine.		Tang.		Secant.	M

60 *Degrees.*

30 Degrees.

M	Sine.		Tang.		Secant.		
0	9.6989700	9.9375306	9.7614394	10.2385606	10.0624694	10.3010300	60
1	9.6991887	9.9374577	9.7617311	10.2382689	10.0625423	10.3008113	59
2	9.6994073	9.9373847	9.7620227	10.2379773	10.0626153	10.3005927	58
3	9.6996258	9.9373116	9.7623142	10.2376858	10.0626884	10.3003742	57
4	9.6998441	9.9372385	9.7626056	10.2373944	10.0627615	10.3001559	56
5	9.7000622	9.9371653	9.7628969	10.2371031	10.0528347	10.2999378	55
6	9.7002802	9.9370921	9.7631881	10.2368119	10.0629079	10.2997198	54
7	9.7004981	9.9370189	9.7634792	10.2365208	10.0629811	10.2995019	53
8	9.7007158	9.9369456	9.7637702	10.2362298	10.0630544	10.2992842	52
9	9.7009334	9.9368722	9.7640612	10.2359388	10.0631278	10.2990666	51
10	9.7011508	9.9367988	9.7643520	10 2359480	10.0632012	10.2988492	50
11	9.7013681	9.9367254	9.7646427	10.2353573	10.0632746	10.2986319	49
12	9.7015852	9.9366519	9.7649334	10.2350666	10.0633481	10.2984148	48
	9.7018022	9.9365783	9.7652239	10.2347761	10.0634217	10.2981978	47
14	9.7020190	9.9365047	9.7655143	10.2344857	10.0634953	10.2979810	46
15	9.7022357	9.9364311	9.7658047	10.2341953	10.0635689	10.2977643	45
16	9.7024523	9.9363574	9.7660949	10.2339051	10.0636426	10.2975477	44
17	9.7026687	9.9362836	9.7663851	10.2336149	10.0637164	10.2973313	43
18	9.7028849	9.9362098	9.7666751	10.2333249	10.0637902	10.2971151	42
19	9.7031011	9.9361360	9.7669651	10.2330349	10.0638640	10.2968989	41
20	9.7033170	9.9360621	9.7672550	10.2327450	10.0639379	10.2966830	40
21	9.7035329	9.9359881	9.7675448	10.2324552	10.0640119	10.2964671	39
22	9.7037486	9.9359141	9.7678344	10.2321656	10.0640859	10.2962514	38
23	9.7039641	9.9358401	9.7681240	10.2318760	10.0641599	10.2960359	37
24	9.7041795	9.9357660	9.7684135	10.2315865	10.0642340	10.2958205	36
25	9.7043947	9.9356918	9.7687029	10.2312971	10.0643082	10.2956053	35
26	9.7046099	9.9356177	9.7689922	10.2310078	10.0643823	10.2953901	34
27	9.7048248	9.9355434	9.7692814	10.2307186	10.0644566	10.2951752	33
28	9.7050397	9.9354691	9.7695705	10.2304295	10.0645309	10.2949603	32
29	9.7052543	9.9353948	9.7698596	10.2301404	10.0646052	10.2947457	31
30	9.7054689	9.9353204	9.7701485	10.2298515	10.0646796	10.2945311	30
		Sine.		Tang.		Secant.	M

59 Degrees.

30 Degrees.

Sine.		Tang.		Secant.		
9.7054689	9.9353204	9.7701485	10.2298515	10.0646796	10.2945311	30
9.7056833	9.9352459	9.7704373	10.2295627	10.0647541	10.2943167	29
9.7058975	9.9351715	9.7707261	10.2292739	10.0648285	10.2941025	28
9.7061116	9.9350969	9.7710147	10.2289853	10.0649031	10.2938884	27
9.7063256	9.9350223	9.7713033	10.2286967	10.0649777	10.2936744	26
9.7065394	9.9349477	9.7715917	10.2284083	10.0650523	10.2934606	25
9.7067531	9.9348730	9.7718801	10.2281199	10.0651270	10.2932469	24
9.7069667	9.9347983	9.7721684	10.2278316	10.0652017	10.2930333	23
9.7071801	9.9347235	9.7724566	10.2275434	10.0652765	10.2928199	22
9.7073933	9.9346486	9.7727447	10.2272553	10.0653514	10.2926067	21
9.7076064	9.9345738	9.7730327	10.2269673	10.0654262	10.2923936	20
9.7078194	9.9344988	9.7733206	10.2266794	10.0655012	10.2921806	19
9.7080323	9.9344238	9.7736084	10.2263916	10.0655762	10.2919677	18
9.7082450	9.9343488	9.7738961	10.2261039	10.0656512	10.2917550	17
9.7084575	9.9342737	9.7741838	10.2258162	10.0657263	10.2915425	16
9.7086699	9.9341986	9.7744713	10.2255287	10.0658014	10.2913301	15
9.7088822	9.9341234	9.7747588	10.2252412	10.0658766	10.2911178	14
9.7090943	9.9340482	9.7750462	10.2249538	10.0659518	10.2909057	13
9.7093063	9.9339729	9.7753334	10.2246666	10.0660271	10.2906937	12
9.7095182	9.9338976	9.7756206	10.2243794	10.0661024	10.2904818	11
9.7097299	9.9338222	9.7759077	10.2240923	10.0661778	10.2902701	10
9.7099415	9.9337467	9.7761947	10.2238053	10.0662533	10.2900585	9
9.7101529	9.9336713	9.7764816	10.2235184	10.0663287	10.2898471	8
9.7103642	9.9335957	9.7767685	10.2232315	10.0664043	10.2896358	7
9.7105753	9.9335201	9.7770552	10.2229448	10.0664799	10.2894247	6
9.7107863	9.9334445	9.7773418	10.2226582	10.0665555	10.2892137	5
9.7109972	9.9333688	9.7776284	10.2223716	10.0666312	10.2890028	4
9.7112080	9.9332931	9.7779149	10.2220851	10.0667069	10.2887920	3
9.7114186	9.9332173	9.7782012	10.2217988	10.0667827	10.2885814	2
9.7116290	9.9331415	9.7784875	10.2215125	10.0668585	10.2883710	1
9.7118393	9.9330656	9.7787737	10.2212263	10.0669344	10.2881607	0
	Sine.		Tang.		Secant.	

59 Degrees.

A Table of Artificial Sines,

31 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.7118393	9.9330656	9.7787737	10.2212263	10.0669344	10.2881607	60
1	9.7120495	9.9329897	9.7790599	10.2209401	10.0670103	10.2879505	59
2	9.7122596	9.9329137	9.7793455	10.2206541	10.0670863	10.2877404	58
3	9.7124695	9.9328376	9.7796318	10.2203682	10.0671624	10.2875305	57
4	9.7126792	9.9327616	9.7799177	10.2200823	10.0672384	10.2873208	56
5	9.7128889	9.9326854	9.7802034	10.2197966	10.0673146	10.2871111	55
6	9.7130983	9.9326092	9.7804891	10.2195109	10.0673908	10.2869017	54
7	9.7133077	9.9325330	9.7807747	10.2192253	10.0674670	10.2866923	53
8	9.7135169	9.9324567	9.7810602	10.2189398	10.0675433	10.2864831	52
9	9.7137260	9.9323804	9.7813456	10.2186544	10.0676196	10.2862740	51
10	9.7139349	9.9323040	9.7816309	10.2183691	10.0676960	10.2860651	50
11	9.7141437	9.9322276	9.7819162	10.2180838	10.0677724	10.2858563	49
12	9.7143524	9.9321511	9.7822013	10.2177987	10.0678489	10.2856476	48
13	9.7145609	9.9320746	9.7824864	10.2175136	10.0679254	10.2854391	47
14	9.7147693	9.9319980	9.7827713	10.2172287	10.0680020	10.2852307	46
15	9.7149776	9.9319213	9.7830562	10.2169438	10.0680787	10.2850224	45
16	9.7151857	9.9318447	9.7833410	10.2166590	10.0681553	10.2848143	44
17	9.7153937	9.9317679	9.7836258	10.2163742	10.0682321	10.2846063	43
18	9.7156015	9.9316911	9.7839104	10.2160896	10.0683089	10.2843985	42
19	9.7158092	9.9316143	9.7841949	10.2158051	10.0683857	10.2841908	41
20	9.7160168	9.9315374	9.7844794	10.2155206	10.0684626	10.2839832	40
21	9.7162243	9.9314605	9.7847638	10.2152362	10.0685395	10.2837757	39
22	9.7164316	9.9313835	9.7850481	10.2149519	10.0686165	10.2835684	38
23	9.7166387	9.9313065	9.7853323	10.2146677	10.0686935	10.2833613	37
24	9.7168458	9.9312294	9.7856164	10.2143836	10.0687706	10.2831542	36
25	9.7170526	9.9311522	9.7859004	10.2140996	10.0688478	10.2829474	35
26	9.7172594	9.9310750	9.7861844	10.2138156	10.0689250	10.2827406	34
27	9.7174660	9.9309978	9.7864682	10.2135318	10.0690022	10.2825340	33
28	9.7176725	9.9309205	9.7867520	10.2132480	10.0690795	10.2823275	32
29	9.7178789	9.9308432	9.7870357	10.2129643	10.0691568	10.2821211	31
30	9.7180851	9.9307658	9.7873193	10.2126807	10.0692342	10.2819149	30
		Sine.		Tang.		Secant.	M

58 *Degrees.*

31 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.7180851	9.9307658	9.7873193	10.2126807	10.0692342	10.2819149	30
31	9.7182912	9.9300883	9.7876028	10.2123972	10.0693117	10.2817088	29
32	9.7184971	9.9306109	9.7878863	10.2121137	10.0693891	10.2815029	28
33	9.7187030	9.9305333	9.7881696	10.2118304	10.0694667	10.2812970	27
34	9.7189086	9.9304557	9.7884529	10.2115471	10.0695443	10.2810914	26
35	9.7191142	9.9303781	9.7887361	10.2112639	10.0696219	10.2808858	25
36	9.7193196	9.9303004	9.7890192	10.2109808	10.0696995	10.2806804	24
37	9.7195249	9.9302226	9.7893023	10.2106977	10.0697774	10.2804751	23
38	9.7197300	9.9301448	9.7895852	10.2104148	10.0698552	10.2802700	22
39	9.7199350	9.9300670	9.7898681	10.2101319	10.0699330	10.2800650	21
40	9.7201395	9.9299891	9.7901508	10.2098492	10.0700109	10.2798601	20
41	9.7203447	9.9299112	9.7904335	10.2095665	10.0700888	10.2796553	19
42	9.7205493	9.9298332	9.7907161	10.2092839	10.0701668	10.2794507	18
43	9.7207538	9.9297551	9.7909987	10.2090013	10.0702449	10.2792464	17
44	9.7209581	9.9296770	9.7912811	10.2087189	10.0703230	10.2790419	16
45	9.7211623	9.9295989	9.7915635	10.2084365	10.0704011	10.2788377	15
46	9.7213664	9.9295207	9.7918458	10.2081542	10.0704793	10.2786336	14
47	9.7215704	9.9294424	9.7921280	10.2078720	10.0705576	10.2784296	13
48	9.7217742	9.9293641	9.7924101	10.2075899	10.0706359	10.2782258	12
49	9.7219779	9.9292857	9.7926921	10.2073079	10.0707143	10.2780221	11
50	9.7221814	9.9292073	9.7929741	10.2070259	10.0707927	10.2778186	10
51	9.7223848	9.9291289	9.7932560	10.206744	10.0708711	10.2776152	9
52	9.7225881	9.9290504	9.7935378	10.2064622	10.0709496	10.2774119	8
53	9.7227913	9.9289718	9.7938195	10.2061805	10.0710282	10.2772087	7
54	9.7229943	9.9288932	9.7941011	10.2058989	10.0711068	10.2770057	6
55	9.7231972	9.9288145	9.7943827	10.2056173	10.0711855	10.2768028	5
56	9.7234000	9.9287358	9.7946641	10.2053359	10.0712642	10.2766000	4
57	9.7236026	9.9286571	9.7949455	10.2050545	10.0713429	10.2763974	3
58	9.7238051	9.9285783	9.7952268	10.2047732	10.0714217	10.2761949	2
59	9.7240075	9.9284994	9.7955081	10.2044919	10.0715006	10.2759925	1
60	9.7242097	9.9284205	9.7957892	10.2042108	10.0715795	10.2757903	0
		Sine.		Tang.		Secant.	M

58 *Degrees.*

A Table of Artificial Sines,

32 Degrees.

M	Sine.		Tang.		Secant.		
0	9.7242097	9.9284205	9.7957892	10.2042108	10.0715795	10.2757903	60
1	9.7244113	9.9283415	9.7960703	10.2039297	10.0716585	10.2755882	59
2	9.7246138	9.9282625	9.7963513	10.2036487	10.0717375	10.2753862	58
3	9.7248156	9.9281834	9.7966322	10.2033678	10.0718166	10.2751844	57
4	9.7250174	9.9281043	9.7969130	10.2030870	10.0718957	10.2749826	56
5	9.7252189	9.9280251	9.7071938	10.2028062	10.0719749	10.2747811	55
6	9.7254204	9.9279459	9.7974745	10.2025255	10.0720541	10.2745796	54
7	9.7256217	9.9278666	9.7977551	10.2022449	10.0721334	10.2743783	53
8	9.7258229	9.9277873	9.7980356	10.2019644	10.0722127	10.2741771	52
9	9.7260240	9.9277079	9.7983160	10.2016840	10.0722921	10.2739760	51
10	9.7262249	9.9276285	9.7985964	10.2014036	10.0723715	10.2737751	50
11	9.7264257	9.9275490	9.7988767	10.2011233	10.0724510	10.2735743	49
12	9.7266264	9.9274695	9.7991569	10.2008431	10.0725305	10.2733736	48
13	9.7268269	9.9273899	9.7994370	10.2005630	10.0726101	10.2731731	47
14	9.7270273	9.9273103	9.7997170	10.2002830	10.0726897	10.2729727	46
15	9.7272276	9.9272306	9.7999970	10.2000030	10.0727694	10.2727724	45
16	9.7274278	9.9271509	9.8002769	10.1997231	10.0728491	10.2725722	44
17	9.7276278	9.9270711	9.8005567	10.1994433	10.0729289	10.2723722	43
18	9.7278277	9.9269913	9.8008365	10.1991635	10.0730087	10.2721723	42
19	9.7280275	9.9269114	9.8011161	10.1988839	10.0730886	10.2719725	41
20	9.7282271	9.9268314	9.8013957	10.1986043	10.0731686	10 2717729	40
21	9.7284267	9.9267514	9.8016752	10.1983248	10.0732486	10.2715733	39
22	9.7286260	9.9266714	9.8019540	10.1980454	10.0733286	10.2713740	38
23	9.7288253	9.9265913	9.8022340	10.1977660	10.0734087	10.2711747	37
24	9.7290244	9.9265112	9.8025133	10.1974867	10.0734888	10.2709756	36
25	9.7292234	9.9264310	9.8027925	10.1972075	10.0735690	10.2707766	35
26	9.7294223	9.9263507	9.8030716	10.1969284	10 0736493	10.2705777	34
27	9.7296211	9.9262704	9.8033506	10.1966494	10.0737296	10.2703790	33
28	9.7298197	9.9261901	9.8036296	10.1963704	10.0738099	10.2701803	32
29	9.7300187	9.9261096	9.8039085	10.1960915	10.0738904	10.2699818	31
30	9.7302165	9.9260292	9.8041873	10.1958127	10.0739708	0.2697835	30
		Sine.		Tang.		Secant.	M

57 Degrees.

32 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.7302165	9.9260292	9.8041873	10.1958127	10.0739708	10.2697835	30
1	9.7304148	9.9259487	9.8044661	10.1955339	10.0740513	10.2695852	29
2	9.7306129	9.9258681	9.8047447	10.1952553	10.0741319	10.2693871	28
3	9.7308109	9.9257875	9.8050233	10.1949767	10.0742125	10.2691891	27
4	9.7310087	9.9257069	9.8053019	10.1946981	10.0742931	10.2689913	26
5	9.7312064	9.9256261	9.8055803	10.1944197	10.0743739	10.2687936	25
6	9.7314040	9.9255454	9.8058587	10.1941413	10.0744546	10.2685960	24
7	9.7316015	9.9254646	9.8061370	10.1938630	10.0745354	10.2683985	23
8	9.7317989	9.9253837	9.8064152	10.1935848	10.0746163	10.2682011	22
9	9.7319961	9.9253028	9.8066933	10.1933067	10.0746972	10.2680039	21
	9.7321932	9.9252218	9.8069714	10.1930286	10.0747782	10.2678068	20
1	9.7323902	9.9251408	9.8072494	10.1927506	10.0748592	10.2676098	19
2	9.7325870	9.9250597	9.8075273	10.1924727	10.0749403	10.2674130	18
3	9.7327837	9.9249786	9.8078052	10.1921948	10.0750214	10.2672163	17
4	9.7329803	9.9248974	9.8080829	10.1919171	10.0751026	10.2670197	16
5	9.7331768	9.9248161	9.8083606	10.1916394	10.0751839	10.2668232	15
6	9.7333731	9.9247349	9.8086383	10.1913617	10.0752651	10.2666269	14
7	9.7335693	9.9246535	9.8089158	10.1910842	10.0753465	10.2664307	13
8	9.7337654	9.9245721	9.8091933	10.1908067	10.0754279	10.2662346	12
9	9.7339614	9.9244907	9.8094707	10.1905293	10.0755093	10.2660386	11
	9.7341572	9.9244092	9.8097480	10.1902520	10.0755908	10.2658428	10
1	9.7343529	9.9243277	9.8100253	10.1899747	10.0756723	10.2656471	9
2	9.7345485	9.9242461	9.8103025	10.1896975	10.0757539	10.2654515	8
3	9.7347440	9.9241644	9.8105796	10.1894204	10.0758356	10.2652560	7
	9.7349393	9.9240827	9.8108566	10.1891434	10.0759173	10.2650607	6
	9.7351345	9.9240010	9.8111336	10.1888664	10.0759990	10.2648655	5
	9.7353296	9.9239191	.8114105	01.1885895	10.0760809	10.2646704	4
	9.7355246	9.9238373	.8116873	10.1883127	10.0761627	10.2644754	3
	9.7357195	9.9237554	.8119641	10.1880359	10.0762446	10.2642805	2
	9.7359142	9.9236734	.8122408	10.1877592	10.0763265	10.2640858	1
	9.7361088	9.9235914	.8125174	10.1874826	10.0764086	10.2638912	0
		Sine.		Tang.		Secant.	M

57 *Degrees.*

A Table of Artificial Sines,

33 *Degrees.*

M.	Sine.		Tang.		Secant.		
0	9.7361088	9.9235914	9.8125174	10.1874826	10.0764086	10.2638912	6
1	9.7363032	9.9235093	9.8127939	10.1872061	10.0764907	10.2636968	59
2	9.7364976	9-9234272	9.8130704	10.1869296	10.0765728	10.2635024	58
3	9.736691	9.9233450	9 8133468	10.1866532	10.0766550	10.2633082	57
4	9.7368859	9.9232628	9.8136231	10.1863769	10.0767372	10.2631141	56
	9.7370799	9.9231805	9.8138993	10.1861007	10.0768195	10.2629201	55
6	9.7372737	9.9230982	9,8141755	10.1858245	10.0769018	10.2627263	54
7	9.7374675	9.9230158	9.8144516	10.1855484	10.0769842	10.2625325	53
8	9.7376611	9.9229334	9.8147277	10.1852723	10.0770666	10.2623389	52
9	9.7378546	9.9228509	9.8150036	10.1849964	10.0771491	10.2621454	51
10	9.7380479	9.9227684	9.8152795	10.1847205	10 0772316	10.2619521	50
11	9-7382412	9.9226858	9.8155554	10.1844446	10.0773142	10.2617588	49
12	9.7384343	9,9226032	9.8158311	10.1841689	10.0773969	10.2615657	48
13	9.7386277	9.9225205	9.8161068	10.1838932	10.0774795	10.2613727	47
14	9.7388201	9.9224377	9.8163824	10.1836176	10.0775623	10.2611799	46
15	9.7390129	9.9223549	9.8166580	10.1833420	10.0776451	10.2609871	45
16	9.7392055	9.9222721	9.8169335	10.1830665	10.0777279	10.2607945	44
17	9.7393980	9.9221891	9,8172089	10.1827911	10.0778109	10.2606020	43
18	9,7395904	9.9221062	9.8174842	10.1825158	10.0778938	10.2604096	42
19	9.7397827	9.9220232	9.8177595	10.1822405	10.0779768	10.2602173	41
20	9.7399748	9.9219401	9.8180347	10.1819653	10.0780595	10 2600252	40
21	9.7401668	9.9218570	9.8183098	10.1816902	10.0781430	10.2598332	39
22	9.7403587	9.9217738	9.8185849	10.1814151	10.0782262	10.2596413	38
23	9.7405505	9.9216906	9,8188595	10.1811401	10.0783094	10.2594495	37
24	9.7407421	9.9216073	9.8191348	10.1808652	10.0783927	10.2592579	36
25	9.7409337	9.9215240	9.8194096	10.1805904	10.0784760	10.2590663	35
26	9.7411251	9.9214406	9.8196844	10.1803156	10 0785594	10.2588749	34
27	9.7413164	9.9213572	9.8199592	10.1800408	10.0786428	10.2586836	33
28	9.7415075	9.9212737	9.8202338	10.1797662	10.0787263	10.2584925	32
29	9.7416986	9.9211902	9.8205084	10,1794916	10.0788098	10.2583014	31
30	9.7418895	9.9211066	9.8207829	10.1792171	10.0788934	10.2581105	30
		Sine.		Tang.		Secant.	M

56 *Degrees.*

33 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.7418895	9.9211066	9.8207829	10.1792171	10.0788934	10.2581105	30
31	9.7420803	9.9210229	9.8210574	10.1789426	10.0789771	10.2579197	29
32	9.7422710	9.9209393	9.8213317	10.1786683	10.0790607	10.2577290	28
33	9.7424616	9.9208555	9.8216060	10.1783940	10.0791445	10.2575384	27
34	9.7426520	9.9207717	9.8218803	10.1781197	10.0792283	10.2573480	26
35	9.7428423	9.9206878	9.8221545	10.1778455	10.0793122	10.2571577	25
36	9.7430325	9.9206039	9.8224286	10.1775714	10.0793961	10.2569675	24
37	9.7432226	9.9205200	9.8227026	10.1772974	10.0794800	10.2567774	23
38	9.7434126	9.9204360	9.8229766	10.1770234	10.0795640	10.2565874	22
39	9.7436024	9.9203519	9.8232505	10.1767495	10.0796481	10.2563976	21
40	9.7437921	9.9202678	9.8235244	10.1764756	10.0797322	10.2562079	20
41	9.7439817	9.9201836	9.8237981	10.1762019	10.0798164	10.2560183	19
42	9.7441712	9.9200994	9.8240719	10.1759281	10.0799006	10.2558288	18
43	9.7443606	9.9200151	9.8243455	10.1756545	10.0799849	10.2556394	17
44	9.7445498	9.9199308	9.8246191	10.1753809	10.0800692	10.2554502	16
45	9.7447390	9.9198464	9.8248926	10.1751074	10.0801536	10.2552610	15
46	9.7449280	9.9197619	9.8251660	10.1748340	10.0802381	10.2550720	14
47	9.7451169	9.9196775	9.8254394	10.1745606	10.0803225	10.2548831	13
48	9.7453056	9.9195929	9.8257127	10.1742873	10.0804071	10.2546944	12
49	9.7454943	9.9195083	9.8259860	10.1740140	10.0804917	10.2545057	11
50	9.7456828	9.9194237	9.8262592	10.1737408	10.0805763	10.2543172	10
51	9.7458712	9.9193390	9.8265323	10.1734677	10.0806610	10.2541288	9
52	9.7460595	9.9192543	9.8268053	10.1731947	10.0807458	10.2539405	8
53	9.7462477	9.9191694	9.8270783	10.1729217	10.0808306	10.2537523	7
54	9.7464358	9.9190845	9.8273513	10.1726487	10.0809155	10.2535642	6
55	9.7466237	9.9189996	9.8276241	10.1723759	10.0810004	10.2533763	5
56	9.7468115	9.9189146	9.8278969	10.1721031	10.0810854	10.2531885	4
57	9.7469992	9.9188296	9.8281696	10.1718304	10.0811704	10.2530008	3
58	9.7471868	9.9187445	9.8284423	10.1715577	10.0812555	10.2528132	2
59	9.7473743	9.9186594	9.8287149	10.1712851	10.0813406	10.2526257	1
60	9.7475617	9.9185742	9.8289874	10.1710126	10.0814258	10.2524383	0
		Sine.		Tang.		Secant.	M

56 *Degrees.*

O

34 Degrees.

M	Sine.		Tang.		Secant.		
0	9.7475617	9.9185742	9.8289874	10.1710126	10.0814258	10.2524383	60
1	9.7477489	9.9184890	9.8292595	10.1707401	10.0815110	10.2522511	59
2	9.7479360	9.9184037	9.8295322	10.1704677	10.0815963	10.2520640	58
3	9.7481230	9.9183183	9.8298047	10.1701953	10.0816817	10.2518770	57
4	9.7483099	9.9182329	9.8300765	10.1699231	10.0817671	10.2516901	56
5	9.7484967	9.9181475	9.8303475	10.1696508	10.0818525	10.2515033	55
6	9.7486833	9.9180620	9.8306213	10.1693787	10.0819380	10.2513167	54
7	9.7488698	9.9179764	9.8308934	10.1691066	10.0820236	10.2511302	53
8	9.7490562	9.9178908	9.8311654	10.1688346	10.0821092	10.2509438	52
9	9.7492425	9.9178051	9.8314374	10.1685626	10.0821949	10.2507575	51
10	9.7494287	9.9177194	9.8317093	10.1682907	10.0822806	10.2505713	50
11	9.7496148	9.9176336	9.831981	10.1680189	10.0823664	10.2503852	49
12	9.7498007	9.9175478	9.8322529	10.1677471	10.0824522	10.2501993	48
13	9.7499866	9.9174619	9.8325246	10.1674754	10.0825381	10.2500134	47
14	9.7501723	9.9173760	9.8327963	10.1672037	10.0826240	10.2498277	46
15	9.7503579	9.9172900	9.8330679	10.1669321	10.0827100	10.2496421	45
16	9.7505434	9.9172040	9.8333394	10.1666606	10.0827960	10.2494566	44
17	9.7507287	9.9171179	9.8336109	10.1663891	10.0828821	10.2492713	43
18	9.7509140	9.9170317	9.8338823	10.1661177	10.0829683	10.2490860	42
19	9.7510991	9.9169455	9.8341536	10.1658464	10.0830545	10.2489009	41
20	9.7512842	9.9168593	9.8344249	10.1655751	10.0831407	10.2487158	40
21	9.7514691	9.9167730	9.8346961	0.1653039	10.0832270	10.2485309	39
22	9.7516538	9.9166866	9.8349673	10.1650327	10.0833134	10.2483462	38
23	9.7518385	9.9166002	9.8352384	10,1647616	10.0833998	10.2481615	37
24	9.7520231	9.9165137	9.8355094	10.1644906	10.0834863	10.2479769	36
25	9.7522075	9.9164272	9.8357804	10.1642196	10.0835728	10.2477925	35
26	9.7523919	9.9163406	9.8360513	10.1639487	10.0836594	10.2476081	34
27	9.7525761	9.9162539	9.8363221	10.1636779	10.0837461	10.2474239	33
28	9,7527602	9.9161673	9.8365929	10.1634071	10.0838327	10.2472398	32
29	9.7529442	9.9160805	9.8368636	10.1631364	10.0839195	10.2470558	31
30	9.7531280	9.9159937	9.8371343	10.1628657	10.0840063	10.2468720	30
		Sine.		Tang.		Secant.	M

55 Degrees.

34 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.7531280	9.9159937	9.8371343	10.1628657	10.0840063	10,2468720	30
31	9.7533118	9.9159069	9.8374049	10.1625951	10.0840931	10.2466882	29
32	9.7534954	9.9158200	9.8376755	10.1623245	10.0841800	10.2465046	28
33	9.7536790	9.9157330	9.8379460	10.1620540	10.0842670	10.2463210	27
34	9.7538624	9.9156460	9.8382164	10.1617836	10.0843540	10.2461376	26
35	9.7540457	9.9155589	9.8384867	10.1615133	10.0844411	10.2459543	25
36	9.7542288	9.9154718	9.8387571	10.1612429	10.0845282	10.2457712	24
37	9.7544119	9.9153846	9.8390273	10.1609727	10.0846154	10.2455881	23
38	9.7545949	9.9152974	9.8392975	10.1607025	10.0847026	10.2454051	22
39	9.7547777	9.9152101	9.8395676	10.1604324	10.0847899	10.2452223	21
40	9.7549604	9.9151228	9.8398377	10.1601623	10.0848772	10.2450396	20
41	9.7551431	9.9150354	9.8401077	10.1598923	10.0849646	10.2448569	19
42	9.7553256	9.9149479	9.8403776	10.1596224	10.0850521	10.2446744	18
43	9.7555080	9.9148604	9.8406475	10.1593525	10.0851396	10.2444920	17
44	9.7556902	9.9147729	9.8409174	10.1590826	10.0852271	10.2443098	16
45	9.7558724	9.9146852	9.8411871	10.1588129	10.0853148	10.2441276	15
46	9.7560544	9.9145976	9.8414569	10.1585431	10.0854024	10.2439456	14
47	9.7562364	9.9145099	9.8417265	10.1582735	10.0854901	10.2437636	13
48	9.7564182	9,9144221	9.8419961	10.1580039	10.0855779	10.2435818	12
49	9.7565999	9.9143342	9.8422657	10.1577343	10.0856658	10.2434001	11
50	9,7567815	9.9142464	9.8425351	10,1574649	10.0857536	10.2432185	10
51	9,7569630	9.9141584	9.8428046	10.1571954	10.0858416	10.2430370	9
52	9.7571444	9.9140704	9.8430739	10.1569261	10.0859296	10.2428556	8
53	9.7573256	9.9139824	9.8433432	10.1566568	10.0860176	10.2426744	7
54	9.7575068	9.9138943	9.8436125	10.1563875	10.0861057	10.2424932	6
55	9.7576878	9.9138061	9.8438817	10.1561183	10.0861939	10.2423122	5
56	9.7578687	9.9137179	9.8441508	10.1558492	10.0862821	10.2421313	4
57	9.7580495	9.9136296	9.8444199	10.1555801	10.0863704	10.2419505	3
58	9.7582302	9.9135413	9.8446889	10.1553111	10.0864587	10.2417698	2
59	9.7584108	9.9134530	9.8449579	10.1550421	10.0865471	10.2415892	1
60	9.7585913	9.9133645	9.8452268	10.1547732	10.0866355	10.2414087	0
		Sine.		Tang.		Secant.	M

55 *Degrees.*

A Table of Artificial Sines,

35 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.7585913	9.9133645	9.8452268	10.1547732	10.0866355	10.2414087	60
1	9.7587717	9.9132760	9.8454956	10.1545044	10.0867240	10.2412283	59
2	9.7589519	9.9131875	9.8457644	10.1542356	10.0868125	10.2410481	58
3	9.7591321	9.9130989	9.8460332	10.1539668	10.0869011	10.2408679	57
4	9.7593121	9.9130102	9.8463018	10.1536982	10.0869898	10.2406879	56
5	9.7594920	9.9129215	9.8465705	10.1534295	10.0870785	10.2405080	55
6	9.7596718	9.9128328	9.8468390	10.1531610	10.0871672	10.2403282	54
7	9.7598515	9.9127440	9.8471075	10.1528925	10.0872560	10.2401485	53
8	9.7600311	9.9126551	9.8473760	10.1526240	10.0873449	10.2399689	52
9	9.7602100	9.9125662	9.8476444	10.1523556	10.0874338	10.2397894	51
10	9.7603899	9.9124772	9.8479127	10.1520873	10.0875228	10.2396101	50
11	9.7605692	9.9123882	9.8481810	10.1518190	10.0876118	10.2394308	49
12	9.7607483	9.9122991	9.8484492	10.1515508	10.0877009	10.2392517	48
13	9.7609274	9.9122099	9.8487174	10.1512826	10.0877901	10.2390726	47
14	9.7611063	9.9121207	9.8489855	10.1510145	10.0878793	10.2388937	46
15	9.7612851	9.9120315	9.8492536	10.1507464	10.0879685	10.2387149	45
16	9.7614638	9.9119422	9.8495216	10.1504784	10.0880578	10.2385362	44
17	9.7616424	9.9118528	9.8497896	10.1502104	10.0881472	10.2383576	43
18	9.7618208	9.9117634	9.8500575	10.1499425	10.0882360	10.2381792	42
19	9.7619992	9.9116739	9.8503253	10.1496747	10.0883261	10.2380008	41
20	9.7621775	9.9115844	9.8505931	10.1494069	10.0884156	0.2378225	40
21	9.7623556	9.9114948	9.8508608	10.1491392	10.0885052	10.2376444	39
22	9.7625337	9.9114051	9.8511285	10.1488715	10.0885949	10.2374663	38
23	9.7627116	9.9113155	9.8513961	10.1486039	10.0886845	10.2372884	37
24	9.7628894	9.9112257	9.8516637	10.1483363	10.0887743	10.2371106	36
25	9.7630671	9.9111359	9.8519312	10.1480688	10.0888641	10.2369329	35
26	9.7632447	9.9110460	9.8521987	10.1478013	10.0889540	10.2367553	34
27	9.7634222	9.9109561	9.8524661	10.1475339	10.0890439	10.2365778	33
28	9.7635996	9.9108661	9.8527335	10.1472665	10.0891339	10.2364004	32
29	9.7637769	9.9107761	9.8530008	10.1469992	10.0892239	10.2362231	31
30	9.7639540	9.9106860	9.8532680	10.1467320	10.0893140	10.2360460	30
		Sine.		Tang.		Secant.	M

54 *Degrees.*

35 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.7639540	9.9106860	9.8532680	10.1467320	10.0893140	10.2360460	30
31	9.7641311	9.9105959	9.8535352	10.1464648	10.0894041	10.2358689	29
32	9.7643080	9.9105057	9.8538023	10.1461977	10.0894943	10.2356920	28
33	9.7644849	9.9104155	9.8540694	10.1459306	10.0895845	10.2355151	27
34	9.7646616	9.9103251	9.8543365	10.1456635	10.0896749	10.2353384	26
35	9.7648382	9.9102348	9.8546034	10.1453966	10.0897652	10.2351618	25
36	9.7650147	9.9101444	9.8548704	10.1451296	10.0898556	10.2349853	24
37	9.7651911	9.9100539	9.8551372	10.1448628	10.0899461	10.2348089	23
38	9.7653674	9.9099634	9.8554041	10.1445959	10.0900366	10.2346326	22
39	9.7655436	9.9098728	9.8556708	10.1443292	10.0901272	10.2344564	21
40	9.7657197	9.9097821	9.8559376	10.1440624	10.0902179	10.2342803	20
41	9.7658[illegible]	9.9096915	9.8562042	10.1437958	10.0903085	10.2341043	19
42	9.7660715	9.9096007	9.8564708	10.1435292	10.0903993	10.2339285	18
43	9.7662473	9.9095099	9.8567374	10.1432626	10.0904901	10.2337527	17
44	9.7664229	9.9094190	9.8570039	10.1429961	10.0905810	10.2335771	16
45	9.7665985	9.9093281	9.8572704	10.1427296	10.0906719	10.2334015	15
46	9.7667739	9.9092371	9.8575368	10.1424632	10.0907619	10.2332261	14
47	9.7669492	9.9091461	9.8578031	10.1421969	10.0908539	10.2330508	13
48	9.7671244	9.9090550	9.8580694	10.1419306	10.0909450	10.2328756	12
49	9.7672996	9.9089639	9.8583357	10.1416643	10.0910361	10.2327004	11
50	9.7674746	9.9088727	9.8586019	10.1413981	10.0911273	10.2325254	10
51	9.7676494	9.9087814	9.8588680	10.1411320	10.0912186	10.2323506	9
52	9.7678242	9.9086901	9.8591341	10.1408659	10.0913099	10.2321758	8
53	9.7679989	9.9085988	9.8594002	10.1405998	10.0914012	10.2320011	7
54	9.7681735	9.9085073	9.8596661	10.1403339	10.0914927	10.2318265	6
55	9.7683480	9.9084159	9.8599321	10.1400679	10.0915841	10.2316520	5
56	9.7685223	9.9083243	9.8601980	01.1398020	10.0916757	10.2314777	4
57	9.7686966	9.9082327	9.8604638	10.1395362	10.0917673	10.2313034	3
58	9.7688707	9.9081411	9.8607296	10.1392704	10.0918589	10.2311293	2
59	9.7690448	9.9080494	9.8609954	10.1390046	10.0919506	10.2309552	1
60	9.7692187	9.9079576	9.8612610	10.1387390	10.0920424	10.2307813	0
		Sine.		Tang.		Secant.	M

54 *Degrees.*

A Table of Artificial Sines,

36 *Degrees.*

M	Sine.		Tang.		Secant.	
0	9.7692187	9.9079576	9.8612610	10.1387390	10.0920424	10.2307813
1	9.7693925	9.9078658	9.8615267	10.1384733	10.0921342	10.2306075
2	9.7695662	9.9077740	9.8617923	10.1382077	10.0922260	10.2304338
3	9.7697398	9.9076820	9.8620578	10.1379422	10.0923180	10.2302602
4	9.7699134	9.9075901	9.8623233	10.1376767	10.0924099	10.2300866
5	9.7700868	9.9074980	9.8625887	10.1374113	10.0925020	10.2299132
6	9,7702601	9.9074059	9,8628541	10.1371459	10.0925941	10.2297399
7	9.7704332	9.9073138	9 8631195	10.1368805	10.0926862	10.2295668
8	9.7706063	9.9072216	9.8633848	10.1366152	10.0927784	10.2293937
9	9.7707793	9.9071293	9.8636500	10.1363500	10.0928707	10.2292207
10	9.7709522	9.9070370	9.8639152	10.1360848	10 0929630	10.2290478
11	9.7711249	9.9069446	9.8641803	10.1358197	10.0930554	[illegible]288751
12	9.7712976	9.9068522	9.8644454	10.1355546	10.0931478	10.2287024
13	9.7714702	9.9067597	9.8647105	10.1352895	10.0932403	10.2285298
14	9.7716426	9.9066671	9.8649755	10.1350245	10.0933329	10.2283574
15	9.7718150	9.9065745	9.8652404	10.1347596	10.0934255	10.2281850
16	9.7719872	9.9064819	9.8655053	10 1344947	10.0935181	10.2280128
17	9.7721593	9.9063892	9.8657702	10.1342298	10.0936108	10.2278407
18	9,7723314	9.9062964	9.8660350	10.1339650	10.0937036	10.2276686
19	9.7725033	9.9062036	9,86[illegible]997	10.1337003	10.0937964	10.2274967
20	9.7726751	9.9061107	9.8665644	10.1334356	10.0938893	10.2273249
21	9.7728468	9.9060177	9.8668291	10.1331709	10.0939823	10.2271532
22	9.7730185	9.9059247	9,8670937	10.1329063	10.0940753	10.2269815
23	9.7731900	9.9058317	9.8673583	10.1326417	10.0941683	10.2268100
24	9.7733614	9.9057386	9.8676228	10.1323772	10.0942614	10.2266386
25	9.7735327	9.9056454	9.8678873	10.1321127	10.0943546	10.2264673
26	9.7737039	9.9055522	9.8681517	10.1318483	10 0944478	10.2262961
27	9.7738749	9.9054589	9.8684160	10.1315840	10.0945411	10.2261251
28	9.7740459	9.9053656	9.8686804	10.1313196	10.0946344	10.2259541
29	9.7742168	9.9052722	9.8689446	10,1310554	10.0947278	10.2257832
30	9.7743876	9.9051787	9.8692089	10.1307911	10.0948213	10.2256124
		Sine.		Tang.		Secant.

53 *Degrees.*

36 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.7743876	9.9051787	9.8692089	10.1307911	10.0948213	10.2256124	30
31	9.7745583	9.9050852	9.8694731	10.1305269	10.0949148	10.2254417	29
32	9.7747288	9.9049916	9.8697372	10.1302628	10.0950084	10.2252712	28
33	9.7748993	9.9048980	9.8700013	10.1299987	10.0951020	10.2251007	27
34	9.7750697	9.9048043	9.8702653	10.1297347	10.0951957	10 2249303	26
35	9.7752399	9.9047106	9.8705293	10.1294707	10.0952894	10.2247601	25
36	9.7754101	9.9046168	9.8707933	10.1292067	10.0953832	10.2245899	24
37	9.7755801	9.9045230	9.8710572	10.1289428	10.0954770	10.2244199	23
38	9.7757501	9.9044291	9.8713210	10.1286790	10.0955709	10.2242499	22
39	9.7759199	9.9043351	9.8715848	10.1284152	10.0956649	10 2240801	21
40	9.7760897	9.9042411	9.8718486	10.1281514	10.0957589	10.2239103	20
41	9.7762[illegible]	9.9041470	9.8721123	10.1278877	10 0958530	10.2237407	19
42	9.7764289	9.9040529	9.8723760	10.1276240	10.0959471	10.2235711	18
43	9.7765983	9.9039587	9.8726396	10.1273604	10.0960413	10.2234017	17
44	9.7767676	9.9038644	9.8729032	10.1270968	10.0961356	10.2232324	16
45	9.7769369	9 9037701	9.8731668	10.1268332	10.0962299	10.2230631	15
46	9.7771060	9.9036757	9.8734302	10.1265698	10.0963243	10.2228940	14
47	9.7772750	9.9035813	9 8736937	10.1263063	10 0964187	10.2227250	13
48	9.7774439	9.9034868	9.8739571	10.1260429	10.0965132	10.2225561	12
49	9.7776128	9.9033923	9.8742204	10.1257[illegible]06	10.0966077	10.2223872	11
50	9.7777815	9.9032977	9.8744838	10.1255162	10.0967023	10.2222185	10
51	9.7779501	9.9032031	9.8747470	10.1252530	10.0967969	10.2220499	9
52	9.7781186	9.9031084	9.8750102	10.1249898	10.0968916	10.2218814	8
53	9.7782870	9.9030136	9.8752734	10.1247266	10.0969864	10.2217130	7
54	9.7784553	9.9029188	9.8755365	10.1244635	10.0970812	10.2215447	6
55	9.7786235	9.9028239	9.8757996	10.1242004	10.0971761	10.2213765	5
56	9.7787916	9.9027289	9.8760627	01.1239373	10.0972711	10.2212084	4
57	9.7789596	9.9026339	9.8763257	10.1236743	10.0973661	10.2210404	3
58	9.7791275	9.9025389	9.8765886	10.1234114	10.0974611	10.2208725	2
59	9,7792953	9.9024438	9.8768515	10.1231485	10.0975562	10.2207047	1
60	9.7794630	9.9023486	9.8771144	10.1228856	10.0976514	10.2205370	0
		Sine.		Tang.		Secant.	M

53 *Degrees.*

A Table of Artificial Sines,

37 Degrees.

M	Sine.		Tang.		Secant.	
0	9.7794630	9.9023486	9.8771144	10.1228856	10.0976514	10.2205370
1	9.7796306	9.9022534	9.8773772	10.1226228	10.0977466	10.2203694
2	9.7797981	9.9021581	9.8776400	10.1223600	10.0978419	10.2202019
3	9.7799655	9.9020628	9.8779027	10.1220973	10.0979372	10.2200345
4	9.7801328	9.9019674	9.8781654	10.1218346	10.0980326	10.2198672
5	9.7803000	9.9018719	9.8784281	10.1215719	10.0981281	10.2197000
6	9.7804671	9.9017764	9.8786907	10.1213093	10.0982236	10.2195329
7	9.7806341	9.9016808	9.8789533	10.1210467	10.0983192	10.2193659
8	9.7808010	9.9015852	9.8792158	10.1207842	10.0984148	10.2191990
9	9.7809677	9.9014895	9.8794782	10.1205218	10.0985105	10.2190323
10	9.7811344	9.9013938	9.8797407	10.1202593	10.0986062	10.2188656
11	9.7813010	9.9012980	9.8800031	10.1199969	10.0987020	1[illegible]186990
12	9.7814675	9.9012021	9.8802654	10.1197346	10.0987979	10.2185325
13	9.7816339	9.9011062	9.8805277	10.1194723	10.0988938	10.2183661
14	9.7818002	9.9010102	9.8807900	10.1192100	10.0989898	10.2181995
15	9.7819664	9.9009142	9.8810522	10.1189478	10.0990858	10.218033
16	9.7821324	9.9008181	9.8813144	10.1186856	10.0991819	10.21786
17	9.7822984	9.9007219	9.8815765	10.1184235	10.0992781	10.217701
18	9.7824643	9.9006257	9.8818386	10.1181614	10.0993743	10.217535
19	9.7826301	9.9005294	9.8821007	10.1178993	10.0994706	10.21735
20	9.7827958	9.9004331	9.88[illegible]3627	10.1176373	10.0995669	10.217204
21	9.7829614	9.9003367	9.8826246	10.1173754	10.0996633	10.21705
22	9.7831268	9.9002403	9.8828866	10.1171134	10.0997597	10.216873
23	9.7832,92	9.9001438	9.8831484	10.1168516	10.0998562	10.21670
24	9.7834575	9.9000472	9.8834103	10.1165897	10.0999528	10.216542
25	9.783622	9.8999506	9.8836721	10.1163279	10.100049	10.21637
26	9.7837878	9.8998539	9.8839338	10.1160662	10.1001461	10.216212
27	9.7839528	9.8997572	9.8841956	10.1158044	10.1002428	10.21607
28	9.7841177	9.8996604	9.8844572	10.1155428	10.1003396	10.21558
29	9.7842824	9.8995636	9.8847189	10.1152811	10.1004364	10.21571
30	9.7844471	9.8994667	9.8849805	10.1150195	10.1005333	10.21555
		Sine.		Tang.		Secant.

52 Degrees.

37 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.7844471	9.8994667	9.8849805	10.1150195	10.1005333	10,2155529	30
31	9.7846117	9.8993697	9.8852420	10.1147580	10.1006303	10.2153883	29
32	9.7847762	9.8992727	9.8855035	10.1144965	10.1007273	10.2152238	28
33	9.7849406	9.8991756	9.8857650	10.1142350	10.1008244	10.2150594	27
34	9.7851049	9.8990784	9.8860264	10.1139736	10.1009216	10.2148951	26
35	9.7852691	9.8989812	9.8862878	10.1137122	10.1010188	10.2147309	25
36	9.7854332	9.8988840	9.8865492	10.1134508	10.1011160	10.2145668	24
37	9.7855972	9.8987867	9.8868105	10.1131895	10.1012133	10.2144028	23
38	9.7857611	9.8986893	9.8870718	10.1129282	10.1013107	10.2142389	22
39	9.7859249	9.8985919	9.8873330	10.1126670	10.1014081	10.2140751	21
40	9.7860886	9.8984944	9.8875942	10.1124058	10.1015056	10.2139114	20
41	9.7862522	9.8983968	9.8878554	10.1121446	10.1016032	10.2137478	19
42	9.7864157	9.8982992	9.8881165	10.1118835	10.1017008	10.2135843	18
43	9.7865791	9.8982015	9.8883775	10.1116225	10.1017985	10.2134209	17
44	9.7867424	9.8981038	9.8886386	10.1113614	10.1018962	10.2132576	16
45	9.7869056	9.8980060	9.8888996	10.1111004	10.1019940	10.2130944	15
46	9.7870687	9.8979082	9.8891605	10.1108395	10.1020918	10.2129313	14
47	9.7872317	9.8978103	9.8894214	10.1105786	10.1021897	10.2127683	13
48	9.7873946	9,8977123	9.8896823	10.1103177	10.1022877	10.2126054	12
49	9.7875574	9.8976143	9.8899432	10.1100568	10.1023857	10.2124426	11
50	9,7877202	9.8975162	9.8902040	10.1097960	10.1024838	10.2122798	10
51	9,7878828	9.8974181	9.8904647	10.1095353	10.1025819	10.2121172	9
52	9.7880453	9.8973199	9.8907254	10.1092746	10.1026801	10.2119547	8
53	9.7882077	9.8972216	9.8909861	10.1090139	10.1027784	10.2117923	7
54	9.7883701	9.8971233	9.8912468	10.1087532	10.1028767	10.2116299	6
55	9.7885323	9.8970249	9.8915074	10.1084926	10.1029751	10.2114677	5
56	9.7886944	9.8969265	9.8917679	10.1082321	10.1030735	10.2113056	4
57	9.7888565	9.8968280	9.8920285	10.1079715	10.1031720	10.2111435	3
58	9.7890184	9.8967294	9.8922890	10.1077110	10.1032706	10.2109816	2
59	9.7891802	9.8966308	9.8925494	10.1074506	10.1033692	10.2108198	1
60	9.7893420	9.8965321	9.8928098	10.1071902	10.1034679	10.2106580	0
		Sine.		Tang.		Secant.	M

52 *Degrees.*

A Table of Artificial Sines.

38 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.7893420	9.8965321	9.8928098	10.1071902	10.1034679	10.2106580	60
1	9.7895036	9.8964334	9.893070	10.1069298	10.1035666	10.2104964	59
2	9.7896652	9.8963346	9.8933306	10.1066694	10.1036654	10.2103348	58
3	9.7898266	9.8962358	9.8935905	10.1064091	10.1037642	10.2101734	57
4	9.7899880	9.8961369	9.8938511	10.1061489	10.1038631	10.2100120	56
5	9.7901493	9.8960379	9.8941114	10.1058886	10.1039621	10.2098507	55
6	9.7903104	9.8959389	9.8943715	10.1056285	10.1040611	10.2096896	54
7	9.7904715	9.8958398	9.8946317	10.1053683	10.1041602	10.2095285	53
8	9.7906325	9.8957406	9.8948918	10.1051082	10.1042594	10.2093675	52
9	9.7907930	9.8956414	9.8951519	10.1048481	10.1043586	10.2092007	51
10	9.7909541	9.8955422	9.8954119	10.1045881	10.1044578	10.2090459	50
11	9.7911140	9.8954429	9.8956719	10.1043281	10.1045572	10.2088852	49
12	9.7912754	9.8953435	9.8959319	10.1040681	10.1046565	10.2087246	48
13	9.7914359	9.8952440	9.8961918	10.1038082	10.1047560	10.2085641	47
14	9.7915963	9.8951445	9.8964517	10.1035483	10.1048555	10.2084037	46
15	9.7917566	9.8950450	9.8967116	10.1032884	10.1049550	10.2082434	45
16	9.7919168	9.8949453	9.8969714	10.1030286	10.1050547	10.2080832	44
17	9.7920769	9.8948457	9.8972312	10.1027688	10.1051543	10.2079231	43
18	9.7922369	9.8947459	9.8974910	10.1025090	10.1052541	10.2077631	42
19	9.7923968	9.8946461	9.8977507	10.1022493	10.1053539	10.2076032	41
20	9.7925566	9.8945463	9.8980104	10.1019896	10.1054537	10.2074434	40
21	.7927163	9.8944463	9.8982700	10.1017300	10.1055537	10.2072837	39
22	9.7928760	9.8943464	9.8985296	10.1014704	10.1056536	10.2071240	38
23	9.7930355	9.8942463	9.8987892	10.1012108	10.1057537	10.2069645	37
24	9.793194	9.8941462	9.8990487	10.1009513	10.1058538	10.2068051	36
25	9.7933543	9.8940461	9.8993082	10.1006918	10.1059539	10.2066457	35
26	9.7935135	9.8939458	9.8995677	10.1004323	10.1060542	10.2064865	34
27	9.7936727	9.8938456	9.8998271	10.1001729	10.1061544	10.2063273	33
28	9.7938317	9.8937452	9.9000865	10.0999135	10.1062548	10.2061683	32
29	9.7939907	9.8936448	9.9003459	10.0996541	10.1063552	10.2060093	31
30	9.7941496	9.8935444	9.9006052	10.0993948	10.1064556	10.2058504	30
		Sine.		Tang.		Secant.	M

51 *Degrees.*

38 Degrees.

M	Sine.		Tang.		Secant.		
30	9.7941496	9.8935444	9.9006052	10.0993948	10.1064556	10.2058504	30
31	9.7943083	9.8934439	9.9008645	10.0991355	10.1065561	10.2056917	29
32	9.7944670	9.8933433	9.9011237	10.0988763	10.1066567	10.2055330	28
33	9.7946256	9.8932426	9.9013830	10.0986170	10.1067574	10.2053744	27
34	9.7947841	9.8931419	9.9016422	10.0983578	10.1068581	10.2052159	26
35	9.7949425	9.8930412	9.9019013	10.0980987	10.1069588	10.2050575	25
36	9.7951008	9.8929404	9.9021604	10.0978396	10.1070596	10.2048992	24
37	9.7952590	9.8928395	9.9024195	10.0975805	10.1071605	10.2047410	23
38	9.7954171	9.8927385	9.9026786	10.0973214	10.1072615	10.2045829	22
39	9.7955751	9.8926375	9.9029376	10.0970624	10.1073625	10 2044249	21
40	9.7957330	9.8925365	9.9031966	10.0998034	10.1074635	10.2042670	20
41	9.7958909	9.8924354	9.9034555	10.0965445	10.1075646	10.2041091	19
42	9.7960486	9.8923342	9.9037144	10.0962856	10.1076658	10.2039514	18
43	9.7962062	9.8922329	9.9039733	10.0960267	10.1077671	10.2037938	17
44	9.7963638	9.8921316	9.9042321	10.0957679	10.1078684	10.2036362	16
45	9.7965212	9.8920303	9.9044910	10.0955090	10.1079697	10.2034788	15
46	9.7966786	9.8919289	9.9047497	10.0952503	10.1080711	10.2033214	14
47	9.7968359	9.8918274	9.9050085	10.0949915	10.1081726	10.2031641	13
48	9.7969930	9.8917258	9.9052672	10.0947328	10.1082742	10.2030070	12
49	9.7971501	9.8916242	9.9055259	10.0944741	10.1083758	10.2028499	11
50	9.7973071	9.8915226	9.9057845	10.0942155	10.1084774	10.2026929	10
51	9.7974640	9.8914208	9.9060431	10.0939569	10.1085792	10.2025360	9
52	9.7976208	9.8913191	9.9063017	10.0936983	10.1086809	10.2023792	8
53	9.7977775	9.8912172	9.9065603	10.0934397	10.1087828	10.2022225	7
54	9.7979341	9.8911153	9.9068188	10.0931812	10.1088847	10.2020659	6
55	9.7980906	9.8910133	9.9070773	10.0929227	10.1089867	10.2019094	5
56	9.7982470	9.8909113	9.9073357	10.0926643	10.1090887	10.2017530	4
57	9.7984034	9.8908092	9.9075941	10.0924059	10.1091908	10.2015966	3
58	9.7985596	9.8907071	9.9078525	10.0921475	10.1092929	10.2014404	2
59	9.7987158	9.8906049	9.9081109	10.0918891	10.1093951	10.2012842	1
60	9.7988718	9.8905026	9.9083692	10.0916308	10.1094974	10.2011282	0
		Sine.		Tang.		Secant.	M

51 Degrees.

39 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.7988718	9.8905026	9.9083697	10.0916308	10.1094974	10.2011282	60
1	9.7990278	9.8904003	9.9086275	10.0913725	10.1095997	10.2009722	59
2	9.7991836	9.8902979	9.9088858	10.0911142	10.1097021	10.2008164	58
3	9.7993394	9.8901954	9.909144	10.0908560	10.1098046	10.2006606	57
4	9.7994951	9.8900929	9.909402	10.0905978	10.1099071	10.2005049	56
5	9.7996507	9.8899903	9.9096603	10.0903397	10.1100097	10.2003493	55
6	9.7998062	9.8898877	9.9099185	10.0900815	10.1101123	10.2001938	54
7	9.7999616	9.8897850	9.9101766	10.0898234	10.1102150	10.2000384	53
8	9.8001169	9.8896822	9.9104347	10.0895653	10.1103178	10.1998831	52
9	9.8002721	9.8895794	9.9106927	10.0893073	10.1104206	10.1997279	51
10	9.8004272	9.8894765	9.9109507	10.0890493	10.1105235	10.1995728	50
11	9.8005823	9.8893736	9.9112087	10.0887913	10.1106264	10.1994177	49
12	9.8007372	9.8892706	9.9114666	10.0885334	10.1107294	10.1992628	48
13	9.8008921	9.8891675	9.9117245	10.0882755	10.1108325	10.1991079	47
14	9.8010468	9.8890644	9.9119824	10.0880176	10.1109356	10.1989532	46
15	9.8012015	9.8889612	9.9122403	10.0877597	10.1110388	10.1987985	45
16	9.8013561	9.8888580	9.9124981	10.0875019	10.1111420	10.1986439	44
17	9.8015106	9.8887547	9.9127559	10.0872441	10.1112453	10.1984894	43
18	9.8016649	9.8886513	9.9130137	10.0869863	10.1113487	10.1983351	42
19	9.8018192	9.8885479	9.9132714	10.0867286	10.1114521	10.1981808	41
20	9.8019735	9.8884444	9.913529	10.0864709	10.1115556	10.1980265	40
21	9.8021276	9.8883408	9.9137868	10.0862132	10.1116592	10.1978724	39
22	9.8022816	9.8882372	9.9140444	10.0859556	10.1117628	10.1977184	38
23	9.8024355	9.8881335	9.9143020	10.0856980	10,1118665	10.1975645	37
24	9.8025894	9.8880298	9.9145596	10.0854404	10.1119702	10.1974106	36
25	9.8027431	9.8879260	9.9148171	10.0851829	10.1120740	10.1972569	35
26	9.8028968	9.8878221	9.9150747	10.0849253	10.1121779	10.1971032	34
27	9.8030504	9.8877182	9.9153322	10.0846678	10.1122818	10.1969496	33
28	9,8032038	9.8876142	9.9155896	10.0844104	10.1123858	10.1967962	32
29	9.8033572	9.8875102	9.9158471	10.0841529	10.1124898	10.1966428	31
30	9.8035105	9.8874061	9.9161045	10.0838955	10.1125939	10.1964895	30
		Sine.		Tang.		Secant.	M

50 *Degrees.*

39 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.8035105	9.8874061	9.9161045	10.0838955	10.1125939	10.1964895	30
31	9.8036637	9.8873019	9.9163618	10.0836382	10.1126981	10.1963363	29
32	9.8038168	9.8871977	9.9166192	10.0833808	10.1128023	10.1961832	28
33	9.8039699	9.8870934	9.9168765	10.0831235	10.1129066	10.1960301	27
34	9.8041228	9.8869890	9.9171338	10.0828662	10.1130110	10.1958772	26
35	9.8042757	9.8868846	9.9173911	10.0826089	10.1131154	10.1957243	25
36	9.8044284	9.8867801	9.9176483	10.0823517	10.1132199	10.1955716	24
37	9.8045811	9.8866756	9.9179055	10.0820945	10.1133244	10.1954189	23
38	9.8047336	9.8865710	9.9181627	10.0818373	10.1134290	10.1952664	22
39	9.8048861	9.8864663	9.9184198	10.0815802	10.1135337	10.1951139	21
40	9.8050385	9.8863616	9.9186769	10.0813231	10.1136384	10.1949615	20
41	9.8051908	9.8862568	9.9189340	10.0810660	10.1137432	10.1948092	19
42	9.8053430	9.8861519	9.9191911	10.0808089	10.1138481	10.1946570	18
43	9.8054951	9.8860470	9.9194481	10.0805519	10.1139530	10.1945049	17
44	9.8056472	9.8859420	9.9197051	10.0802949	10.1140580	10.1943528	16
45	9.8057991	9.8858370	9.9199621	10.0800379	10.1141630	10.1942009	15
46	9.8059510	9.8857319	9.9202191	10.0797809	10.1142681	10.1940490	14
47	9.8061027	9.8856267	9.9204760	10.0795240	10.1143733	10.1938973	13
48	9.8062544	9.8855215	9.9207329	10.0792671	10.1144785	10.1937456	12
49	9.8064060	9.8854162	9.9209898	10.0790102	10.1145838	10.1935940	11
50	9.8065575	9.8853109	9.9212466	10.0787534	10.1146891	10.1934425	10
51	9.8067089	9.8852055	9.9215034	10.0784966	10.1147945	10.1932911	9
52	9.8068602	9.8851000	9.9217602	10.0782398	10.1149000	10.1931398	8
53	9.8070114	9.8849945	9.9220170	10.0779830	10.1150055	10.1929886	7
54	9.8071626	9.8848889	9.9222737	10.0777263	10.1151111	10.1928374	6
55	9.8073136	9.8847832	9.9225304	10.0774696	10.1152168	10.1926864	5
56	9.8074646	9.8846775	9.9227871	10.0772129	01.1153225	10.1925354	4
57	9.8076154	9.8845717	9.9230437	10.0769563	10.1154283	10.1923846	3
58	9.8077662	9.8844659	9.9233004	10.0766996	10.1155341	10.1922338	2
59	9.8079169	9.8843599	9.9235570	10.0764430	10.1156401	10.1920831	1
60	9.8080675	9.8842540	9.9238135	10.0761865	10.1157460	10.1919325	0
		Sine.		Tang.		Secant.	M

50 *Degrees.*

40 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.8080675	9.8842540	9.9238135	10.0761865	10.1157460	10.1919325	60
1	9.8082180	9.8841475	9.9240701	10.0759299	10.1158521	10.1917820	59
2	9.8083684	9.8840418	9.9243266	10.0756734	10.1159582	10.1916316	58
3	9.8085188	9.8839357	9.9245831	10.0754169	10.1160643	10.1914812	57
4	9.8086690	9.8838294	9.9248396	19.0751604	10.1161706	10.1913310	56
5	9.8088192	9.8837232	9.9250960	10.0749040	10.1162768	10.1911808	55
6	9,8089692	9.8836168	9,9253524	10.0746476	10.1163832	10.1910308	54
7	9.8091192	9-8835104	9.9256088	10.0743912	10.1164896	10.1908808	53
8	9.8092691	9.8834039	9.9258652	10.0741348	10.1165961	10.1907309	52
9	9.8094189	9.8832974	9.9261215	10.0738785	10.1167026	10.1905811	51
10	9.8095686	9.8831908	9.9263778	10.0736222	10.1168092	10.1904314	50
11	9-8097182	9.8830841	9.9266341	10.0733659	10.1169159	10.1902818	49
12	9.8098678	9,8829774	9.9268904	10.0731096	10.1170226	10.1901322	48
13	9.8100172	9.8828706	9.9271466	10.0728534	10.1171294	10.1899828	47
14	9.8101666	9.8827638	9.9274028	10.0725972	10.1172362	10.1898334	46
15	9.8103155	9.8826568	9.9276590	10.0723410	10.1173432	10.1896841	45
16	9.8104650	9.8825499	9.9279152	10.0720848	10.1174501	10.1895350	44
17	9.8106141	9.8824428	9.9281713	10.0718287	10.1175572	10.1893859	43
18	9,8107631	9.8823357	9.9284274	10.0715726	10.1176643	10.1892369	42
19	9.8109121	9.8822285	9,9286835	10.0713165	10.1177715	10.1890879	41
20	9.8110609	9.8821213	9.9289396	10.0710604	10.1178787	10.1889391	40
21	9.8112096	9.8820140	9.9291956	10.0708044	10.1179860	10.1887904	39
22	9.8113583	9.8819067	9,9294516	10.0705484	10.1180933	10.1886417	38
23	9.8115069	9.8817992	9.9297076	10.0702924	10.1182008	10.1884931	37
24	9.8116554	9.8816918	9.9299636	10.0700364	10.1183082	10.1883446	36
25	9.8118038	9.8815842	9.9302195	10.0697805	10.1184158	10.1881962	35
26	9.8119521	9.8814766	.9304755	10.0695245	10.1185234	10.1880479	34
27	9.8121003	9.8813689	9.9307314	10.0692686	10.1186311	10.1878997	33
28	9.8122484	9.8812612	9.9309872	10.0690128	10.1187388	10.1877516	32
29	9.8123965	9.8811534	9.9312431	10,0687569	10.1188466	10.1876035	31
30	9.8125444	9.8810455	9.9314989	10.0685011	10.1189545	10.1874556	30
		Sine.		Tang.		Secant.	

49 *Degrees.*

40 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.8125444	9.8810455	9.9314989	10.0685011	10.1189545	10.1874550	30
31	9.8126923	9.8809376	9.9317547	10.0682453	10.1190624	10.1873077	29
32	9.8128401	9.8808296	9.9320105	10.0679895	10.1191704	10.1871599	28
33	9.8129878	9.8807215	9.9322662	10.0677338	10.1192785	10.1870122	27
34	9.8131354	9.8806134	9.9325220	10.0674780	10.1193866	10 1868646	26
35	9.8132829	9.8805052	9.9327777	10.0672223	10.1194948	10.1867171	25
36	9.8134303	9.8803970	9.9330334	10.0669666	10.1196030	10.1865697	24
37	9.8135777	9.8802887	9.9332890	10.0667110	10.1197113	10.1864223	23
38	9.8137250	9.8801803	9.9335446	10.0664554	10.1198197	10.1862750	22
39	9.8138721	9.8800719	9.9338003	10.0661997	10.1199281	10.1861279	21
40	9.8140192	9.8799634	9.9340559	10.0659441	10.1200366	10.1859808	20
41	9.8141662	9.8798548	9.9343114	10.0656886	10.1201452	10.1858338	19
42	9.8143131	9.8797462	9.9345670	10.0654330	10.1202538	10.1856869	18
43	9.8144600	9.8796375	9.9348225	10.0651775	10.1203625	10.1855400	17
44	9.8146067	9.8795287	9.9350780	10.0649220	10.1204713	10.1853933	16
45	9.8147534	9.8794199	9.9353335	10.0646665	10.1205801	10.1852466	15
46	9.8148999	9.8793110	9.9355889	10.0644111	10.1206890	10.1851001	14
47	9.8150464	9.8792021	9.9358444	10.0641556	10.1207979	10.1849536	13
48	9.8151928	9.8790930	9.9360998	10.0639002	10.1209070	10.1848072	12
49	9.8153391	9.8789840	9.9363552	10.0636448	10.1210160	10.1846609	11
50	9.8154854	9.8788748	9.9366105	10.0633895	10.1211252	10.1845140	10
51	9.8156315	9.8787656	9.9368659	10.0631341	10.1212344	10.1843685	9
52	9.8157776	9.8786563	9.9371212	10.0628788	10.1213437	10.1842224	8
53	9.8159235	9.8785470	9.9373765	10.0626235	10.1214530	10.1840765	7
54	9.8160694	9.8784376	9.9376318	10.0623682	10.1215624	10.1839306	6
55	9.8162152	9.8783281	9.9378871	10.0621129	10.1216719	10.1837848	5
56	9.8163609	9.8782186	9.9381423	10 0618577	01.1217814	10.1836391	4
57	9.8165066	9.8781090	9.9383975	10.0616025	10.1218910	10.1834934	3
58	9.8166521	9.8779994	9.9386527	10.0613473	10.1220006	10.1833479	2
59	9.8167975	9.8778896	9.9389079	10.0610921	10.1221104	10.1832025	1
60	9.8169429	9.8777799	9.9391631	10.0608369	10.1222201	10.1830571	0
		Sine.		Tang.		Secant.	M

49 *Degrees.*

A Table of Artificial Sines,

41 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.8169429	9.8777799	9.9391631	10.0608369	10.1222201	10.1830571	60
1	9.8170882	9.8776700	9.9394182	10.0605818	10.1223300	10.1829118	59
2	9.8172334	9.8775601	9.9396733	10.0603267	10.1224399	10.1827666	58
3	9.8173785	9.8774501	9.9399284	10.0600716	10.1225499	10.1826215	57
4	9.8175235	9.8773401	9.9401835	10.0598165	10.1226599	10.1824765	56
5	9.8176685	9.8772300	9.9404385	10.0595615	10.1227700	10.1823315	55
6	9,8178133	9.8771198	9,9406936	10.0593064	10.1228802	10.1821867	54
7	9.8179581	9.8770096	9 9409486	10.0590514	10.1229904	10.1820419	53
8	9.8181028	9.8768993	9.9412036	10.0587964	10.1231007	10.1818972	52
9	9.8182474	9.8767889	9.9414585	10.0585415	10.1232111	10.1817526	51
10	9.8183919	9.8766785	9.9417135	10.0582865	10 1233215	10.1816081	50
11	9.8185364	9.8765680	9.9419684	10.0580316	10.1234320	10.1814636	49
12	9.8186807	9.8764574	9.9422233	10.0577767	10.1235426	10.1813193	48
13	9.8188250	9.8763468	9.9424782	10.0575218	10.1236532	10.1811750	47
14	9.8189692	9.8762361	9.9427331	10.0572669	10.1237639	10.1810308	46
15	9.8191133	9.8761253	9 9429879	10.0570121	10.1238747	10.1808867	45
16	9.8192573	9.8760145	9.9432428	10 0567572	10.1239855	10.1807427	44
17	9.8194012	9 8759036	9.9434976	10.0565024	10.1240964	10.1805985	43
18	9,8195450	9.8757927	9.9437524	10.0562476	10.1242073	10.1804550	42
19	9.8196888	9.8756816	9,9440072	10.0559928	10.1243184	10.1803112	41
20	9.8198325	9.8755706	9.9442619	10.0557381	10.1244294	10 1801675	40
21	9.8199761	9.8754594	9.9445166	10.0554834	10.1245406	10.1800237	39
22	9.8201196	9.8753482	9,9447714	10.0552286	10.1246518	10.1798804	38
23	9.8202630	9.8752369	9.9450261	10.0549739	10.1247631	10.1797370	37
24	9.8204063	9.8751256	9.9452807	10.0547193	10.1248744	10.1795937	36
25	9.8205496	9.8750142	9.9455354	10.0544646	10.1249858	10.1794504	35
26	9.8206927	9.8749027	9.9457900	10.0542100	10 1250973	10.1793073	34
27	9.8208358	9.8747912	9.9460447	10.0539553	10.1252088	10.1791642	33
28	9.8209788	9.8746795	9.9462993	10.0537007	10.1253205	10.1790212	32
29	9.8211217	9.8745679	9.9465539	10,0534461	10.1254321	10.1788783	31
30	9.8212646	9.8744561	9.9468084	10.0531916	10.1255439	10.1787354	30
		Sine.		Tang.		Secant.	

48 *Degrees.*

41 Degrees.

M	Sine.		Tang.		Secant.		
30	9.8212646	9.8744561	9.9468084	10.0531916	10.1255439	10.1787354	30
31	9.8214073	9.8743443	9.9470630	10.0529370	10.1256557	10.1785927	29
32	9.8215500	9.8742325	9.9473175	10.0526825	10.1257675	10.1784500	28
33	9.8216926	9.8741205	9.9475720	10.0524280	10.1258795	10.1783074	27
34	9.8218351	9.8740085	9.9478265	10.0521735	10.1259915	10.1781649	26
35	9.8219775	9.8738965	9.9480810	10.0519190	10.1261035	10.1780225	25
36	9.8221198	9.8737844	9.9483355	10.0516645	10.1262156	10.1778802	24
37	9.8222621	9.8736722	9.9485899	10.0514101	10.1263278	10.1777379	23
38	9.8224042	9.8735599	9.9488443	10.0511557	10.1264401	10.1775958	22
39	9.8225463	9.8734476	9.9490987	10.0509013	10.1265524	10.1774537	21
40	9.8226883	9.8733352	9.9493531	10.0506469	10.1266648	10.1773117	20
41	9.8228302	9.8732227	9.9496075	10.0503925	10.1267773	10.1771698	19
42	9.8229721	9.8731102	9.9498619	10.0501381	10.1268898	10.1770279	18
43	9.8231138	9.8729976	9.9501162	10.0498838	10.1270024	10.1768862	17
44	9.8232555	9.8728849	9.9503705	10.0496295	10.1271151	10.1767445	16
45	9.8233971	9.8727722	9.9506248	10.0493752	10.1272278	10.1766029	15
46	9.8235386	9.8726594	9.9508791	10.0491209	10.1273406	10.1764614	14
47	9.8236800	9.8725466	9.9511334	10.0488666	10.1274534	10.1763200	13
48	9.8238213	9.8724337	9.9513876	10.0486124	10.1275663	10.1761787	12
49	9.8239626	9.8723207	9.9516419	10.0483581	10.1276793	10.1760374	11
50	9.8241037	9.8722076	9.9518961	10.0481039	10.1277924	10.1758963	10
51	9.8242448	9.8720945	9.9521503	10.0478497	10.1279055	10.1757552	9
52	9.8243858	9.8719813	9.9524045	10.0475955	10.1280187	10.1756142	8
53	9.8245267	9.8718681	9.9526587	10.0473413	10.1281319	10.1754733	7
54	9.8246676	9.8717548	9.9529128	10.0470872	01.1282452	10.1753324	6
55	9.8248083	9.8716414	9.9531670	10.0468330	10.1283586	10.1751917	5
56	9.8249490	9.8715275	9.9534211	10.0465789	10.1284721	10.1750510	4
57	9.8250896	9.8714144	9.9536752	10.0463248	10.1285856	10.1749104	3
58	9.8252301	9.8713008	9.9539293	10.0460707	10.1286992	10.1747699	2
59	9.8253705	9.8711872	9.9541834	10.0458166	10.1288128	10.1746295	1
60	9.8255109	9.8710735	9.9544374	10.0455626	10.1289265	10.1744891	0
		Sine.		Tang.		Secant.	M

48 Degrees.

A Table of Artificial Sines.

42 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.8255109	9.8710735	9.9544374	10.0455626	10.1289265	10.1744891	60
1	9.8256512	9.8709597	9.9546915	10.0453085	10.1290403	10.1743488	59
2	9.8257913	9.8708458	9.9549455	10.0450545	10.1291542	10.1742087	58
3	9.8259314	9.8707319	9.9551995	10.0448005	10.1292681	10.1740686	57
4	9.8260715	9.8706179	9.9554535	10.0445465	10.1293821	10.1739285	56
5	9.8262114	9.8705039	9.9557075	10.0442925	10.1294961	10.1737886	55
6	9.8263512	9.8703898	9.9559615	10.0440385	10.1296102	10.1736488	54
7	9.8264910	9.8702756	9.9562154	10.0437846	10.1297244	10.1735090	53
8	9.8266307	9.8701613	9.9564694	10.0435306	10.1298387	10.1733693	52
9	9.8267703	9.8700470	9.9567233	10.0432767	10.1299530	10.1732297	51
10	9.8269098	9.8699326	9.9569772	10.0430228	10.1300674	10.1730902	50
11	9.8270493	9.8698182	9.9572311	10.0427689	10.1301815	10.1729507	49
12	9.8271887	9.8697037	9.9574850	10.0425150	10.1302963	10.1728113	48
13	9.8273279	9.8695891	9.9577389	10.0422611	10.1304109	10.1726721	47
14	9.8274671	9.8694744	9.9579927	10.0420073	10.1305256	10.1725329	46
15	9.8276063	9.8693597	9.9582465	10.0417535	10.1306403	10.1723937	45
16	9.8277453	9.8692445	9.9585004	10.0414996	10.1307551	10.1722547	44
17	9.8278843	9.8691301	9.9587542	10.0412458	10.1308699	10.1721157	43
18	9.8280231	9.8690152	9.9590080	10.0409920	10.1309848	10.1719769	42
19	9.8281619	9.8689002	9.9592618	10.0407382	10.1310998	10.1718381	41
20	9.8283006	9.8687851	9.9595155	10.0404845	10.1312147	10.1716994	40
21	9.8284393	9.8686700	9.9597693	10.0402307	10.1313300	10.1715607	39
22	9.8285778	9.8685548	9.9600230	10.0399770	10.1314452	10.1714222	38
23	9.8287163	9.8684396	9.9602767	10.0397233	10.1315604	10.1712837	37
24	9.8288547	9.8683242	9.9605305	10.0394695	10.1316758	10.1711453	36
25	9.8289930	9.8682088	9.9607842	10.9392158	10.1317912	10.1710070	35
26	9.8291312	9.8680934	9.9610378	10.0389622	10.1319066	10.1708688	34
27	9.8292694	9.8679779	9.9612915	10.0387085	10.1320221	10.1707306	33
28	9,8294075	9.8678623	9.9615452	10.0384548	10.1321377	10.1705925	32
29	9.8295454	9.8677466	9.9617988	10.0382012	10.1322534	10.1704546	31
30	9.8296833	9.8676309	9.9620525	10.0379475	10.1323691	10.1703167	30
		Sine.		Tang.		Secant.	M

47 *Degrees.*

42 *Degrees.*

M	Sine.		Tang.		Secant.		
30	9.8296833	9.8676309	9.9620525	10.0379475	10.1323691	10.1703167	30
31	9.8298212	9.8675151	9.9623061	10.0376939	10.1324849	10.1701788	29
32	9.8299589	9,8673992	9.9625597	10.0374403	10.1326008	10.1700411	28
33	9.8300966	9.8672833	9.9628133	10.0371867	10.1327167	10.1699034	27
34	9.8302342	9.8671673	9,9630669	10.0369331	10.1328327	10.1697658	26
35	9.8303717	9.8670512	9.9633204	10.0366796	10.1329488	10.1696283	25
36	9.8305091	9.8669351	9.9635740	10.0364260	10.1330649	10.1694909	24
37	9.8306464	9.8668189	9.9638275	10.0361725	10.1331811	10.1693536	23
38	9.8307837	9.8667026	9.9640811	10.0359185	10.1332974	10.1692163	22
39	9.8309209	9.8665863	9.9643346	10.0356654	10.1334137	10.1690791	21
40	9.8310580	9.8664699	9.9645881	10.0354119	10.1335301	10.1689420	20
41	9.8311950	9.8663534	9.9648416	10.0351584	10.1336466	10.1688050	19
42	9.8313320	9.8662369	9.9650951	10.0349049	10.1337631	10.1686680	18
43	9.8314688	9 8661203	9.9653486	10.0346514	10.1338797	10.1685312	17
44	9.8316056	9.8660036	9.9656020	10.0343980	10.1339964	10.1683944	16
45	9.8317423	9.8658868	9.9658555	10.0341445	10.1341132	10.1682577	15
46	9.8318789	9.8657700	9.9661089	10.0338911	10.1342300	10.1681211	14
47	9.8320155	9.8656531	9.9663623	10 0336377	10.1343469	10.1679845	13
48	9.8321519	9.8655362	9.9666157	10.0333843	10.1344638	10.1678481	12
49	9.8322883	9.8654192	9.9668692	10.0331308	10.1345808	10.1677117	11
50	9.8324246	9.8653021	9.9671225	10.0328775	10.1346979	10.1675754	10
51	9.8325609	9.8651849	9.9673759	10.0326241	10.1348151	10.1674391	9
52	9.8326970	9.8650677	9.9676293	10.0323707	10.1349323	10.1673030	8
53	9.8328331	9.8649504	9.9678827	10.0321173	10.1350496	10.1671669	7
54	9.8329691	9.8648331	9.9681360	10.0318640	10.1351669	10.1670309	6
55	9.8331050	9.8647156	9.9683893	10.0316107	10.1352844	10.1668950	5
56	9.8332408	9.8645981	9.9686427	10.0313573	10.1354019	10.1667592	4
57	9.8333766	9.8644806	9.9688960	10.0311040	10.1355194	10.1666234	3
58	9.8335122	9.8643629	9.9691493	10.0308507	10.1356371	10.1664878	2
59	9,8336478	9.8642452	9.9694026	10.0305974	10.1357548	10.1663522	1
60	9.8337833	9.8641275	9.9696559	10.0303441	10.1358725	10.1662167	0
		Sine.		Tang.		Secant.	M

47 *Degrees.*

A Table of Artificial Sines.

43 *Degrees.*

M	Sine.		Tang.		Secant.		
0	9.8337833	9.8641275	9.9696555	10.0303441	10.1358725	10.1662167	60
1	9.8339188	9.8640096	9.9699091	10.0300905	10.1359904	10.1660812	59
2	9.8340541	9.8638917	9.9701624	10.0298376	10.1361083	10.1659459	58
3	9.8341894	9.8637737	9.9704157	10.0295843	10.1362263	10.1658106	57
4	9.8343246	9.8636557	9.9706689	10.0293311	10.1363443	10.1656754	56
5	9.8344597	9.8635376	9.9709221	10.0290779	10.1364624	10.1655403	55
6	9.8345948	9.8634194	9.9711754	10.0288246	10.1365806	10.1654052	54
7	9.8347297	9.8633011	9.9714286	10.0285714	10.1366989	10.1652703	53
8	9.8348647	9.8631828	9.9716818	10.0283182	10.1368172	10.1651354	52
9	9.8349994	9.8630644	9.9719350	10.0280650	10.1369356	10.1650006	51
10	9.835134	9.8629460	9.9721882	10.0278118	10.1370540	10.1648659	50
11	9.8352688	9.8628274	9.9724413	10.0275587	10.1371726	10.1647312	49
12	9.8354033	9.8627088	9.9726945	10.0273055	10.1372912	10.1645967	48
13	9.8355378	9.8625902	9.9729477	0.0270523	10.1374098	10.1644622	47
14	9.8356722	9.8624714	9.9732008	10.0267992	10.1375286	10.1643278	46
15	9.8355066	9.8623526	9.9734539	10.0265461	10.1376474	10.1641934	45
16	9.8359408	9.8622338	9.9737071	10.0262929	10.1377662	10.1640592	44
17	9.8360750	9.8621148	9.9739602	10.0260398	10.1378852	10.1639250	43
18	9.8362091	9.8619958	9.9742133	10.0257867	10.1380042	10.1637909	42
19	9.8363431	9.8618767	9.9744664	0.0255336	10.1381233	10.1636569	41
20	9.8364771	9.8617576	9.9747196	10.0252805	10.1382424	10.1635229	40
21	9.8366109	9.8616383	9.9749726	10.0250274	10.1383617	10.1633891	39
22	9.8367447	9.8615190	9.9752257	10.0247743	10.1384810	10.1632553	38
23	9.8368784	9.8613997	9.9754787	10.0245213	10.1386003	10.1631216	37
24	9.8370121	9.8612803	9.9757318	10.0242682	10.1387197	10.1629879	36
25	9.8371456	9.8611608	9.9759848	10.0240151	10.1388392	10.1628544	35
26	9.8372791	9.8610412	9.9762379	10.0237621	10.1389588	10.1627209	34
27	9.8374125	9.8609215	9.9764909	10.0235091	10.1390785	10.1625875	33
28	9.8375458	9.8608018	9.9767440	10.0232560	10.1391982	10.1624542	32
29	9.8376790	9.8606821	9.9769970	10.0230030	10.1393179	10.1623210	31
30	9.8378122	9.8605622	9.9772500	10.0227500	10.1394378	10.1621878	30
		Sine.		Tang.		Secant.	M

46 *Degrees.*

43 Degrees.

M	Sine.		Tang.		Secant.		
30	9.8378122	9.8605622	9.9772500	10.0227500	10.1394378	10.1621878	30
31	9.8379453	9.8604423	9.9775030	10.0224970	10.1395577	10.1620547	29
32	9.8380783	9.8603223	9.9777560	10.0222440	10.1396777	10.1619217	28
33	9.8382112	9.8602022	9.9780090	10.0219910	10.1397978	10.1617888	27
34	9.8383441	9.8600821	9.9782620	10.0217380	10.1399179	10.1616559	26
35	9.8384769	9.8599619	9.9785149	10.0214851	10.1400381	10.1615231	25
36	9.8386096	9.8598416	9.9787679	10.0212321	10.1401584	10.1613904	24
37	9.8387422	9.8597213	9.9790209	10.0209791	10.1402787	10.1612578	23
38	9.8388747	9.8596009	9.9792738	10.0202762	10.1403991	10.1611253	22
39	9.8390072	9.8594804	9.9795268	10.0204732	10.1405496	0.1609928	21
40	9.8391396	9.8593599	9.9797797	10.0202203	10.1406401	10.1608604	20
41	9.8392719	9.8592393	9.9800326	10.0199674	10.1407607	10.1607281	19
42	9.8394041	9.8591186	9.9802856	10.0197144	10.1408814	10.1605959	18
43	9.8395363	9.8589978	9.9805385	10.0194615	10.1410022	10.1604637	17
44	9.8396684	9.8588770	9.9807914	10.0192086	10.1411230	10.1603316	16
45	9.8398004	9.8587561	9.9810443	10.0189557	10.1412439	10.1601996	15
46	9.8399323	9.8586351	9.9812972	10.0187028	10.1413649	10.1600677	14
47	9.8400642	9.8585141	9.9815501	10.0184499	10.1414859	10.1599358	13
48	9.8401959	9.8583929	9.9818030	10.0181970	10.1416071	10.1598041	12
49	9.8403276	9.8582718	9.9820559	10.0179441	10.1412282	10.1596724	11
50	9.8404593	9.8581505	9.9823087	10.0176913	10.1418495	10.1595407	10
51	9.8405908	9.8580292	9.9825616	10.0174384	10.1419708	10.1594092	9
52	9.8407223	9.8579078	9.9828145	10.0171855	10.1420922	10.1592777	8
53	9.8408537	9.8577863	9.9830673	10.0169327	10.1422137	10.1591463	7
54	9.8409850	9.8576648	9.9833202	10.0166798	10.1423352	10.1590150	6
55	9.8411162	9.8575432	9.9835730	10.0164270	10.1424568	10.1588838	5
56	9.8412474	9.8574215	9.9838259	10.0161741	01.1425785	10.1537526	4
57	9.8413785	9.8572998	9.9840787	10.0159213	10.1427002	10.1586215	3
58	9.8415095	9.8571779	9.9843315	10.0156685	10.1428221	10.1584905	2
59	9,8416404	9.8570561	9.9845844	10.0154156	10.1429439	10.1583596	1
60	9.8417713	9.8569341	9.9848372	10.0151628	10.1430659	10.1582287	0
		Sine.		Tang.		Secant.	M

46 Degrees.

44 Degrees.

M	Sine.		Tang.		Secant.		
0	9.8417713	9.8569341	9.9848372	10.0151628	10.1430659	10.1582287	60
1	9.8419021	9.8568121	9.9850900	10.0149100	10.1431879	10.1580979	59
2	9.8420328	9.8566900	9.9853428	10.0146572	10.1433100	10.1579672	58
3	9.8421634	9.8565678	9.9855956	10.0144044	10.1434322	10.1578366	57
4	9.8422935	9.8564455	9.9858484	19.0141510	10.1435545	10.1577061	56
5	9.8424244	9.8563232	9.9861012	10.0138988	10.1436768	10.1575756	55
6	9.8425548	9.8562008	9.9863540	10.0136460	10.143799	10.1574452	54
7	9.8426851	9.8560784	9.9866068	10.0133932	10.1439216	10.1573149	53
8	9.8428154	9.8559558	9.9868596	10.0131404	10.1440442	10.1571846	52
9	9.8429456	9.8558332	9.9871123	10.0128877	10.1441668	10.1570544	51
10	9.8430757	9.8557106	9.9873651	10.0126349	10.1442894	10.1569243	50
11	9.8432057	9.8555878	9.9876179	10.0123821	10.1444122	10.1567943	49
12	9.8433356	9.8554650	9.9878706	10.0121294	10.1445350	10.1566644	48
13	9.8434655	9.8553421	9.9881234	10.0118766	10.1446575	10.1565345	47
14	9.8435953	9.8552192	9.9883761	10.0116239	10.1447808	10.1564047	46
15	9.8437250	9.8550961	9.9886289	10.0113711	10.1449039	10.1562750	45
16	9.8438547	9.8549730	9.9888816	10.0111184	10.1450270	10.1561453	44
17	9.8439842	9.8548499	9.9891344	10.0108656	10.1451501	10.1560158	43
18	9.8441137	9.8547266	9.9893871	10.0106129	10.1452734	10.1558863	42
19	9.8442432	9.8546033	9.9896399	10.0103601	10.1453967	10.1557568	41
20	9.8443725	9.8544799	9.9898926	10.0101074	10.1455201	10.1556275	40
21	9.8445018	9.8543564	9.9901453	10.0098547	10.1456436	10.1554982	39
22	9.8446310	9.8542329	9.9903981	10.0096019	10.1457671	10.1553690	38
23	9.8447601	9.8541093	9.9906508	10.0093492	10.1458907	10.1552399	37
24	9.8448891	9.8539856	9.9909035	10.0090965	10.1460144	10.1551109	36
25	9.8450181	9.8538619	9.9911562	10.0088438	10.1461381	10.1549819	35
26	9.8451470	9.8537381	9.9914089	10.0085911	10.1462619	10.1548530	34
27	9.8452758	9.8536142	9.9916616	10.0083384	10.1463858	10.1547242	33
28	9.8454045	9.8534902	9.9919143	10.0080857	10.1465098	10.1545955	32
29	9.8455332	9.8533662	9.9921670	10,0078330	10.1466338	10.1544668	31
30	9.8456618	9.8532421	9.9924197	10.0075803	10.1467579	10.1543382	30
		Sine.		Tang.		Secant.	

45 Degrees.

44 Degrees.

M	Sine.		Tang.		Secant.		
30	9.8456618	9,8532421	9.9924197	10.0075803	10.1467579	10,1543382	30
31	9.8457903	9.8531179	9.9926724	10.0073276	10.1468821	10.1542097	29
32	9.8459188	9.8529936	9.9929251	10.0070749	10.1470064	10.1540812	28
33	9.8460471	9.8528693	9.9931778	10.0068222	10.1471307	15.1539529	27
34	9.8461754	9.8527449	9 9934305	10.0065695	10.1472551	10.1538246	26
35	9.8463036	9.8526204	9.9936832	10.0063168	10.1473796	10.1536964	25
36	9.8464318	9.8524959	9.9939359	10.0060641	10.1475041	10.1535682	24
37	9.8465599	9.8523713	9.9941886	10.0058114	10.1476287	10.1534401	23
38	9.8466879	9.8522466	9.9944413	10.0055587	10.1477534	10.1533121	22
39	9.8468158	9.8521218	9.9946940	10.0053060	10.1478782	10.1531842	21
40	9.8469436	9.8519970	9.9949466	10.0050534	10.1480030	10.1530564	20
41	9.8470714	9.8518721	9.9951993	10.0048007	10.1481279	10.1529286	19
42	9.8471991	9.8517471	9.9954520	10.0045480	10.1482529	10.1528009	18
43	9.8473267	9.8516220	9.9957047	10.0042953	10.1483780	10.1526733	17
44	9.8474543	9.8514969	9.9959573	10.0040427	10.1485031	10.1525457	16
45	9.8475817	9.8513717	9.9962100	10.0037900	10.1486283	10.1524183	15
46	9.8477091	9.8512465	9.9964627	10.0035373	10.1487535	10.1522909	14
47	9.8478365	9.8511211	9.9967154	10.0032846	10.1488789	10.1521635	13
48	9.8479637	9,8509957	9.9969680	10.0030320	10.1490043	10.1520363	12
49	9.8480909	9.8508702	9.9972207	10.0027793	10.149129[illegible]	10.1519091	11
50	9,8482180	9.8507446	9.9974734	10.0025266	10.1492554	10.1517820	10
51	9,8483450	9.8506190	9.9977260	10.0022740	10.1493810	10.1516550	9
52	9.8484720	9.8504933	9.9979787	10.0020213	10.1495067	10.1515280	8
53	9.8485989	9.8503675	9.9982314	10.0017686	10.1496325	10.1514011	7
54	9.8487257	9.8502417	9.9984840	10.0015160	10.1497583	10.1512743	6
55	9.8488524	9 8501157	9.9987367	10.0012633	10.1498843	10.1511476	5
56	9.8489791	9.8499897	9.9989893	10.0010107	10.1500103	10.1510209	4
57	9.8491057	9.8498637	9.9992420	10.0007580	10.1501363	10,1508943	3
58	9.8492322	9.8497375	9.9994947	10.0005053	10.1502625	10.1507678	2
59	9.8493586	9.8496113	9.9997473	10.0002527	10.1503887	10.1506414	1
60	9 8494850	9.8494850	10.0000000	10.0000000	10.1505150	10.1505150	0
	Sine.	Sine.		Tang.		Secant.	M

45 Degrees.